Critical line of QCD with four degenerate quarks

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work done in collaboration with:
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Outline

- Introduction
- Numerical results:
  - SU(3) Nf=4 staggered fermions at finite baryon density
- Conclusions

based on:

P. Cea, L.C., M. D'Elia, A.Papa, Phys. Rev. D81 (2010) 094502 [arXiv:1004.0184]

Introduction

- Understanding the phase diagram of QCD on the temperature–chemical potential (T,μ) plane has many important implications in cosmology, in astrophysics and in the phenomenology of heavy ion collisions.

- Unfortunately, the study of QCD at nonzero baryonic density by numerical simulations on a space-time lattice is plagued by the well-known ***sign problem*** : the fermion determinant is complex and the Monte Carlo sampling becomes unfeasible.

- One of the possibilities to circumvent the sign problem is to perform Monte Carlo numerical simulations for ***imaginary values*** of the chemical potential (where the fermionic determinant is real and the sign problem is absent) and to infer the behavior at real chemical potential by ***analytic continuation***.

Theory at imaginary μ

first suggestion:

[1] M. G. Alford, A. Kapustin, and F. Wilczek, Phys. Rev. D **59**, 054502 (1999).

applications to QCD:

[2] M.-P. Lombardo, Nucl. Phys. B, Proc. Suppl. **83**, 375 (2000) [arXiv:hep-lat/9908006].

[3] Ph. de Forcrand and O. Philipsen, Nucl. Phys. **B642**, 290 (2002); **B673**, 170 (2003).

[4] M. D'Elia and M. P. Lombardo, Phys. Rev. D **67**, 014505 (2003); **70**, 074509 (2004).

[5] V. Azcoiti, G. Di Carlo, A. Galante, and V. Laliena, Nucl. Phys. **B723**, 77 (2005).

[6] H. S. Chen, X. Q. Luo, Phys. Rev. D **72**, 034504 (2005).

[7] Ph. de Forcrand and O. Philipsen, J. High Energy Phys. 01 (2007) 077; Proc. Sci., LAT2007 (2007) 178 [arXiv:0711.0262].

[8] L. K. Wu, X. Q. Luo, and H. S. Chen, Phys. Rev. D **76**, 034505 (2007).

[9] M. D'Elia, F. Di Renzo, and M. P. Lombardo, Phys. Rev. D **76**, 114509 (2007).

[10] M. D'Elia and F. Sanfilippo, Phys. Rev. D **80**, 014502 (2009).

test in QCD-like theories free of the sign problem:

[11] A. Hart, M. Laine, and O. Philipsen, Phys. Lett. B **505**, 141 (2001).

[12] P. Giudice and A. Papa, Phys. Rev. D **69**, 094509 (2004); Nucl. Phys. B, Proc. Suppl. **140**, 529 (2005).

[13] P. Cea, L. Cosmai, M. D'Elia, and A. Papa, J. High Energy Phys. 02 (2007) 066; Proc. Sci., LAT2006 (2006) 143 [arXiv:hep-lat/0610088].

[14] P. Cea, L. Cosmai, M. D'Elia, and A. Papa, Phys. Rev. D **77**, 051501 (2008); Proc. Sci., LAT2007 (2007) 214 [arXiv:0710.2068]; Proc. Sci., LAT2009 (2009) 192 [arXiv:1001.4390].

[15] S. Conradi and M. D'Elia, Phys. Rev. D **76**, 074501 (2007).

[16] Y. Shinno and H. Yoneyama, arXiv:0903.0922.

[17] P. Cea, L. Cosmai, M. D'Elia, C. Manneschi, and A. Papa, Phys. Rev. D **80**, 034501 (2009); Proc. Sci., LAT2009 (2009) 161 [arXiv:1001.4439].

[18] S. Kim, P. de Forcrand, S. Kratochvila, and T. Takaishi, Proc. Sci., LAT2005 (2006) 166 [arXiv:hep-lat/0510069].

[19] F. Karbstein and M. Thies, Phys. Rev. D **75**, 025003 (2007).

The method of analytic continuation

Advantages:

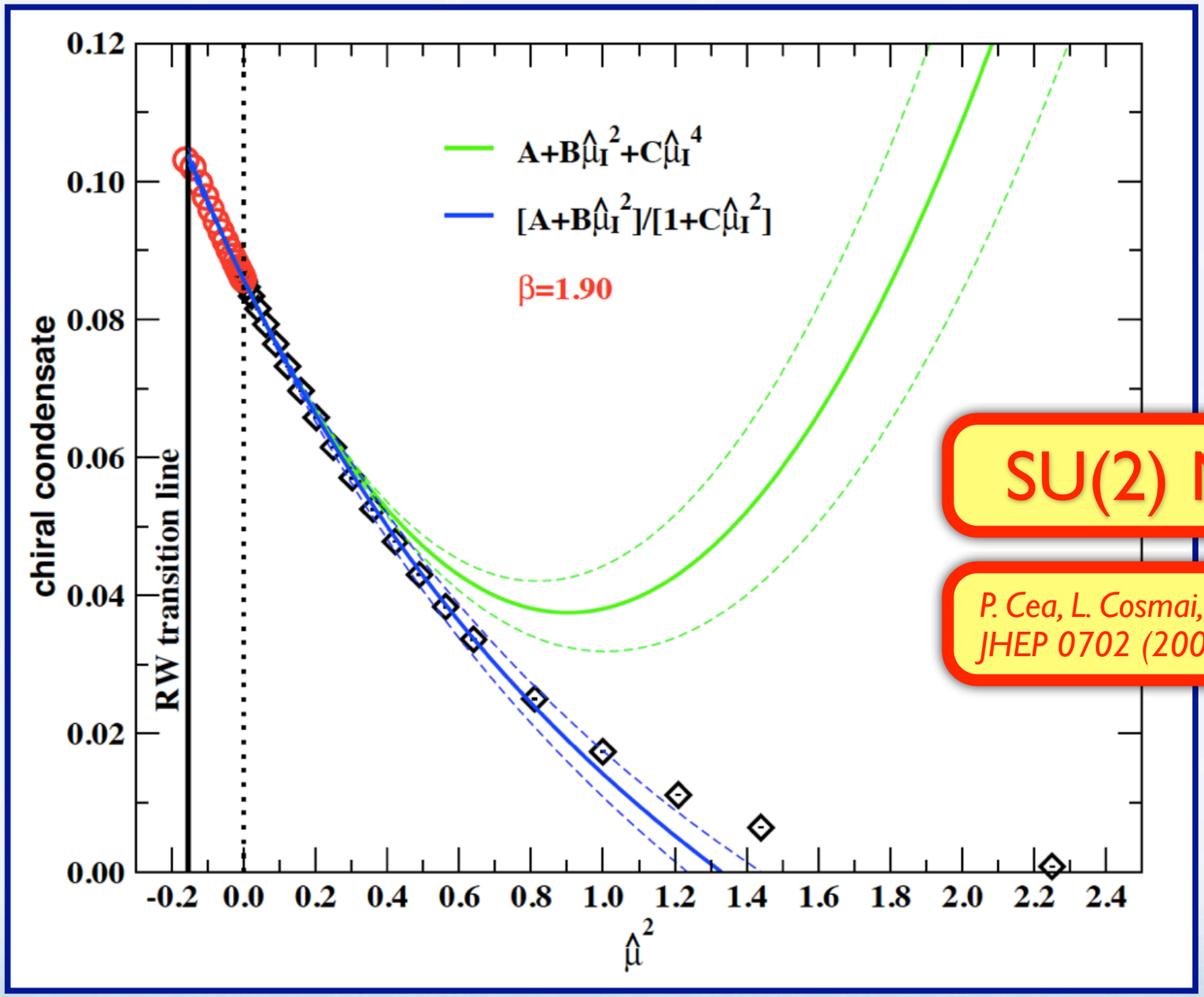
- coupling β and chemical potential μ can be varied independently
- no limitation for increasing lattice size

Drawbacks:

- extent of the attainable domain with real μ is limited by:
  - the periodicity and non-analyticities
  - the accuracy of the interpolation of data for imaginary μ

Analytic continuation of physical observables

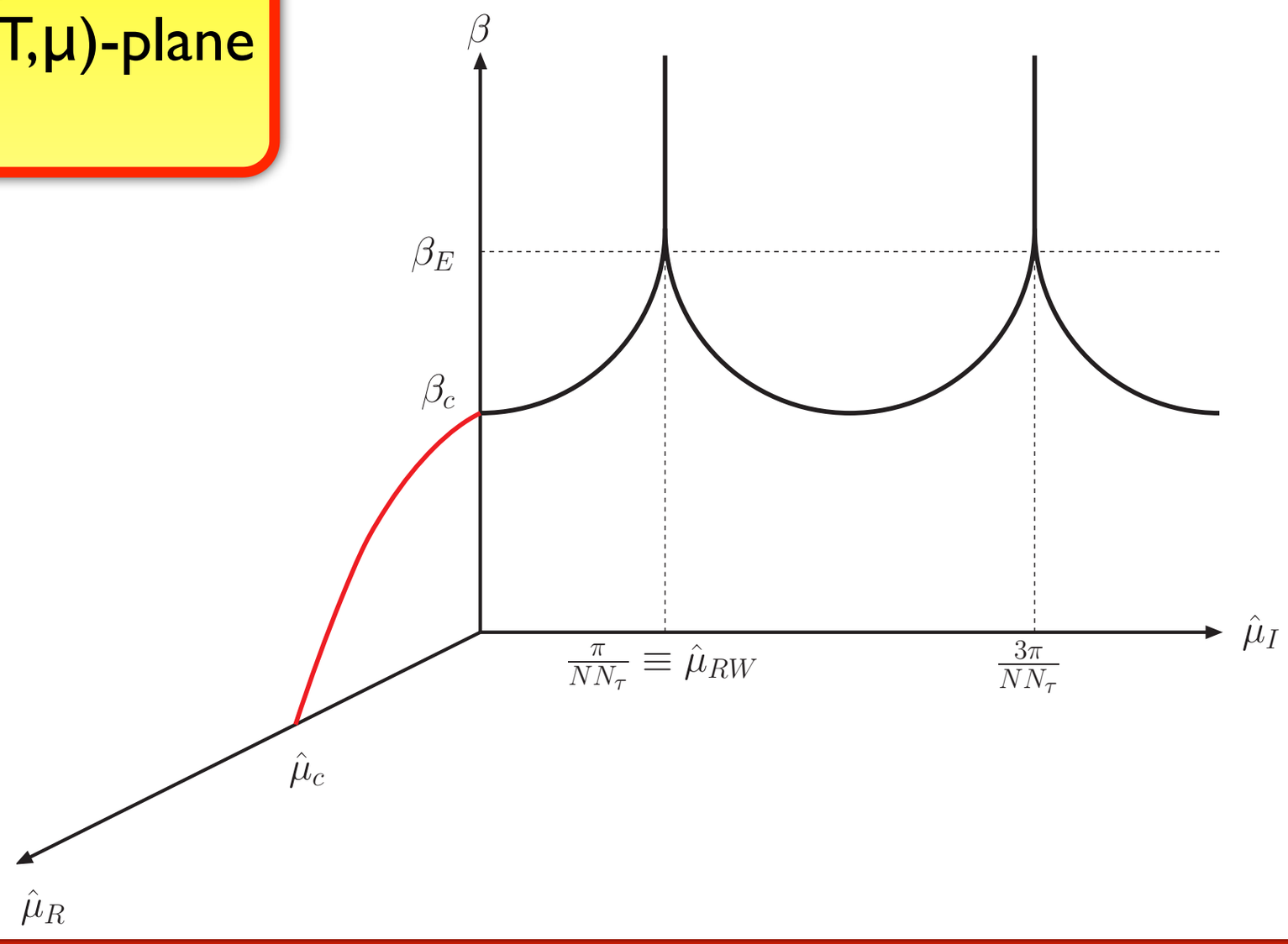
A careful numerical analysis in SU(2) has shown that a considerable improvement can be achieved if ***ratio of polynomials*** are used as interpolating function



SU(2) Nf=8

P. Cea, L. Cosmai, M. D'Elia, and A. Papa, JHEP 0702 (2007) 066 [arXiv:hep-lat/0612018].

The main goal of the application of the method of analytic continuation is locating the **critical line** on the (T,μ)-plane for real μ



Analytic continuation of the critical line: HOW TO

- **locate** the (pseudo-)critical β's for several fixed values of the imaginary chemical potential, by looking for peaks in the susceptibilities of a given observable.

- **interpolate** the critical β's obtained at imaginary chemical potential with an analytic function of μ, to be then extrapolated to real chemical potential

- *if the theory is free from the sign problem, compare the extrapolated curve with the direct determinations of the critical β's at real chemical potential.*

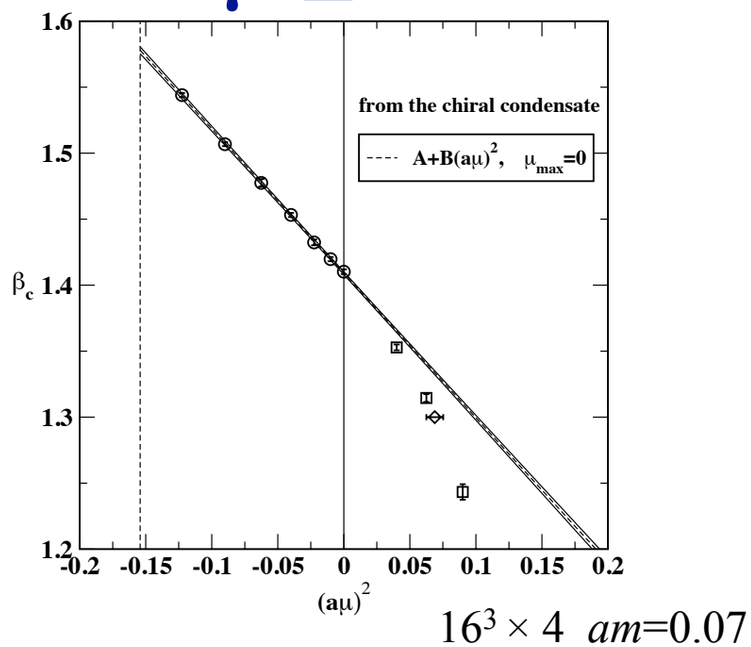
Observables: chiral condensate, Polyakov loop, plaquette.

On a finite volume there are no true non analyticities at the transition line. The location of the critical line may be dependent on the observable chosen to probe the transition.

Test of the method of analytic continuation in QCD-like theories

SU(2) N_f=8

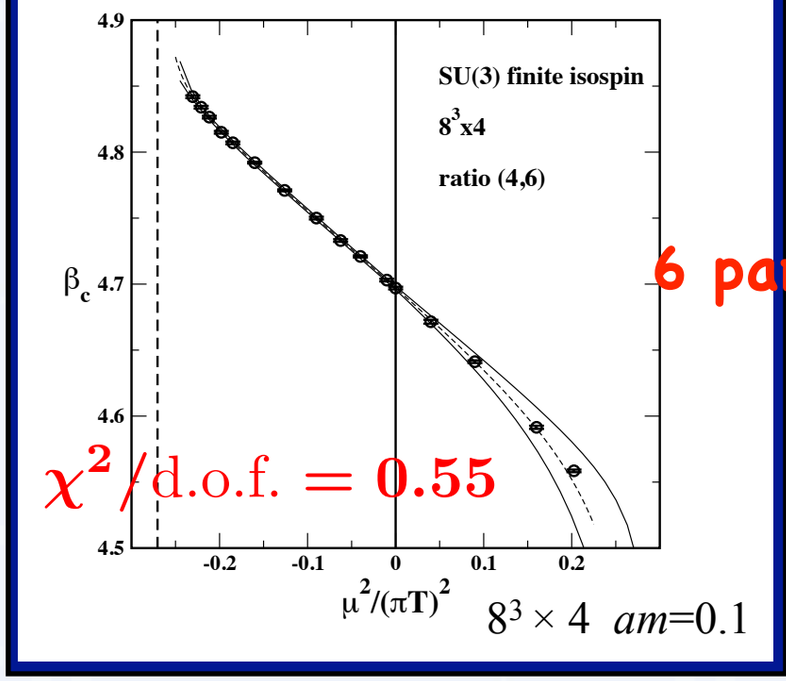
$\mu^2 \leq 0$



P. Cea, L.C., M. D'Elia, A.Papa,
Phys. Rev. D77 (2008) 051501 [arXiv:0712.3755]

SU(3) N_f=8 finite isospin

polynomial ratio $O(\mu^4)/O(\mu^6)$



6 parameters

P. Cea, L.C., M. D'Elia, C. Manneschi, A.Papa,
Phys. Rev. D80 (2009) 034501 [arXiv:905.1292]

Non-linear terms in the dependence of β_c on μ^2 in general cannot be neglected: the prediction for the pseudocritical couplings at real chemical potentials may be wrong if data at imaginary μ are fitted according to a linear dependence.

The *aim* of the present work is *to apply the experience acquired through the study of sign-problem-free-theories to the*

determination of the pseudocritical line in SU(3) N_f=4 at finite baryon density

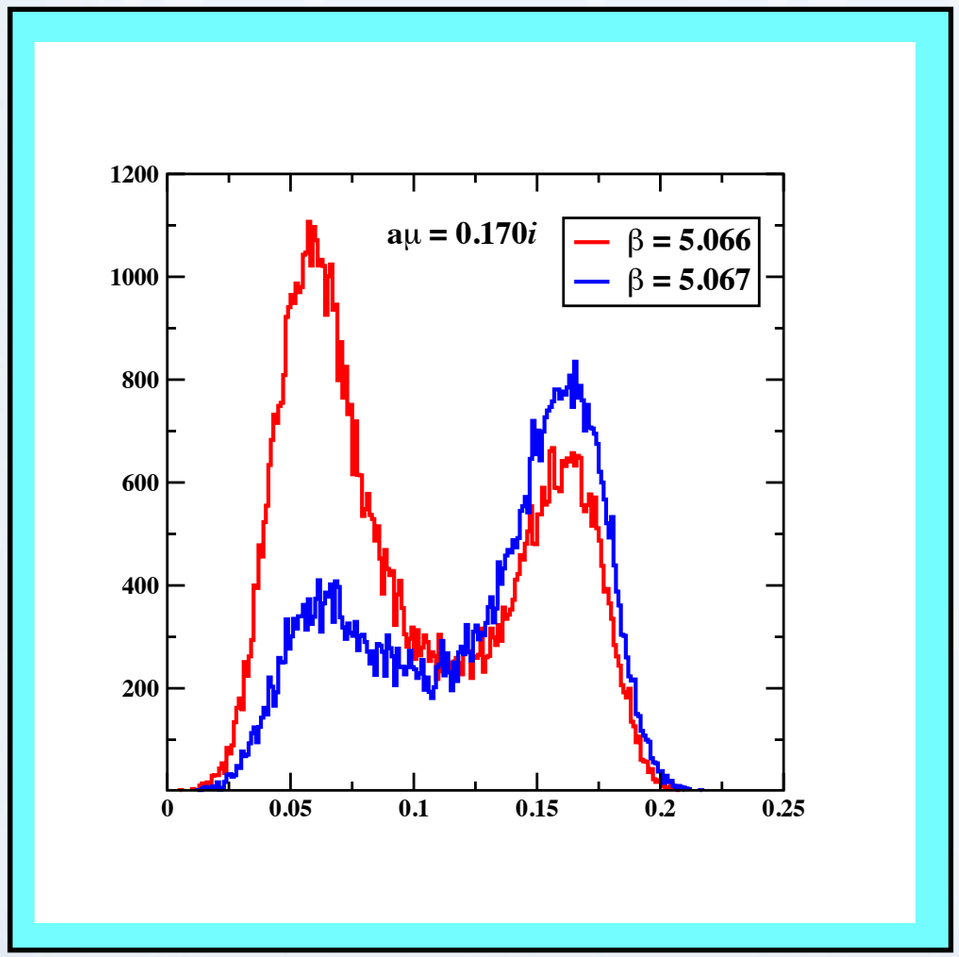
NUMERICAL SET-UP

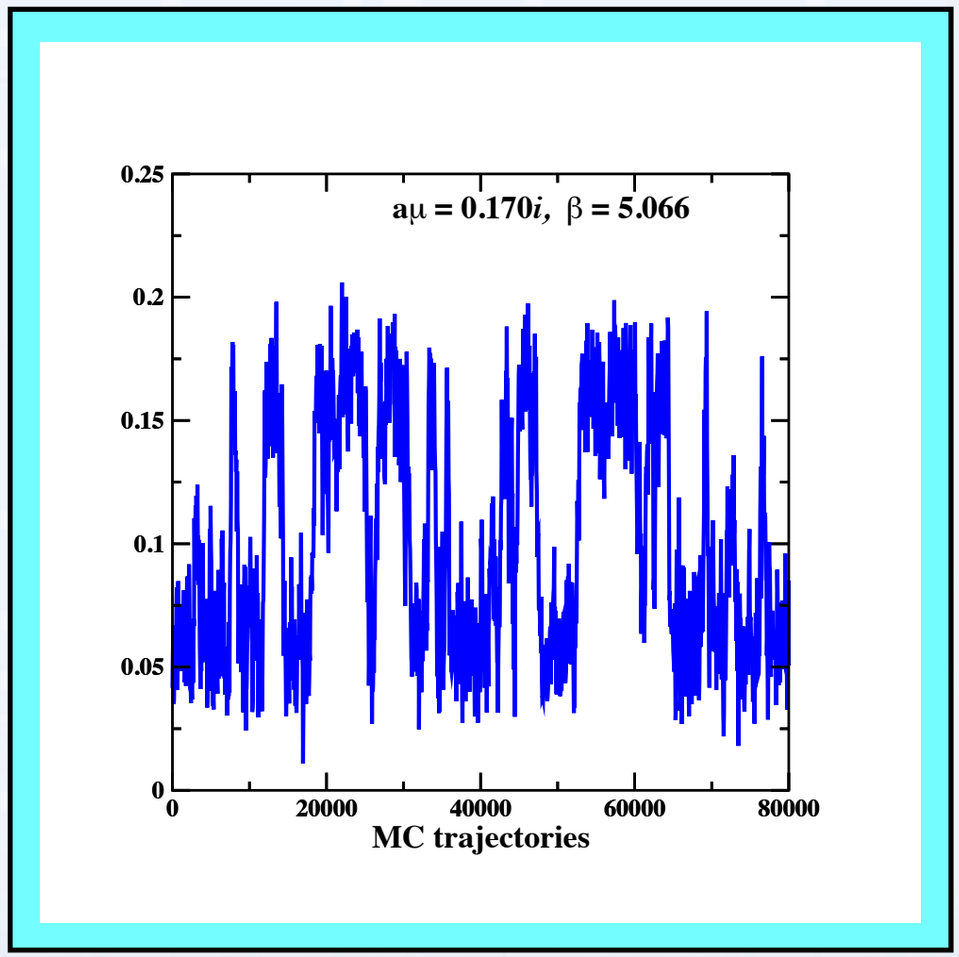
- SU(3) N_f=4 degenerate standard staggered fermions of mass *am=0.05*
- 12^3 x 4 lattice
- Monte Carlo simulation using the exact Φ algorithm, properly modified for the inclusion of a finite chemical potential
- Typical statistics: ~10k trajectories of 1 molecular dynamics unit, growing up to 100k trajectories for 4-5 β values around the peak
- Simulations performed using the computer facilities at the *INFN apeNEXT Computing Center in Rome* and of the PC clusters of the *INFN Bari Computer Center for Science*

The critical coupling β_c at a given chemical potential μ

- SU(3) N_f=4 : the critical line is a line of first order transitions in the first *Roberge-Weiss sector* $-(\pi/3)^2 \leq (\mu/T)^2 \leq 0$

- *Tunneling* between the different phases every few thousands trajectories:

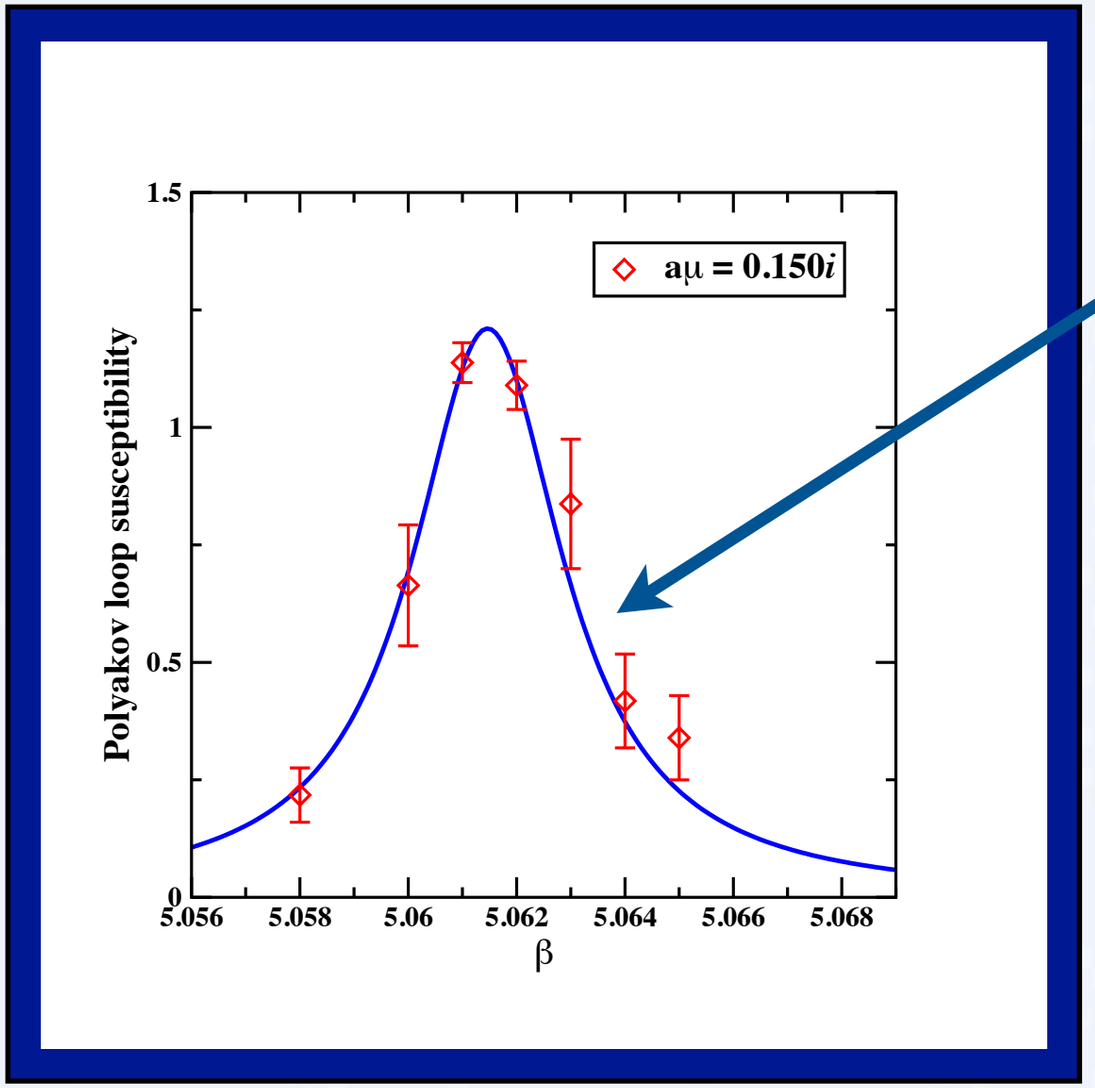




Real (Polyakov) around the transition

Determination of $\beta_c(\mu^2)$

The critical $\beta(\mu^2)$ is determined as the value for which the susceptibility of (the real part of) the Polyakov loop exhibits a peak



Lorentzian interpolation to locate the peak

check:

- β_c determined by means of Ferrenberg-Swendsen method
- β_c determined by estimating the point where the peaks in the distribution of Re(Polyakov) have equal height

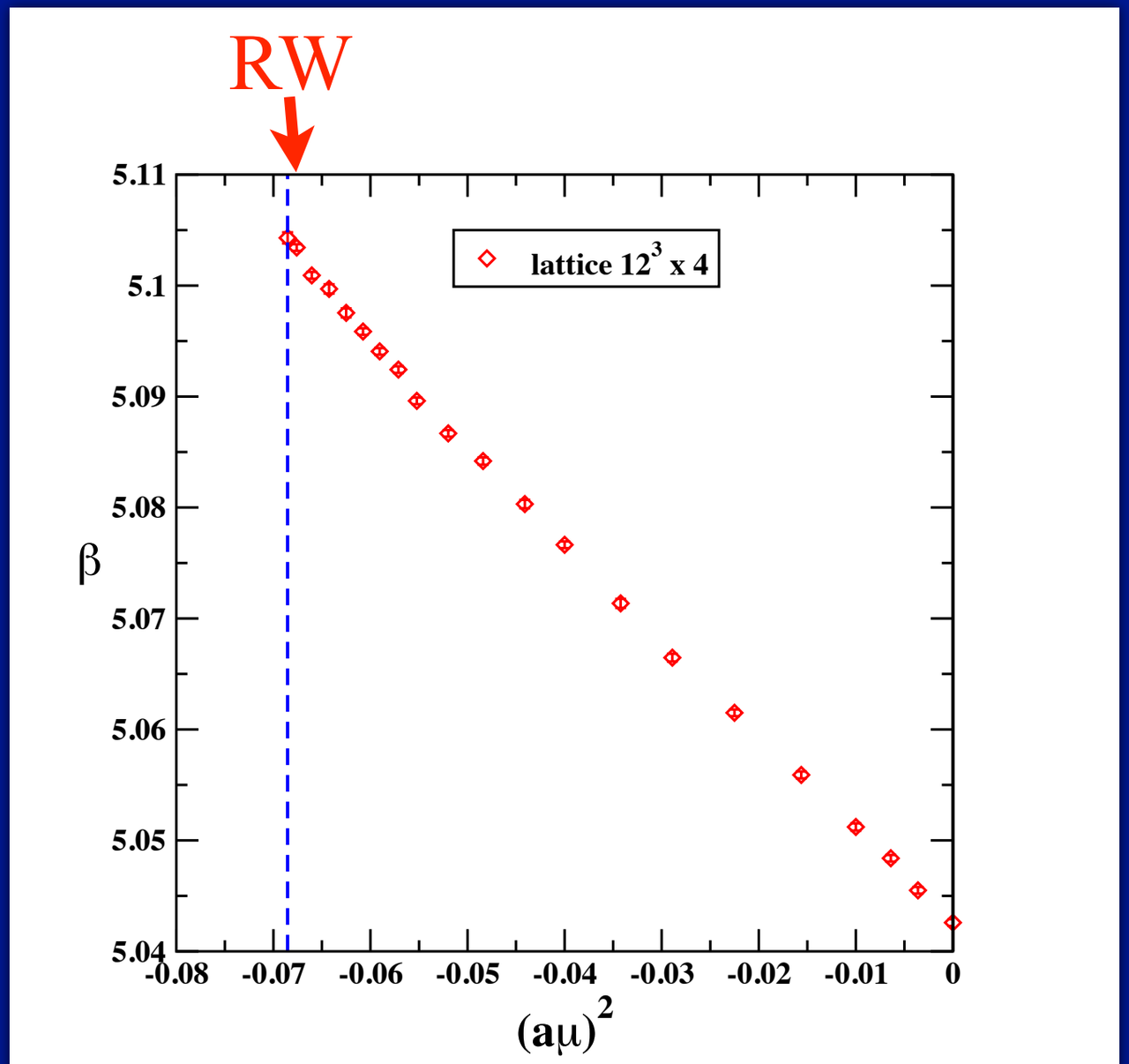
β_c determinations do not change if the susceptibility of another observable is used

SU(3) N_f =4 $12^3 \times 4$ lattice, *am=0.05* (*)

(*) in a few cases check on a $16^3 \times 4$ lattice: negligible corrections within the reported errors

β_c vs $(a\mu)^2$

$a\mathrm{Im}(\mu)$	β_c
0.	5.042 59(30)
0.060	5.045 50(30)
0.080	5.048 39(30)
0.100	5.051 21(33)
0.125	5.055 90(31)
0.150	5.061 50(30)
0.170	5.066 47(35)
0.185	5.071 36(40)
0.200	5.076 64(30)
0.210	5.080 31(38)
0.220	5.084 19(33)
0.228	5.086 68(30)
0.235	5.089 61(30)
0.239	5.092 43(30)
0.243	5.094 07(30)
0.2465	5.095 86(30)
0.250	5.097 54(40)
0.2535	5.099 70(42)
0.257	5.100 92(31)
0.260	5.103 43(30)
$\pi/12$	5.1043(5)



data do not line up along a straight line!

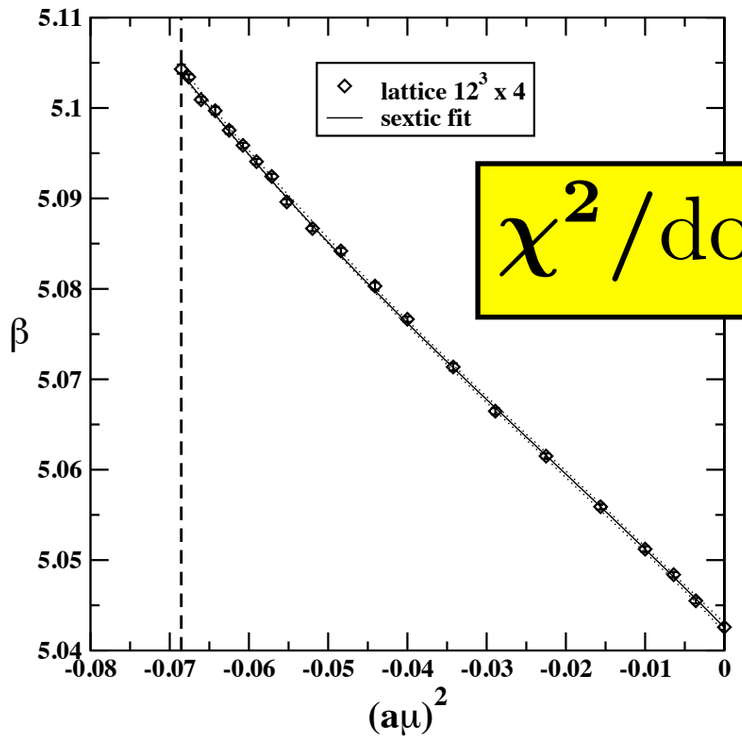
$\beta_c(\mu^2)$ cannot be parametrized by a polynomial of order μ^2

fit function:

$$\frac{a_0 + a_1\mu^2 + a_2\mu^4 + a_3\mu^6}{1 + a_4\mu^2 + a_5\mu^4}$$

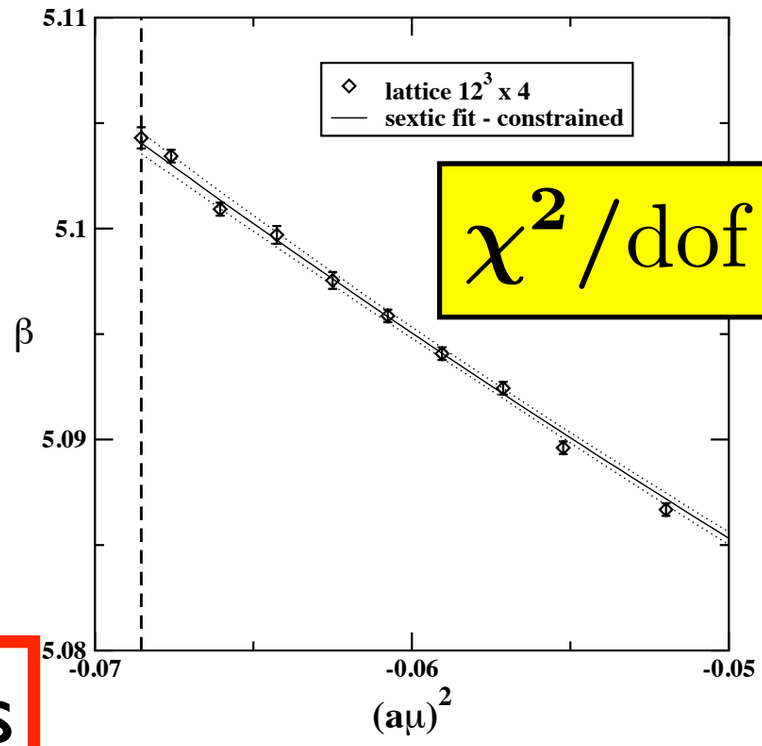
a_0	a_1	a_2	a_3	a_4	a_5	χ^2/d.o.f.	$(a\mu_{\rm min})^2$
5.041 98(22)	−0.8839(48)					6.63	$-(\pi/12)^2$
5.042 56(24)	−0.8509(71)					0.85	-0.235^2
5.043 11(36)	−0.761(26)	1.77(36)				2.13	$-(\pi/12)^2$
5.042 54(50)	−0.892(72)	−3.1(2.4)	−46.(23.)			1.10	$-(\pi/12)^2$
5.042 77(27)	−0.8509*	−1.70(55)	−34.0(8.2)			1.20	$-(\pi/12)^2$
5.042 84(28)	55.799(14)			11.2266(27)	1.741(29)	1.13	$-(\pi/12)^2$
5.042 76(27)	58.196(12)	−9.46(13)		11.7044(24)		1.09	$-(\pi/12)^2$

sextic polynomial fit



$\chi^2/\mathrm{dof} = 1.10$

sextic pol. fit (constrained)



$\chi^2/\mathrm{dof} = 1.20$

4 parameters

ratio of polynomials fit: $O(\mu^4)/O(\mu^2)$

a_0	a_1	a_2	a_3	a_4	a_5	χ^2/d.o.f.	$(a\mu_{\rm min})^2$
5.041 98(22)	−0.8839(48)					6.63	$-(\pi/12)^2$
5.042 56(24)	−0.8509(71)					0.85	-0.235^2
5.043 11(36)	−0.761(26)	1.77(36)				2.13	$-(\pi/12)^2$
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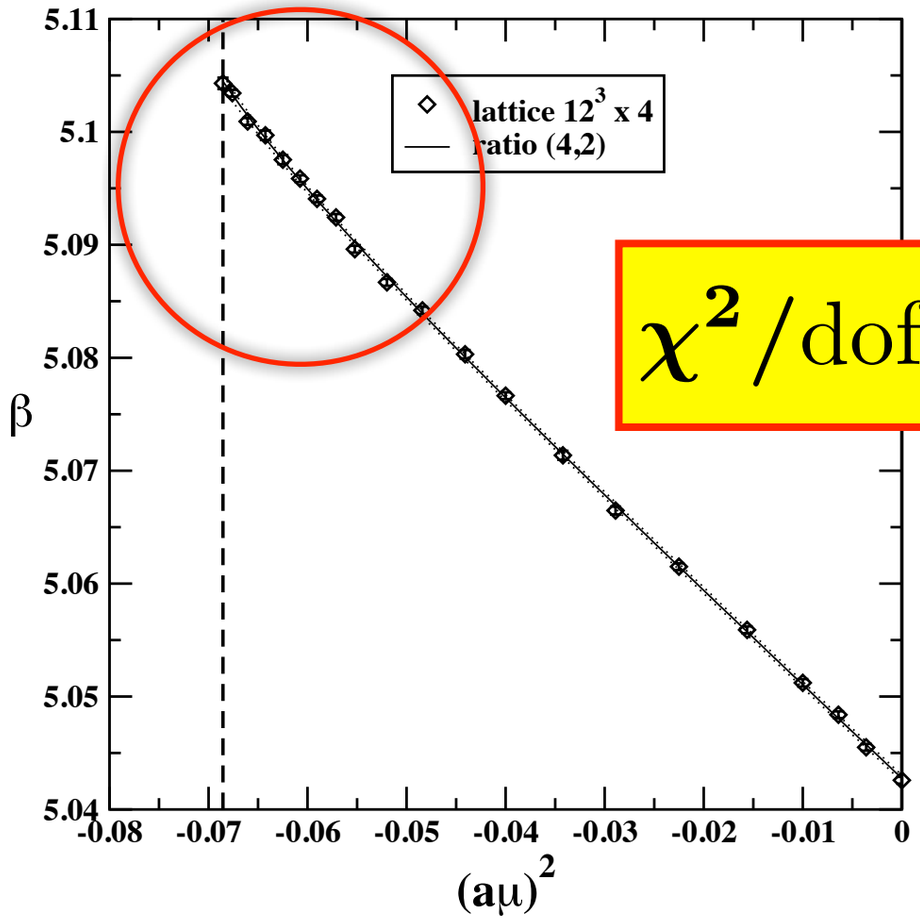
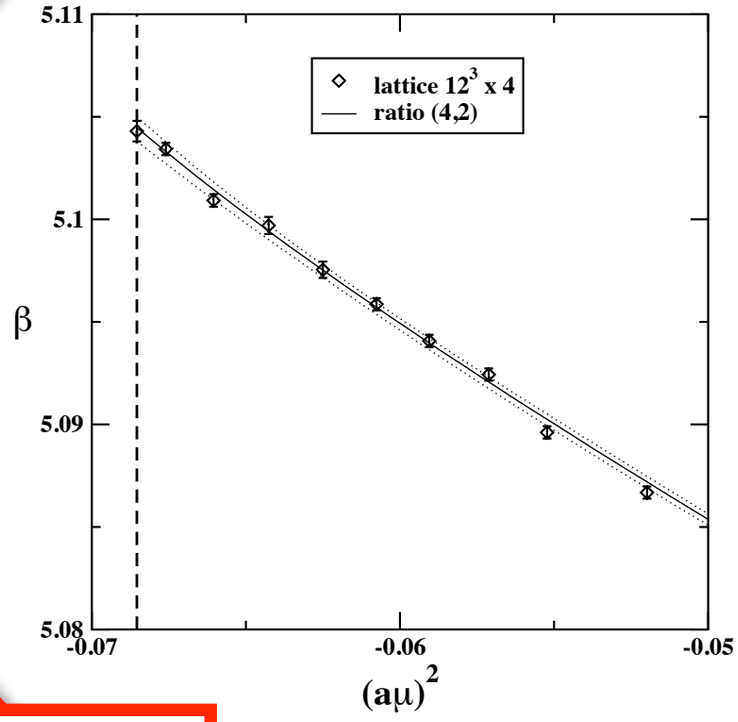


$\chi^2/\mathrm{dof} = 1.09$



4 parameters

A new fit strategy

- write down the interpolating function in **physical units**

$$\left[\frac{T_c(\mu)}{T_c(0)}\right]^2 = \frac{1 + C\mu^2/T_c^2(\mu)}{1 + A\mu^2/T_c^2(\mu) + B\mu^4/T_c^4(\mu)}$$

- implicit relation between β_c on μ^2 can be obtained using(*)

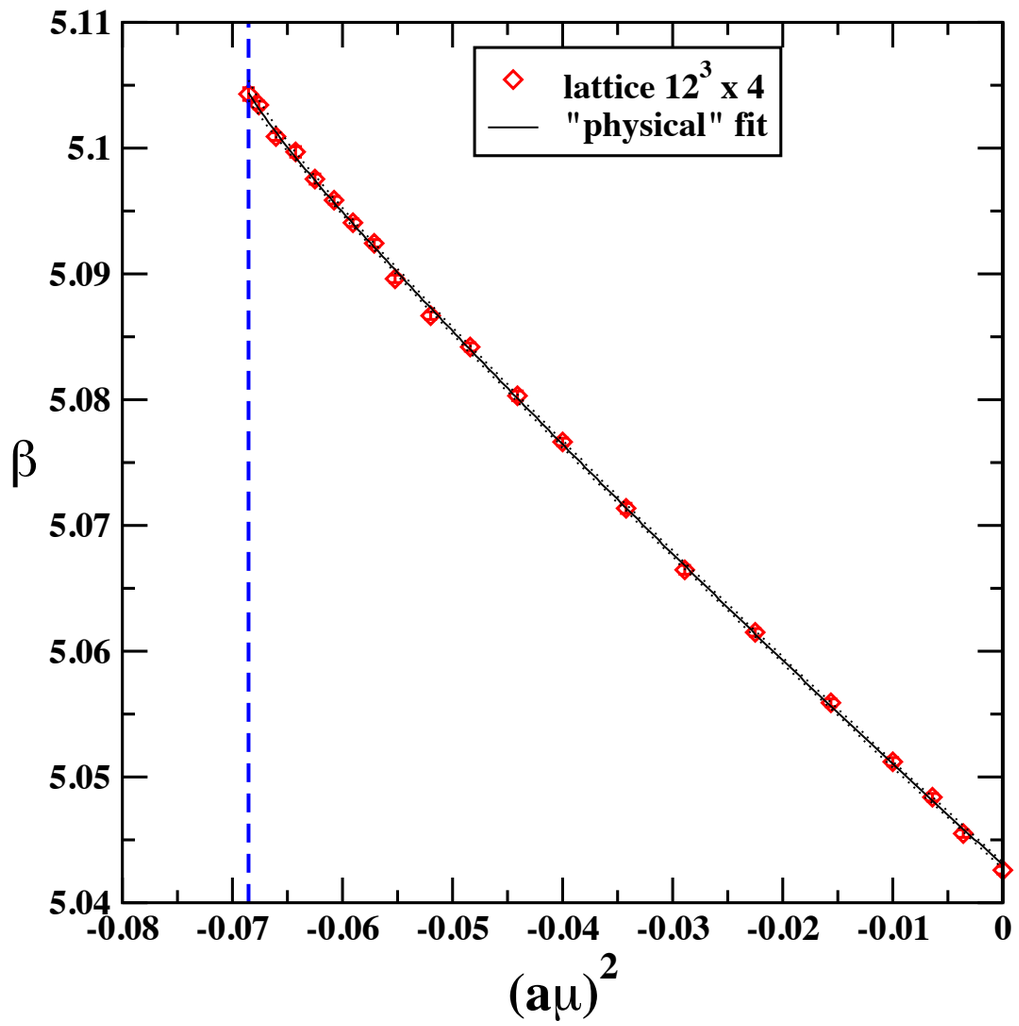
$$T = \frac{1}{a(\beta) L_t}$$

$$a^2(\beta_c(\mu^2))|_{2-\mathrm{loop}} = a^2(\beta_c(0))|_{2-\mathrm{loop}} \times \frac{1+A\mu^2/T_c^2+B\mu^4/T_c^4}{1+C\mu^2/T_c^2}$$

(*) Strictly speaking the lattice spacing depends also on the bare quark mass **am**, which we fix. This means that the physical quark mass in our runs changes slightly as we change β. However in the following evaluation, which is only based on the perturbative 2-loop β-function, we shall neglect such dependence.

Interpolating function in physical units:

$$a^2(\beta_c(\mu^2))|_{2-\mathrm{loop}} = a^2(\beta_c(0))|_{2-\mathrm{loop}} \quad \times \quad \frac{1 + A\mu^2/T_c^2 + B\mu^4/T_c^4}{1 + C\mu^2/T_c^2}$$

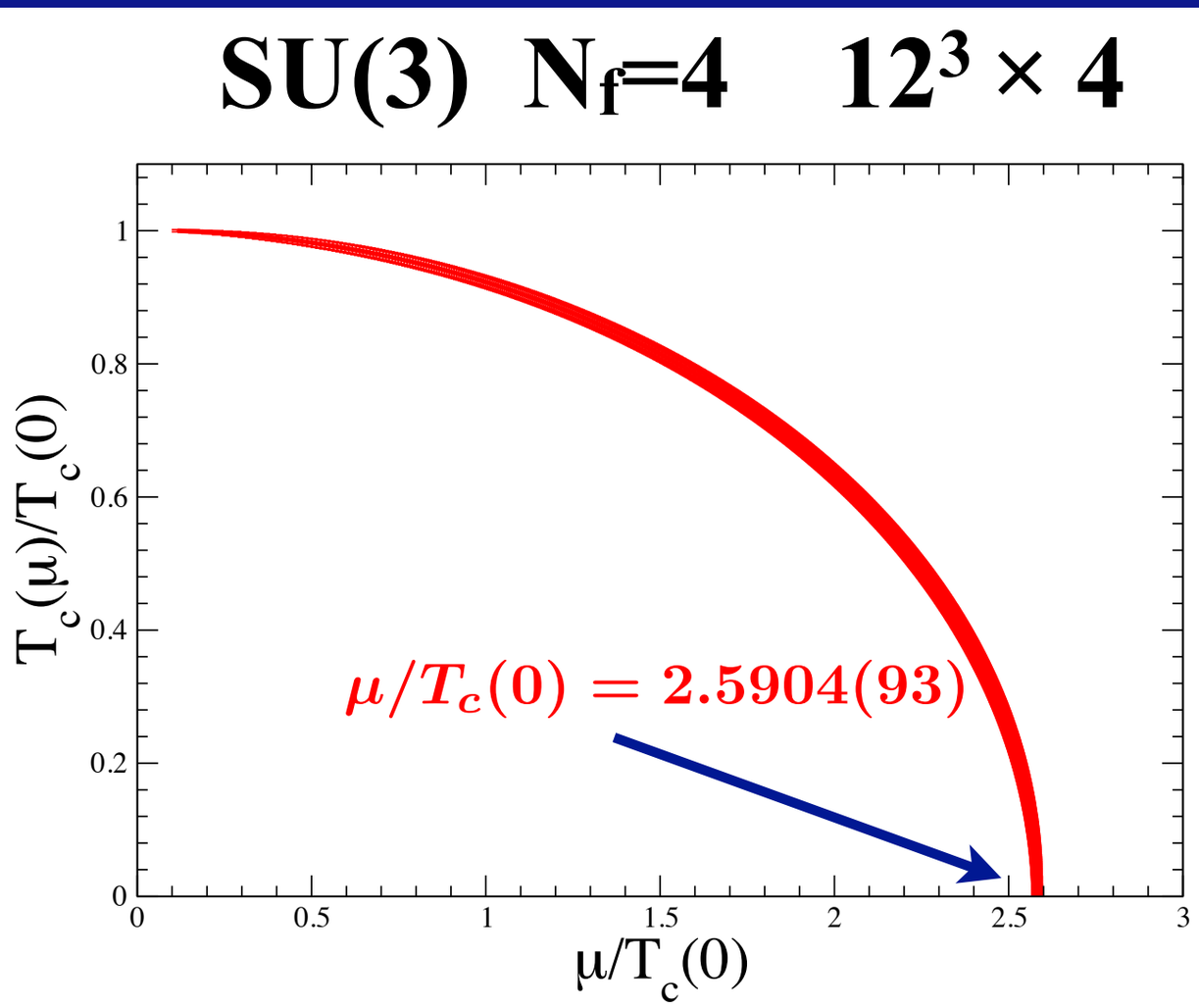


4 parameters fit

$$\begin{aligned} \beta_c(0) &= 5.04295(25) \\ A &= 1.00315(95) \\ B &= 0.12724(75) \\ C &= 0.8538(11) \\ \chi^2/d.o.f. &= 1.26 \end{aligned}$$

Estrapolation of the critical line

$$\left[\frac{T_c(\mu)}{T_c(0)}\right]^2 = \frac{1 + C\mu^2/T_c^2(\mu)}{1 + A\mu^2/T_c^2(\mu) + B\mu^4/T_c^4(\mu)}$$



Under the assumption (*) that:

the physical fit gives the correct behavior of the critical line at real μ down to T=0

estimate of the critical value of μ on the T=0 axis

$$\mu = \sqrt{C/B}\, T_c(0) = 2.5904(93)\, T_c(0)$$

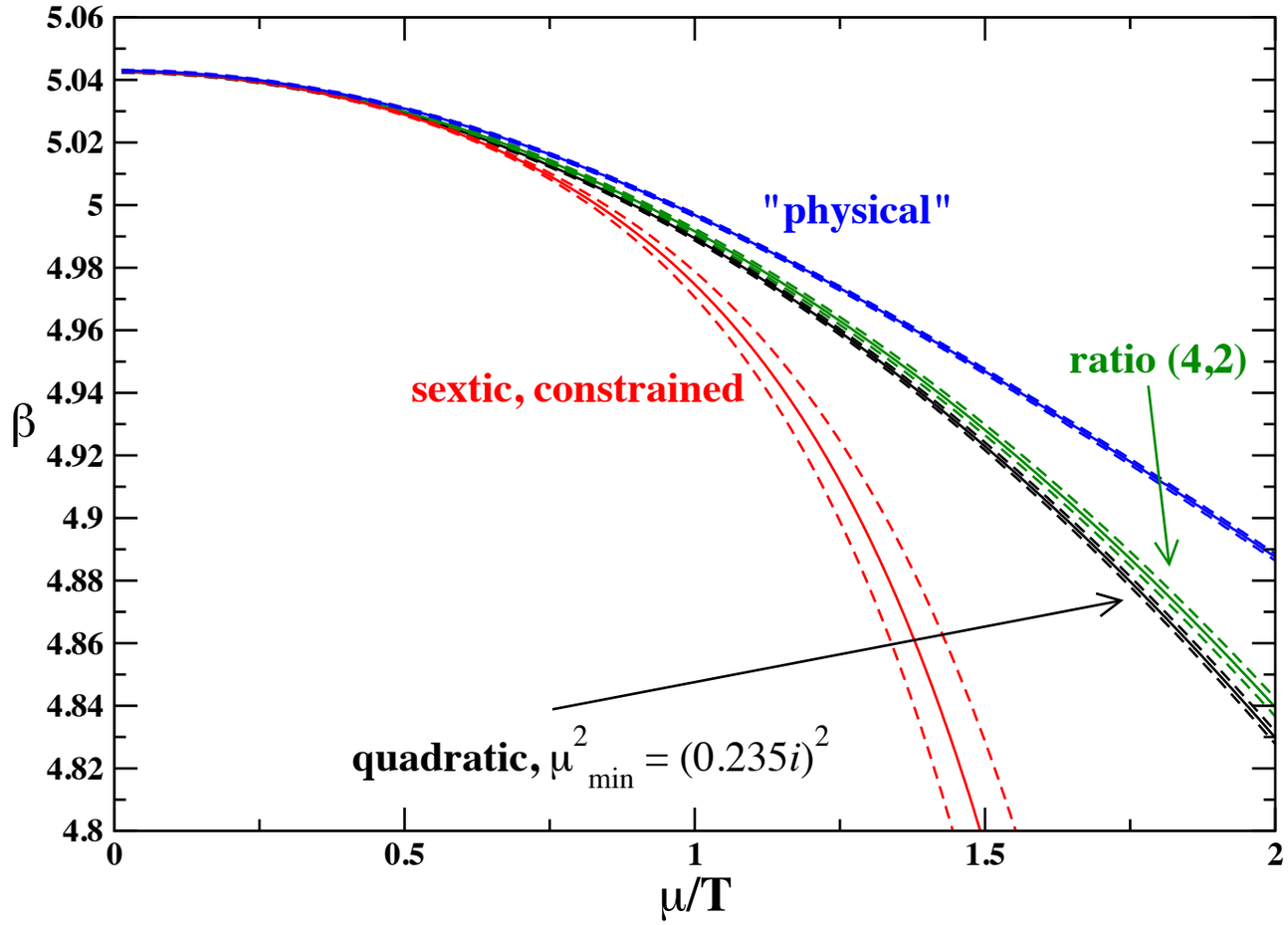
(*) *(no valid argument can support this assumption)*

QUESTION:

Do the successful interpolations we found in the $\mu^2 \leq 0$ region have a consistent extrapolation to $\mu^2 > 0$

?

Different interpolations lead to somewhat distinct extrapolations (one cannot rely on a unique extrapolation, except in the region $\mu/T \leq 0.6$)



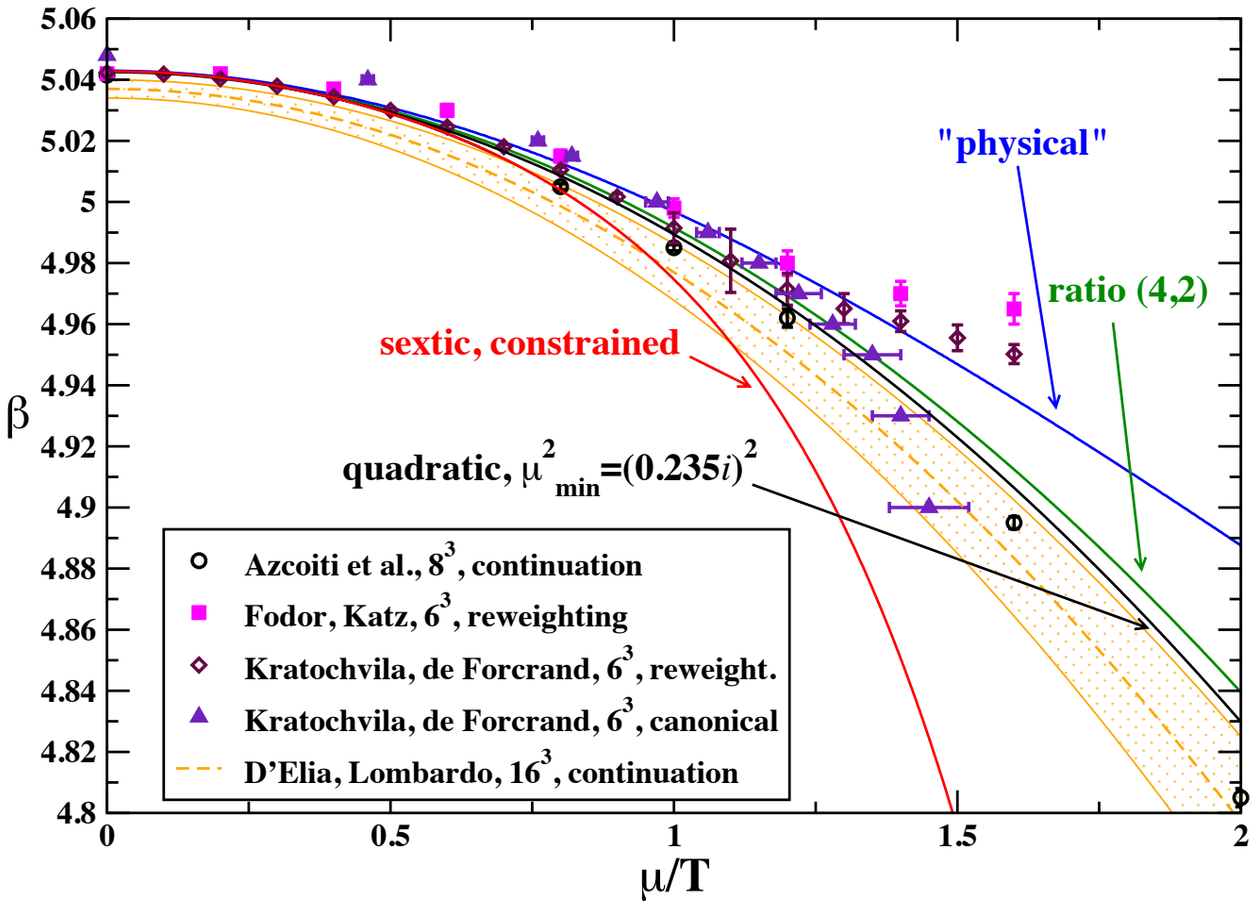
Shortcoming of analytic continuation ?

could be less severe in the more physical case of 2 flavors or 2+1 flavors (*)

(thanks to Ph. De Forcrand)

(*) where the curvature of the critical line at $\mu=0$ (i.e. the linear term in μ^2) is smaller than in the $N_f = 4$ case and larger non-linear contributions should be needed to bend the critical line towards a critical baryon chemical potential of the order of 1 GeV at $T = 0$, so that the sensitivity to such non-linear contributions could be hopefully enhanced

Determinations of the critical line in the literature together with our results



S. Kratochvila and P. de Forcrand, [arXiv:hep-lat/0509143]

"physical fit" ~ data from reweighting

"sextic constrained" ~ strong coupling behavior

K. Miura, T. Z. Nakano, A. Ohnishi, and N. Kawamoto, Phys. Rev. D 80, 074034 (2009).

"ratio (4,2)" ~ in fair agreement up to $\mu/T \sim 1.2$

a "combined approach": include in our fit data at real chemical potential available from the literature

?

limitation due to the inhomogeneity of the data presently available

acceptable value of χ^2/d.o.f. only for $\mu/T \leq 0.6$

Summary & Conclusions

We have revisited the application of the method of analytic continuation from imaginary to real chemical potential in QCD with $N_f=4$ degenerate flavors:

- to determine precisely the pseudo-critical line $\beta_c(\mu^2)$ in the region of negative μ^2 (20 data points almost uniformly distributed in the region $-(\pi/12)^2 \leq (a\mu)^2 \leq 0$)
- to exploit interpolating functions sensitive to possible deviations of the critical line from the quadratic behavior in μ for larger absolute values of μ (these deviations were clearly seen in QCD-like theories, such as 2-color QCD and finite isospin QCD, where it was given compelling evidence that their neglect could mislead the analytic continuation to real chemical potential)
- to extrapolate the newly adopted interpolations to the region of real μ and to re-determine, therefore, the critical line in QCD.

Outcome

- Deviations from the quadratic behavior in μ of $\beta_c(\mu^2)$ at negative μ^2 visible in QCD with $N_f=4$
- Several kinds of functions able to interpolate them, leading to extrapolations to real μ which start diverging from each other for $\mu/T \geq 0.6$

Outlook

- The shortcomings of the method of analytic continuation could be less severe for $N_f=2$ or $N_f=2+1$ (sensitivity to nonlinear terms in μ^2 could be enhanced).
- Possible improvement by theoretical development able to discriminate between interpolations, or by a combined numerical strategy aimed at gathering information from different approaches (i.e. rewght., canonical,...) applied so far independently from each other.