One-loop Matching Factors for Staggered Four-Fermion Operators with Improved gluons

Jongjeong Kim, Weonjong Lee, and Stephen R. Sharpe

Lattice 2010

Motivation

- We want to obtain the 1-loop matching factors for B_K.
- Mixed action:
  - HYP staggered valence quarks
  - MILC Lattices (Symanzik improved gluons)
- This work is a direct extension of the previous work using HYP staggered valence quarks with Wilson gluons, done by Lee & Sharpe in '03. [Phys. Rev. D68 (2003) 054510, hep-lat/0306016]

Actions

- Sea quarks: Asqtad fermions (They do not contribute to the renormalization of the four-fermion operators at one loop order.)
- Valence quarks:
  1. unimproved staggered fermions
  2. HYP staggered fermions
- Gluons:
  1. Wilson plaquette gluons
  2. Symanzik improved gluons
- Four choices of actions:
  (a) unimproved staggered fermions + Wilson plaquette gluons

  (b) HYP staggered fermions + Wilson plaquette gluons

  (c) unimproved staggered fermions + Symanzik improved gluons

  (d) HYP staggered fermions + Symanzik improved gluons

HYP Staggered Fermions

$$S_f = \sum_{x,y} \bar{\chi}(x) \left[\sum_{\mu} \eta_\mu(x) D_\mu^{\mathrm{HYP}}(x,y) + m\delta_{x,y} \right] \chi(y)$$

- HYP covariant derivative:
  - $V_\mu(x)$ = HYP links

$$D_\mu^{\mathrm{HYP}}(x,y) = \frac{1}{2}\left(\delta_{y,x+a\hat{\mu}} V_\mu(x) - \delta_{y,x-a\hat{\mu}} V_\mu^\dagger(x-a\hat{\mu})\right)$$

- Unimproved covariant derivative:
  - $U_\mu(x)$ = Thin links

$$D_\mu(x,y) = \frac{1}{2}\left(\delta_{y,x+a\hat{\mu}} U_\mu(x) - \delta_{y,x-a\hat{\mu}} U_\mu^\dagger(x-a\hat{\mu})\right)$$

- HYP links $V_\mu(n)$ are constructed through three steps of blocking with the SU(3) projection using the links only within the hypercube.

[A. Hasenfratz & F. Knechtli]

Smeared Gauge Fields

- The thin links $U_\mu(x)$ are related with the gauge fields:

$$U_\mu(x) = e^{iagA_\mu(x+a\hat{\mu}/2)}$$

- The HYP links $V_\mu(x)$ also can be written in the same manner

$$V_\mu(x) = e^{iagB_\mu(x+a\hat{\mu}/2)}$$

  in terms of HYP smeared gauge fields $B_\mu(x)$.

- The smeared gauge fields are functions of the original gauge fields:

$$B_\mu[A_\nu] = \sum_{n=1}^{\infty} B_\mu^{(n)}[A_\nu] \quad (\text{e.g. } B^{(1)} \propto A,\ B^{(2)} \propto AA)$$

- At one loop order, only $B_\mu^{(1)}$ contributes to the renormalization of four-fermion operators. [PRD66 (2002) 114504, hep-lat/0208032, by W. Lee]

Smearing Kernel

$$B_\mu^{(1)}(k) = \sum_\nu h_{\mu\nu}(k) A_\nu(k)$$

(linear part of)

Smeared gauge fields

original gauge fields

- $h_{\mu\nu}(k)$ contains all information of HYP smearing.
- HYP Links with the coefficients chosen to remove O(a^2) taste symmetry breaking term at tree-level:

$$h_{\mu\nu}(k) = \delta_{\mu\nu} D_\mu(k) + (1 - \delta_{\mu\nu}) G_{\mu\nu}(k)$$

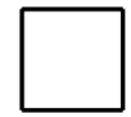
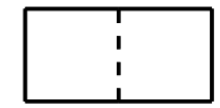
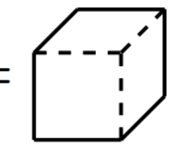
$$D_\mu(k) = 1 - \sum_{\nu \neq \mu} \bar{s}_\nu^2 + \sum_{\substack{\nu < \rho \\ \nu, \rho \neq \mu}} \bar{s}_\nu^2 \bar{s}_\rho^2 - \bar{s}_\nu^2 \bar{s}_\rho^2 \bar{s}_\sigma^2$$

$$G_{\mu\nu}(k) = \bar{s}_\mu \bar{s}_\nu \left[1 - \frac{(\bar{s}_\rho^2 + \bar{s}_\sigma^2)}{2} + \frac{\bar{s}_\rho^2 \bar{s}_\sigma^2}{3} \right] \qquad \bar{s}_\mu = \sin(k_\mu/2)$$

- Unimproved Links: $h_{\mu\nu}(k) = \delta_{\mu\nu}$

Improved Gluon Action

$$S_g = \frac{2}{g^2}\left[c_{\rm pl}\sum_{\rm pl}{\rm ReTr}(1-U_{\rm pl}) + c_{\rm rt}\sum_{\rm rt}{\rm ReTr}(1-U_{\rm rt}) + c_{\rm pg}\sum_{\rm pg}{\rm ReTr}(1-U_{\rm pg})\right]$$

pl =  rt =  pg = 

- Iwasaki: [Iwasaki]

$$c_{\rm pl} = (1-8c_{\rm rt})\,,\;\; c_{\rm rt} = -0.331\,,\;\; c_{\rm pg} = 0\,.$$

- DBW2: [Takaishi]

$$c_{\rm pl} = (1-8c_{\rm rt})\,,\;\; c_{\rm rt} = -1.4067\,,\;\; c_{\rm pg} = 0\,.$$

- Symanzik: [Lüscher & Weisz]

$$c_{\rm pl} = \frac{5}{3}\,,\;\; c_{\rm rt} = -\frac{1}{12}\,,\;\; c_{\rm pg} = 0\,.$$ Tree level

$$c_{\rm pl} = \frac{5}{3} + [0.2370]g^2\,,\;\; c_{\rm rt} = -\frac{1}{12} - [0.02521]g^2\,,\;\; c_{\rm pg} = -[0.00441]g_0^2\,.$$ 1-loop level

- Wilson plaquette action:

$$c_{\rm pl} = 1\,,\;\; c_{\rm rt} = 0\,,\;\; c_{\rm pg} = 0\,.$$

Bilinear Operators

In the continuum:

- Quark field : $Q^{a,f}_{\alpha,t}(x)$ (Color index: a; flavor index: f; Spin index: α; Taste index: t)

$$\gamma_S = \gamma_1^{S_1}\gamma_2^{S_2}\gamma_3^{S_3}\gamma_4^{S_4}$$
$$\xi_F = \xi_1^{F_1}\xi_2^{F_2}\xi_3^{F_3}\xi_4^{F_4}$$
$$\xi_\mu = \gamma_\mu^*$$

- Bilinear Operator having spin “S” and taste “F”:

$$\bar{Q}(x)(\gamma_S \otimes \xi_F)Q(x) = \bar{Q}^{a,f}_{\alpha,t}(x)[\gamma_S]^{\alpha\beta}[\xi_F]^{th}Q^{a,f'}_{\beta,h}(x)$$

On the Lattice:

[Kluberg-Stern]

- A,B,S,F : hypercube indices
- a,b,c : color indices

$$[S \times F](y) = \frac{1}{16}\sum_{A,B}[\bar{\chi}_b(y+A)\ (\overline{\gamma_S \otimes \xi_F})_{AB}\ \chi_c(y+B)]\ \mathcal{V}^{bc}(y+A, y+B)$$

- $\mathcal{V}^{bc}(y+A, y+B)$ ensures the gauge invariance.
- $\mathcal{V}^{bc}(y+A, y+B)$ is the Wilson line made of HYP links averaged over all the possible shortest paths connecting $y+A$ and $y+B$.

Continuum Four-Fermion Operators

- There are two types of four-fermion operators according to how the color indices are contracted.

one-color-trace:

- The color index of the quark in one of bilinears is contracted with that of the anti-quark in the other bilinear.

$$\bar{Q}^a(x)(\gamma_S \otimes \xi_F)Q^b(x)\bar{Q}^b(x)(\gamma_{S'} \otimes \xi_{F'})Q^a(x)$$

two-color-trace:

- Color indices of the quark and the anti-quark within each bilinear are contracted

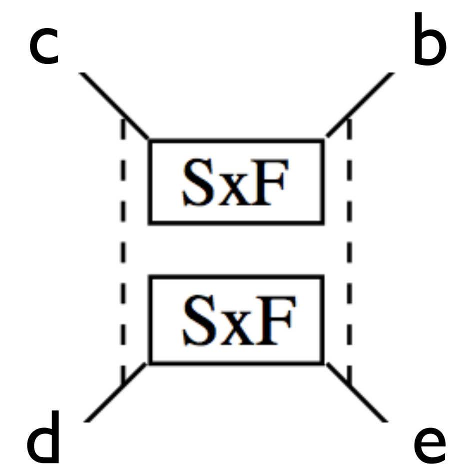
$$\bar{Q}^a(x)(\gamma_S \otimes \xi_F)Q^a(x)\bar{Q}^b(x)(\gamma_{S'} \otimes \xi_{F'})Q^b(x)$$

Lattice Four-Fermion Operators

Flavor → $\chi_a^{(1)}(y+A)$ ← Hypercube
↑ Color

one-color-trace:

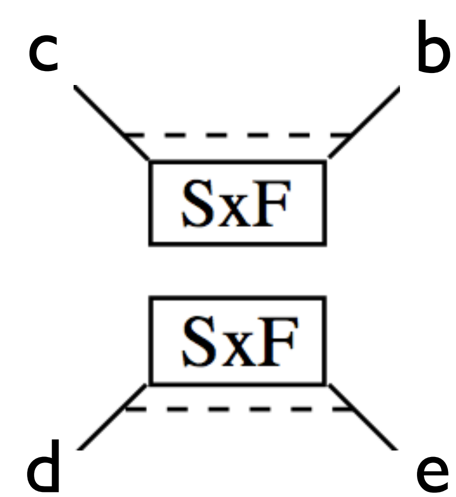
$$[S\times F][S'\times F']_I(y) = \frac{1}{N_f^4}\sum_{A,B,C,D}[\bar\chi_b^{(1)}(y+A)\ (\overline{\gamma_S\otimes\xi_F})_{AB}\ \chi_c^{(2)}(y+B)]\ [\bar\chi_d^{(3)}(y+C)\ (\overline{\gamma_{S'}\otimes\xi_{F'}})_{CD}\ \chi_e^{(4)}(y+D)]\ \mathcal{V}^{be}(y+A,y+D)\ \mathcal{V}^{dc}(y+C,y+B)$$



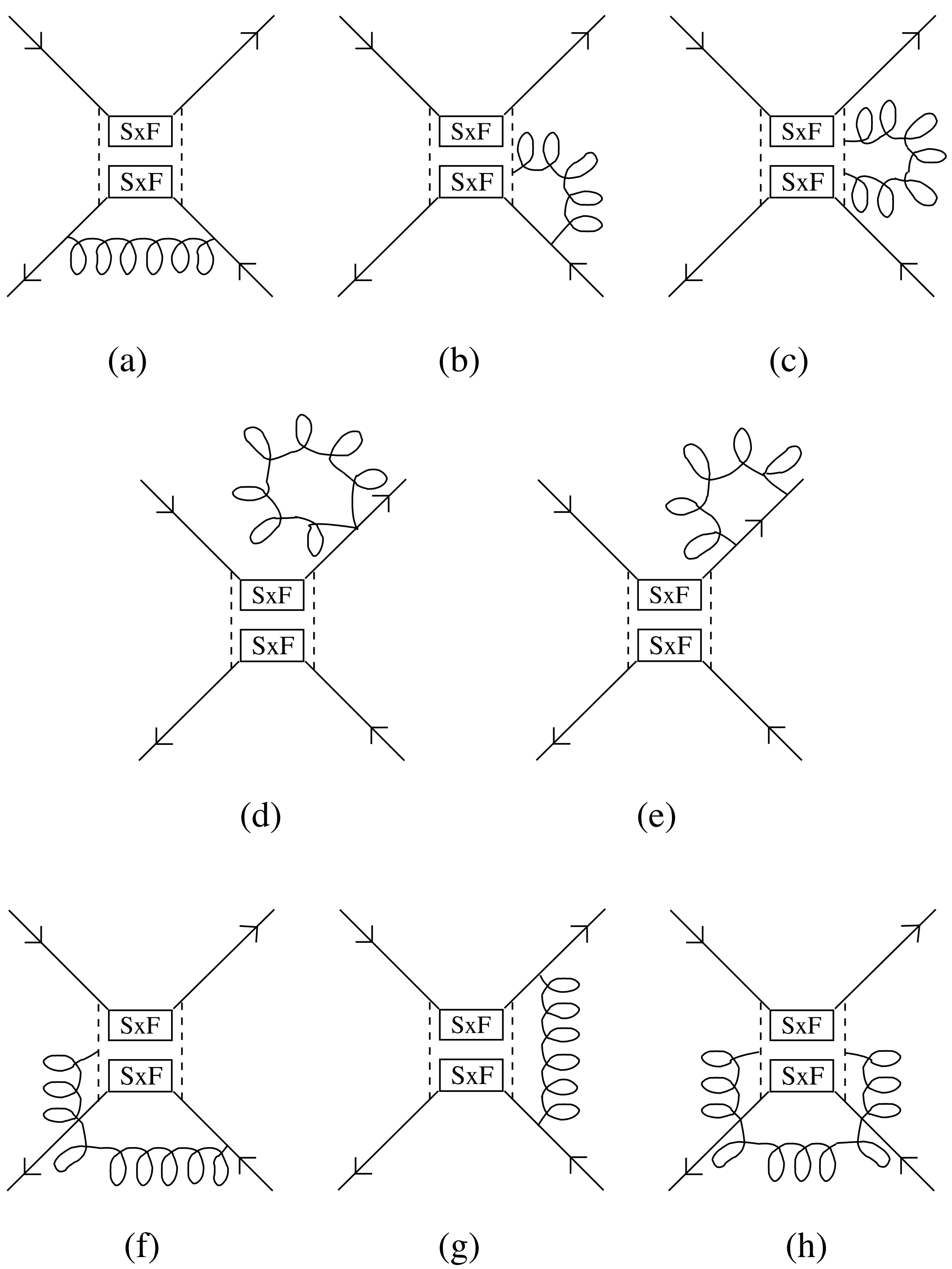
- $\mathcal{V}^{bc}(y+A,y+B)$ is the Wilson line made of HYP links averaged over all the possible shortest paths connecting $y+A$ and $y+B$.

two-color-trace:

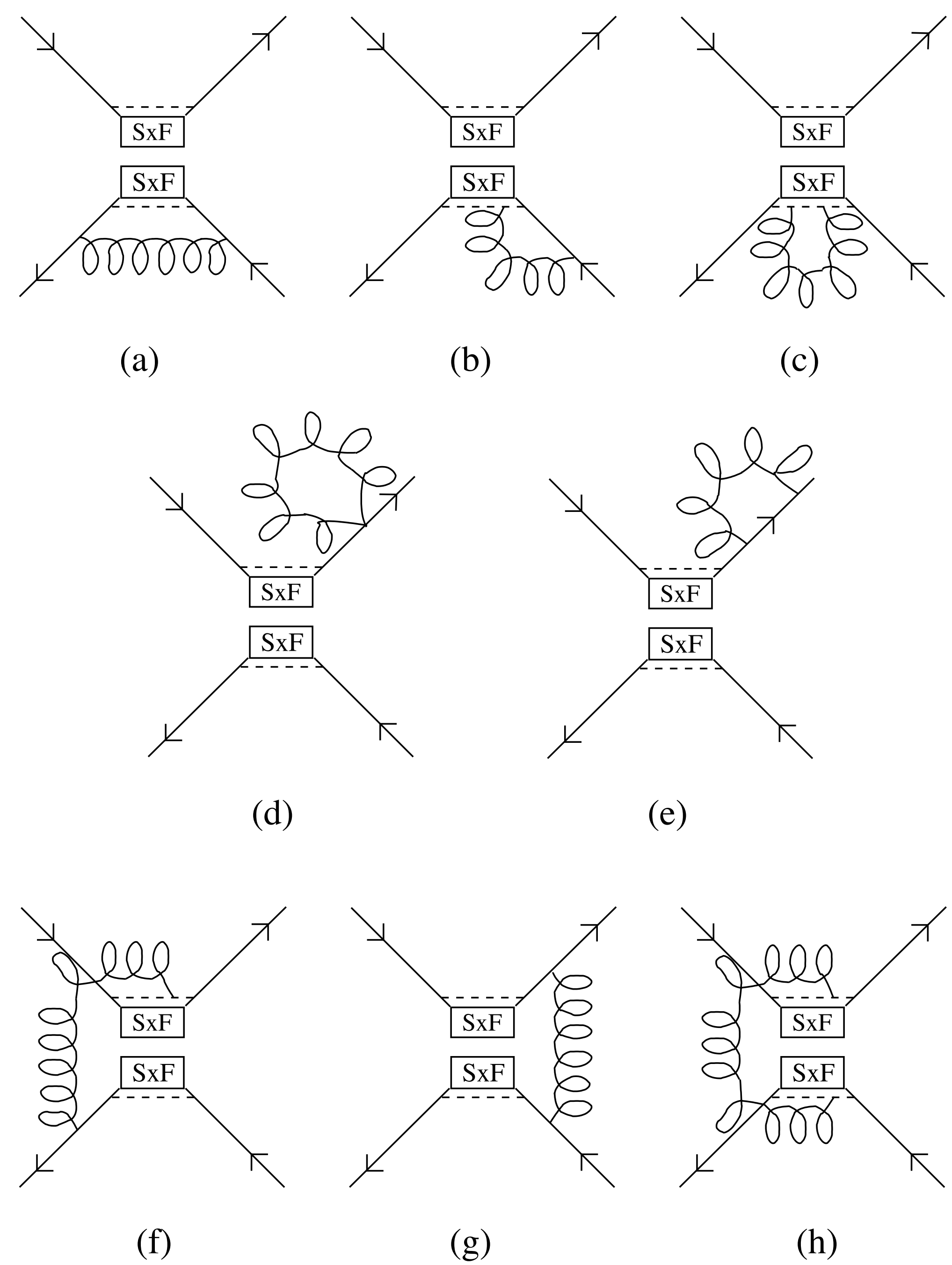
$$[S\times F][S'\times F']_{II}(y) = \frac{1}{N_f^4}\sum_{A,B,C,D}[\bar\chi_b^{(1)}(y+A)\ (\overline{\gamma_S\otimes\xi_F})_{AB}\ \chi_c^{(2)}(y+B)]\ [\bar\chi_d^{(3)}(y+C)\ (\overline{\gamma_{S'}\otimes\xi_{F'}})_{CD}\ \chi_e^{(4)}(y+D)]\ \mathcal{V}^{bc}(y+A,y+B)\ \mathcal{V}^{de}(y+C,y+D)$$



Feynman Diagrams



one-color-trace



two-color-trace

[PRD68 (2003) 054510, hep-lat/0306016, by Lee & Sharpe]

Smeared Gluon Propagator

[Lee & Sharpe]

- Gluon propagator (in Feynman gauge):

$$\langle A^b_\alpha(k) A^c_\beta(-k)\rangle = \begin{cases} \delta^{bc}\delta_{\alpha\beta}B(k) & \text{for Wilson gluons} \\ \delta^{bc}D^{\rm Imp}_{\alpha\beta}(k) & \text{for Improved gluons} \end{cases} \qquad B(k) = \frac{1}{4\sin(k/2)^2}$$

- Smeared gluon propagator

$$\langle B^{(1),b}_\mu(k) B^{(1),c}_\nu(-k)\rangle$$

$$= \sum_{\alpha,\beta} h_{\mu\alpha}(k)\ h_{\nu\beta}(-k)\ \langle A^b_\alpha(k) A^c_\beta(-k)\rangle = \begin{cases} \delta^{bc}\sum_\lambda h_{\mu\lambda}(k)\ h_{\nu\lambda}(k) B(k) & \text{for Wilson gluons} \\ \delta^{bc}\sum_{\alpha,\beta} h_{\mu\alpha}(k)\ h_{\nu\beta}(k) D^{\rm Imp}_{\alpha\beta}(k) & \text{for Improved gluons} \end{cases}$$

- The results of using the improved gluons can be obtained from those of the previous work by a simple replacement of the smeared gluon propagator:

$$\sum_\lambda h_{\mu\lambda}h_{\nu\lambda}B \rightarrow \sum_{\alpha\beta} h_{\mu\alpha}h_{\nu\beta}D^{\rm Imp}_{\alpha\beta}$$

Fierz Transformation

$$(\overline{\gamma_S \otimes \xi_F})_{AB}(\overline{\gamma_{S'} \otimes \xi_{F'}})_{A'B'} = \frac{1}{16}\sum_{DE}(\overline{\gamma_S \gamma_D^\dagger \otimes \xi_E^\dagger \xi_{F'}})_{AB'}(\overline{\gamma_{S'}\gamma_D \otimes \xi_E \xi_F})_{A'B} \quad .$$

one-color-trace op. $\Longleftrightarrow$ two-color-trace op.

Non-trivial check

- One can obtain the results of one-color-trace operators from those of two-color-trace operators and vice versa.
- We calculate the 1-loop renormalization of one and two color-trace operators directly and use this Fierz transformation to check our results.
- We've confirmed this Fierz transformation holds for the our results.

Mean-Field Improvement (I)

[Lepage & Mackenzie]

- Fields rescaling:

$$\chi \longrightarrow \psi = \sqrt{v_0}\chi$$
$$\bar{\chi} \longrightarrow \bar{\psi} = \sqrt{v_0}\bar{\chi}$$
$$V_\mu \longrightarrow \frac{V_\mu}{v_0}$$

- Mean-field factor:

$$v_0 = \left[\frac{1}{3}\mathrm{Re}\langle \mathrm{Tr} V_\Box \rangle\right]^{1/4}$$
$$= 1 - \frac{g^2}{(4\pi)^2} C_F I_{\mathrm{MF}} + O(g^4)$$

- $C_F = \frac{4}{3}$

- I_{MF} :

(a)	(b)	(c)	(d)
9.8696	1.0538	7.2297	0.7228

Mean-Field Improvement (II)

two-color-trace:

- The effect of the mean-field improvement is a shift of the diagonal components of the renormalization factor.

$$[S \times F][S' \times F']_{II} \mathrel{+}= \frac{g^2}{(4\pi)^2} C_F I_{\rm MF} (\Delta_{SF} + \Delta_{S'F'} - 2)[S \times F][S' \times F']_{II}$$

- $\Delta_{SF} = \sum_{\mu} (S - F)^2_{\mu}$ is the distance between quark and anti-quark in the bilinear having spin "S" and taste "F".

one-color-trace:

- We calculate the mean-field improvement factor using the Fierz transformation.

1-loop Renormalization to B_K

In the continuum(NDR):

- λ "gluon mass"

$$\mathcal{O}_{B_K}^{\rm Cont} = [\bar{s}\gamma_\mu(1-\gamma_5)d][\bar{s}\gamma_\mu(1-\gamma_5)d]$$

$$\mathcal{O}_{B_K}^{\rm Cont,(1)} = \left(1 + \frac{g^2}{(4\pi)^2}\left[4\log(\lambda/\mu) - \frac{11}{3}\right]\right)\mathcal{O}_{B_K}^{\rm Cont,(0)}$$

On the Lattice:

spin

$$[S \times F][S' \times F']_I$$

taste

color-trace

- V_μ : vector,
- A_μ : axial-vector
- P : pseudo-scalar

$$\mathcal{V}_1 = [V_\mu \times P][V_\mu \times P]_I\,, \qquad \mathcal{V}_2 = [V_\mu \times P][V_\mu \times P]_{II}$$

$$\mathcal{A}_1 = [A_\mu \times P][A_\mu \times P]_I\,, \qquad \mathcal{A}_2 = [A_\mu \times P][A_\mu \times P]_{II}$$

$$\mathcal{O}_{B_K}^{\rm Latt} = \mathcal{V}_1 + \mathcal{V}_2 + \mathcal{A}_1 + \mathcal{A}_2$$

$$\mathcal{O}_{B_K}^{\rm Latt,(1)} = \left(1 + \frac{g^2}{(4\pi)^2}[4\log(a\lambda)]\right)\mathcal{O}_{B_K}^{\rm Latt,(0)} + \frac{g^2}{(4\pi)^2}\mathcal{F} + \mathcal{O}(a)$$

- Finite Corrections: $\mathcal{F} = c_1\mathcal{V}_1^{(0)} + c_2\mathcal{V}_2^{(0)} + c_3\mathcal{A}_1^{(0)} + c_4\mathcal{A}_2^{(0)}$

Finite Corrections

$$\mathcal{F} = c_1 \mathcal{V}_1^{(0)} + c_2 \mathcal{V}_2^{(0)} + c_3 \mathcal{A}_1^{(0)} + c_4 \mathcal{A}_2^{(0)}$$

$$\mathcal{V}_1 = [V_\mu \times P][V_\mu \times P]_I \,, \qquad \mathcal{V}_2 = [V_\mu \times P][V_\mu \times P]_{II}$$
$$\mathcal{A}_1 = [A_\mu \times P][A_\mu \times P]_I \,, \qquad \mathcal{A}_2 = [A_\mu \times P][A_\mu \times P]_{II}$$

- Four choices of lattice actions:
  - (a) unimproved staggered quarks + Wilson plaquette gluons
  - (b) HYP staggered quarks + Wilson plaquette gluons
  - (c) unimproved staggered quarks + Symanzik improved gluons
  - (d) HYP staggered quarks + Symanzik improved gluons

	(a)	(b)	(c)	(d)
c_1	-2.35(9)	-2.18(8)	-2.49(8)	-1.73(8)
c_2	-12.8(1)	-5.43(9)	-11.5(1)	-4.67(9)
c_3	-2.95(8)	-2.69(8)	-3.07(8)	-2.19(8)
c_4	-3.73(8)	0.98(5)	-2.91(7)	1.05(5)

- The HYP smearing is significantly more efficient to reduce the size of 1-loop perturbative corrections than the improvement of gluon action.
- Combining both improvements, we obtain the smallest size of 1-loop corrections. They are about 28% ~ 74% level of the unimproved values.

Finite Corrections

$$\mathcal{F} = c_1 \mathcal{V}_1^{(0)} + c_2 \mathcal{V}_2^{(0)} + c_3 \mathcal{A}_1^{(0)} + c_4 \mathcal{A}_2^{(0)}$$

$$\mathcal{V}_1 = [V_\mu \times P][V_\mu \times P]_I\,, \qquad \mathcal{V}_2 = [V_\mu \times P][V_\mu \times P]_{II}$$

$$\mathcal{A}_1 = [A_\mu \times P][A_\mu \times P]_I\,, \qquad \mathcal{A}_2 = [A_\mu \times P][A_\mu \times P]_{II}$$

- Four choices of lattice actions:
  - (a) unimproved staggered quarks + Wilson plaquette gluons
  - (b) HYP staggered quarks + Wilson plaquette gluons
  - (c) unimproved staggered quarks + Symanzik improved gluons
  - (d) HYP staggered quarks + Symanzik improved gluons

	(a)	(b)	(c)	(d)
c_1	-2.35(9)	-2.18(8)	-2.49(8)	-1.73(8)
c_2	-12.8(1)	5.43([illegible])	-11.5(1)	-4.67(9)
c_3	-2.95(8)	-2.69(8)	-3.07(8)	-2.19(8)
c_4	-3.73(8)	0.98(5)	-2.91(7)	1.05(5)

- The HYP smearing is significantly more efficient to reduce the size of 1-loop perturbative corrections than the improvement of gluon action.
- Combining both improvements, we obtain the smallest size of 1-loop corrections. They are about 28% ~ 74% level of the unimproved values.

Finite Corrections

$$\mathcal{F} = c_1 \mathcal{V}_1^{(0)} + c_2 \mathcal{V}_2^{(0)} + c_3 \mathcal{A}_1^{(0)} + c_4 \mathcal{A}_2^{(0)}$$

$$\mathcal{V}_1 = [V_\mu \times P][V_\mu \times P]_I\,, \qquad \mathcal{V}_2 = [V_\mu \times P][V_\mu \times P]_{II}$$

$$\mathcal{A}_1 = [A_\mu \times P][A_\mu \times P]_I\,, \qquad \mathcal{A}_2 = [A_\mu \times P][A_\mu \times P]_{II}$$

- Four choices of lattice actions:
  - (a) unimproved staggered quarks + Wilson plaquette gluons
  - (b) HYP staggered quarks + Wilson plaquette gluons
  - (c) unimproved staggered quarks + Symanzik improved gluons
  - (d) HYP staggered quarks + Symanzik improved gluons

	(a)	(b)	(c)	(d)
c_1	-2.35(9)	-2.18(8)	-2.49(8)	-1.73(8)
c_2	-12.8(1)	-5.43(9)	-11.5(1)	-4.67(9)
c_3	-2.95(8)	-2.69(8)	-3.07(8)	-2.19(8)
c_4	-3.73(8)	0.98(5)	-2.91(7)	1.05(5)

- The HYP smearing is significantly more efficient to reduce the size of 1-loop perturbative corrections than the improvement of gluon action.
- Combining both improvements, we obtain the smallest size of 1-loop corrections. They are about 28% ~ 74% level of the unimproved values.

Finite Corrections

$$\mathcal{F} = c_1 \mathcal{V}_1^{(0)} + c_2 \mathcal{V}_2^{(0)} + c_3 \mathcal{A}_1^{(0)} + c_4 \mathcal{A}_2^{(0)}$$

$$\mathcal{V}_1 = [V_\mu \times P][V_\mu \times P]_I \,, \qquad \mathcal{V}_2 = [V_\mu \times P][V_\mu \times P]_{II}$$

$$\mathcal{A}_1 = [A_\mu \times P][A_\mu \times P]_I \,, \qquad \mathcal{A}_2 = [A_\mu \times P][A_\mu \times P]_{II}$$

- Four choices of lattice actions:
  - (a) unimproved staggered quarks + Wilson plaquette gluons
  - (b) HYP staggered quarks + Wilson plaquette gluons
  - (c) unimproved staggered quarks + Symanzik improved gluons
  - (d) HYP staggered quarks + Symanzik improved gluons

	(a)	(b)	(c)	(d)
c_1	-2.35(9)	-2.18(8)	-2.49(8)	-1.73(8)
c_2	-12.8(1)	-5.43(9)	-11.5(1)	-4.67(9)
c_3	-2.95(8)	-2.69(8)	-3.07(8)	-2.19(8)
c_4	-3.73(8)	0.98(5)	-2.91(7)	1.05(5)

- The HYP smearing is significantly more efficient to reduce the size of 1-loop perturbative corrections than the improvement of gluon action.
- Combining both improvements, we obtain the smallest size of 1-loop corrections. They are about 28% ~ 74% level of the unimproved values.

Finite Corrections

$$\mathcal{F} = c_1 \mathcal{V}_1^{(0)} + c_2 \mathcal{V}_2^{(0)} + c_3 \mathcal{A}_1^{(0)} + c_4 \mathcal{A}_2^{(0)}$$

$$\mathcal{V}_1 = [V_\mu \times P][V_\mu \times P]_I\,, \qquad \mathcal{V}_2 = [V_\mu \times P][V_\mu \times P]_{II}$$

$$\mathcal{A}_1 = [A_\mu \times P][A_\mu \times P]_I\,, \qquad \mathcal{A}_2 = [A_\mu \times P][A_\mu \times P]_{II}$$

- Four choices of lattice actions:
  - (a) unimproved staggered quarks + Wilson plaquette gluons
  - (b) HYP staggered quarks + Wilson plaquette gluons
  - (c) unimproved staggered quarks + Symanzik improved gluons
  - (d) HYP staggered quarks + Symanzik improved gluons

	(a)	(b)	(c)	(d)
c_1	-2.35(9)	-2.18(8)	-2.49(8)	-1.73(8)
c_2	-12.8(1)	-5.43(9)	-11.5(1)	-4.67(9)
c_3	-2.95(8)	-2.69(8)	-3.07(8)	-2.19(8)
c_4	-3.73(8)	0.98(5)	-2.91(7)	1.05(5)

- The HYP smearing is significantly more efficient to reduce the size of 1-loop perturbative corrections than the improvement of gluon action.
- Combining both improvements, we obtain the smallest size of 1-loop corrections. They are about 28% ~ 74% level of the unimproved values.

1-loop Matching Factor for B_K

$$\mathcal{O}_{B_K}^{\text{Cont},(1)} = \bar{c}_1 \mathcal{V}_1 + \bar{c}_2 \mathcal{V}_2 + \bar{c}_3 \mathcal{A}_1 + \bar{c}_4 \mathcal{A}_2$$

$$\bar{c}_i \equiv 1 + \frac{\alpha_s}{(4\pi)} \Big[-4 \log(\mu a) + (-\frac{11}{3} - c_i) \Big] \,, \quad \text{for } i = 1,2,3,4$$

- For MILC coarse lattice $(a = 0.12 \text{ fm})$ using the parallel (or horizontal) matching and using the value of $g^2_{\overline{\text{MS}}}(\mu = 1/a; N_f = 3, \text{4-loop}) = 4.1861\ldots$

	(a)	(b)	(c)	(d)
$\bar{c}_1$	$0.965(2)$	$0.961(2)$	$0.969(2)$	$0.949(2)$
$\bar{c}_2$	$1.243(3)$	$1.047(2)$	$1.207(3)$	$1.027(2)$
$\bar{c}_3$	$0.981(2)$	$0.974(2)$	$0.984(2)$	$0.961(2)$
$\bar{c}_4$	$1.002(2)$	$0.877(1)$	$0.980(2)$	$0.875(1)$

1-loop Matching Factor for B_K

$$\mathcal{O}_{B_K}^{\mathrm{Cont},(1)} = \bar{c}_1 \mathcal{V}_1 + \bar{c}_2 \mathcal{V}_2 + \bar{c}_3 \mathcal{A}_1 + \bar{c}_4 \mathcal{A}_2$$

$$\bar{c}_i \equiv 1 + \frac{\alpha_s}{(4\pi)} \Big[-4 \log(\mu a) + (-\frac{11}{3} - c_i) \Big] \,, \quad \text{for } i = 1, 2, 3, 4$$

- For MILC coarse lattice $(a = 0.12 \text{ fm})$ using the parallel (or horizontal) matching and using the value of $g^2_{\overline{\mathrm{MS}}}(\mu = 1/a; N_f = 3, \text{4-loop}) = 4.1861 \ldots$

	(a)	(b)	(c)	(d)
$\bar{c}_1$	0.965(2)	0.961(2)	0.969(2)	0.949(2)
$\bar{c}_2$	1.243(3)	1.047(2)	1.207(3)	1.027(2)
$\bar{c}_3$	0.981(2)	0.974(2)	0.984(2)	0.961(2)
$\bar{c}_4$	1.002(2)	0.877(1)	0.980(2)	0.875(1)

1-loop Matching Factor for B_K

$$\mathcal{O}^{\text{Cont},(1)}_{B_K} = \bar{c}_1 \mathcal{V}_1 + \bar{c}_2 \mathcal{V}_2 + \bar{c}_3 \mathcal{A}_1 + \bar{c}_4 \mathcal{A}_2$$

$$\bar{c}_i \equiv 1 + \frac{\alpha_s}{(4\pi)} \Big[-4 \log(\mu a) + (-\frac{11}{3} - c_i) \Big] , \quad \text{for } i = 1, 2, 3, 4$$

- For MILC coarse lattice $(a = 0.12 \text{ fm})$ using the parallel (or horizontal) matching and using the value of $g^2_{\overline{\text{MS}}}(\mu = 1/a; N_f = 3, 4\text{-loop}) = 4.1861\ldots$

	(a)	(b)	(c)	(d)
$\bar{c}_1$	0.965(2)	0.961(2)	0.969(2)	0.949(2)
$\bar{c}_2$	1.243(3)	1.047(2)	1.207(3)	1.027(2)
$\bar{c}_3$	0.981(2)	0.974(2)	0.984(2)	0.961(2)
$\bar{c}_4$	1.002(2)	0.877(1)	0.980(2)	0.875(1)

1-loop Matching Factor for B_K

$$\mathcal{O}_{B_K}^{\text{Cont},(1)} = \bar{c}_1 \mathcal{V}_1 + \bar{c}_2 \mathcal{V}_2 + \bar{c}_3 \mathcal{A}_1 + \bar{c}_4 \mathcal{A}_2$$

$$\bar{c}_i \equiv 1 + \frac{\alpha_s}{(4\pi)} \Big[-4\log(\mu a) + (-\frac{11}{3} - c_i) \Big] \,, \quad \text{for } i = 1,2,3,4$$

- For MILC coarse lattice $(a = 0.12 \text{ fm})$ using the parallel (or horizontal) matching and using the value of $g^2_{\overline{\text{MS}}}(\mu = 1/a; N_f = 3, \text{4-loop}) = 4.1861\ldots$

	(a)	(b)	(c)	(d)
$\bar{c}_1$	0.965(2)	0.961(2)	0.969(2)	0.949(2)
$\bar{c}_2$	1.243(3)	1.047(2)	1.207(3)	1.027(2)
$\bar{c}_3$	0.981(2)	0.974(2)	0.984(2)	0.961(2)
$\bar{c}_4$	1.002(2)	0.877(1)	0.980(2)	0.875(1)

1-loop Matching Factor for B_K

$$\mathcal{O}_{B_K}^{\text{Cont},(1)} = \bar{c}_1 \mathcal{V}_1 + \bar{c}_2 \mathcal{V}_2 + \bar{c}_3 \mathcal{A}_1 + \bar{c}_4 \mathcal{A}_2$$

$$\bar{c}_i \equiv 1 + \frac{\alpha_s}{(4\pi)} \Big[-4\log(\mu a) + (-\frac{11}{3} - c_i) \Big] , \quad \text{for } i = 1, 2, 3, 4$$

- For MILC coarse lattice $(a = 0.12 \text{ fm})$ using the parallel (or horizontal) matching and using the value of $g^2_{\overline{\text{MS}}}(\mu = 1/a; N_f = 3, \text{4-loop}) = 4.1861\ldots$

	(a)	(b)	(c)	(d)
$\bar{c}_1$	0.965(2)	0.961(2)	0.969(2)	0.949(2)
$\bar{c}_2$	1.243(3)	1.047(2)	1.207(3)	1.027(2)
$\bar{c}_3$	0.981(2)	0.974(2)	0.984(2)	0.961(2)
$\bar{c}_4$	1.002(2)	0.877(1)	0.980(2)	0.875(1)

1-loop Matching Factor for B_K

$$\mathcal{O}_{B_K}^{\text{Cont},(1)} = \bar{c}_1 \mathcal{V}_1 + \bar{c}_2 \mathcal{V}_2 + \bar{c}_3 \mathcal{A}_1 + \bar{c}_4 \mathcal{A}_2$$

$$\bar{c}_i \equiv 1 + \frac{\alpha_s}{(4\pi)} \Big[-4 \log(\mu a) + (-\frac{11}{3} - c_i) \Big] \,, \quad \text{for } i = 1, 2, 3, 4$$

- For MILC coarse lattice $(a = 0.12 \text{ fm})$ using the parallel (or horizontal) matching and using the value of $g^2_{\overline{\text{MS}}}(\mu = 1/a; N_f = 3, 4\text{-loop}) = 4.1861\ldots$

	(a)	(b)	(c)	(d)
$\bar{c}_1$	0.965(2)	0.961(2)	0.969(2)	0.949(2)
$\bar{c}_2$	1.243(3)	1.047(2)	1.207(3)	1.027(2)
$\bar{c}_3$	0.981(2)	0.974(2)	0.984(2)	0.961(2)
$\bar{c}_4$	1.002(2)	0.877([illegible])	0.980(2)	0.875(1)

1-loop Matching Factor for B_K

$$\mathcal{O}^{\text{Cont},(1)}_{B_K} = \bar{c}_1\mathcal{V}_1 + \bar{c}_2\mathcal{V}_2 + \bar{c}_3\mathcal{A}_1 + \bar{c}_4\mathcal{A}_2$$

$$\bar{c}_i \equiv 1 + \frac{\alpha_s}{(4\pi)}\Big[-4\log(\mu a) + (-\frac{11}{3} - c_i)\Big]\,, \quad \text{for } i = 1,2,3,4$$

- For MILC coarse lattice $(a = 0.12\ \text{fm})$ using the parallel (or horizontal) matching and using the value of $g^2_{\overline{\text{MS}}}(\mu = 1/a; N_f = 3, 4\text{-loop}) = 4.1861\ldots$

	(a)	(b)	(c)	(d)
$\bar{c}_1$	0.965(2)	0.961(2)	0.969(2)	0.949(2)
$\bar{c}_2$	1.243(3)	1.047(2)	1.207(3)	1.027(2)
$\bar{c}_3$	0.981(2)	0.974(2)	0.984(2)	0.961(2)
$\bar{c}_4$	1.002(2)	0.877(1)	0.980(2)	0.875(1)

1-loop Matching Factor for B_K

$$\mathcal{O}^{\text{Cont},(1)}_{B_K} = \bar{c}_1 \mathcal{V}_1 + \bar{c}_2 \mathcal{V}_2 + \bar{c}_3 \mathcal{A}_1 + \bar{c}_4 \mathcal{A}_2$$

$$\bar{c}_i \equiv 1 + \frac{\alpha_s}{(4\pi)} \Big[-4\log(\mu a) + (-\frac{11}{3} - c_i) \Big] , \quad \text{for } i = 1,2,3,4$$

- For MILC coarse lattice $(a = 0.12 \text{ fm})$ using the parallel (or horizontal) matching and using the value of $g^2_{\overline{\text{MS}}}(\mu = 1/a; N_f = 3, \text{4-loop}) = 4.1861\ldots$

	(a)	(b)	(c)	(d)
$\bar{c}_1$	0.965(2)	0.961(2)	0.969(2)	0.949(2)
$\bar{c}_2$	1.243(3)	1.047(2)	1.207(3)	1.027(2)
$\bar{c}_3$	0.981(2)	0.974(2)	0.984(2)	0.961(2)
$\bar{c}_4$	1.002(2)	0.877(1)	0.980(2)	0.875(1)

1-loop Matching Factor for B_K

$$\mathcal{O}_{B_K}^{\text{Cont},(1)} = \bar{c}_1 \mathcal{V}_1 + \bar{c}_2 \mathcal{V}_2 + \bar{c}_3 \mathcal{A}_1 + \bar{c}_4 \mathcal{A}_2$$

$$\bar{c}_i \equiv 1 + \frac{\alpha_s}{(4\pi)} \Big[-4 \log(\mu a) + (-\frac{11}{3} - c_i) \Big] \,, \quad \text{for } i = 1, 2, 3, 4$$

- For MILC coarse lattice $(a = 0.12 \text{ fm})$ using the parallel (or horizontal) matching and using the value of $g^2_{\overline{\text{MS}}}(\mu = 1/a; N_f = 3, \text{4-loop}) = 4.1861\ldots$

	(a)	(b)	(c)	(d)
$\bar{c}_1$	0.965(2)	0.961(2)	0.969(2)	0.949(2)
$\bar{c}_2$	1.243(3)	1.047(2)	1.207(3)	1.027(2)
$\bar{c}_3$	0.981(2)	0.974(2)	0.984(2)	0.961(2)
$\bar{c}_4$	1.002(2)	0.877(1)	0.980(2)	0.875(1)

Summary

- We've calculated the one-loop matching factors for four-fermion operators using the HYP improved staggered fermions with the Symanzik improved gluons.
- The results of using the improved gluons can be obtained from those of the previous work just by the simple replacement of the smeared gluon propagator.
- As a direct application, we've obtained the one-loop matching factors for B_K relevant to our numerical work.
- We've found that the 1-loop corrections to matching factors for B_K is 3% ~ 12% level for the MILC coarse lattice using the parallel matching.