Update on charm annihilation contribution to the hyperfine splitting in charmonium

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[Lattice 2010, Cagliari]

Outline

- Motivation and history
- Analytic framework
- Modeling and fitting the disconnected propagator
- Results
- Conclusions

Motivation and history

- Lattice calculations of the hyperfine splitting in charmonium usually ignore the contributions of the annihilation (disconnected) diagrams to both the vector J/Ψ and the pseudoscalar η_c states.

- Our goal is to determine the actual value of the contributions.

- Perturbatively, the contribution of these diagrams in charmonium is expected to be small due to the OZI suppression.

- Previous calculations (C. McNeile and C. Michael, Phys. Rev. D70 (2004) 034506, P. de Forcrand *et al.*, JHEP 0408 (2004) 004) using two-flavor gauge ensembles very roughly estimate the contribution to be within ± 20 MeV.

- Our previous work with clover charm quarks (Lattice 2007 and 2008) estimated the contribution to be around -3 MeV.
- NEW: The charm quark mass is tuned using the rest mass of the η_c. Previously it was done using the kinetic mass of D_s. The new tuning accounts better for the possible mixing with glueballs.

Analytic framework

- The full meson propagator is:

$$F(p^2) \;=\; C(p^2) + D(p^2) = \frac{A}{p^2+m_c^2} + \frac{\sqrt{A}}{p^2+m_c^2}\lambda(p^2)\frac{\sqrt{A}}{p^2+m_c^2} + \frac{\sqrt{A}}{p^2+m_c^2}\lambda(p^2)\frac{1}{p^2+m_c^2}\lambda(p^2)\frac{\sqrt{A}}{p^2+m_c^2} + \cdots$$

- Thus

$$F(p^2) = \frac{A}{p^2+m_c^2-\lambda(p^2)} = \frac{A}{p^2+m_f^2}\,,$$

- Then the contribution we want to determine is:

$$\delta m = m_c - m_f \approx \frac{\lambda(-m_c^2)}{2m_c}\,.$$

Properties of the disconnected propagator

- In this study we work with the point-to-point disconnected propagator, which hugely improves our statistics and signal.

- The asymptotic behavior at large times t of the full charmonium propagator, $F(t)$, is

$$F(t) = C(t) + D(t) = \sum_n \langle 0|O|n\rangle\langle n|O|0\rangle e^{-E_n t} \;\; \overrightarrow{t \to \infty} \;\; \langle 0|O|0\rangle^2 e^{-E_0 t}\,.$$

  If O is Hermitian, then $F(t) \geq 0$. This is also true for the point-to-point propagator $F(r)$.

- If $D(r)$ dominates in $F(r)$ at large r (which will happen if there are light glueballs and light hadronic states coupling to the disconnected diagram) then $D(r) \geq 0$ at large r.

- Thus we can predict that if the operators are Hermitian, $D(r)$ for the η_c will change its sign form $D(0) < 0$ to $D(r) > 0$ at large r. Indeed we observed that (see lattice 2007 and 2008).

Fitting the disconnected propagator

- Choosing a fitting model:
  - The model should treat $D(r)$ as a composite object, which has contributions not only from the studied charmonium ground state, but also possible effects from excited charmonium states, states lighter than the charmonium ground state and possibly the $U_A(1)$ anomaly.
  - We also have to take into account that our data exhibits rotational symmetry violations at short distances, due to the finite lattice spacing.
- A simplified form that describes $D(r)$ in momentum space is:

$$D(p^2) = \lambda(p^2) \left(\frac{\sqrt{A}}{p^2 + m_c^2} + \sum_{n=1}^{N} \frac{a_n}{p^2 + (m_c^n)^2} \right)^2 .$$

- What is the form of $\lambda(p^2)$? How many excited states we should keep ?

Modeling the disconnected propagator

- In the fully quenched case we model $\lambda(p^2)$ as:

$$\lambda(p^2) = U + \frac{f}{p^2 + m_g^2},$$

where U stands for possible effects of the $U_A(1)$ anomaly, and the second term is a light glueball term with m_g – the glueball mass.

- In the dynamical 2+1 flavor case:

$$\lambda(p^2) = U + \frac{f}{p^2 + m_g^2} + \frac{l}{p^2 + m_l^2} + \cdots .$$

We keep only one light hadronic mode with an effective mass m_l that (we hope) describes well the long distance behavior of the point-to-point propagator.

- We want to limit the number of free parameters in our model to as few as possible. Thus we determine all masses here from $C(t)$, the long distance behavior of $D(r)$ or other literature, and keep them constant.

Fitting the disconnected propagator

- From fits to $C(t)$ we determine with high accuracy m_c, $m_c^\star$, A_t and $A_t^\star$. We obtain $A = 2m_c A_t$ and $a_1 = 2m_c^\star A_t^\star$. We use the central values of all of the above parameters as constants in our model function.

- We fix m_g using results from Y. Chen *et al.*, Phys. Rev. D73:014516 (2006).
- In our model we replace we replace p^2 with $\sum_i 2(1-\cos(p_i))$ and all the masses with $\sqrt{2(\cosh(m_c)-1)}$ to account for the discretization effects.
- In the quenched case, it is convenient to write:

  $$D_{\text{fit}}(p^2) = UT_1(p^2) + fT_2(p^2).$$

  We Fourier transform the functions $T_{1,2}(r)$ and tabulate them. Thus our disconnected propagator is fitted to a linear model

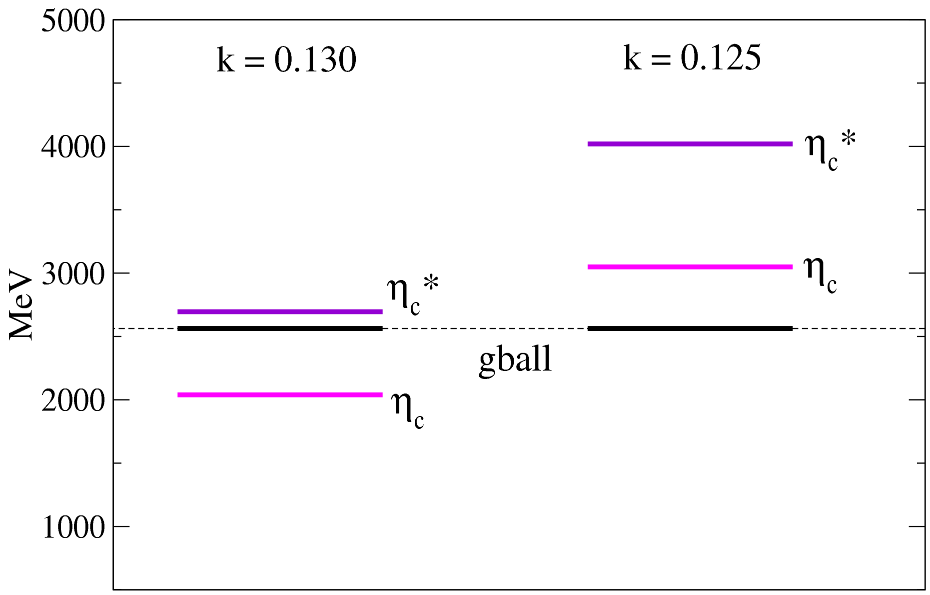
  $$D_{\text{fit}}(r) = UT_1(r) + fT_2(r).$$

  We determine U and f and thus $\lambda(-m_c^2)$ and δm.
- In the dynamical case same is accomplished by fitting to:

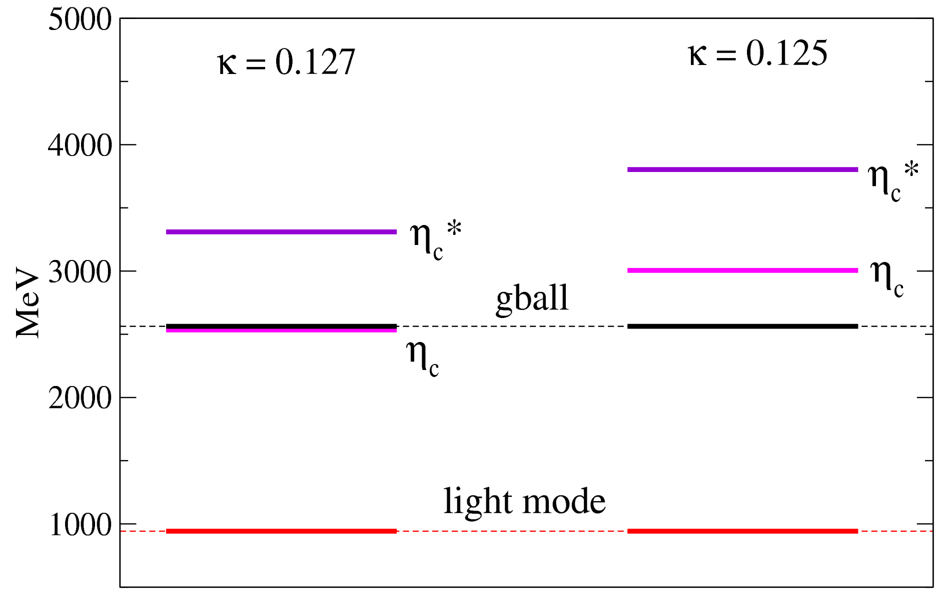
  $$D_{\text{fit}}(r) = UT_1(r) + fT_2(r) + lT_3(r).$$

Tuning the charm quark mass

- Previously we used the Fermilab interpretation of clover fermions to tune the κ_c. We used matching to the kinetic mass of D_s.
- But to position the η_c state correctly in the spectrum we now tune to its the rest mass.



- Quenched fine ensemble $a = 0.085$ fm.



- Dynamical fine ensemble $a = 0.085$ fm.

Calculation details

- Ensembles:

ensemble	a [fm]	m_l/m_s	volume	κ_c	# config.
quenched fine	≈ 0.085	$\cdots$	$28^3 \times 96$	**0.120**, 0.127	410
quenched s.fine	≈ 0.063	$\cdots$	$48^3 \times 144$	**0.125**, 0.130	415
2+1 flavors, fine	≈ 0.086	$0.0031/0.031$	$40^3 \times 96$	**0.125**, 0.127	766

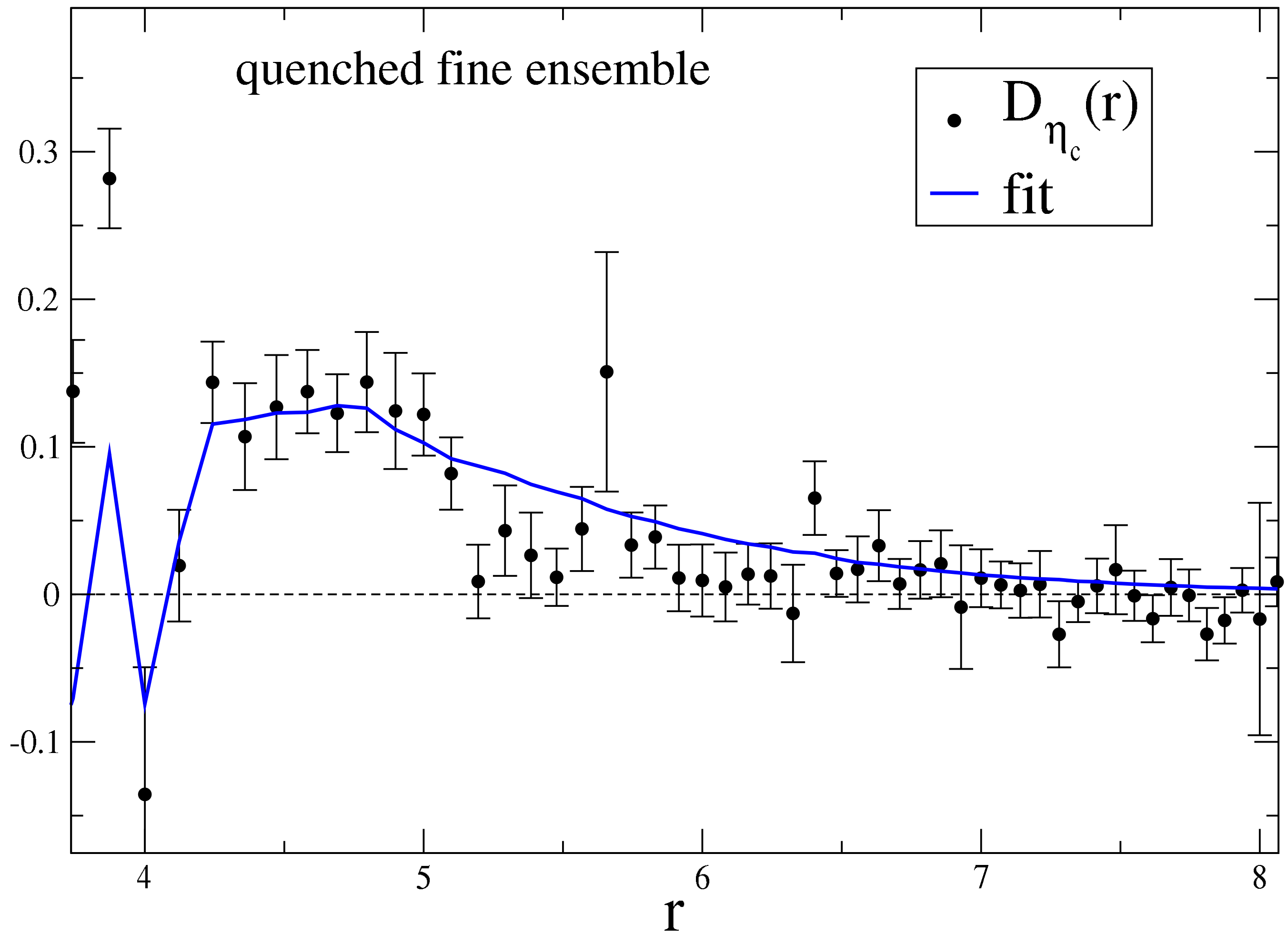
Boldface κ_c - current work.

- To calculate the disconnected diagrams we employ:
  - Point-to-point propagators.
  - 72 Z_2 spin-color-diluted random sources (72×12 matrix inversions per lattice.
  - Unbiased subtraction technique in the stochastic estimators (up to third order in κ_c).

Results: Quenched fine ensemble

- Fit model:

$$D_{\text{fit}}(p^2) = (U + \frac{f}{p^2 + m_g^2}) \left(\frac{\sqrt{A}}{p^2 + m_c^2} + \sum_{n=1}^{2} \frac{a_n}{p^2 + (m_c^n)^2} \right)^2$$

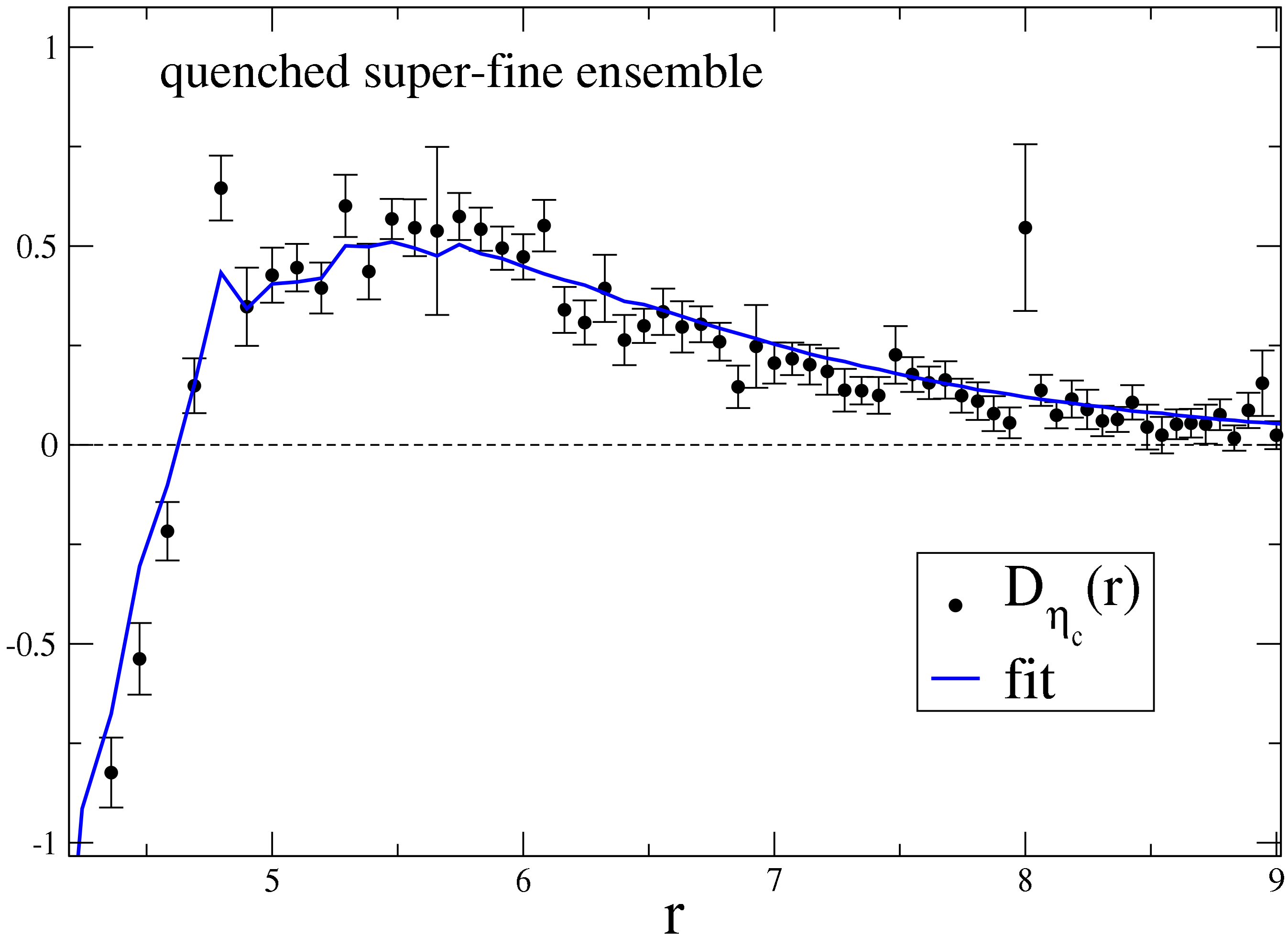


- There is strong model dependence (number of excited states and amplitude values): $\delta m \in [-7, -1.6]$ MeV.

Results:Quenched superfine ensemble

- Fit model:

$$D_{\rm fit}(r) = (U + \frac{f}{p^2+m_g^2})\left(\frac{\sqrt{A}}{p^2+m_c^2} + \sum_{n=1}^{2}\frac{a_n}{p^2+(m_c^n)^2}\right)^2$$

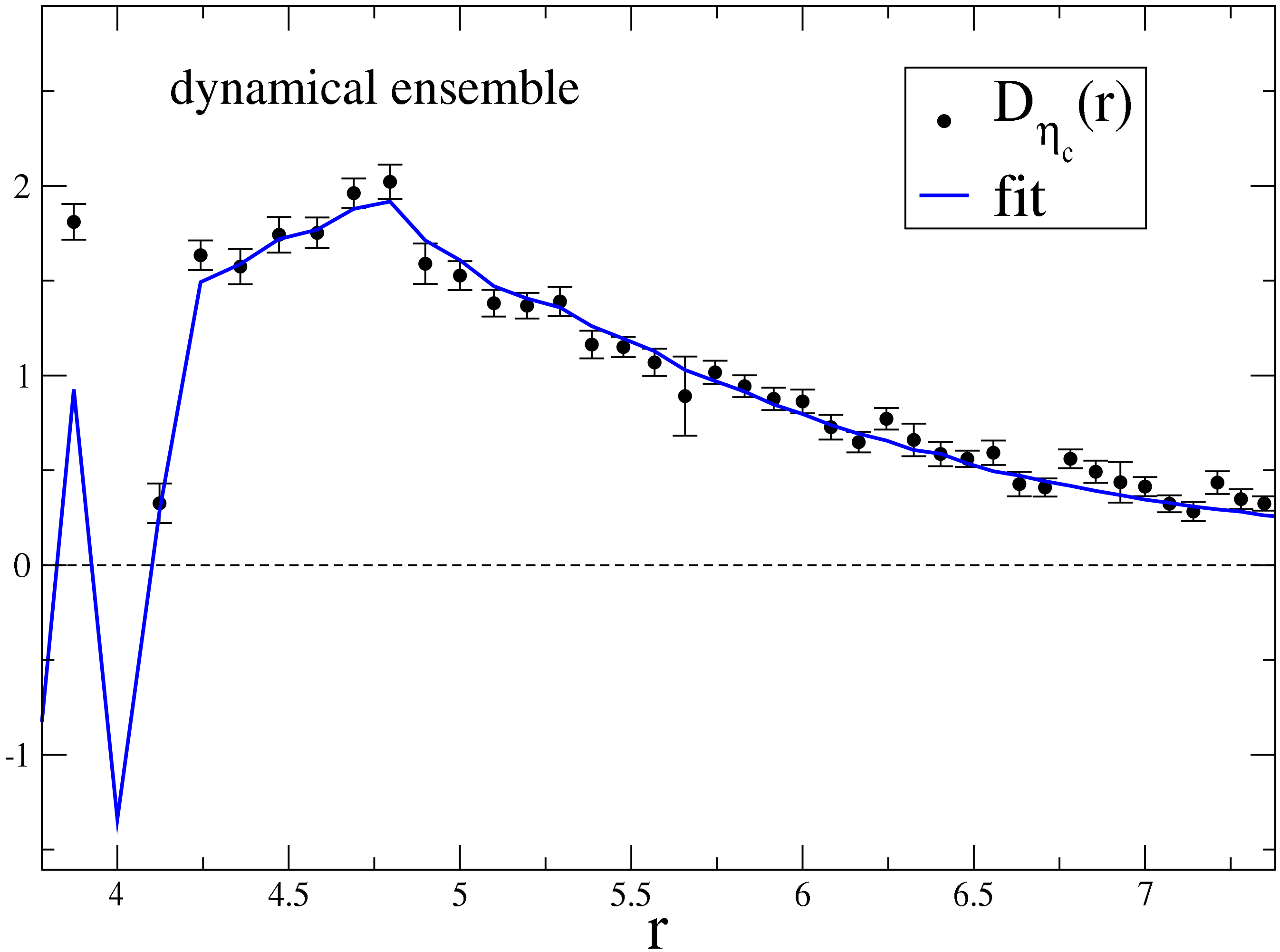


- There is strong model dependence: $\delta m \in [-10, -1.6]$ MeV.

Results: Quenched dynamical ensemble

- Fit model:

$$D_{\rm fit}(r) = (U + \frac{f}{p^2+m_g^2} + \frac{l}{p^2+m_l^2}) \left(\frac{\sqrt{A}}{p^2+m_c^2} + \sum_{n=1}^{2} \frac{a_n}{p^2+(m_c^n)^2} \right)^2$$



- There is strong model dependence: $\delta m \in [-20, -6]$ MeV.

Conclusions

- We retuned k_c to better take into account the mixing with glueballs.
- We gained deeper understanding of the importance of excited states and model assumptions for the final result.
- Future: Improve the fitting model using more sophisticated studies of the excited states of charmonium.