Pairs of massless quarks on the lattice from staggered fermions

David Adams

Nanyang Technological University
Singapore

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Numerically investigated by Ph. de Forcrand & collaborators (next talk).

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  - Describe massless fermions in staggered framework
  - Exact zero-modes, index theorem, η' mass...

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Overlap Dirac operator [Neuberger, PLB 1998]:

$$D_{ov} = \frac{\rho}{a}\left(1 + \frac{D_W - M}{\sqrt{(D_W - M)^\dagger (D_W - M)}}\right)$$

has exact zero-modes, index theorem, axial anomaly etc.

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$D_{st} = \eta_\mu \nabla_\mu$: staggered Dirac operator

- has purely imaginary spectrum
- $\Rightarrow$ would-be zero-modes do not become real negative eigenvalue modes for $D_{st} - M$
- $\Rightarrow$ Overlap construction based on $D_{st} - M$ fails: e.g. no exact zero-modes or index theorem

Reason to still hope for a staggered overlap construction

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Consider free field staggered Dirac operator in flavor space rep [Gliozzi, Kluberg-Stern et al.]:

$$D_{st} = (\gamma_\mu \otimes 1)\nabla_\mu + \frac{1}{2}(\gamma_5 \otimes \gamma_5\gamma_\nu)\Delta_\nu$$

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Then $\tilde{\gamma}_5 = \gamma_5$ and

$$i(\gamma_5 \otimes 1)D_{st} = (\tilde{\gamma}_\mu \otimes 1)\nabla_\mu + \tfrac{1}{2}(1 \otimes \tilde{\gamma}_\nu)\Delta_\nu \;\equiv\; D_{stW}$$

– has structure of Wilson-Dirac operator

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$\Rightarrow$ Can expect the spectral flow of $H_{st}(m)$ in the interacting case determined by gauge field topology.

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$\Rightarrow$ Cannot simply replace $\gamma_5 \rightarrow \Gamma_5$ in overlap construction.

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Go back and start the staggered overlap construction from $D_{st} - M$, but with

$$M = \frac{\rho}{a}\Gamma_{55}\Gamma_5$$

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$\Rightarrow$ M is a "flavor-chiral" mass term

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$\Rightarrow$ get 2 physical fermion flavours!

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Take the staggered overlap Dirac operator to be

$$D_{sov} = \frac{\rho}{a}\left(1 + \frac{D_{st} - M}{\sqrt{(D_{st} - M)^\dagger (D_{st} - M)}}\right) \qquad M = \frac{\rho}{a}\Gamma_{55}\Gamma_5$$

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$$\Rightarrow \; \boxed{\text{Can use } \;\Gamma_{55}\; \text{ for role of } \;\gamma_5\;!}$$

The " $\Gamma_5^2 \neq 1$ problem" is hereby solved.

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$\therefore$ The exact flavored chiral symm of D_{st} has been turned into an exact *unflavored* GW chiral symm of D_{sov}.

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Its spectral flow is already known to be determined by gauge field topology, giving $2Q$.
[D.A., PRL (2010)]

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Note that the staggered overlap operator,

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where the hermitian operator $\widetilde{H}_{st}$ is

$$\widetilde{H}_{st} = \Gamma_{55}(D_{st} - M) = \Gamma_{55} D_{st} - \frac{\rho}{a}\Gamma_5$$

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Theorem:

$$\widetilde{H}_{st}(m) = \Gamma_{55} D_{st} - m\Gamma_5 \qquad \text{and} \qquad H_{st}(m) = iD_{st} - m\Gamma_5$$

have the same eigenvalue spectrum.

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$\Rightarrow$ D_{sov} has exact zero-modes satisfying 2-flavor index theorem.

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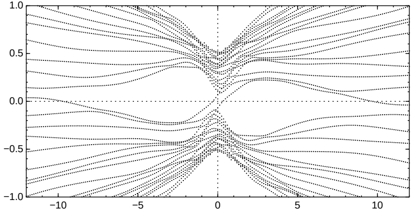
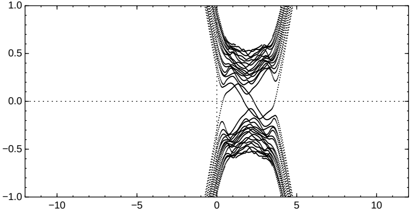


staggered



Wilson

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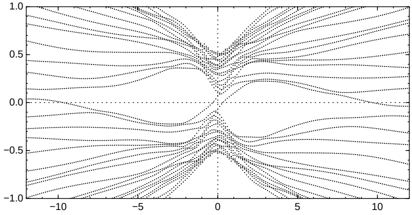

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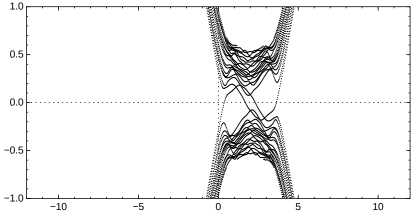

Wilson

Results in 4 dimensions will be shown in Ph. de Forcrand's talk.

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- Again 2 physical (almost massless) flavors
- Gives truncation of staggered overlap, just like in Wilson case [Neuberger, PRD (1998)].
- Completely unrelated to a previous proposal for staggered domain wall fermions by G. Flemming and P. Vranas (PRD, 2002).

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- A good option for calculating the η' mass in Lattice QCD?