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Non-locality of the nucleon-nucleon potential from Lattice QCD

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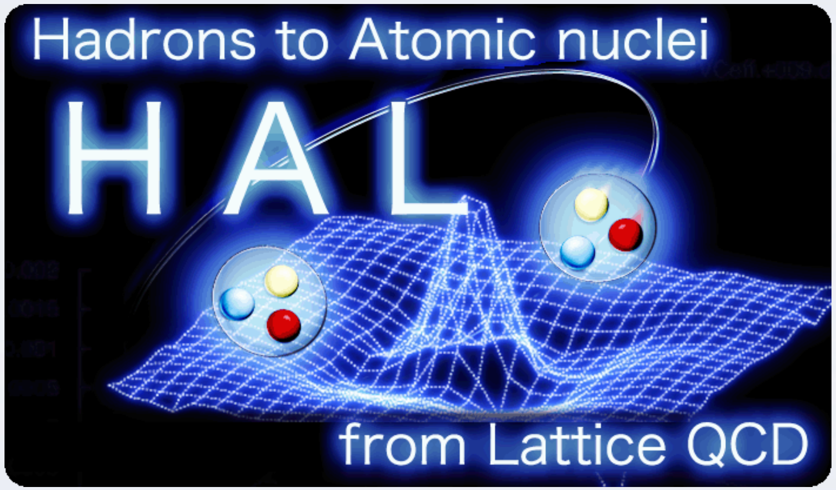
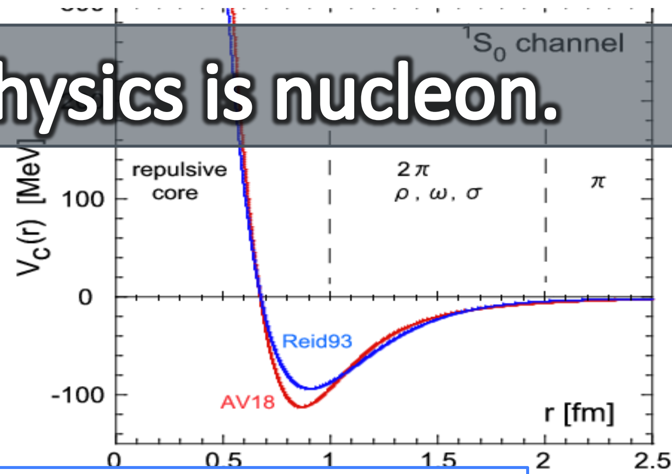
in collaboration with

S.Aoki T.Hatsuda N.Ishii H.Nemura for HAL QCD Coll.



Nuclear potential

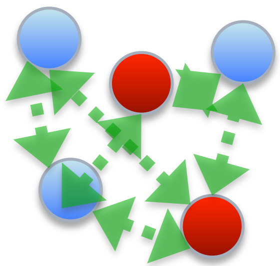
the fundamental particle in nuclear physics is nucleon.



Nuclear potential

Once the nucleon-nucleon potential (NN potential) is obtained, various features of nucleus can be studied via the Schrodinger equation.

Many-body system



Few-body system

high density

equation of state nuclear matter

→ neutron stars

hyper nucleus,

exotic nucleus

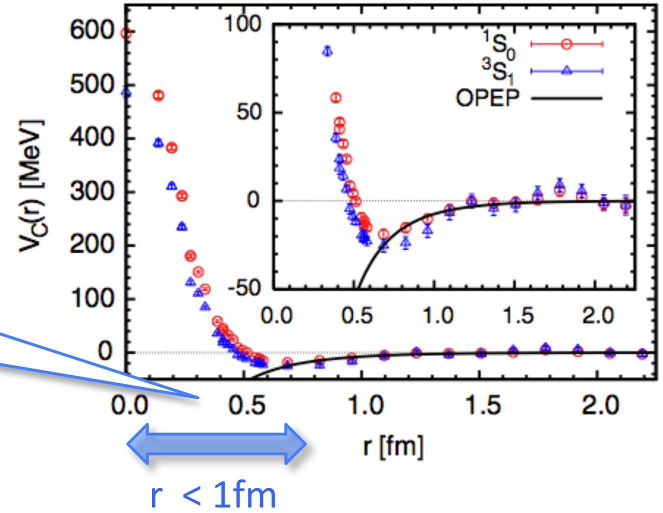
Nuclear potential from Lattice QCD

Recently, a method is proposed to extract the NN potential from Lattice QCD.

N. Ishii, S. Aoki and T. Hatsuda, Phys. Rev. Lett. **99, 022001 (2007)**

Their method reproduced the short range part of the NN potential ,"repulsive core".

The region nucleons are overlapped.
Study from the QCD is needed.



This approach has following advantages:

- Scattering experimental data are not needed:

  ➡ hyperon-nucleon, hyperon-hyperon interaction

  H. Nemura, N. Ishii, S. Aoki and T. Hatsuda, Phys. Lett. B **673, 136** (2009)

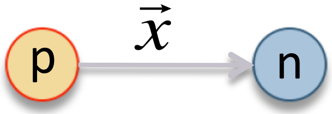
- It is also possible to investigate three nucleon force systematically. (18 June room1 15:50~ T.Doi)

NN potential from Lattice QCD: HAL's method

pioneering work: *N. Ishizuka* Phys.Rev.**D71**:094504,2005.

derivation: S.Aoki, H.Hatsuda, N.Ishii, PTP123(2010)89

BS wave function



$$\equiv \phi(\vec{x})$$

Effective Scrodinger Equation:

$$\left(\Delta + k^2\right)\phi(\vec{x};k) = m_N \int d^3y\, U(\vec{x};\vec{y})\,\phi(\vec{y};k)$$

Energy independent non-local potential

obtain the BS wave function ⇨ calculate back to the potential.

$$\phi(\vec{r};E) = \lim_{t\to\infty} \sum_{\vec{x}} \langle 0 | \hat{N}(\vec{x}+\vec{r};t)\hat{N}(\vec{x};t)\overline{NN}(t_0) | 0 \rangle$$

For the moment, U is obtained by LO of derivative expansion:

$$U(\vec{x};\vec{y}) = [V_0(\vec{x}) + V_\sigma(\vec{x})(\vec{\sigma}_1 \cdot \vec{\sigma}_2) + V_T(r)S_{12} + V_{LS}(r)\vec{L}\cdot\vec{S} + O(\nabla^2)]\,\delta(\vec{x}-\vec{y})$$

LO: $V_0(\vec{x}) + V_\sigma(\vec{x})(\vec{\sigma}_1 \cdot \vec{\sigma}_2)$, $V_T(r)S_{12}$

The purpose of this work:

Check the convergence of the derivative expansion

(Is LO enough?)

The convergence is related to E and L dependence of LO potential

ex) S=0 case : (S_{12}=0)

$$\left(\Delta + k^2\right)\phi^{S=0}(\vec{x};k) = m_N \int d^3y U(\vec{x};\vec{y})\phi^{S=0}(\vec{y};k)$$

$$= m_N \Big[V_C(r) + V_L(r)\ L^2 + V_{LL}(r)\ L^4 + \ \ldots$$

$$+\{V_p(r)\ ,\nabla^2\} + \{V_{pp}(r),\nabla^4\} + \ldots \Big]\phi^{S=0}(\vec{x};k)$$

The convergence is related to E and L dependence of LO potential

ex) S=0 case : (S_{12}=0)

$$\left(\Delta + k^2\right)\phi^{S=0}(\vec{x};k) = m_N \int d^3y U(\vec{x};\vec{y})\phi^{S=0}(\vec{y};k)$$

$$= m_N\left[V_C(r) + V_L(r)\ L^2 + V_{LL}(r)\ L^4 + \ \ldots \right.$$

$$\left. + \{V_p(r)\ ,\nabla^2\} + \{V_{pp}(r),\nabla^4\} + \ldots\right]\phi^{S=0}(\vec{x};k)$$

LO

$$\left(\Delta + k^2\right)\phi^{S=0}(\vec{x};k) = m_N Vc(r)\ \phi^{S=0}(\vec{x};k)$$

the potential obtained by LO:

$$V_C^{LO}(r;L,E) = \frac{1}{m_N}\left(\frac{\Delta\phi^{S=0}(r;E;L)}{\phi^{S=0}(r;E;L)} + k^2\right)$$

The convergence is related to E and L dependence of LO potential

ex) S=0 case : (S_{12}=0)

$$\left(\Delta + k^2\right)\phi^{S=0}(\vec{x};k) = m_N \int d^3y U(\vec{x};\vec{y})\phi^{S=0}(\vec{y};k)$$

$$= m_N \left[V_C(r) + V_L(r)\ L^2 + V_{LL}(r)\ L^4 + \ \ldots \right.$$

$$\left. + \{V_p(r)\ ,\nabla^2\} + \{V_{pp}(r),\nabla^4\} + \ldots \right]\phi^{S=0}(\vec{x};k)$$

LO

$$\left(\Delta + k^2\right)\phi^{S=0}(\vec{x};k) = m_N Vc(r)\ \phi^{S=0}(\vec{x};k)$$

$\vec{k} = -i\nabla$

$\vec{L} = -\vec{r} \times i\vec{\nabla}$

$k^2 / m_N = E,$

the potential obtained by LO:

$$V_C^{LO}(r;L,E) = \frac{1}{m_N}\left(\frac{\Delta\phi^{S=0}(r;E;L)}{\phi^{S=0}(r;E;L)} + k^2\right)$$

$$= V_C(r) + V_L(r)L^2 + V_{LL}(r)L^4 + \cdots + \{V_p(r)\ ,\nabla^2\} + \{V_{pp}(r),\nabla^4\} + \ldots$$

k^2 k^4

L dependent

Energy dependent

Size of the higher order terms can be estimate from the examination of the E and L dependence of LO potential.

Energy dependence

Strategy

previous study of energy dependence:

2d Ising analytical study

(S. Aoki, J. Balog P. Weisz arXiv:0805.3098)

□ momentum are discritized in a finite box of size L.

PBC $\vec{p} = \left(\frac{2n_x\pi}{L}, \frac{2n_y\pi}{L}, \frac{2n_z\pi}{L} \right)$

APBC $\vec{p} = \left(\frac{(2n_x+1)\pi}{L}, \frac{(2n_y+1)\pi}{L}, \frac{(2n_z+1)\pi}{L} \right)$

ground state

$E_0 \sim 0$

$E_0 \sim \frac{3}{m_N} \times \left(\frac{\pi}{L} \right)^2$

compare

~ 45 MeV in this work.

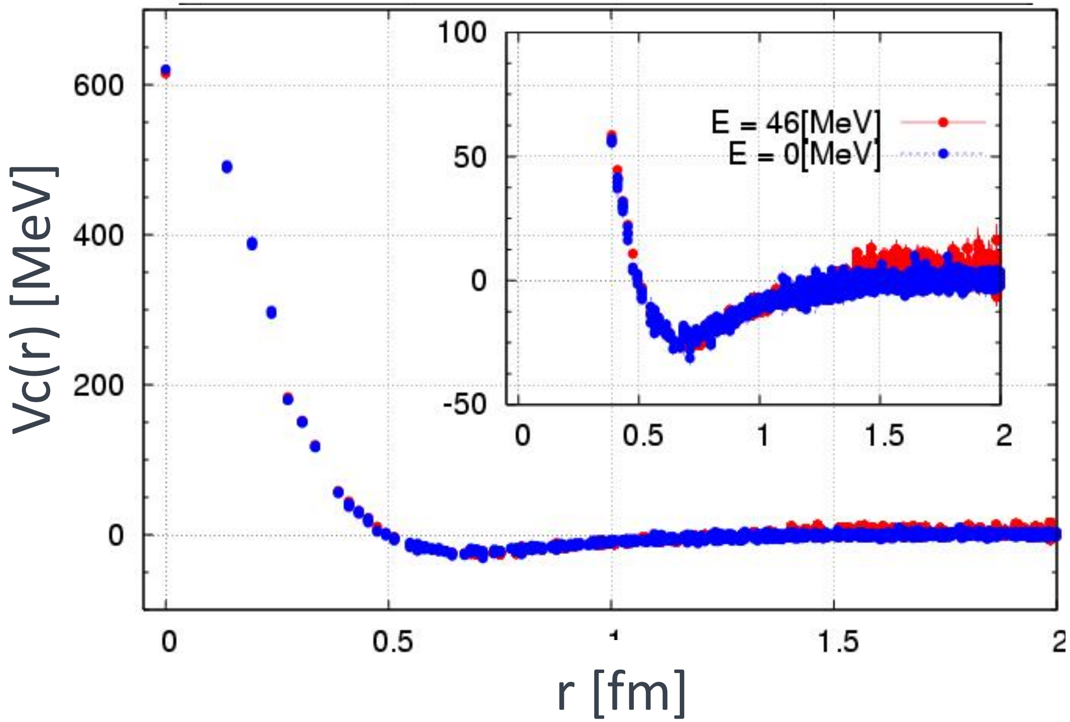
$$\sum_{\vec{x}} \langle 0 | \hat{N}^i{}_\alpha(\vec{x}+\vec{r},t) \hat{N}^j{}_\beta(\vec{x},t) \overline{NN}(t_0) | 0 \rangle$$

$$= \phi(\vec{r};E_0) e^{-E_0(t-t_0)} + \phi(\vec{r};E_1) e^{-E_1(t-t_0)} + \cdots \xrightarrow{t\to\infty} \phi(\vec{r};E_0) e^{-E_0(t-t_0)}$$

ground state

comparison of potentials : 0 MeV and 45 MeV

$^1S_0$ Vc (+-5)



45MeV and 0MeV are consistent

Energy dependence is weak.

Set up:

Lattice size: 32^3 x 48 ~4.5 [fm]

$\beta = 5.7$

1/a=1.44 GeV (a∼0.14[fm])

$\kappa_{ud} = 0.1665$

mπ=529.0(4) MeV m_N=1334 MeV

Heat bath quenched

Plaquette gauge action

boundary condition:

PBC for 0 MeV, APBC for 45 MeV

Blue Gene/L

KEK 2048 PU

comparison of potentials : 0 MeV and 45 MeV

$$V_C^{LO}(r;L,E) = V_C(r) + V_L(r)L^2 + V_{LL}(r)L^4 + \cdots + \{V_p(r), \nabla^2\} + \{V_{pp}(r), \nabla^4\} + \ldots$$

Energy dependent part

$$\left(\Delta + k^2\right)\phi^{S=0}(\vec{x};k) = m_N \int d^3y\, U(\vec{x};\vec{y})\phi^{S=0}(\vec{y};k)$$

$$= m_N \Big[V_C(r) + V_L(r)\, L^2 + V_{LL}(r)\, L^4 + \ \ldots$$

$$+ \{V_p(r), \nabla^2\} + \{V_{pp}(r), \nabla^4\} + \ldots \Big] \phi^{S=0}(\vec{x};k)$$

45MeV and 0MeV are consistent

Energy dependence is weak.

→

The size of higher derivative terms are significant weak.

L dependence

L, S, J with representation on the cubic group

symmetry under the rotation is broken => (cubic group)

L (orbital angular mom.), S (spin), J (total angular mom.)

➡ The representation of cubic group

$$\not{L}, \not{S}, \not{J} \longrightarrow A_1, A_2, E, T_1, T_2$$

L, S, J

representation with spin

$l = 0$	1	2	3	4	...
A_1	T_1	$E + T_2$	$A_2 + T_1 + T_2$	$A_1 + E + T_1 + T_2$	...

Source

$$\phi(\vec{r}) = \sum_{\vec{x}} \langle 0 | \hat{N}^i_\alpha(\vec{x}+\vec{r},t)\hat{N}^j_\beta(\vec{x},t)\overline{NN}(t_0)|0\rangle \qquad \overline{N} = \varepsilon_{abc}(\overline{Q}_a C\gamma_5 \overline{Q}_b)\overline{Q}_c$$

For simplicity, we will restrict this talk to S=0 state.

wall source

$$Q = \sum_{\vec{x}} q(t,\vec{x})$$

It was difficult to extract $L \neq 0$ states.

schematic figure

$= A_1$

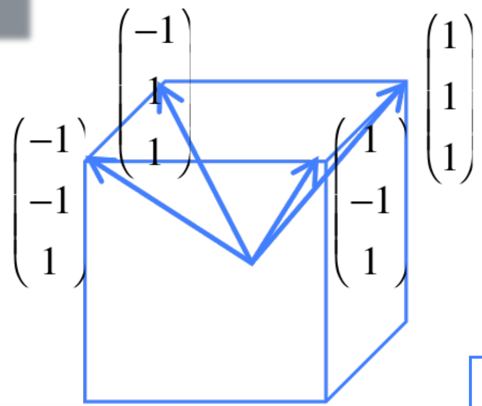
dominant

$J = 0 \quad (L = 0)$

four momentum wall sources

$$Q = \sum_{\vec{x}} q(t,\vec{x}) \cos((\pm x \pm y + z)\pi / L)$$

Break the rotational symmetry with source and extract $L \neq 0$ states.



$= A_1 + T_2$

dominant

$J = 2 \quad (L = 2)$

$L \neq 0$ BS wave functions are extracted with the projection operator.

Projection operator

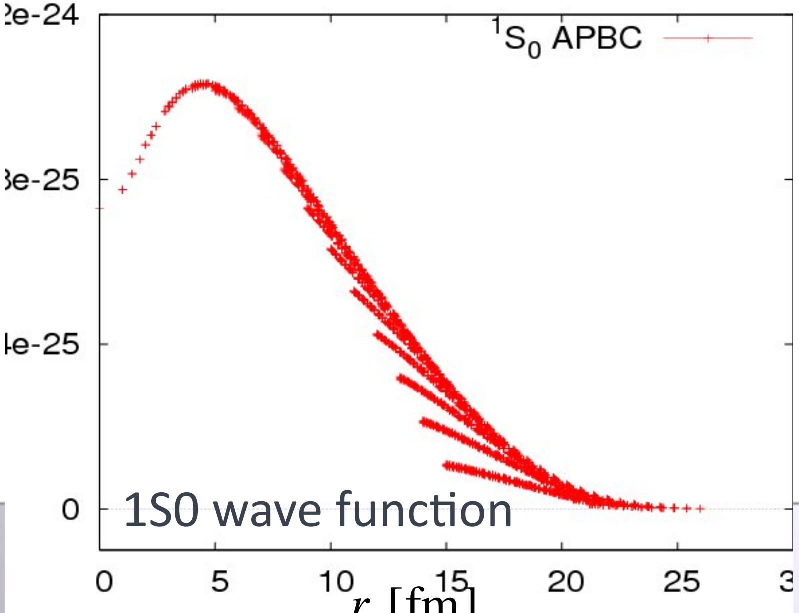
$$P^{(\Gamma)}\phi(\vec{r}) = \frac{d_\Gamma}{24}\sum_{i=1}^{24}\chi^{(\Gamma)}[R_i]\,\phi(R_i\vec{r})$$

$(\Gamma = A_1, T_2$ in this work)

- R_i elements of cubic group
- $\chi^{(\Gamma)}$ characters of representation Γ
- d_Γ dimension of the representation Γ

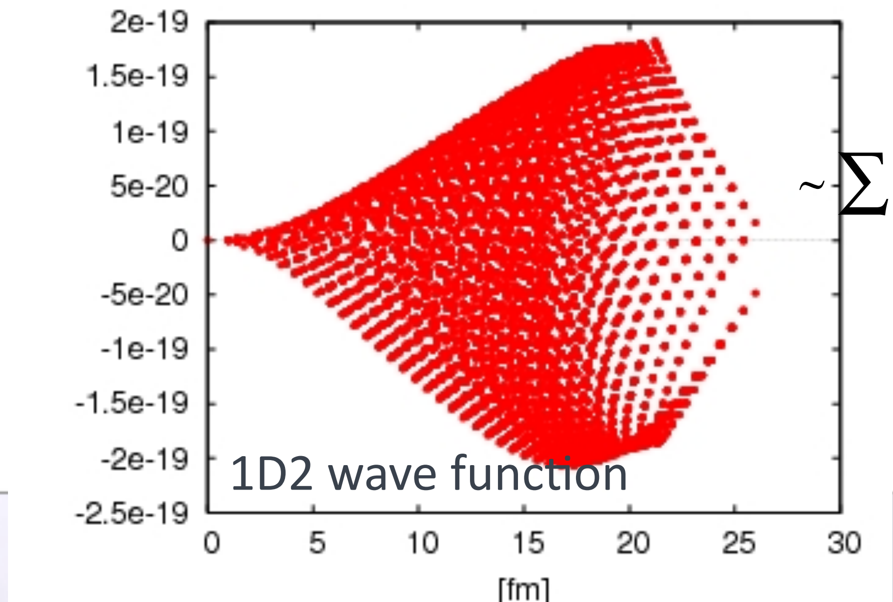
$P^{(A_1)}\phi(\vec{r})$

→ ${}^1S_0 : \quad (S=0)\otimes(L=0)$



$P^{(T_2)}\phi(\vec{r})$

→ ${}^1D_2 : \quad (S=0)\otimes(L=2)$



L dependence of the potential

S=0

$$\left(\nabla^2 + k^2\right)\phi^{S=0}(\vec{x};\vec{L}) = m_N[Vc(r) + V_L(r)L^2 + V_{LL}(r)L^4 + \cdots]\phi^{S=0}(\vec{x};\vec{L})$$

L=0 S=0 : 1S0

$$\phi^{{}^1S_0}(\vec{x};k) = \mathcal{P}^{(A_1)}\phi_{S=0}(\vec{x};k)$$

$$\Rightarrow V_C^{LO}(r) = \frac{1}{m_N}\left(\frac{\Delta\phi^{{}^1S_0}(r)}{\phi^{{}^1S_0}(r)} + k^2\right) = Vc(r)$$

compare

L=2 S=0 : 1D2

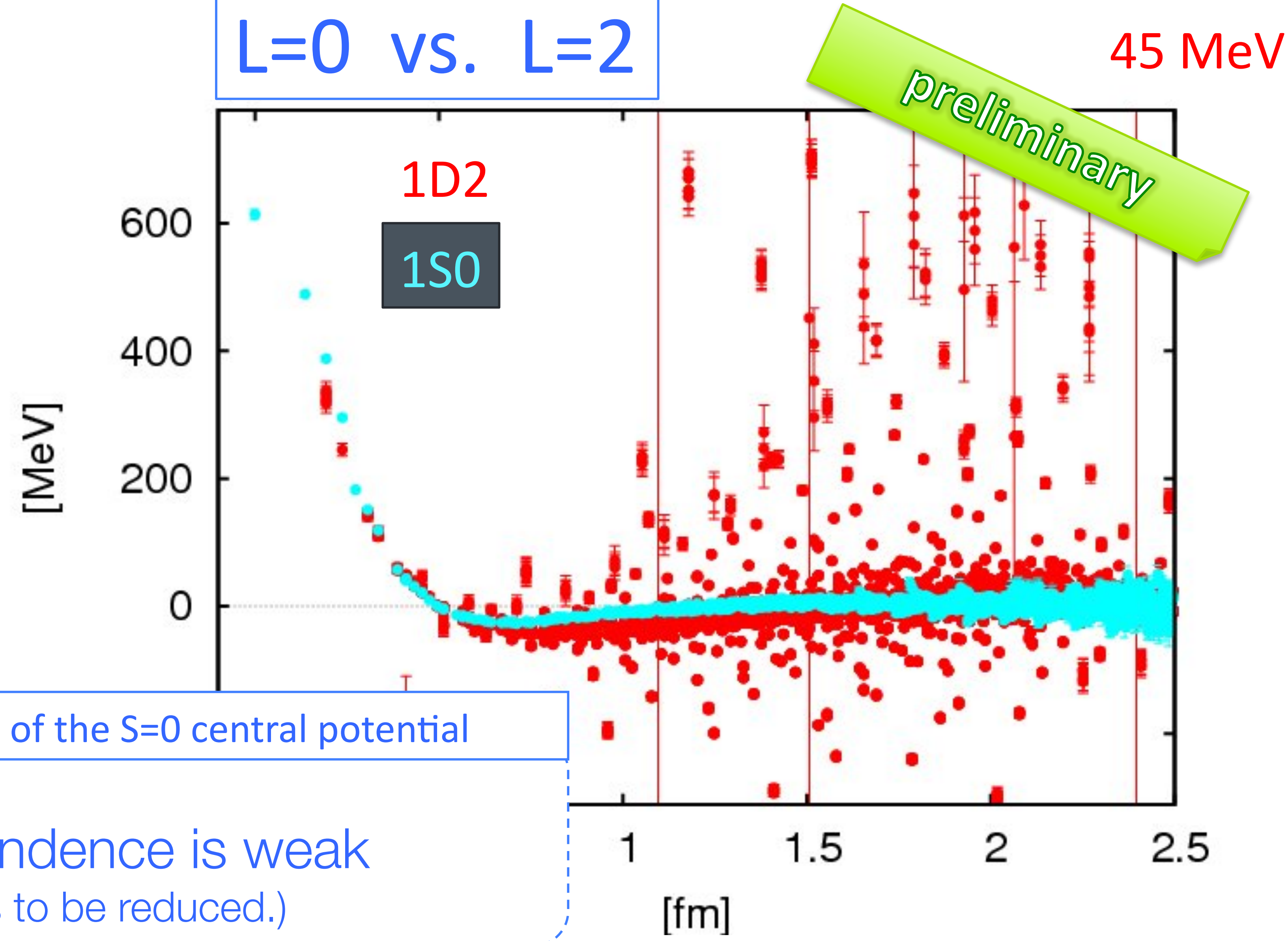
$$\phi^{{}^1D_2}(\vec{x};k) = \mathcal{P}^{(T_2)}\phi_{S=0}(\vec{x};k)$$

$$\Rightarrow V_C^{LO}(r) = \frac{1}{m_N}\left(\frac{\Delta\phi^{{}^1D_2}(r)}{\phi^{{}^1D_2}(r)} + k^2\right) = Vc(r) + V_L(r)2(2+1) + \cdots$$

the size of higher order derivative terms

L dependence

L=0 vs. L=2



L dependence of the S=0 central potential

L dependence is weak

(Noise has to be reduced.)

L dependence

$$V_C^{LO}(r; L, E) = V_C(r) + V_L(r)L^2 + V_{LL}(r)L^4 + \cdots + \{V_p(r), \nabla^2\} + \{V_{pp}(r), \nabla^4\} + \ldots$$

L dependent part

$$\left(\Delta + k^2\right)\phi^{S=0}(\vec{x};k) = m_N \int d^3y U(\vec{x};\vec{y})\phi^{S=0}(\vec{y};k)$$

$$= m_N \left[V_C(r) + V_L(r)\, L^2 + V_{LL}(r)\, L^4 + \ldots \right.$$

$$\left. + \{V_p(r), \nabla^2\} + \{V_{pp}(r), \nabla^4\} + \ldots \right] \phi^{S=0}(\vec{x};k)$$

L dependence is weak
(Noise has to be reduced.)

The size of higher L terms are small.

Summary and conclusion

Summary of this work:

We have examined the convergence of derivative expansion of non-local potential.
In this purpose, we have caclulated 0 MeV, 45 MeV BS wave with using boundary condition and 1S0 ,1D2 BS wave with using cubic group.

$$V_C^{LO}(r;L,E) = V_C(r) + V_L(r)L^2 + V_{LL}(r)L^4 + \cdots + \{V_p(r), \nabla^2\} + \{V_{pp}(r), \nabla^4\} + \ldots$$

Conclusion:

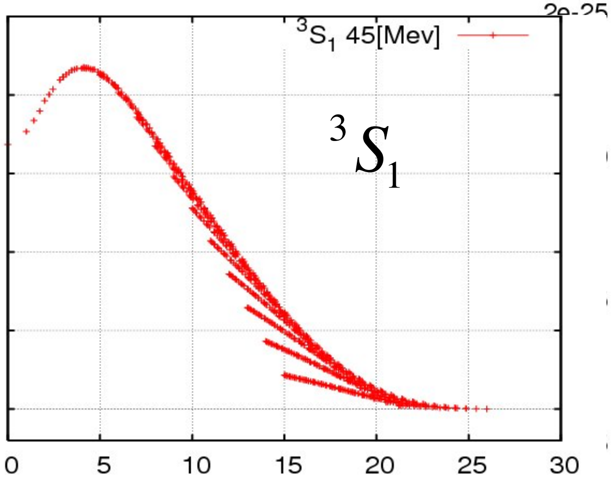
We conclude that the NN potential from the QCD obtained up to LO is valid at E=0 ~ (at least) 45 MeV, and L=0 ~ (at least) 2.

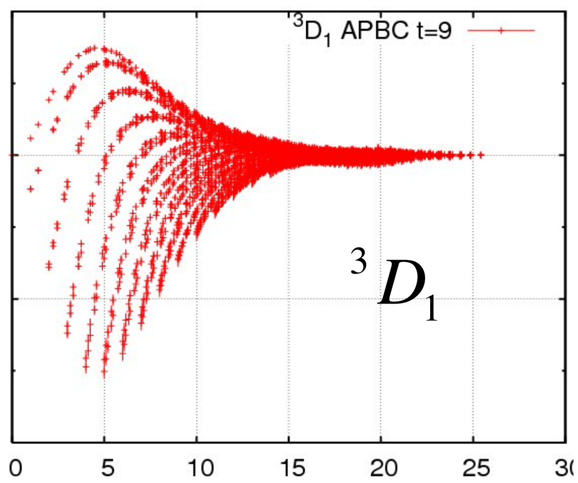
Future work:

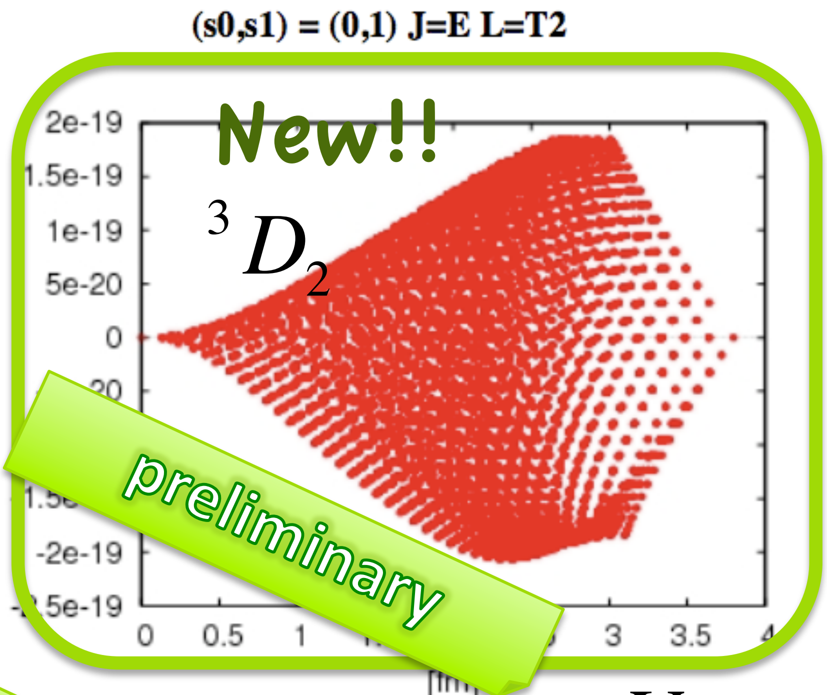
Father check: full QCD, light quark mass, hyperon system...

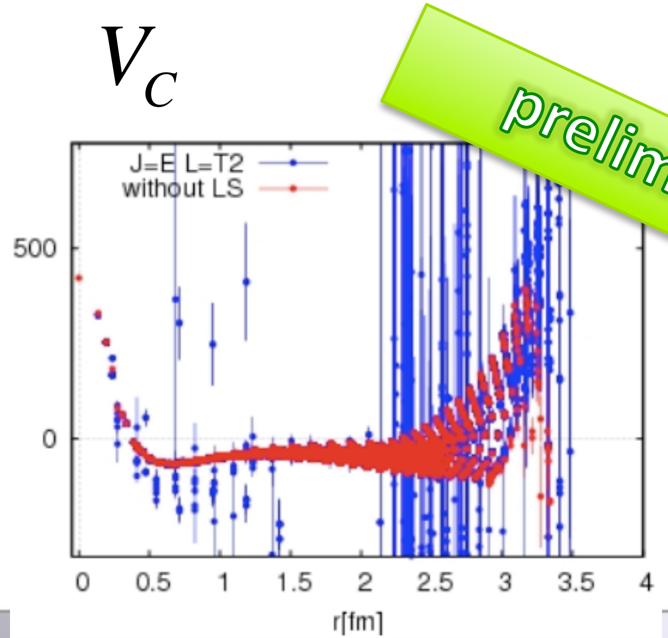
We are applying the technique to S=1 case,
and the calculation of LS force is on going.

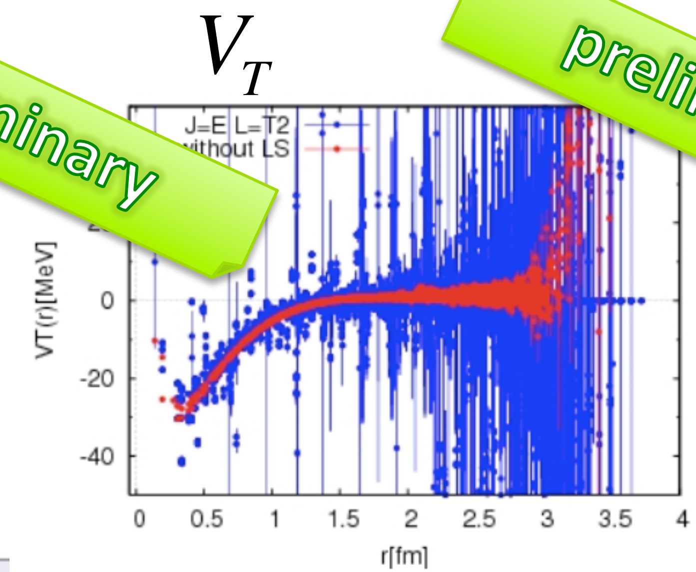
Very preliminary work: LS force

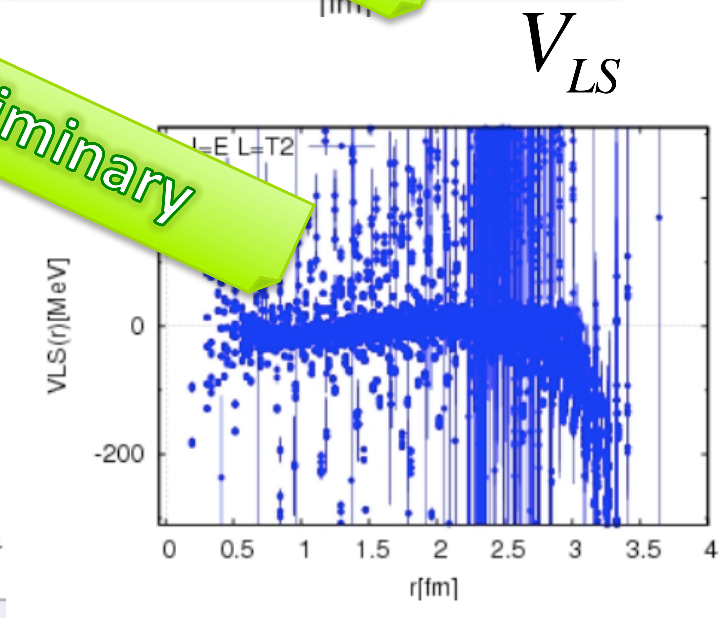








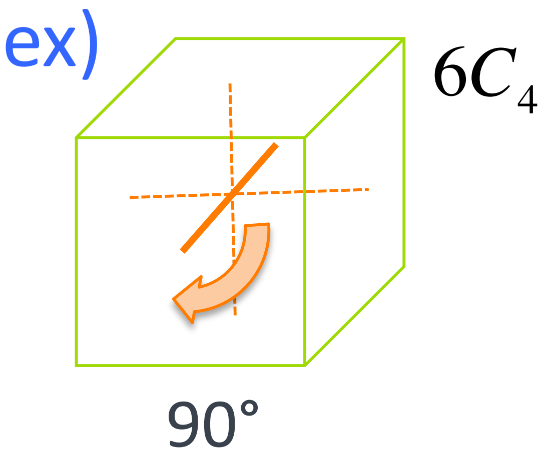




End of slides

What L is included in this system can be understood by the cubic group analysis

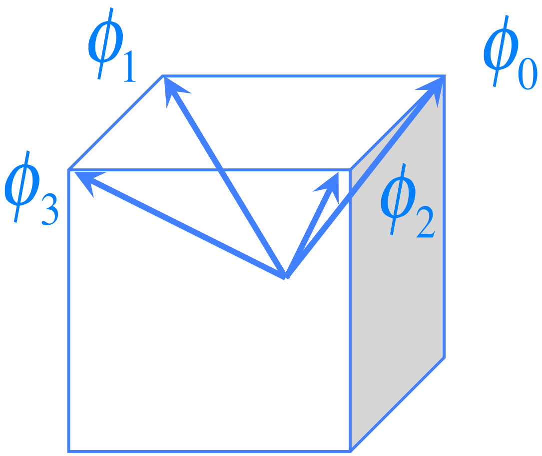
The representation matrix, and charactor can be obtained easily.



$$\begin{pmatrix}\phi 0\\ \phi 1\\ \phi 2\\ \phi 3\end{pmatrix} \longrightarrow \begin{bmatrix} & 1 & & \\ & & 1 & \\ & & & 1\\ 1 & & & \end{bmatrix}\begin{pmatrix}\phi 0\\ \phi 1\\ \phi 2\\ \phi 3\end{pmatrix}$$

charactor of this system

	E	$6C_4$	$3C_2$	$8C_3$	$6C_2$
χ	4	0	0	1	2



$$\phi 0 \leftarrow \quad Q = \sum_{\vec{x}} q(t,\vec{x}) \cos((+x + y + z)\pi / L)$$

$$\phi 1 \leftarrow \quad Q = \sum_{\vec{x}} q(t,\vec{x}) \cos((-x + y + z)\pi / L)$$

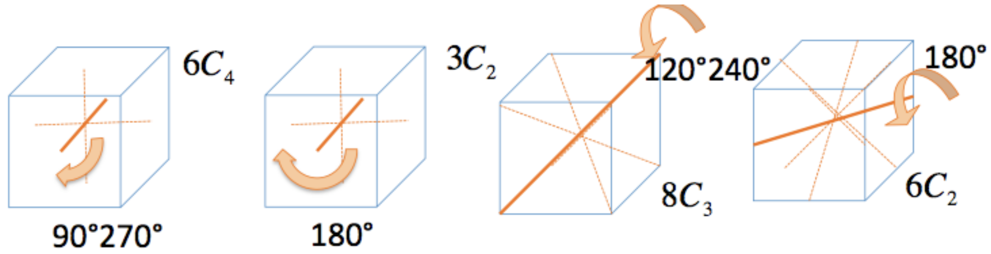
$$\phi 2 \leftarrow \quad Q = \sum_{\vec{x}} q(t,\vec{x}) \cos((+x - y + z)\pi / L)$$

$$\phi 3 \leftarrow \quad Q = \sum_{\vec{x}} q(t,\vec{x}) \cos((-x - y + z)\pi / L)$$

What L is included in this system can be understood by the cubic group analysis

charactor of the cubic group

O	E	$6C_4$	$3C_2$	$8C_3$	$6C_2$
$\chi(A_1)$	1	1	1	1	1
$\chi(A_2)$	1	-1	1	1	-1
$\chi(E)$	2	0	2	-1	0
$\chi(T_1)$	3	1	-1	0	-1
$\chi(T_2)$	3	-1	-1	0	1



charactor of this system

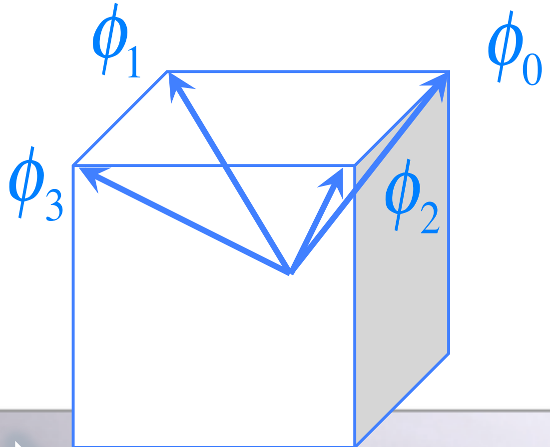
	E	$6C_4$	$3C_2$	$8C_3$	$6C_2$
χ	4	0	0	1	2

irreducible decomposition:

$$\chi(R) = c_{A1}\chi(A_1)[R] + c_{A2}\chi(A_2)[R] + \cdots$$

$$c_\alpha = \sum_{i=1}^{24} \chi(\alpha)[R_i]^* \chi[R_i]$$

R_i : elements of cubic group



L=0

$$= A_1 + T_2$$

L=2 is dominant

This system including L=0 and L=2 states

$$G^{(n)}_{\alpha,\beta;\alpha'\beta'}(\vec{x},\vec{y})$$

$$= \sum_{\vec{x}_1,\ldots,\vec{x}_6} \langle 0 | T\left[N_\alpha(\vec{x},t) N_\beta(\vec{y},t) \overline{\mathcal{N}}_{\bar{\alpha}} \overline{\mathcal{N}}_{\bar{\beta}} \right] | 0 \rangle f^{(n)}(\vec{x}_1,\ldots,\vec{x}_6),$$

$$\mathcal{N}_\alpha = \sum_{x_0,x_1,x_2} \left(q^t(x_0) C\gamma_5\gamma_0 q(x_1) \right) q_\alpha(x_2)$$

$$h_0(\vec{x}) \equiv \cos((+x+y+z)\pi/L),$$
$$h_1(\vec{x}) \equiv \cos((-x+y+z)\pi/L),$$
$$h_2(\vec{x}) \equiv \cos((+x-y+z)\pi/L),$$
$$h_3(\vec{x}) \equiv \cos((-x-y+z)\pi/L)$$

$$f_0(\vec{x}_1,\ldots,\vec{x}_6) \equiv h_0(\vec{x}_1)\ldots h_0(\vec{x}_6),$$
$$f_1(\vec{x}_1,\ldots,\vec{x}_6) \equiv h_1(\vec{x}_1)\ldots h_1(\vec{x}_6),$$
$$f_2(\vec{x}_1,\ldots,\vec{x}_6) \equiv h_2(\vec{x}_1)\ldots h_2(\vec{x}_6),$$
$$f_3(\vec{x}_1,\ldots,\vec{x}_6) \equiv h_3(\vec{x}_1)\ldots h_3(\vec{x}_6)$$

$$G^{(n)}_{\alpha,\beta;\alpha'\beta'}(\vec{x},\vec{y})$$

$$= \sum_{\vec{x}_1,\ldots,\vec{x}_6} \langle 0 | T\ N_\alpha(\vec{x},t) N_\beta(\vec{y},t)\ \bar{q}_{\alpha_1}(\vec{x}_1)\ldots\bar{q}_{\alpha_6}(\vec{x}_6) f^{(n)}(\vec{x}_1,\ldots,\vec{x}_6) | 0\rangle (C\gamma_5\gamma_0)_{\alpha_1\alpha_2} (C\gamma_5\gamma_0)_{\alpha_3,\alpha_4},$$

Projection of J (total angular mom.)

$$\sum_{\vec{x}_1,\ldots,\vec{x}_6} \langle 0 | T\ N_\alpha(\vec{x},t) N_\beta(\vec{y},t)\ \mathcal{P}^{(\Gamma)}\left[\bar{q}_{\alpha_1}(\vec{x}_1)\ldots\bar{q}_{\alpha_6}(\vec{x}_6) f^{(n)}(\vec{x}_1,\ldots,\vec{x}_6)\right] | 0\rangle (C\gamma_5\gamma_0)_{\alpha_1\alpha_2} (C\gamma_5\gamma_0)_{\alpha_3,\alpha_4},$$

$$\sum_{\vec{x}_1,\ldots,\vec{x}_6} \mathcal{P}^{(\Gamma)}\left[\bar{q}_{\alpha_1}(\vec{x}_1)\ldots\bar{q}_{\alpha_6}(\vec{x}_6) f^{(n)}(\vec{x}_1,\ldots,\vec{x}_6)\right] | 0\rangle$$

$$= \sum_{\vec{x}_1,\ldots,\vec{x}_6} \sum_{i=1}^{24} \chi^{(\Gamma)}(R_1)\left[SS\otimes S^t S^t\ \bar{q}_{\alpha_1}(R\ \vec{x}_1)\ldots\bar{q}_{\alpha_6}(R\ \vec{x}_6) f^{(n)}(\vec{x}_1,\ldots,\vec{x}_6)\right] | 0\rangle$$

$R\vec{x} \to \vec{x}$

$$= \sum_{\vec{x}_1,\ldots,\vec{x}_6} \sum_{i=1}^{24} \chi^{(\Gamma)}(R_1)\left[SS\otimes S^t S^t\ \bar{q}_{\alpha_1}(\vec{x}_1)\ldots\bar{q}_{\alpha_6}(\vec{x}_6) f^{(n)}(R^{-1}\vec{x}_1,\ldots,R^{-1}\vec{x}_6)\right] | 0\rangle$$

The representation of J we can obtain

(l)	$\otimes$	(spin=0)						J				
A_1	$\otimes$	A_1	$=$	A_1								
A_2	$\otimes$	A_1	$=$			A_2						
E	$\otimes$	A_1	$=$					E				
T_1	$\otimes$	A_1	$=$							T_1		
T_2	$\otimes$	A_1	$=$									T_2

(l)	$\otimes$	(spin=1)						J				
A_1	$\otimes$	T_1	$=$							T_1		
A_2	$\otimes$	T_1	$=$									T_2
E	$\otimes$	T_1	$=$							T_1	$\oplus$	T_2
T_1	$\otimes$	T_1	$=$	A_1	$\oplus$			E	$\oplus$	T_1	$\oplus$	T_2
T_2	$\otimes$	T_1	$=$		$\oplus$	A_2	$\oplus$	E	$\oplus$	T_1	$\oplus$	T_2

States we can obtain with S=1

S=1 J=A1

T1 x T1=A1

(spin) x (L)=(J)

$^3G_4$ P= + 1

					L
A1					S
			T1		P
		E		T2	D
	A2		T1	T2	F
A1		E	T1	T2	G
		E	T1	T2	H
A1	A2	E	T1	2T2	I

Orbital Angular momentum L

					J
A1					0
			T1		1
		E		T2	2
	A2		T1	T2	3
A1		E	T1	T2	4
		E	T1	T2	5
A1	A2	E	T1	2T2	6

Total Angular momentum J

S=1 J=A2

T1 x T2=A2

(spin) x (L)=(J)

$$^{3}D_{3} + ^{3}G_{3}$$

					L
A1					S
			T1		P
		E		T2	D
	A2		T1	T2	F
A1		E	T1	T2	G
		E	T1	T2	H
A1	A2	E	T1	2T2	I

					J
A1					0
			T1		1
		E		T2	2
	A2		T1	T2	3
A1		E	T1	T2	4
		E	T1	T2	5
A1	A2	E	T1	2T2	6

S=1 J=E

P= + 1

T1 x T1 =E

T1 x T2 =E

(spin) x (L)=(J)

$$^{3}D_{2} \qquad ^{3}G_{4}$$

					L
A1					S
			T1		P
		E		T2	D
	A2		T1	T2	F
A1		E	T1	T2	G
		E	T1	T2	H
A1	A2	E	T1	2T2	I

					J
A1					0
			T1		1
		E		T2	2
	A2		T1	T2	3
A1		E	T1	T2	4
		E	T1	T2	5
A1	A2	E	T1	2T2	6

S=1 J=T1

P= + 1

T1 x A1 =T1

T1 x T1 =T1

T1 x T2 =T1

T1 x E =T1

$$^{3}D_{3} + ^{3}G_{3} \qquad ^{3}S_{1} + ^{3}D_{1}$$

					L
A1					S
			T1		P
		E		T2	D
	A2		T1	T2	F
A1		E	T1	T2	G
		E	T1	T2	H
A1	A2	E	T1	2T2	I

					J
A1					0
			T1		1
		E		T2	2
	A2		T1	T2	3
A1		E	T1	T2	4
		E	T1	T2	5
A1	A2	E	T1	2T2	6

S=1 J=T2

P= + 1

T1 x A2 =T2

T1 x T1 =T2

T1 x T2 =T2

T1 x E =T2

$$^{3}D_{3} + ^{3}G_{3} \qquad ^{3}D_{2}$$

					L
A1					S
			T1		P
		E		T2	D
	A2		T1	T2	F
A1		E	T1	T2	G
		E	T1	T2	H
A1	A2	E	T1	2T2	I

					J
A1					0
			T1		1
		E		T2	2
	A2		T1	T2	3
A1		E	T1	T2	4
		E	T1	T2	5
A1	A2	E	T1	2T2	6