Renormalisation of composite operators in lattice QCD: perturbative versus nonperturbative

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M.G., R. Horsley, Y. Nakamura, H. Perlt, D. Pleiter, P.E.L. Rakow, A. Schäfer, G. Schierholz, A. Schiller, H. Stüben, J.M. Zanotti (QCDSF/UKQCD collaboration), arXiv:1003.5756

Introduction

renormalisation on the lattice: let bare parameters and renormalisation factors Z depend on the lattice spacing a in such a way that the limit $a \to 0$ is finite

$\psi_R(x) = Z_q(\mu, a)^{1/2}\psi(x)$ quark field

$\mathcal{O}_R(\mu) = Z(\mu, a)\mathcal{O}(a)$ composite operator like $\bar{\psi}\psi$

μ: renormalisation scale

direct calculation of physical observables (e.g. hadron masses): Z factors unnecessary (cancel in physical quantities)

↑ scheme and renormalisation scale dependent

Why then worry about Z factors?

It is not always possible to calculate the physical observables directly!

example: deep-inelastic lepton-nucleon scattering

OPE: structure function = Wilson coefficient $\otimes$ hadronic matrix element of a (local) composite operator

structure function	Wilson coefficient	hadronic matrix element
observable	short distance perturbative	long distance non-perturbative

How to evaluate renormalisation factors on the lattice?

perturbative methods:

- standard perturbation theory
  (with or without improvement)
- numerical stochastic perturbation theory
  (higher orders)

nonperturbative methods:

- Schrödinger functional methods
- Rome-Southampton method
  RI-MOM scheme

G. Martinelli, C. Pittori, C.T. Sachrajda, M. Testa, A. Vladikas, NPB 445 (1995) 81 basic ingredients in the RI-MOM scheme:

three-point function of $\mathcal{O} = \bar{\psi} \cdots \psi$ in Landau gauge

$$G^{ij}_{\alpha\beta}(p) = \frac{a^{12}}{V} \sum_{x,y,z} \mathrm{e}^{-\mathrm{i}p\cdot(x-y)} \langle \psi^i_\alpha(x) \mathcal{O}(z) \bar{\psi}^j_\beta(y) \rangle$$

quark propagator

$$S^{ij}_{\alpha\beta}(p) = \frac{a^{8}}{V} \sum_{x,y} \mathrm{e}^{-\mathrm{i}p\cdot(x-y)} \langle \psi^i_\alpha(x) \bar{\psi}^j_\beta(y) \rangle$$

V = lattice volume

vertex function: $\Gamma(p) = S^{-1}(p) G(p) S^{-1}(p)$

$\longrightarrow$ renormalised vertex function: $\Gamma_{\mathrm{R}}(p) = Z_q^{-1} Z \Gamma(p)$

RI-MOM renormalisation condition: $\frac{1}{12} \mathrm{tr}_{DC} \left(\Gamma_{\mathrm{R}}(p) \Gamma_{\mathrm{Born}}(p)^{-1} \right) \Big|_{p^2=\mu^2} = 1$ in the chiral limit

μ = renormalisation scale

renormalisation of the quark fields: $Z_q(p) = \left. \frac{\mathrm{tr}\left(-\mathrm{i} \sum_\lambda \gamma_\lambda \sin(ap_\lambda) a S^{-1}(p)\right)}{12 \sum_\lambda \sin^2(ap_\lambda)} \right|_{p^2=\mu^2}$ RI'-MOM renormalised operator $Z\,\mathcal{O}(a) \equiv Z^{\mathcal{S}}_{\rm bare}(\mu)\mathcal{O}(a)$ (a dependence of Z suppressed)

depends on the renormalisation scale μ and the renormalisation scheme $\mathcal{S}$

scale and scheme independent (for $1/L^2 \ll \Lambda^2_{\rm QCD} \ll \mu^2 \ll 1/a^2$):

$$Z^{\rm RGI} = \left(2\beta_0 \frac{g^{\mathcal{S}}(\mu)^2}{16\pi^2}\right)^{-\gamma_0/(2\beta_0)} \exp\left\{\int_0^{g^{\mathcal{S}}(\mu)} {\rm d}g \left(\frac{\gamma^{\mathcal{S}}(g)}{\beta^{\mathcal{S}}(g)} + \frac{\gamma_0}{\beta_0 g}\right)\right\} Z^{\mathcal{S}}_{\rm bare}(\mu) = \Delta Z^{\mathcal{S}}(\mu) Z^{\mathcal{S}}_{\rm bare}(\mu)$$

intermediate renormalisation scheme $\mathcal{S} = \overline{\rm MS}$, ${\rm MOM}$, ... useful:

$$Z^{\rm RGI} = \Delta Z^{\mathcal{S}}(\mu) Z^{\mathcal{S}}_{\rm RI'-MOM}(\mu) Z^{\rm RI'-MOM}_{\rm bare}(\mu)$$

↑ ↑

continuum perturbation theory

expansion in $g^{\overline{\rm MS}}$, $g^{\widetilde{\rm MOMgg}}$ (K.G. Chetyrkin, A. Rétey, hep-ph/0007088), ...

Perturbative renormalisation on the lattice

- bare perturbation theory: $Z = 1 - \frac{g^2}{16\pi^2}(\gamma_0 \ln(a\mu) + \Delta) + O(g^4)$ (often poorly convergent)
- tadpole-improved perturbation theory (for an operator with n_D covariant derivatives):

$$Z = u_0^{1-n_D}\left[1 - \frac{g_\Box^2}{16\pi^2}\left(\gamma_0 \ln(a\mu) + \Delta + (n_D - 1)\frac{4}{3}\pi^2\right) + O(g^4)\right]$$

$u_0 = \langle \frac{1}{3}\mathrm{tr}U_\Box\rangle^{1/4}$ taken from simulations, $g_\Box^2 = g^2/u_0^4$ boosted coupling

- "tadpole-improved renormalisation-group-improved boosted perturbation theory" (TRB perturbation theory)

combining renormalisation group improvement with tadpole improvement

$$Z^{\mathrm{RGI}} = u_0^{1-n_D}\left(2\beta_0\frac{g_\Box^2}{16\pi^2}\right)^{-\gamma_0/(2\beta_0)}\left(1 + \frac{\beta_1}{\beta_0}\frac{g_\Box^2}{16\pi^2}\right)^{-(\gamma_1^\Box\beta_0-\gamma_0\beta_1)/(2\beta_0\beta_1)+16\pi^2\beta_0(1-n_D)/(12\beta_1)}$$

with $\gamma^\Box(g_\Box) = -a\frac{\mathrm{d}}{\mathrm{d}a}\ln Z^{\mathcal{S}}_{\mathrm{bare}} = \gamma_0\frac{g_\Box^2}{16\pi^2} + \gamma_1^\Box\left(\frac{g_\Box^2}{16\pi^2}\right)^2 + \cdots$

Subtraction of lattice artefacts

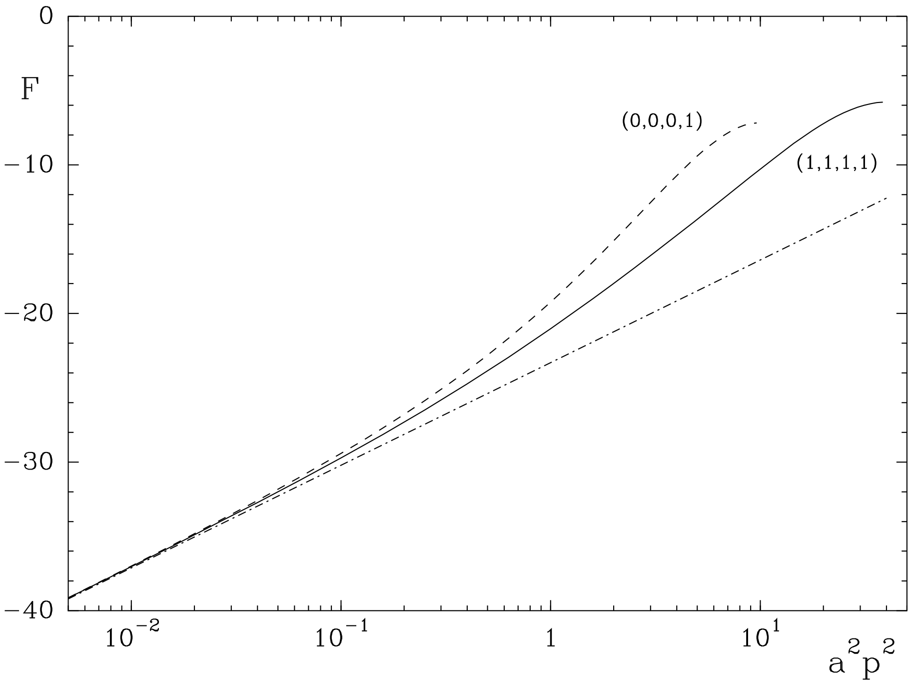
① standard calculation of Z in lattice perturbation theory neglects lattice artefacts: $a^2p^2 \ll 1$

(keeps only logarithmic a dependence)

our momenta usually do not satisfy this condition

② (one-loop) lattice perturbative results for arbitrary a^2p^2:
evaluate the loop integrals numerically (for each p separately)

use the difference between ① and ② to correct for the (perturbative) discretisation errors



scalar density, Landau gauge, $c_{\rm SW} = 1$

lattice artefacts
for p along a lattice axis, $p \propto (0,0,0,1)$,
larger than
for p along a diagonal, $p \propto (1,1,1,1)$,
of the Brillouin zone

The simulations

QCDSF-UKQCD configurations:

two degenerate flavours of clover fermions + plaquette action for the gauge field

four values of β:

β	5.20	5.25	5.29	5.40
a[fm]	0.086	0.079	0.075	0.067

$r_0 = 0.467\,\mathrm{fm}$, $r_0\Lambda_{\overline{\mathrm{MS}}} = 0.617$

3 – 5 quark masses per β

three-point function $$G(p) = \frac{a^{12}}{V}\sum_{x,y,z} \mathrm{e}^{-\mathrm{i}p\cdot(x-y)}\langle\psi(x)\mathcal{O}(z)\bar{\psi}(y)\rangle$$ (quark-line connected)

calculated with the help of momentum sources

☺ reduced statistical fluctuations, arbitrary operators

☹ number of inversions $\propto$ number of momenta

Extracting the renormalisation factors

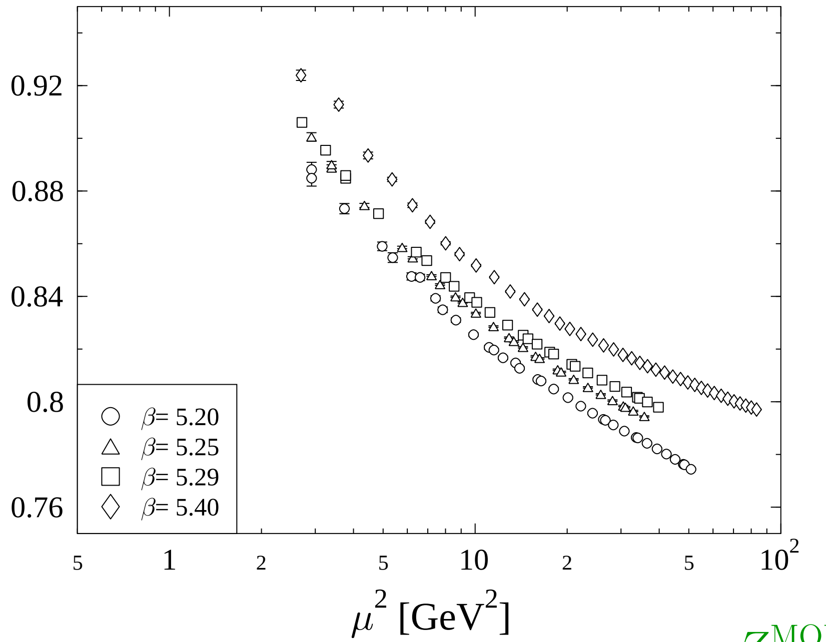
plateau (values independent of μ) in $Z^{\mathrm{RGI}} = \Delta Z^{\mathcal{S}}(\mu) Z^{\mathcal{S}}_{\mathrm{RI'-MOM}}(\mu) Z^{\mathrm{RI'-MOM}}_{\mathrm{bare}}(\mu)$ endangered by:

truncation of perturbative expansion (μ small)   lattice artefacts (μ large)

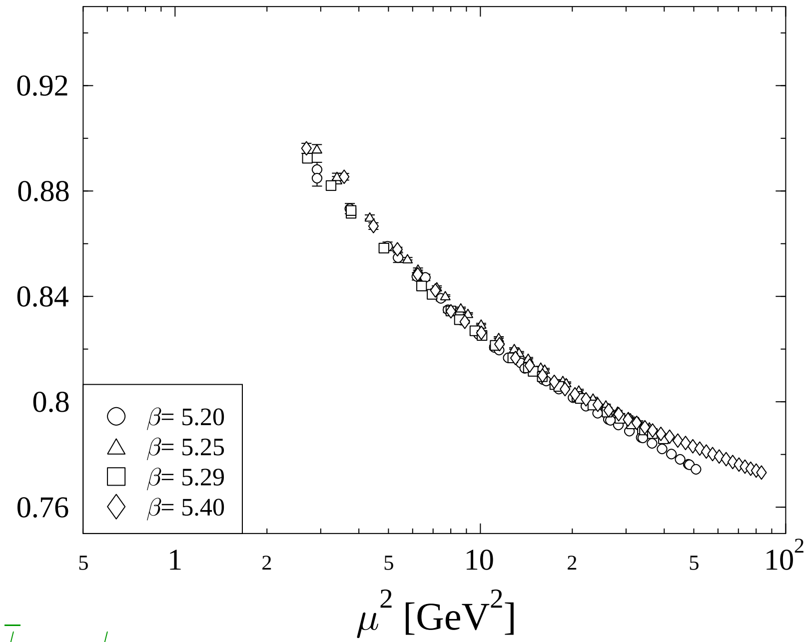
Can we separate truncation effects from lattice artefacts?

$$Z^{\mathcal{S}}_{\mathrm{bare}}(\mu) \equiv Z^{\mathcal{S}}_{\mathrm{RI'-MOM}}(\mu) Z^{\mathrm{RI'-MOM}}_{\mathrm{bare}}(\mu) = \Delta Z^{\mathcal{S}}(\mu)^{-1} Z^{\mathrm{RGI}}$$

rescale $Z^{\mathcal{S}}_{\mathrm{bare}}(\mu)$ data $\longrightarrow$ collapse onto a single curve (up to lattice artefacts)



$\longrightarrow$

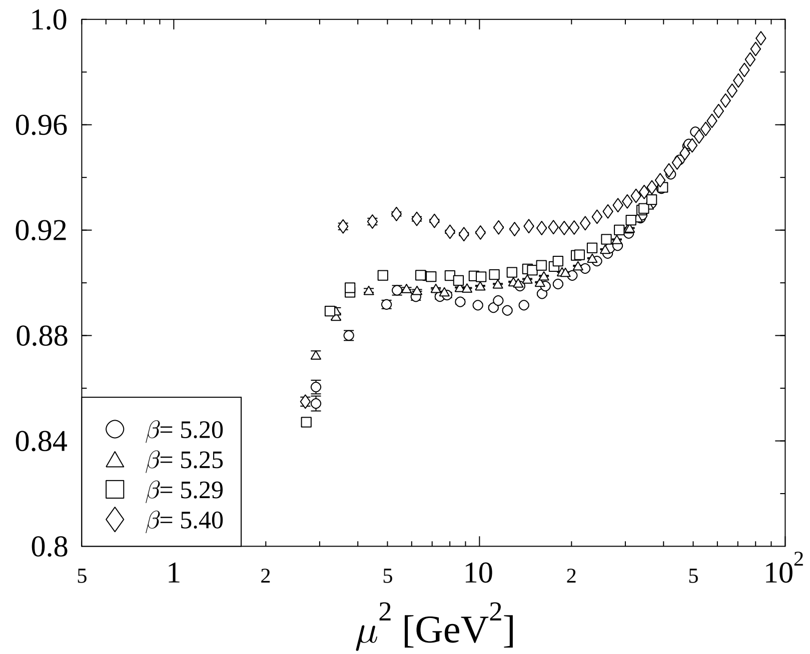


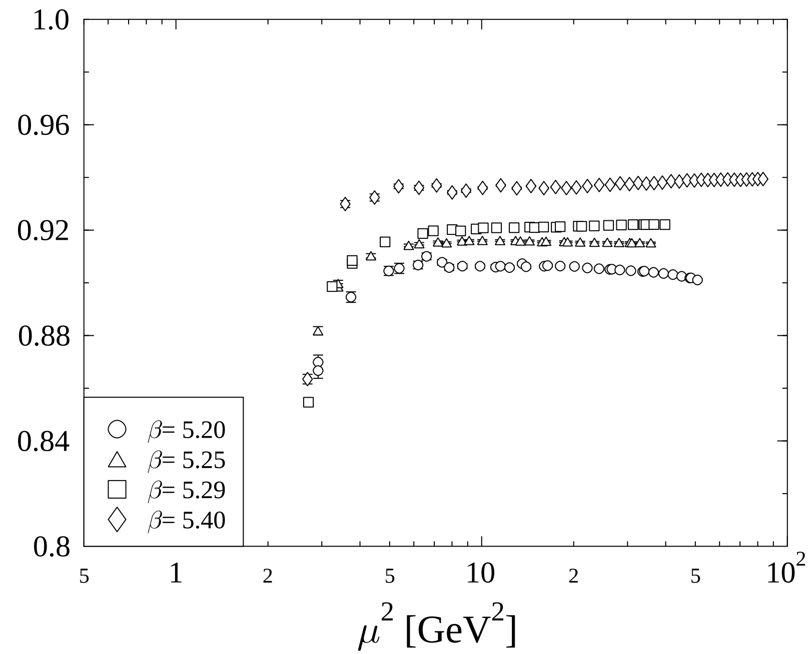
$Z^{\mathrm{MOM}}_{\mathrm{bare}}$ for $\bar{\psi}\sigma_{\mu\nu}\psi$

look for a plateau (values independent of μ) in

$$Z^{\rm RGI} = \Delta Z^{\mathcal{S}}(\mu) Z^{\mathcal{S}}_{\rm RI'-MOM}(\mu) Z^{\rm RI'-MOM}_{\rm bare}(\mu)$$

example: $\bar{\psi}\sigma_{\mu\nu}\psi$





before after

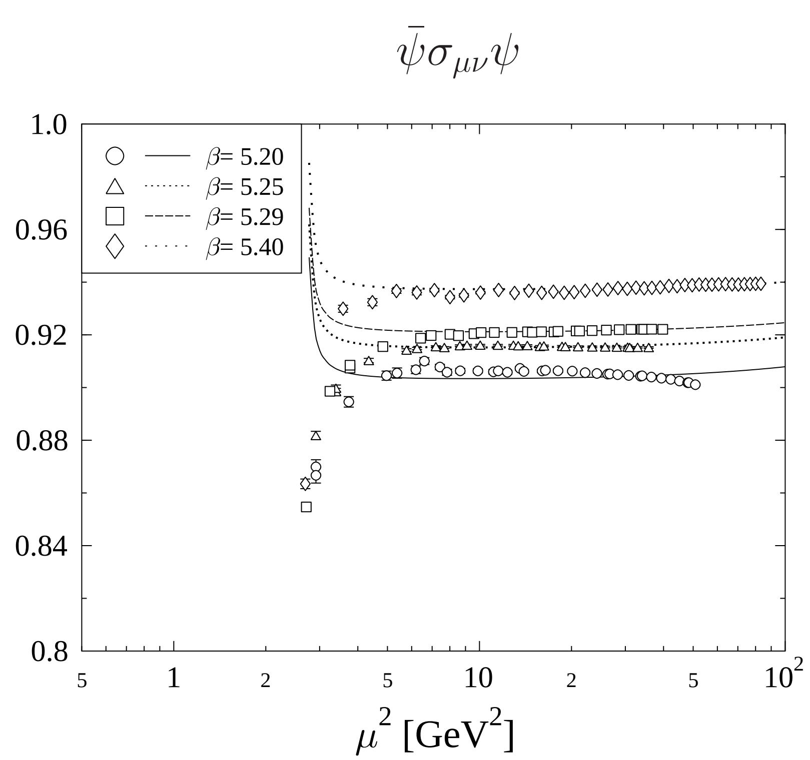
the perturbative subtraction of lattice artefacts

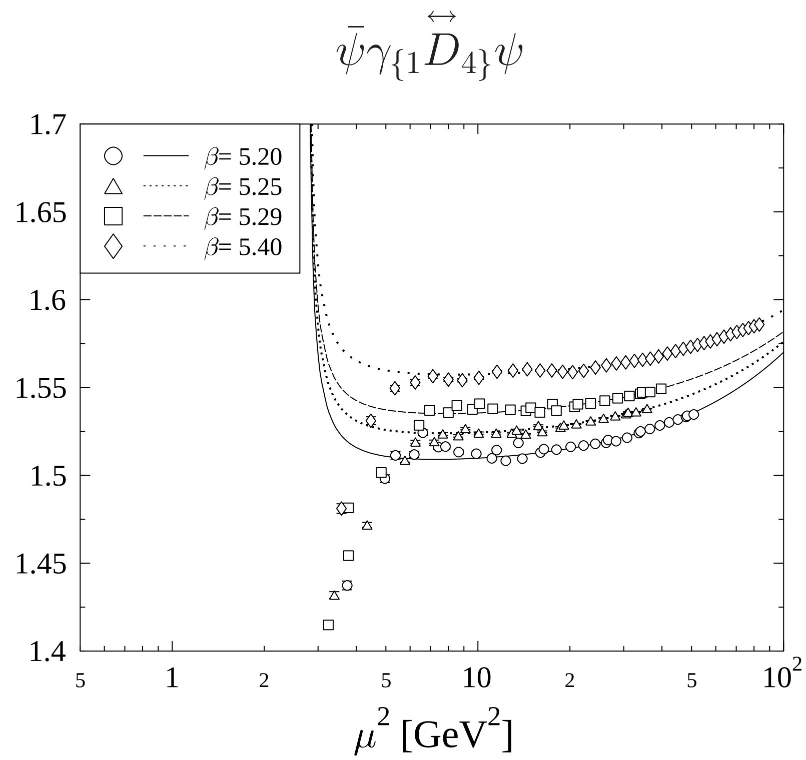
available perturbative results cannot describe the scale dependence below 5 GeV^2

finally: account for deviations from a perfect plateau by a fit for all four β values simultaneously

altogether six fit parameters: $Z^{\rm RGI}$ at the four β values, two "effective" coefficients

lattice artefacts, perturbative expansion





fits work only for subtracted data (exist only for operators with less than two derivatives)

in the other cases: read off $Z^{\rm RGI}$ at $\mu^2 = 20\,{\rm GeV}^2$ (linearly interpolated) "interpolation method"

error: maximum of the differences with the values at $\mu^2 = 10\,{\rm GeV}^2$, $30\,{\rm GeV}^2$

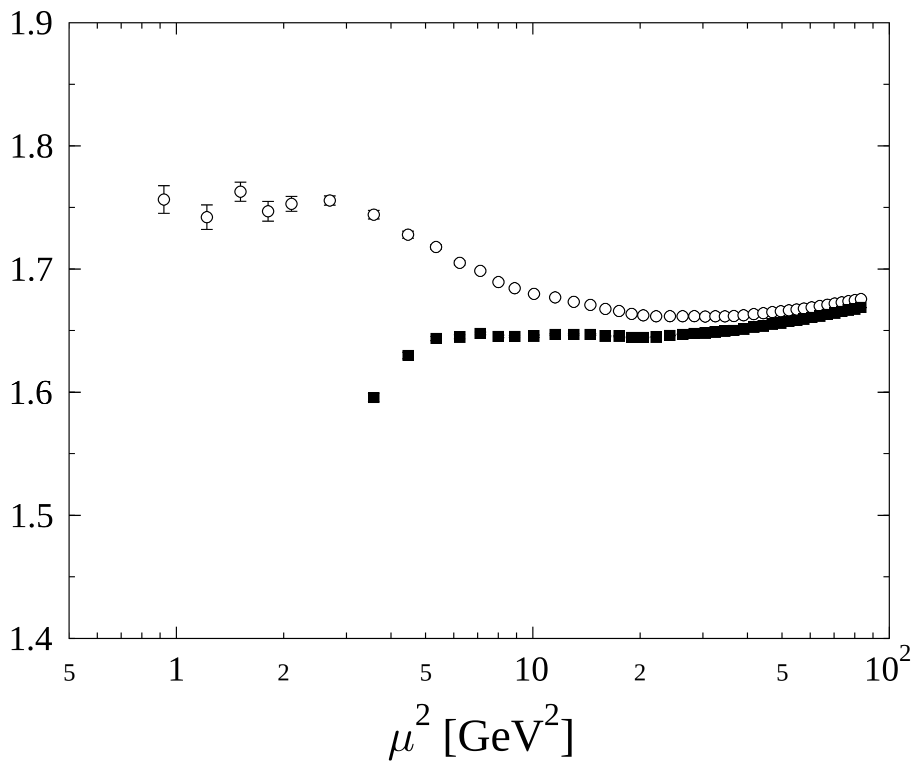
up to now: intermediate scheme $\mathcal{S} = \mathrm{MOM}$, expansion in the $\widetilde{\mathrm{MOMgg}}$ coupling

compare with $\mathcal{S} = \overline{\mathrm{MS}}$, expansion in the $\overline{\mathrm{MS}}$ coupling

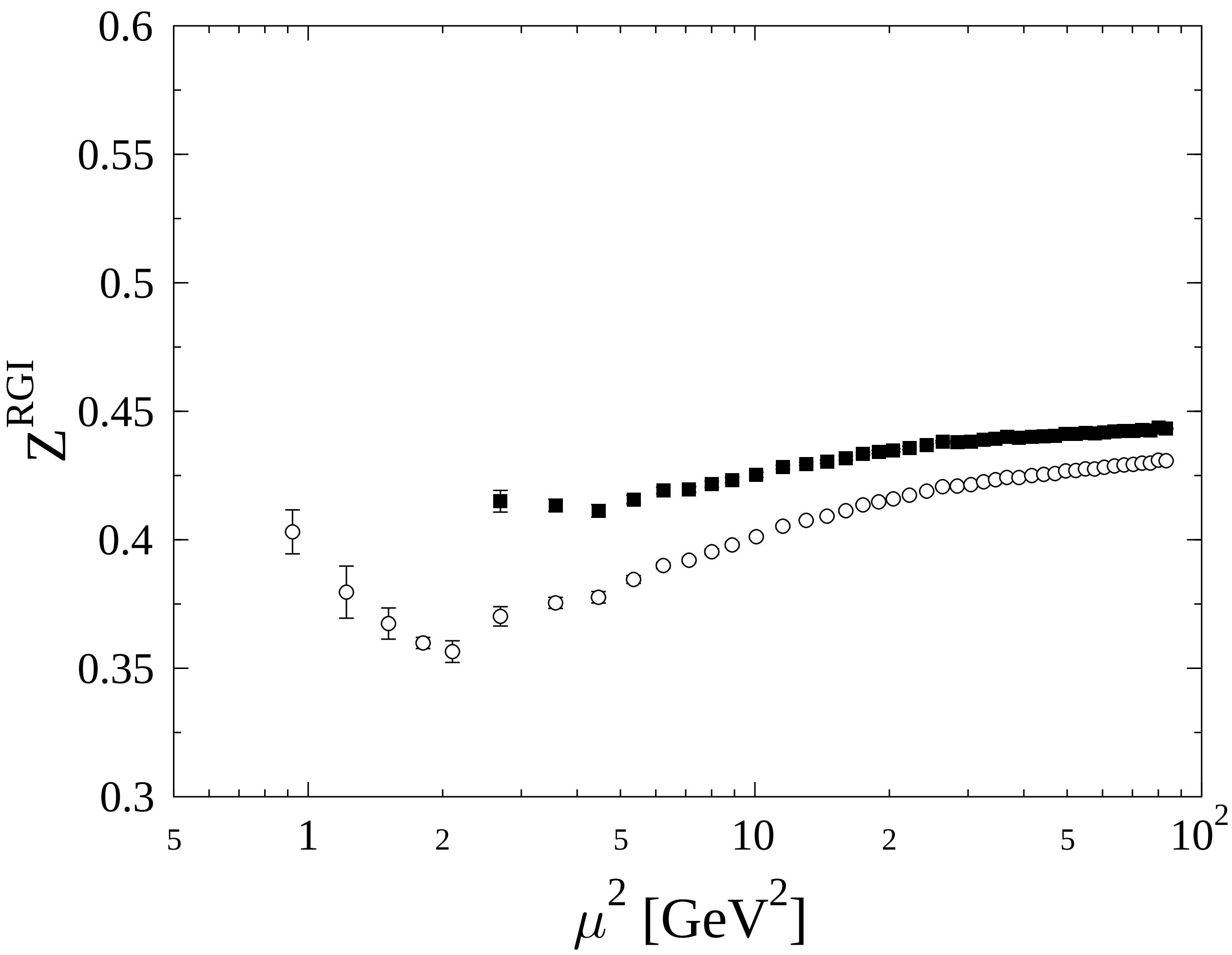
example: $\bar{\psi}\sigma_{1\{2}\overleftrightarrow{D}_{3\}}\psi$

$\mathcal{S} = \mathrm{MOM}$ with $\widetilde{\mathrm{MOMgg}}$ coupling: filled squares

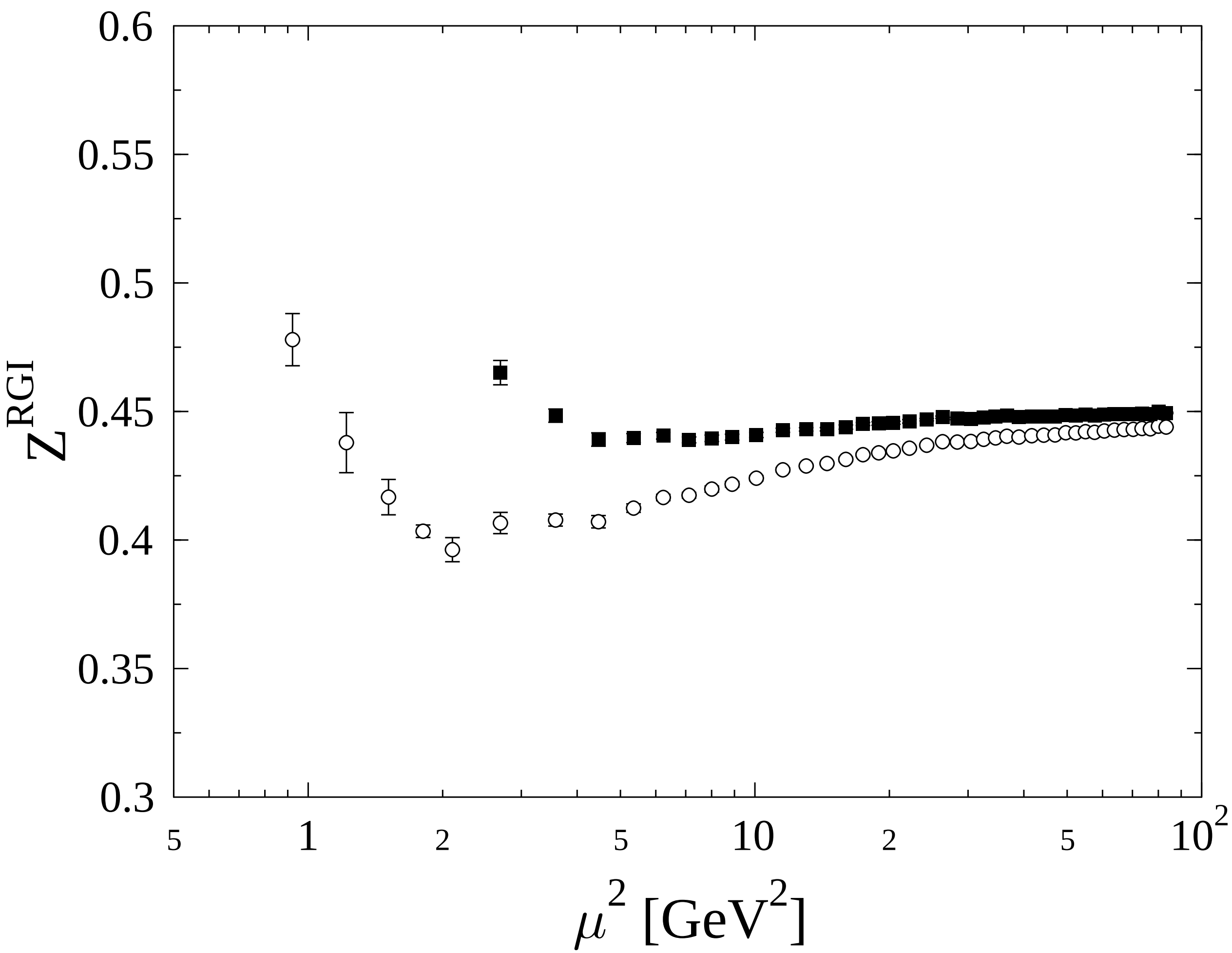
$\mathcal{S} = \overline{\mathrm{MS}}$ with $\overline{\mathrm{MS}}$ coupling: open circles



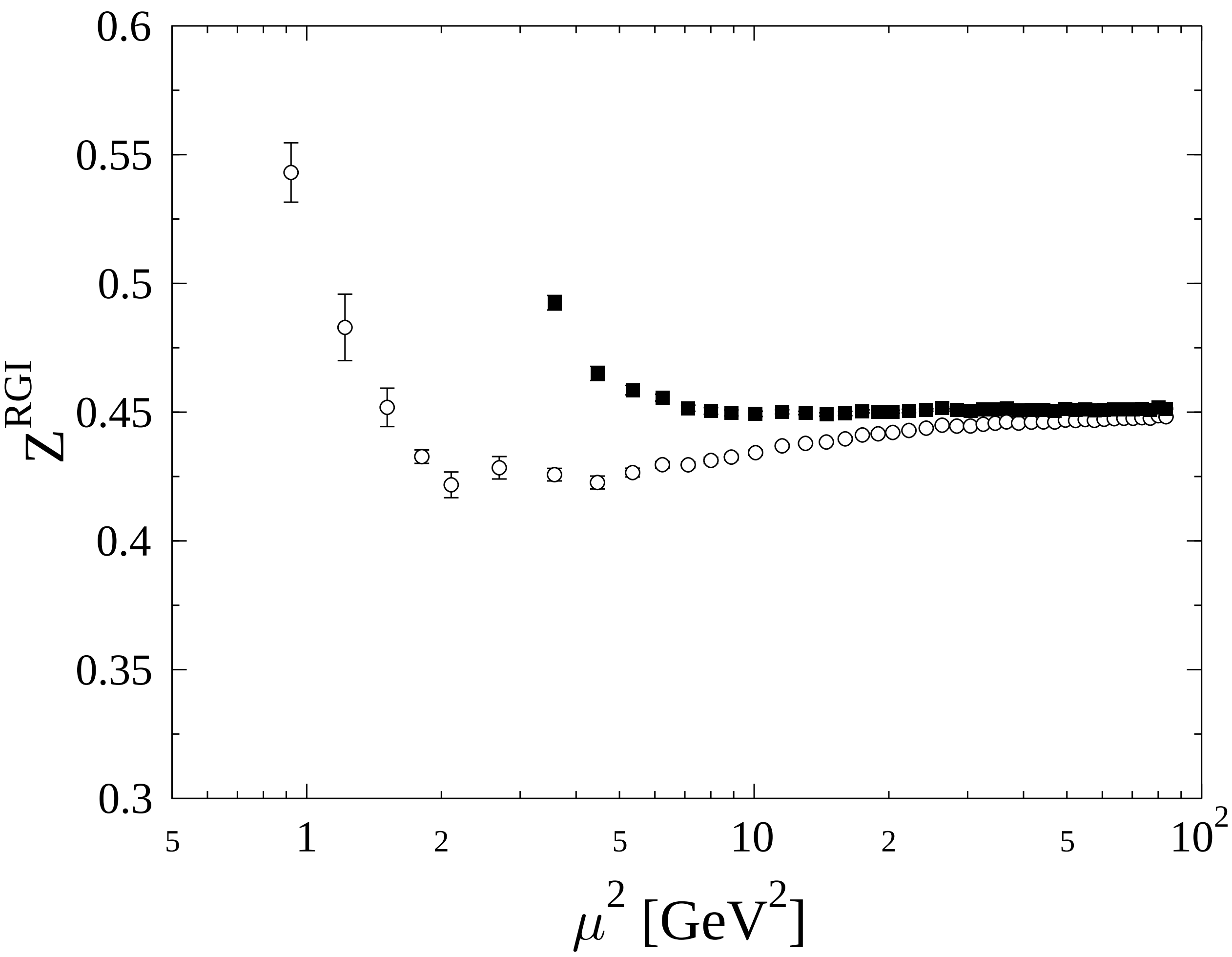
more loops pay off: $\bar{\psi}\psi$ (two-loop anomalous dimension)



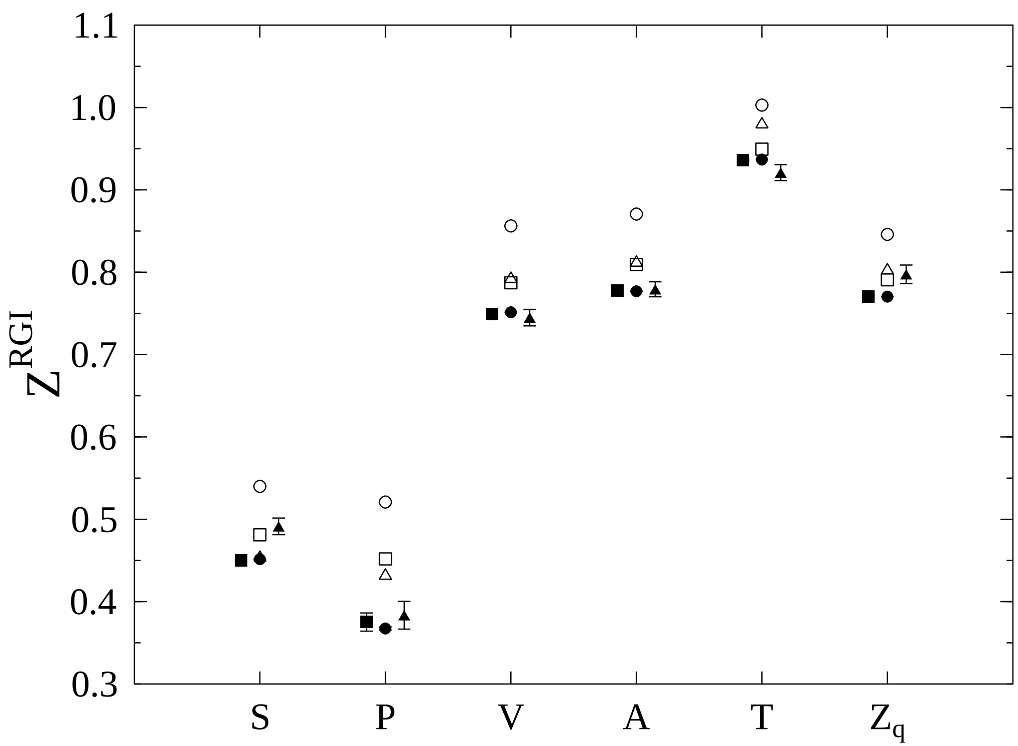
more loops pay off: $\bar{\psi}\psi$ (three-loop anomalous dimension)



more loops pay off: $\bar{\psi}\psi$ (four-loop anomalous dimension)



Results: operators without derivatives

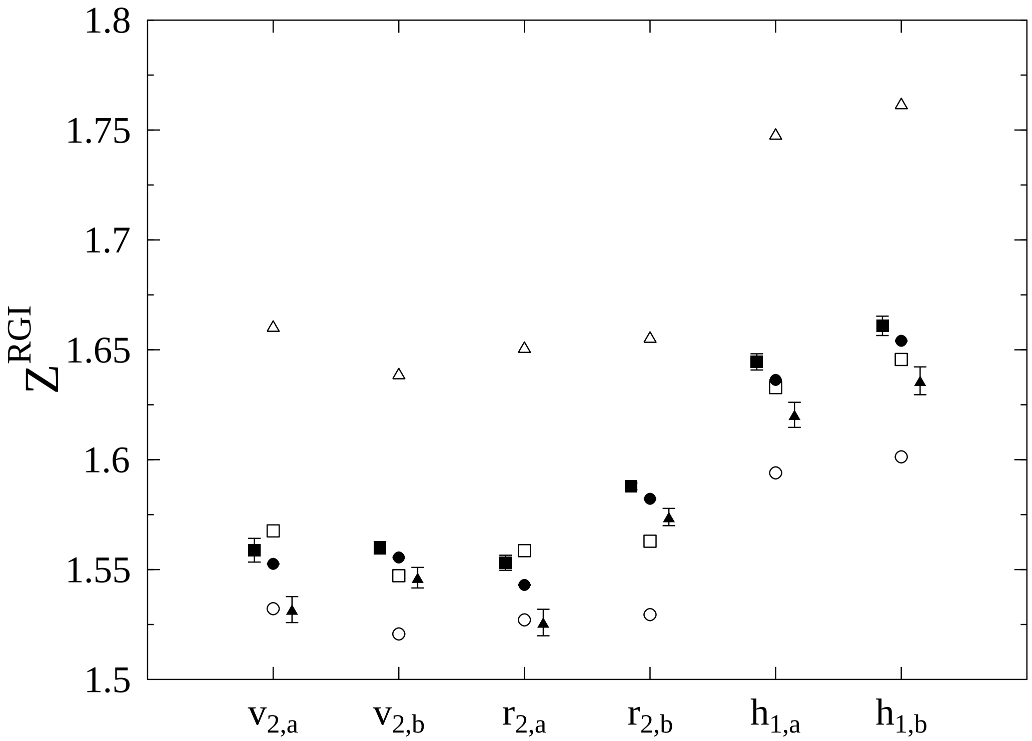


S: $\bar{\psi}\psi$
P: $\bar{\psi}\gamma_5\psi$
V: $\bar{\psi}\gamma_\mu\psi$
A: $\bar{\psi}\gamma_\mu\gamma_5\psi$
T: $\bar{\psi}\sigma_{\mu\nu}\psi$
$Z_q = Z_\psi$

$\beta = 5.40$

filled circles:	fit results (subtracted data)
filled squares:	interpolation results (subtracted data)
filled triangles:	interpolation results (unsubtracted data)
open circles:	bare perturbation theory (one loop)
open squares:	tadpole-improved perturbation theory (one loop)
open triangles:	TRB perturbation theory (one loop)

Results: operators with one derivative



$\beta = 5.40$

filled circles:	fit results (subtracted data)
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open circles:	bare perturbation theory (one loop)
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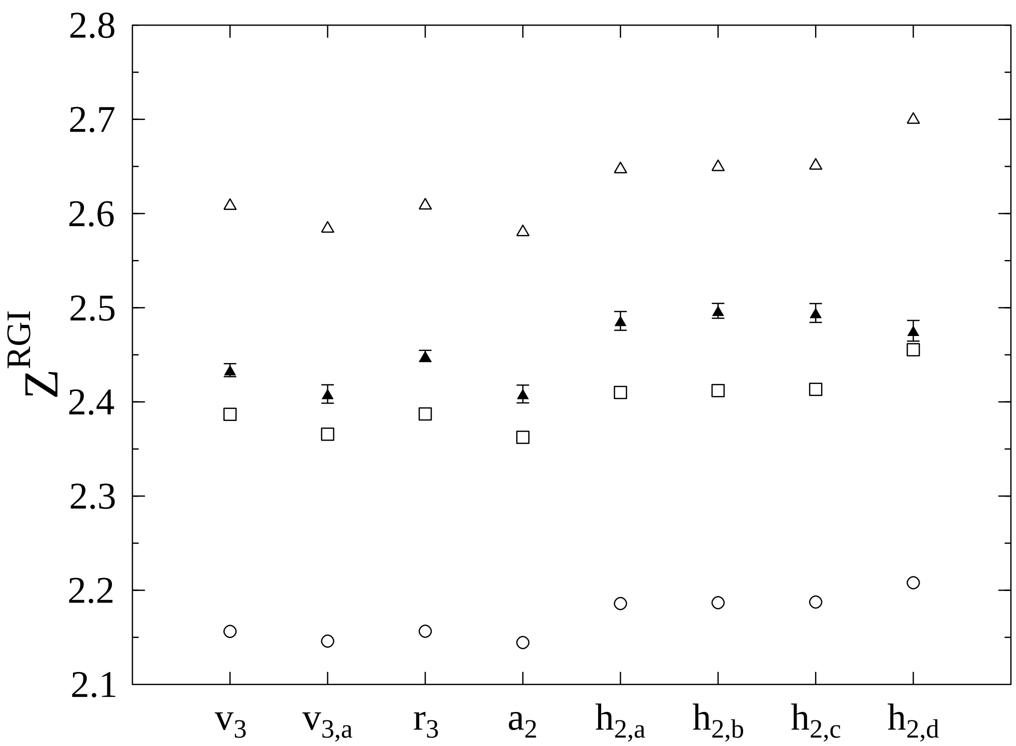
open triangles:	TRB perturbation theory (one loop)

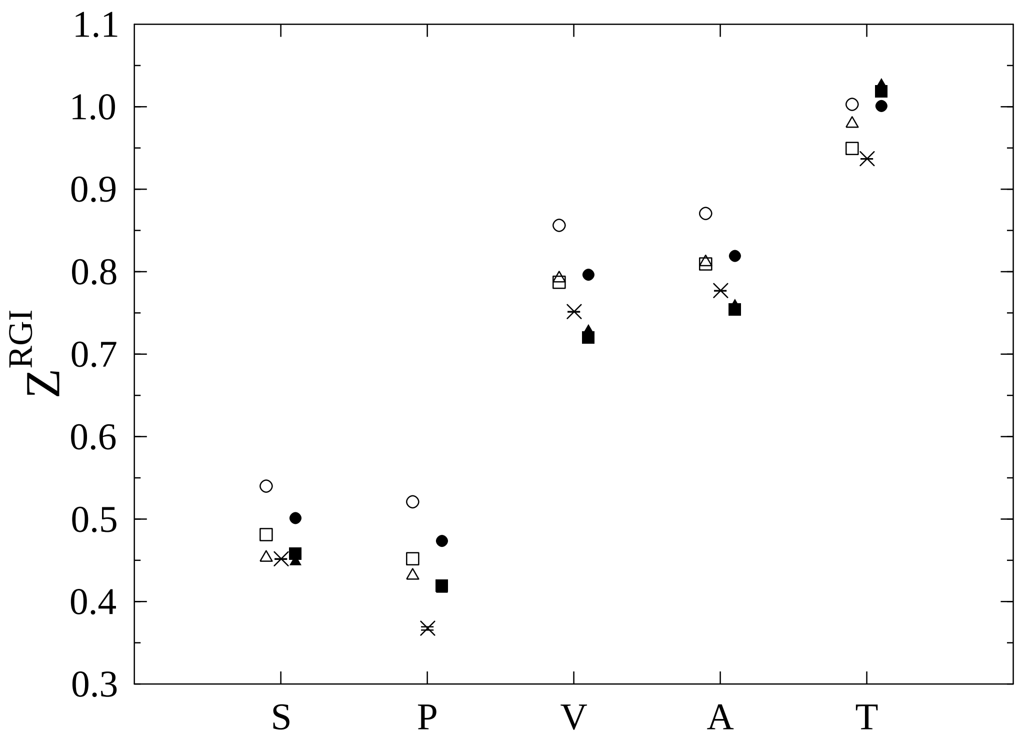
Results: operators with two derivatives



$\beta = 5.40$

filled triangles:	interpolation results (unsubtracted data)
open circles:	bare perturbation theory (one loop)
open squares:	tadpole-improved perturbation theory (one loop)
open triangles:	TRB perturbation theory (one loop)

Results: operators without derivatives compared with two-loop perturbation theory



two-loop results:

A. Skouroupathis, H. Panagopoulos

Phys. Rev. D76 (2007) 094514

Phys. Rev. D79 (2009) 094508

crosses: fit results (subtracted data)

circles: bare perturbation theory

squares: tadpole-improved perturbation theory

triangles: TRB perturbation theory

open symbols: one loop

filled symbols: two loops

Concluding remarks

- NPR in the RI-MOM scheme: (relatively) easy to implement for arbitrary lattice fermions
- momentum sources: many operators in a single simulation
  small statistical errors, but CPU time $\propto$ number of momenta
- choose momenta close to the diagonal of the Brillouin zone (discretisation effects!)
  still: perturbative subtraction of lattice artefacts very helpful, but not sufficient
- continuum perturbation theory needed for the conversion to the $\overline{\text{MS}}$ scheme:
  use as many loops as you can get ($\mu^2 > 5\,\mathrm{GeV}^2$ required?)
- comparison with lattice perturbation theory (one loop and two loops):

  difficult to predict the accuracy of perturbative renormalisation factors
  but improvement seems to work (in most cases)