2 Flavor Chiral Perturbation Theory for Hyperons

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Outline

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Motivation

- Lattice QCD is the only known first principles nonperturbative approach to QCD
- Many lattice simulations done today are restricted to unphysically large quark masses, finite extend of box sizes and nonvanishing lattice spacings
- Chiral perturbation theory (χPT): extrapolate lattice data to the physical point
- The use of lattice QCD in conjunction with 2 flavor χPT enables one to understand nonstrange hadronic physics from first principles calculations quantitatively

- The success of 2-flavor χPT faces a great challenge when extended to include the effects of strange quark
- Various $SU(3)$ predictions for hyperon properties compare poorly to experiment
- p-wave contribution to nonleptonic hyperon decays.
  $\Sigma^+ \to n\pi^+ : 1.81$ (expt), 0.82 (theory)
- $SU(3)$ expansion parameters in the (heavy) baryon sectors contains a term which scales as $m_\eta/M \sim 0.5$: m_η is the mass of η-meson and M is the average of hyperon mass
- Hence the efficacy of 3-flavor χPT is in question

$SU(2)$ Chiral Expansion for Hyperons

- Several approaches are known to improve the $SU(3)$ expansion
- Our approach is altogether different :
  - Based on the observation that it is possible to reorganize the 3-flavor expansion into a 2-flavor one thereby excluding the kaon and eta loops
- The strange quark dependence is resummed:
  - Absorbed into the leading low-energy constants (lec) of the underlying $SU(2)$ theory
  - Arrise through power-law suppressed terms : $\sim (m/m_s)^n$ which are absorbed into lecs of pion mass dependence operators

- $SU(3)$ HBχPT : pion, kaon and eta loops
- $M_\Sigma = M^{SU(3)} + a_\pi\, m_\pi^2 + a_K\, m_K^2 + b_\pi\, m_\pi^3 + b_K\, m_K^3 + b_\eta\, m_\eta^3$

  Convergence : governed by m_ϕ^2/Λ_χ^2 and $m_\phi/M^{SU(3)}$
- Gell-Mann-Oakes-Renner and Gell-Mann-okubo formulae :

$$m_K^2 = \frac{1}{2}m_\pi^2 + \frac{1}{2}m_{\eta_s}^2 , \quad m_\eta^2 = \frac{1}{3}m_\pi^2 + \frac{2}{3}m_{\eta_s}^2$$
$$\epsilon_{SU(2)} = m_\pi^2/m_{\eta_s}^2 \sim 0.04$$

- $M_\Sigma = M_\Sigma^{SU(2)} + a_\pi^{SU(2)} m_\pi^2 + b_\pi^{SU(2)} m_\pi^3 + \ldots$

  Convergence : governed by m_π^2/Λ_χ^2 and $m_\pi/M^{SU(2)}$
- Non-analytic strange quark mass dependence is absorbed into the $SU(2)$ low-energy constants
- The $SU(2)$ expansion of kaon and eta loops generates only analytic contributions to pion mass squared

- Additional expansion parameters related to kaon production thresholds
- For the 2-flavor theory to be effective, the kaon production threshold cannot be reached
- Kaon and eta loops need not appear explicitly in the effective field theory
- Virtual kaon and eta loop contributions can be reordered
- Such a $SU(2)$ theory can be used to describe the virtual strangeness changing transitions provided one is far away from these thresholds

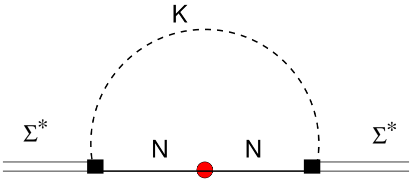
- Consider a strangeness changing process : $B' \to KB\,, \quad \Delta S = -1$
- $\delta_{BB'} = M_{B'} - M_B$:
    - $\delta_{B'B} > m_K$: decay is kinemantically allowed
    - $\delta_{B'B} < m_K$: the process is virtual
- For an $SU(2)$ description of hyperons to be valid, one must stay away from kaon production thresholds : $\delta_{B'B} \ll m_K$
- The largest $\delta_{B'B}$ is $\delta_{N\Sigma^\star} \sim 0.45$GeV
- Must find a relevant expansion parameter so that an $SU(2)$ description of hyperons is valid despite the nearness of thresholds for these physical splittings

- Terms in logarithm can be written as functions of the form :

$$f\left(m_K^2-\delta_{BB'}^2\right) = f\left(\frac{1}{2}m_{\eta_s}^2-\delta_{BB'}^2\right)+\frac{1}{2}m_\pi^2 f'\left(\frac{1}{2}m_{\eta_s}^2-\delta_{BB'}^2\right) +\frac{1}{8}m_\pi^4 f''\left(\frac{1}{2}m_{\eta_s}^2-\delta_{BB'}^2\right)+\ldots$$

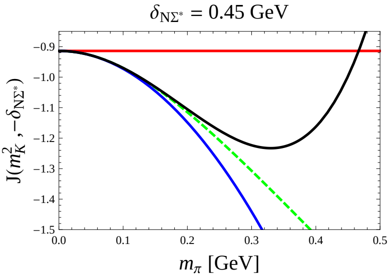
- $\epsilon_{BB'} = \frac{\frac{1}{2}m_\pi^2}{\frac{1}{2}m_{\eta_s}^2-\delta_{B'B}^2}$
- At LO order, using m_{η_s} estimated by GMOR and the physical splittings: $\epsilon_{BB'} < 0.24$
- Beyond LO, one needs to find another relevant expansion parameter around the $SU(2)$ chiral limit: $\epsilon_{BB'} < 0.35$
- $SU(3) : m_\eta/M \sim 0.5$

Effect of Kaon Virtual Thresholds on Hyperon Isovector Axial Charges



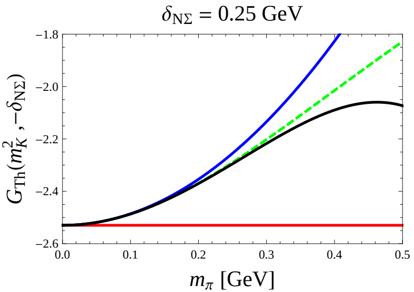
- $\Sigma^\star \to KN \to KN \to \Sigma^\star$
- Kaon-loop contributions are proportional to a non-analytic function $\mathcal{J}$:

$$\mathcal{J}(m_K^2, -\delta_{N\Sigma^\star}) = \mathcal{J}^{(0)} + m_\pi^2 \mathcal{J}^{(2)} + m_\pi^4 \mathcal{J}^{(4)} + \ldots,$$



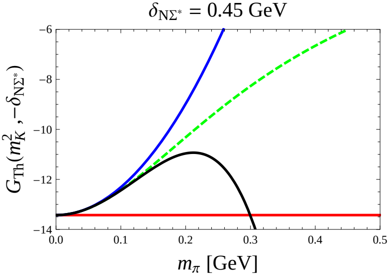
- Red : zeroth ; Blue : first ; Black : second
- Virtual kaon-loop contributions to hyperon axial charges are well-described by a $SU(2)$ expansion
- The same nonanalytic function $\mathcal{J}$ appears in the Kaon-loop contributions to hyperon magnetic moments

Effect of Kaon Virtual Threshold on Hyperon Electric Charge Radii

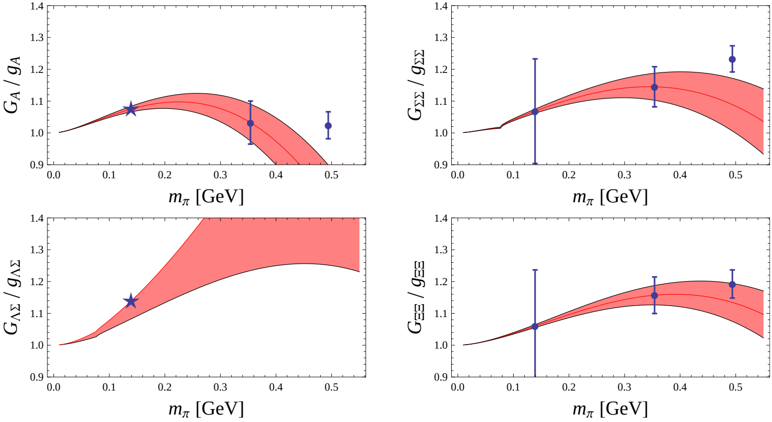


- Other hyperon electromagnetic properties are more sensitive to the nearby kaon thresholds
- A $SU(2)$ expansion for KN-loops is valid up to $m_\pi = 300$MeV for electric charge radius of Σ

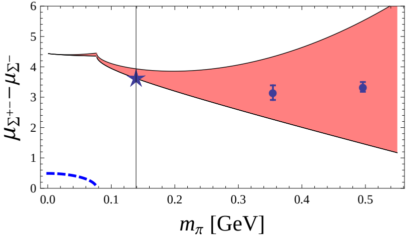
Effect of Kaon Virtual Threshold on Hyperon Electric Charge Radii

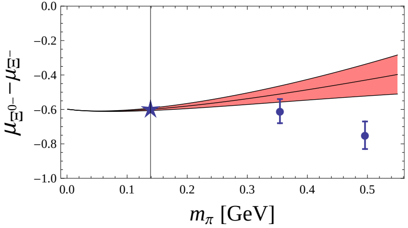


- The efficacy of a $SU(2)$ description for the electric charge radius of $\Sigma^\star$ does not extend considerably far beyond the physical pion mass
- The failure is due to a pole structure related to the thresholds in these nonanalytic functions



- Bands : $A'_{BB}\, g_{BB}\, \frac{\Delta_{BB*}\, m_\pi^2}{M_B\, \Lambda_\chi^2}$ (NNLO)
- $g_A \sim 1.2, \quad g_{\Sigma\Sigma} \sim 0.8, \quad g_{\Xi\Xi} \sim 0.2$ (Lin 2007)
- Lattice data points are better described by our $SU(2)$ chiral formulae as the strangeness quantum number is increased





- Bands : NNLO
- Prediction :
  - $\mu_{\Sigma^0} = 0.65\,, \quad Re(\mu_{\Delta^0}) = -0.74\,, \quad Re(\mu_{\Delta^-}) = -4.2$
  - $2\mu_{\Sigma^0} = \mu_{\Sigma^+} + \mu_{\Sigma^-}, \quad 2\mu_{\Sigma^{*,0}} = \mu_{\Sigma^{*,+}} + \mu_{\Sigma^{*,-}}, \quad \mu_{\Delta^{++}} - \mu_{\Delta^-} = 3(\mu_{\Delta^+} - \mu_{\Delta^0})$

Conclusions

- We show that the convergence of an $SU(2)$ expansion for hyperons is improved over that of an $SU(3)$ one
- We demonstrate that for certain hyperon observables, the virtual kaon-loop contributions are well-described by terms analytic in the pion mass squared
- $m_\pi \sim 330$MeV : Hyperon axial charges, magnetic moments and mass
- $m_\pi \sim 160$MeV : Hyperon electrimagnetic charge radii
- The convergence of our approach is getting better with increasing strangeness quantum number
- p-wave contributions to noleptonic hyperon decays

Table: Relative size of NLO loop contributions compared to experiment in $SU(2)$ and $SU(3)$ HBχPT.

Theory	$\delta\mu_p$	$\delta\mu_n$	$\delta\mu_{\Sigma^+}$	$\delta\mu_{\Sigma^-}$	$\delta\mu_{\Lambda}$	$\delta\mu_{\Sigma\Lambda}$	$\delta\mu_{\Xi^0}$	$\delta\mu_{\Xi^-}$
$SU(3)$ HBχPT	66%	41%	120%	21%	220%	74%	210%	176%
$SU(2)$ HBχPT	39%	57%	17%	35%	0	18%	$< 1\%$	$< 1\%$

- $\Sigma^+ \rightarrow n\pi^+ : 1.81$ (expt) and 0.82 (theory)
- $\Sigma^+ \rightarrow p\pi^0 : 1.17$ (expt) and 0.36 (theory)

Table: Relations between octet baryon magnetic moments. The $SU(3)$ HBχPT results quoted are NLO values for the numerator divided by experimental values for the denominator. NLO expressions for the numerators are provided in the Appendix. The $\Delta\%$ is the relative percent difference of the HBχPT calculation compared to the experimental value.

Relation	Experiment	HBχPT	$\lvert\Delta\%\rvert$
LO (Coleman-Glashow)			
$(\mu_{\Sigma^-} - \mu_{\Xi^-})/(\mu_{\Sigma^-} + \mu_{\Xi^-}) = 0$	0.28	0.50	77%
$(\mu_n - 2\mu_\Lambda)/(\mu_n + 2\mu_\Lambda) = 0$	0.22	0.61	180%
$(\mu_n - \mu_{\Xi^0})/(\mu_n + \mu_{\Xi^0}) = 0$	0.21	0.58	180%
$(\mu_p - \mu_{\Sigma^+})/(\mu_p + \mu_{\Sigma^+}) = 0$	0.064	0.21	230%
$(\mu_n + \mu_{\Sigma^-} + \mu_p)/(\mu_n + \mu_{\Sigma^-} - \mu_p) = 0$	0.048	0.14	190%
$(\sqrt{3}\mu_n + 2\mu_{\Sigma\Lambda})/(\sqrt{3}\mu_n - 2\mu_{\Sigma\Lambda}) = 0$	0.014	0.15	970%
NLO (Caldi-Pagels)			
$\frac{\mu_p+\mu_{\Xi^0}+\mu_{\Xi^-}+\mu_n-2\mu_\Lambda}{\mu_p-\mu_{\Xi^0}-\mu_{\Xi^-}-\mu_n+2\mu_\Lambda} = 0$	0.038	0	–
$\frac{\sqrt{3}\mu_{\Sigma\Lambda}+\mu_{\Xi^0}+\mu_n-\mu_\Lambda}{\sqrt{3}\mu_{\Sigma\Lambda}-\mu_{\Xi^0}-\mu_n-\mu_\Lambda} = 0$	0.036	0	–
$(\mu_{\Sigma^+} + \mu_{\Sigma^-} + 2\mu_\Lambda)/(\mu_{\Sigma^+} - \mu_{\Sigma^-} - 2\mu_\Lambda) = 0$	0.015	0	–
NNLO (Okubo)			
$(\mu_{\Sigma^+} + \mu_{\Sigma^-} - 2\mu_{\Sigma^0})/(\mu_{\Sigma^+} - \mu_{\Sigma^-} + 2\mu_{\Sigma^0}) = 0$	–	0	–
$\frac{6\mu_\Lambda+\mu_{\Sigma^+}+\mu_{\Sigma^-}-4\mu_{\Xi^0}-4\mu_n-4\sqrt{3}\mu_{\Sigma\Lambda}}{6\mu_\Lambda-\mu_{\Sigma^+}+\mu_{\Sigma^-}+4\mu_{\Xi^0}+4\mu_n-4\sqrt{3}\mu_{\Sigma\Lambda}} = 0$	0.028	0	–

$$\mathcal{L}_2^{(S=1)} = \overline{\Lambda}\,(iv\cdot\partial)\,\Lambda + \mathrm{tr}\left[\overline{\Sigma}\,(iv\cdot\mathcal{D}-\Delta_{\Lambda\Sigma})\,\Sigma\right] - \left(\overline{\Sigma}^{*\mu}\left[iv\cdot\mathcal{D}-\Delta_{\Lambda\Sigma^*}\right]\Sigma^*_\mu\right)$$

$$\begin{aligned}\mathcal{L}^{(S=1)} &= g_{\Sigma\Sigma}\mathrm{tr}\left(\overline{\Sigma}S^\mu\left[A_\mu,\Sigma\right]\right) + 2g_{\Sigma^*\Sigma^*}\left(\overline{\Sigma}^{*\mu}S\cdot A\Sigma^*_\mu\right)\\ &+ g_{\Sigma^*\Sigma}\left(\overline{\Sigma}^{*\mu}A_\mu\Sigma + \overline{\Sigma}A^\mu\Sigma^*_\mu\right) + \sqrt{\frac{2}{3}}g_{\Lambda\Sigma}\left[\mathrm{tr}\left(\overline{\Sigma}\,S\cdot A\right)\Lambda + \overline{\Lambda}\mathrm{tr}\left(S\cdot A\Sigma\right)\right]\\ &+ g_{\Sigma^*\Lambda}\left[\left(\overline{\Sigma}^{*\mu}A_\mu\right)\Lambda + \overline{\Lambda}\left(A^\mu\Sigma^*_\mu\right)\right]\end{aligned}$$

$$\mathcal{L}_2^{(S=2)} = \overline{\Xi}\,iv\cdot\mathcal{D}\,\Xi - \overline{\Xi}^{*\mu}\left(iv\cdot\mathcal{D}-\Delta_{\Xi\Xi^*}\right)\Xi^*_\mu$$

$$\mathcal{L}^{(S=2)} = 2g_{\Xi\Xi}\,\overline{\Xi}\,S\cdot A\,\Xi + 2g_{\Xi^*\Xi^*}\,\overline{\Xi}^{*\mu}\,S\cdot A\,\Xi^*_\mu + g_{\Xi^*\Xi}\left(\overline{\Xi}^{*\mu}A_\mu\,\Xi + \overline{\Xi}A^\mu\,\Xi^*_\mu\right)$$

$$F(\delta)\equiv F(\delta,m_\pi) = -\delta\log\left(\frac{m_\pi^2}{4\delta^2}\right) + \sqrt{\delta^2-m_\pi^2}\log\left(\frac{\delta-\sqrt{\delta^2-m_\pi^2+i\varepsilon}}{\delta+\sqrt{\delta^2-m_\pi^2+i\varepsilon}}\right)$$

$$
\begin{aligned}
G_{\Sigma\Sigma} &= g_{\Sigma\Sigma} + \frac{1}{\Lambda_\chi^2}\Big[A_{\Sigma\Sigma}(\mu)m_\pi^2 - (7g_{\Sigma\Sigma}^3 + 4g_{\Sigma\Sigma})\mathcal{J}(0,\mu) + \frac{2}{3}g_{\Sigma\Sigma}g_{\Lambda\Sigma}^2\mathcal{K}(-\Delta_{\Lambda\Sigma},\mu) \\
&\quad -g_{\Sigma\Sigma}g_{\Lambda\Sigma}^2\mathcal{J}(-\Delta_{\Lambda\Sigma},\mu) - \frac{8}{3}\sqrt{\frac{2}{3}}g_{\Sigma^*\Sigma}g_{\Lambda\Sigma}g_{\Lambda\Sigma^*}\mathcal{I}(-\Delta_{\Lambda\Sigma},\Delta_{\Sigma\Sigma^*},\mu) \\
&\quad -\Big(\frac{10}{9}g_{\Sigma^*\Sigma^*} + 4g_{\Sigma\Sigma}\Big)g_{\Sigma^*\Sigma}^2\mathcal{J}(\Delta_{\Sigma\Sigma^*},\mu)\Big], \\
G_{\Lambda\Sigma} &= g_{\Lambda\Sigma} + \frac{1}{\Lambda_\chi^2}\Big[A_{\Lambda\Sigma}(\mu)m_\pi^2 - 4g_{\Lambda\Sigma}\mathcal{J}(0,\mu) - 6g_{\Lambda\Sigma}g_{\Lambda\Sigma^*}^2\mathcal{J}(\Delta_{\Lambda\Sigma^*},\mu) \\
&\quad +2g_{\Lambda\Sigma}g_{\Sigma\Sigma}^2\mathcal{K}(\Delta_{\Lambda\Sigma},\mu) - 3g_{\Lambda\Sigma}g_{\Sigma\Sigma}^2\mathcal{J}(0,\mu) - \frac{1}{3}g_{\Lambda\Sigma}^3\mathcal{I}(\Delta_{\Lambda\Sigma},-\Delta_{\Lambda\Sigma},\mu) \\
&\quad -8\sqrt{\frac{2}{3}}g_{\Sigma\Sigma}g_{\Sigma^*\Sigma}g_{\Lambda\Sigma^*}\mathcal{K}(\Delta_{\Lambda\Sigma^\star},\mu) \\
&\quad +\frac{20}{3}\sqrt{\frac{2}{3}}g_{\Sigma^*\Sigma^*}g_{\Sigma^*\Sigma}g_{\Lambda\Sigma^*}\mathcal{I}(\Delta_{\Lambda\Sigma^*},\Delta_{\Sigma\Sigma^*},\mu) - 2g_{\Lambda\Sigma}g_{\Sigma^*\Sigma}^2\mathcal{J}(\Delta_{\Sigma\Sigma^*},\mu) \\
&\quad -\frac{3}{2}g_{\Lambda\Sigma}^3\mathcal{J}(\Delta_{\Lambda\Sigma},\mu) - \frac{1}{2}g_{\Lambda\Sigma}^3\mathcal{J}(-\Delta_{\Lambda\Sigma},\mu) \\
&\quad +\frac{8}{3}g_{\Lambda\Sigma}g_{\Lambda\Sigma^*}^2\mathcal{I}(\Delta_{\Lambda\Sigma^*},-\Delta_{\Delta\Sigma},\mu) - \frac{8}{3}g_{\Lambda\Sigma}g_{\Sigma^*\Sigma}^2\mathcal{I}(\Delta_{\Lambda\Sigma},\Delta_{\Sigma\Sigma^*},\mu)\Big],
\end{aligned}
$$

$$
\begin{aligned}
G_{\Xi\Xi} &= g_{\Xi\Xi} + \frac{1}{\Lambda_\chi^2}\Big[A_{\Xi\Xi}(\mu)m_\pi^2 - 4(2g_{\Xi\Xi}^3 + g_{\Xi\Xi})\mathcal{J}(0,\mu) \\
&\quad -\Big(6g_{\Xi\Xi} + \frac{10}{9}g_{\Xi^*\Xi^*}\Big)g_{\Xi^*\Xi}^2\mathcal{J}(\Delta_{\Xi\Xi^*},\mu) - \frac{8}{3}g_{\Xi\Xi}g_{\Xi^*\Xi}^2\mathcal{K}(\Delta_{\Xi\Xi^*},\mu)\Big]. \\
G_A &= g_A + \frac{1}{\Lambda_\chi^2}\Big[A_{NN}(\mu)m_\pi^2 - 4(2g_A^3 + g_A)\mathcal{J}(0,\mu) \\
&\quad +\frac{64}{9}g_A g_{\Delta N}^2\mathcal{K}(\Delta,\mu) - 8\Big(g_A + \frac{25}{81}g_{\Delta\Delta}\Big)g_{\Delta N}^2\mathcal{J}(\Delta,\mu)\Big]
\end{aligned}
$$

Input Parameters			Source
$\Delta = 290\,\mathtt{MeV}$		$\Delta_{\Lambda\Sigma} = 77\,\mathtt{MeV}$	Expt.
$\Delta_{\Lambda\Sigma^*} = 270\,\mathtt{MeV}$		$\Delta_{\Xi\Xi^*} = 215\,\mathtt{MeV}$	
$G_A(139\,\mathtt{MeV}) = 1.27$		$G_{\Lambda\Sigma}(139\,\mathtt{MeV}) = 1.47$	Expt.
$g_{\Delta N} = 1.48$		$g_{\Lambda\Sigma^*} = -0.91$	
$g_{\Sigma^*\Sigma} = 0.76$		$g_{\Xi^*\Xi} = 0.69$	
$g_{\Delta\Delta} = -2.2$	$g_{\Sigma^*\Sigma^*} = -1.47$	$g_{\Xi^*\Xi^*} = -0.73$	$SU(3)$ (Butler 1992)
$G_{\Sigma\Sigma}(139\,\mathtt{MeV}) = 0.78$		$G_{\Xi\Xi}(139\,\mathtt{MeV}) = 0.24$	Extrap. (Lin 2007)

Output Parameter Estimates

$g_A = 1.18$	$A_{NN}(\Lambda_\chi) = -12.0$
$g_{\Sigma\Sigma} = 0.73$	$A_{\Sigma\Sigma}(\Lambda_\chi) = -2.9$
$g_{\Lambda\Sigma} = 1.29$	–
$g_{\Xi\Xi} = 0.23$	$A_{\Xi\Xi}(\Lambda_\chi) = -0.22$