Dimensional reduction and confinement from five dimensions

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- Meanfield laboratory for 5d gauge theories
- Continuum limit at fixed anisotropy
- Dimensional reduction and confinement
- Monte Carlo results

Meanfield laboratory for 5d gauge theories

Gauge-Higgs unification

$$\underbrace{A_M}_{\text{5d gauge field}} \quad \xrightarrow{\mathcal{M}_5 = E_4 \times S^1} \quad \{ \underbrace{A_\mu}_{W:\ \text{4d gauge field}} \quad , \quad \underbrace{A_5}_{H:\ \text{4d Higgs}} \}$$

- Higgs potential is generated by quantum corrections and can trigger spontaneous symmetry breaking (Hosotani mechanism)
- 5d gauge symmetry keeps the potential finite
- Triviality requires a cut-off $\longrightarrow$ lattice
- Does a continuum limit exist non-perturbatively? $\longrightarrow$ meanfield, Monte Carlo
- Dimensional reduction?

Meanfield laboratory for 5d gauge theories

Meanfield expansion [Drouffe and Zuber, 1983]
$SU(N)$ gauge links U are replaced by $N \times N$ complex matrices V and Lagrange multipliers H

$$\langle \mathcal{O}[U] \rangle = \frac{1}{Z} \int \mathrm{D}V \int \mathrm{D}H \, \mathcal{O}[V] \mathrm{e}^{-S_{\mathrm{eff}}[V,H]}$$

$$S_{\mathrm{eff}} = S_G[V] + u(H) + (1/N) \mathrm{Re}\, \mathrm{tr}\{HV\} \, , \quad \mathrm{e}^{-u(H)} = \int \mathrm{D}U \, \mathrm{e}^{(1/N)\mathrm{Re}\, \mathrm{tr}\{UH\}}$$

Saddle point solution (background)

$$H \longrightarrow \bar{H} \mathbf{1} \, , \quad V \longrightarrow \bar{V} \mathbf{1} \, , \quad S_{\mathrm{eff}}[\bar{V}, \bar{H}] = \text{minimal}$$

Corrections calculated from Gaussian fluctuations

$$H = \bar{H} + h \quad \text{and} \quad V = \bar{V} + v$$

Covariant gauge fixing is imposed on v [Rühl, 1982]

Meanfield laboratory for 5d gauge theories

Our setup

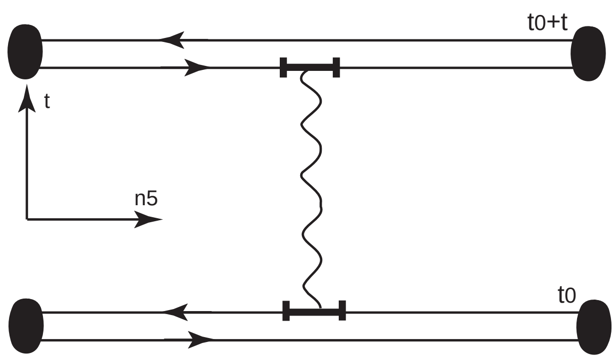
$L_T \times L^3 \times L_5$ lattice, $SU(2)$ gauge theory with anisotropic plaquette action

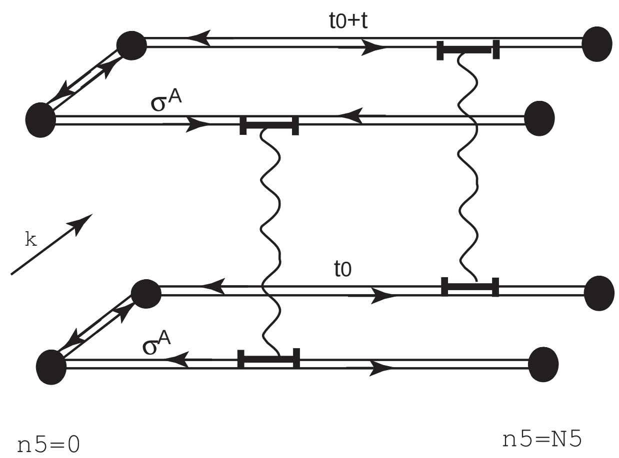
$$S_W = \frac{\beta}{4}\Big[\frac{1}{\gamma}\sum_{\rm 4d-p} {\rm tr}\Big(1 - \{U_p\}\Big) + \gamma \sum_{\rm 5d-p} {\rm tr}\Big(1 - \{U_p\}\Big)\Big]$$

The background is $\overline{v}_0$ along directions $\mu = 0, 1, 2, 3$ and $\overline{v}_5$ along the extra dimension

Observables

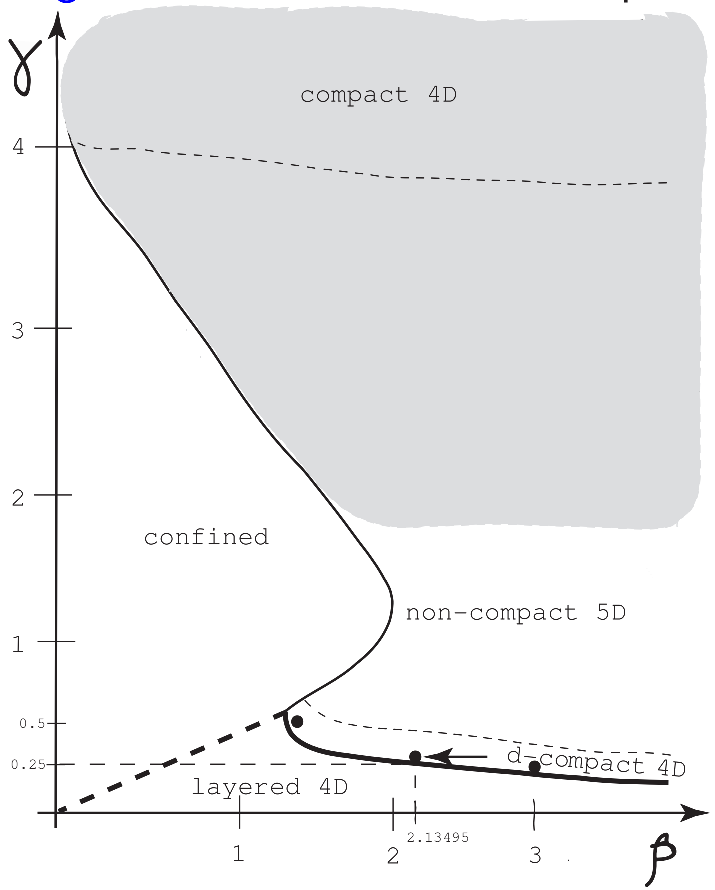
- Static potential V_4 along the 4d hyperplanes and V_5 along the extra dimension
- Higgs (1st order) m_H and gauge boson (2nd order) m_W masses





Meanfield laboratory for 5d gauge theories

Phase diagram based on the computation of the free energy to 1st order



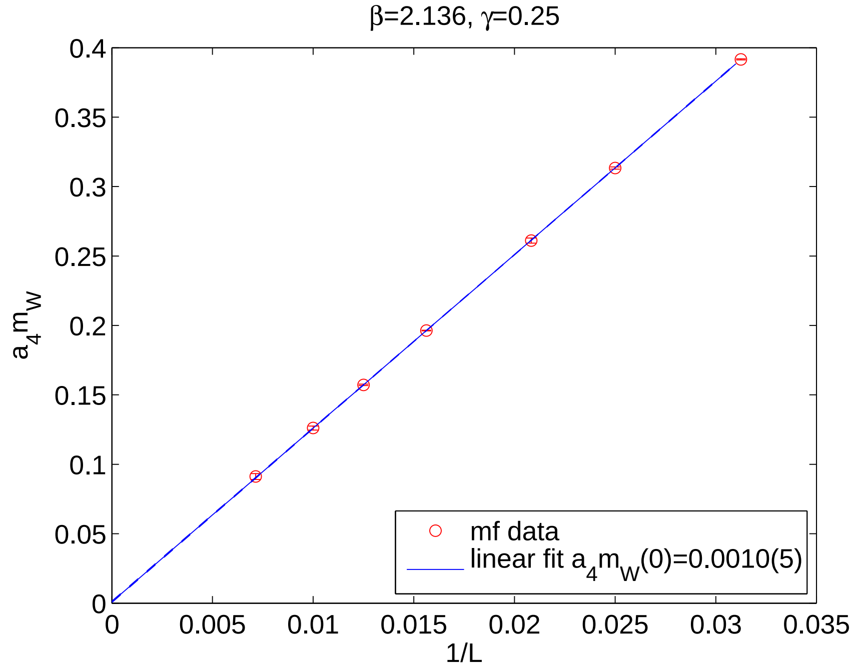
[Irges and Knechtli, 2009]

- The layered phase ($\overline{v}_0 \neq 0$, $\overline{v}_5 = 0$) is unstable
- The deconfined phase ($\overline{v}_0 \neq 0$, $\overline{v}_5 \neq 0$) has a rich structure:
  – at $\gamma >> 1$ (compact phase) it is unstable; V_4 at short distances is 4d Coulomb
  – at $\gamma < 1$ there is a line of 2nd order phase transitions; close to the layered phase V_4 is 4d Coulomb again, we call it the "d-compact" phase

Meanfield laboratory for 5d gauge theories

Spectrum

$a_4 m_H(\beta, \gamma)$ does not depend at 1st order on the geometry



The gauge boson mass at 2nd order depends significantly only on L

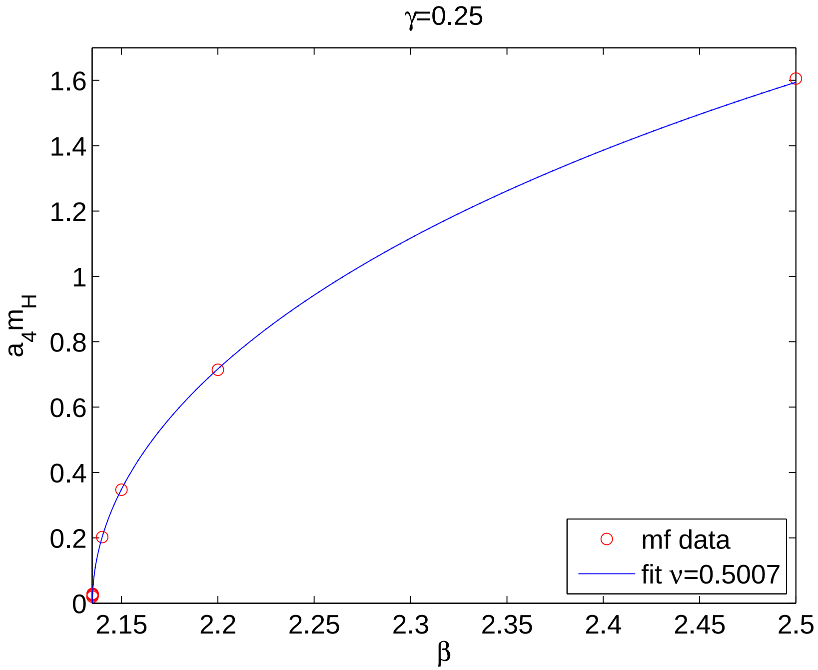
$$a_4 m_W = c_L / L$$

Extrapolation $L \to \infty$ is consistent with zero (we cannot exclude a exponentially small mass)

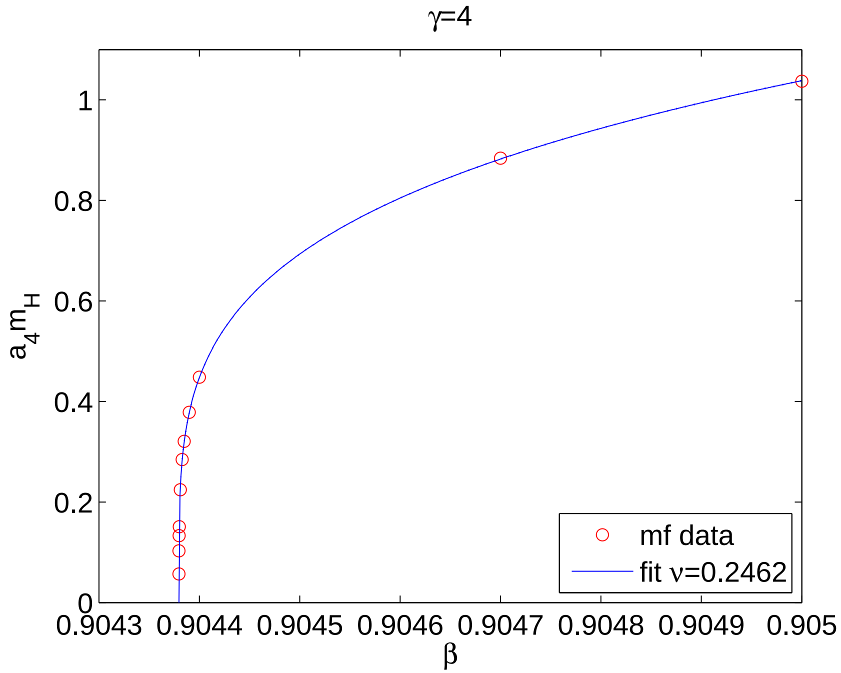
Continuum limit at fixed anisotropy

The second order phase transition separating the d-compact phase from the layered phase:

$$a_4 m_H \sim (1 - \beta_c/\beta)^\nu$$



$\nu = 1/2$: 4d Ising model, confirms [Svetitsky and Yaffe, 1982]



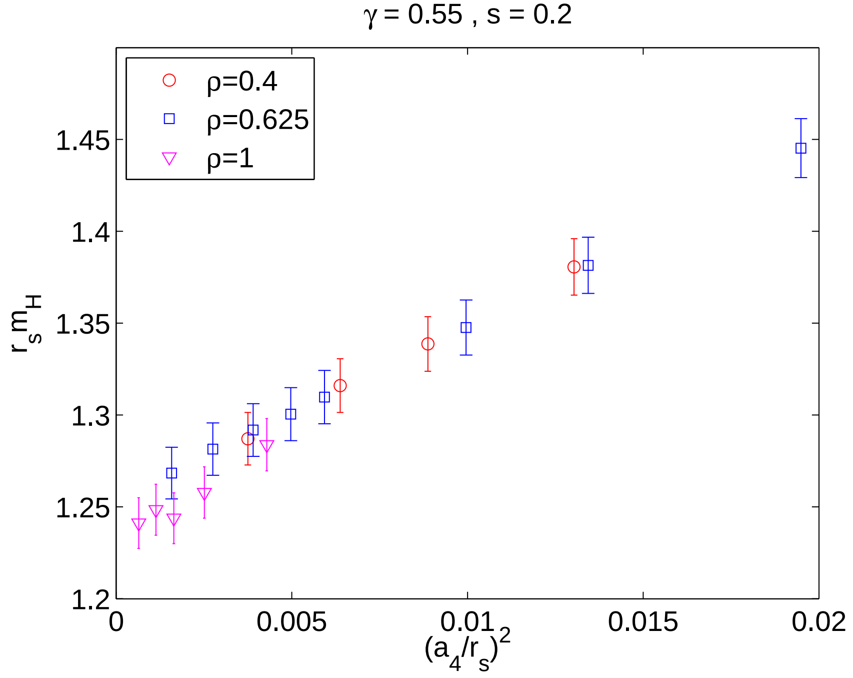
$\nu = 1/4$, the mass does never go to zero

Continuum limit at fixed anisotropy

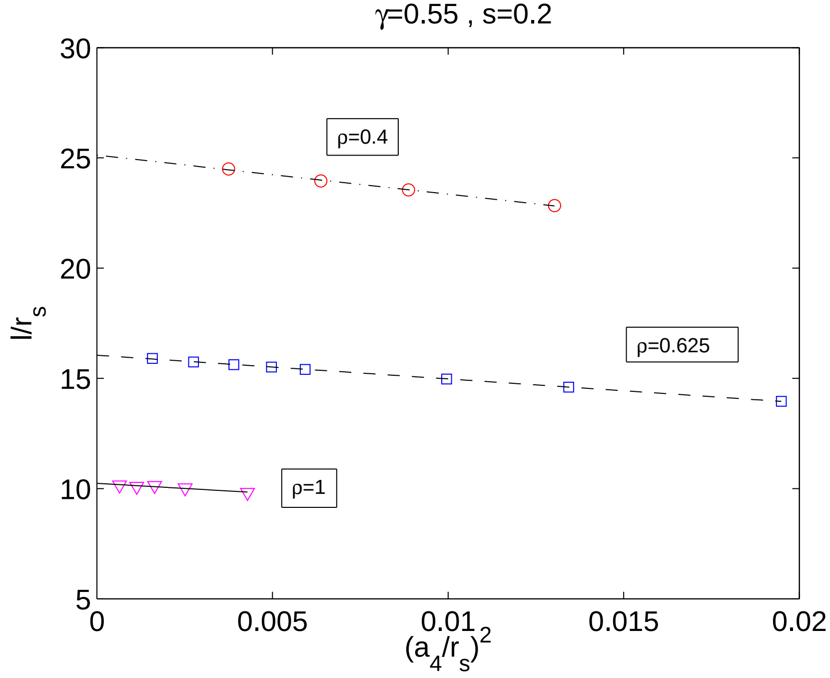
Lines of constant physics [Irges and Knechtli, 2010]

$$(L = L_T = L_5 \longrightarrow \infty, \;\; \beta \longrightarrow \beta_c)|_{\gamma,\rho=m_W/m_H} \;\; \Longleftrightarrow \;\; \text{continuum limit}$$

A physical scale r_s is defined through $r^2 F(r)|_{r=r_s} = s = 0.2$ with $F = V_4'$.
Fixing $\gamma = 0.55$:



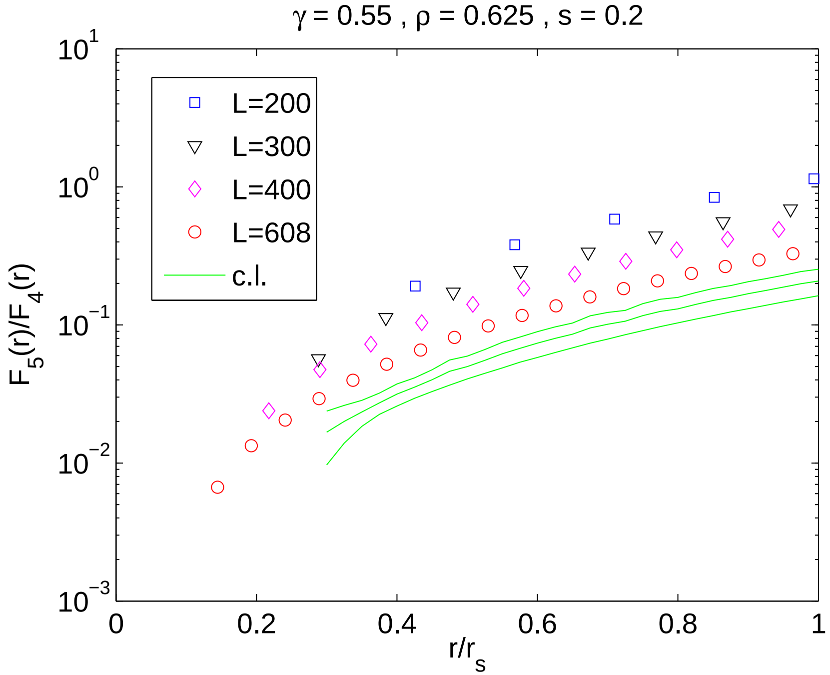
m_H is independent on ρ



ρ determines the physical box size l

Dimensional reduction and confinement

Line of constant physics $\gamma = 0.55$, $\rho = 0.625$: dimensional reduction



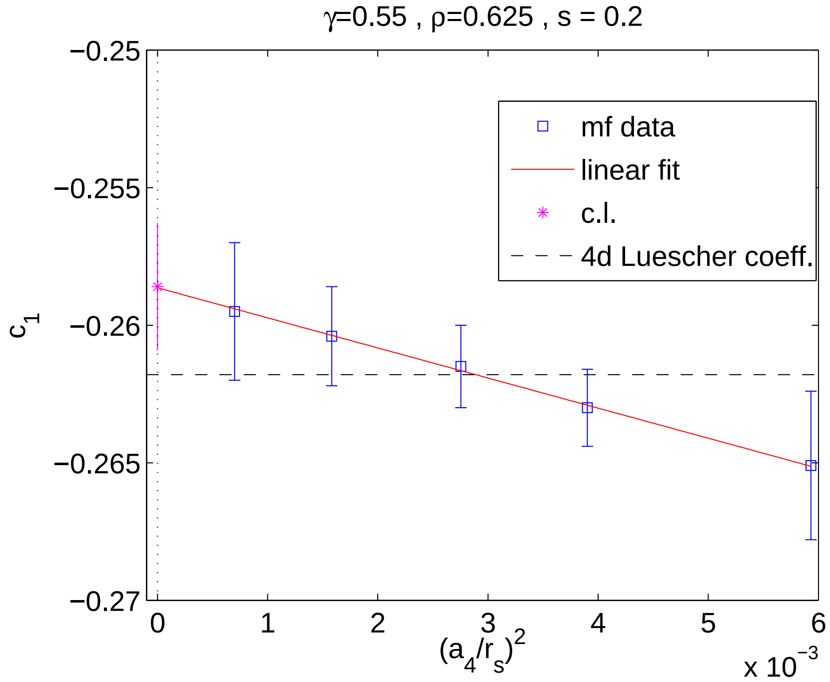
- The force F_4 has a physical nonzero continuum limit
- F_5/F_4 tends to zero in the continuum limit $\Rightarrow$ localization
- Dimensional reduction to 4d Georgi–Glashow model. It must be in the confined phase

Dimensional reduction and confinement

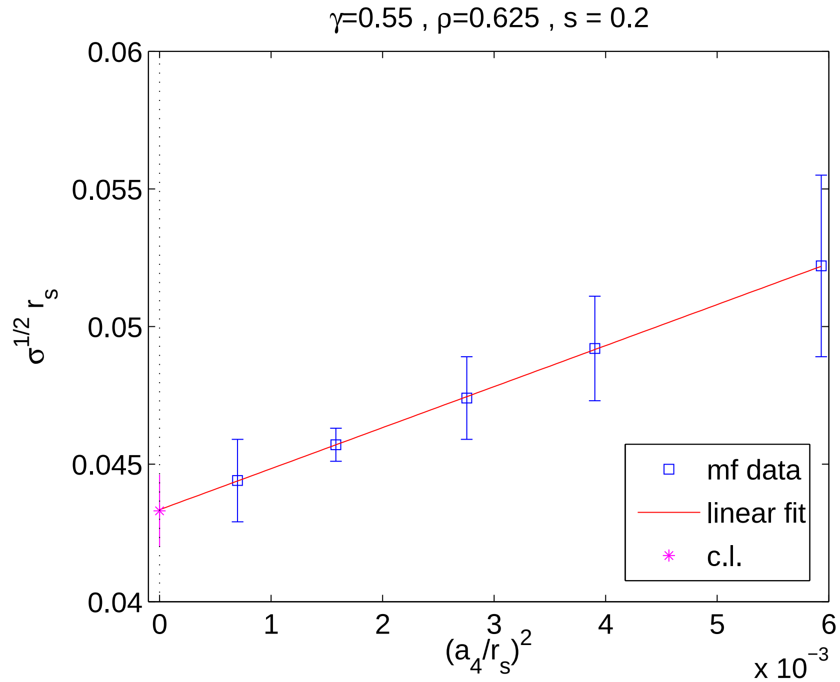
Line of constant physics $\gamma = 0.55$, $\rho = 0.625$: confinement

$$V_4(r) = \mu + \sigma r + c_0 \log(r) + \frac{c_1}{r} + \frac{c_2}{r^2}\,, \quad r/r_s > 1$$

Perform local fits, there are *simultaneous* plateaus for all four coefficients
Continuum limit of plateau values in the range $r/r_s \in [2.15\,,\,2.80]$:



We get the universal 4d value $-\pi/12$!



There is a positive string tension

Dimensional reduction and confinement

Line of constant physics $\gamma = 0.55$, $\rho = 0.625$: confinement

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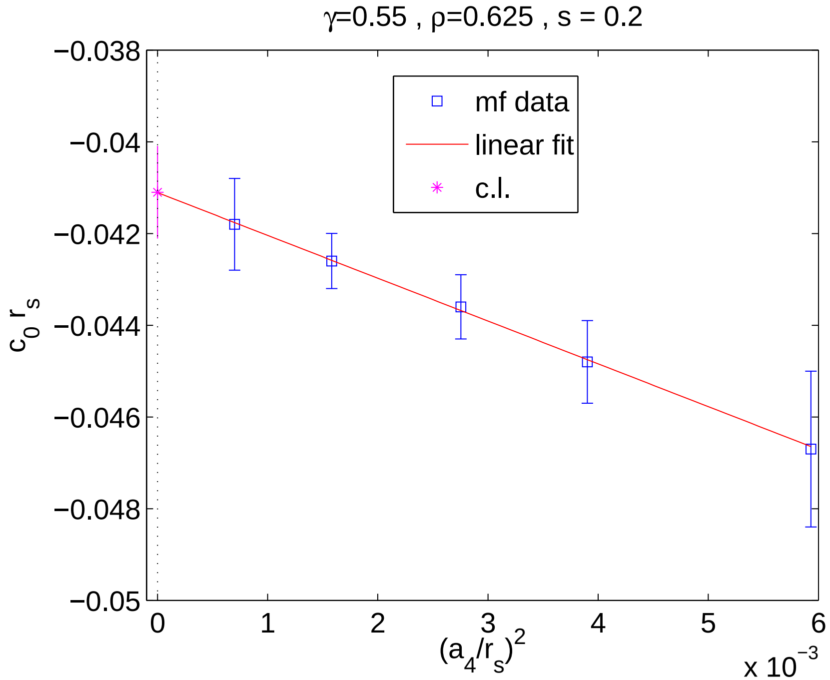
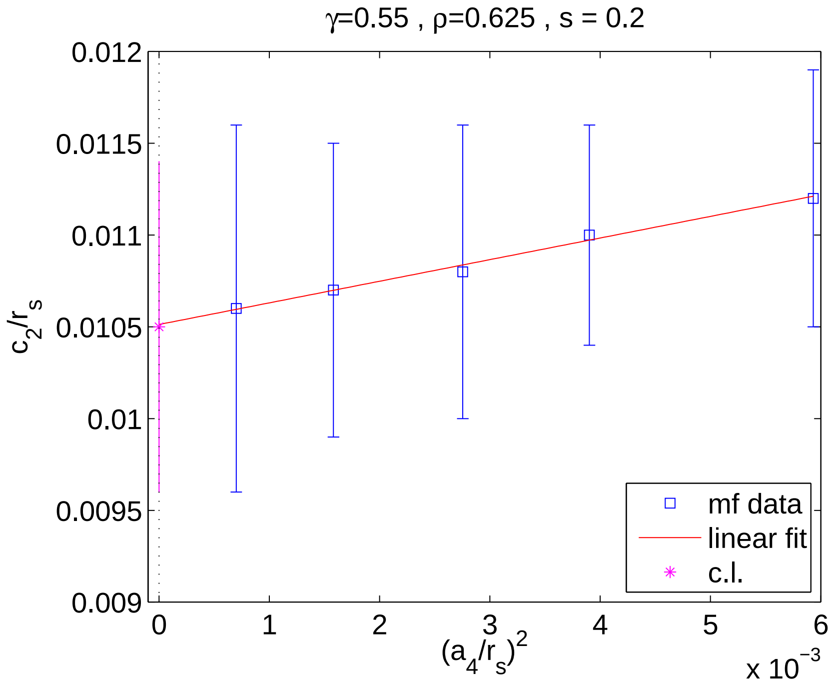
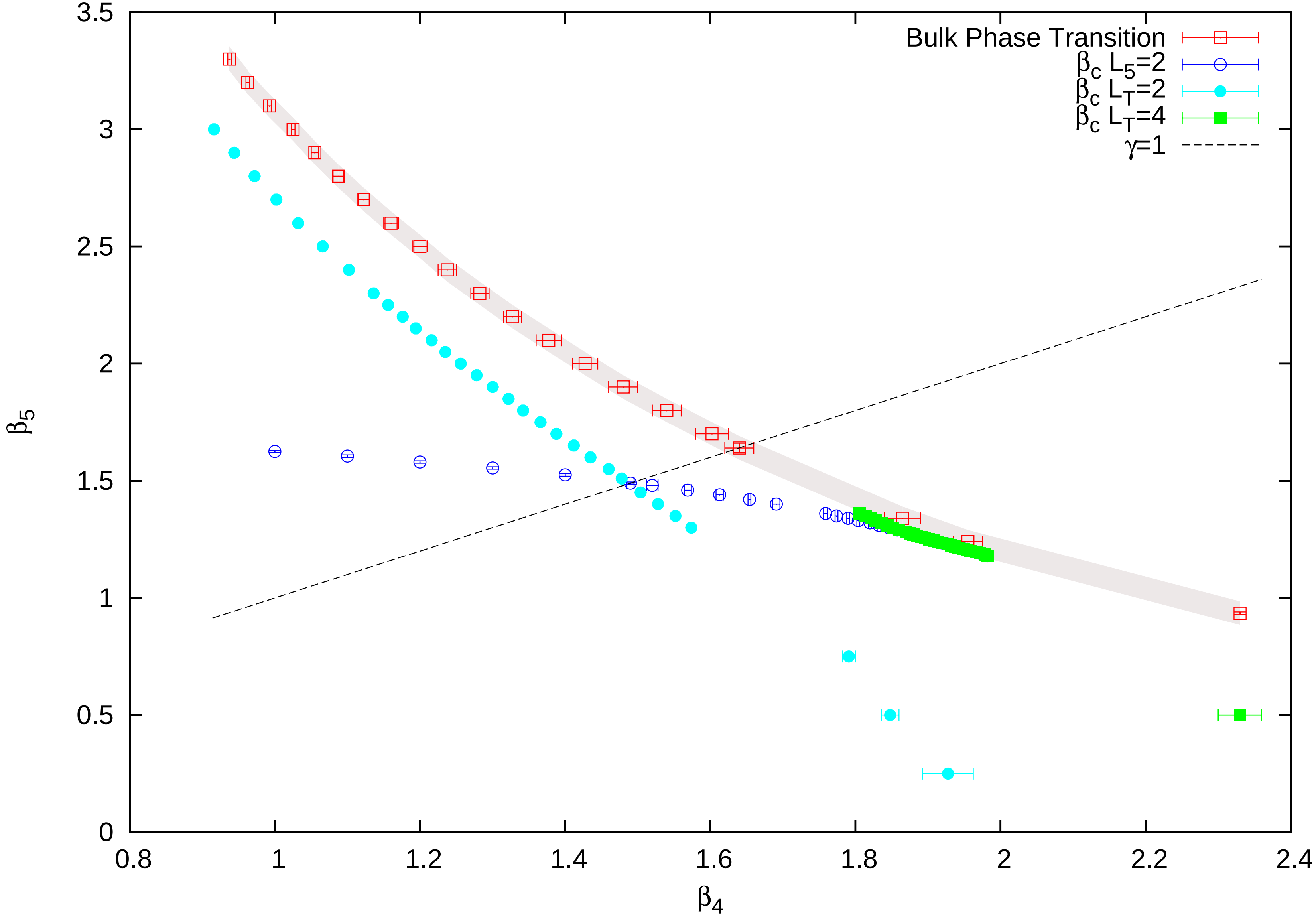


A large negative log term is present (origin?)



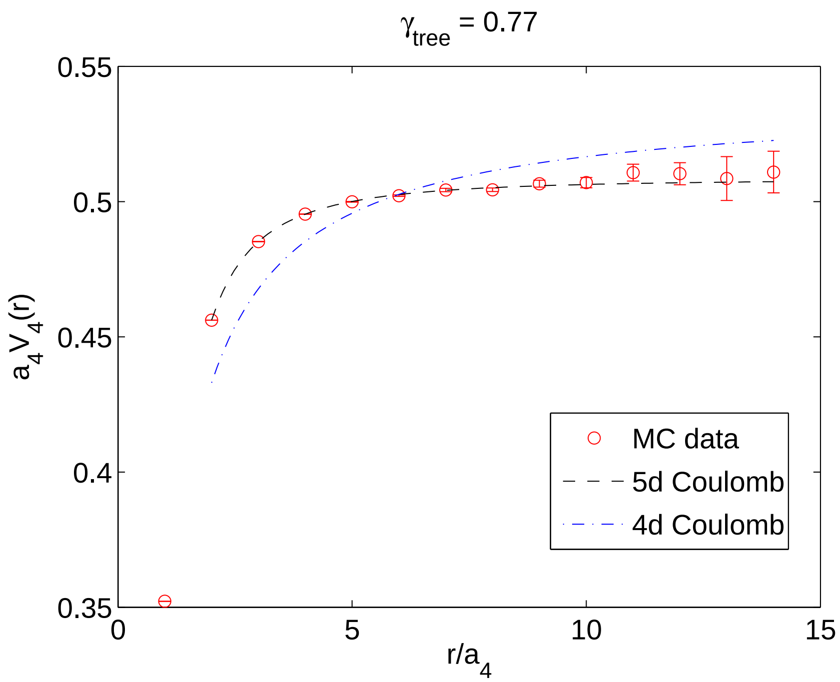
There are also higher order corrections

Monte Carlo results: $\beta_4 = \beta/\gamma$, $\beta_5 = \beta\,\gamma$



Monte Carlo results

- We confirm [Ejiri, Kubo and Murata, 2000; de Forcrand, Kurkela and Panero, 2010]:
  – in infinite volume there is only a first order bulk phase transition (shaded line);
  – at $\gamma > 1$ there are second order phase transitions due to compactification
- We located second order phase transitions when $\gamma < 1$ and $L_T \ll L, L_5$
- We can accurately compute the static potential, example: $32^4 \times L_5 = 16$ lattice in the deconfined phase at $\beta_5 = 1.24$, $\beta_4 = 2.10$



2 levels of 4d spatial HYP smearing

Outlook

Meanfield:

- Check of the convergence of the meanfield expansion: computation of second order correction to the Higgs mass, check finite size effects (ongoing)
- Spontaneous symmetry breaking: implementation of orbifold boundary conditions in the meanfield laboratory (ongoing)
- Generalization to $SU(N)$

Monte Carlo:

- Computation of the static potential and of the order of the phase transition with $L_T = 4$ at $\gamma \simeq 0.5$ (ongoing)
- Study of further geometries (ongoing)
- Computation of the spectrum, orbifold boundary conditions, $SU(3)$

... in order to be ready with predictions when first LHC results will come!