Thermodynamics of SU(N) gauge theories in 2+1 dim in the T<Tc regime

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Plan of the talk

- Introduction
- The integral method
- SU(N) gauge theories in 2+1 dimensions, scaling properties with N
- Description in terms of a gas of Glueballs + String
- Determine the thermodynamical variables
- Conclusions

Introduction

- In this work we show results for the computation of the trace of the energy-momentum tensor in 2+1 dimensions for the groups (SU(N=2,...,6)) basically below the critical temperature (confined phase region) and discuss scaling properties with N.
- To better visualize the kind of results we obtain we then compare them with some possible elementary predictions (gas of free glueballs, and the bosonic string prediction).

Features (I)

The thermodynamics of a field theory is derived from the partition function,

$$T = 1/(aN_t)$$

$$V = (aN_s)^2$$

$$Z(T,V,g^2) = \int \mathcal{D}A_\mu(x) e^{-\int_0^{1/T} dx_0 \int_V d^2x \mathcal{L}(A_\mu(x))}$$

the free energy is

$$F(T,V,g^2) = -T \log Z$$

and the other thermodynamical quantities as pressure, internal energy density and entropy density

$$p(T,g^2) = -f(T,g^2) = -\frac{T}{V}\log Z$$

$$\varepsilon(T,g^2) = T^2 \frac{\partial}{\partial T}\left(\frac{p}{T}\right)$$

$$s(T,g^2) = \frac{\partial p}{\partial T} = \frac{\varepsilon + p}{T}$$

$$\frac{p}{T^3}, \frac{\varepsilon}{T^3}, \frac{s}{T^2}$$

T is the only scale...

Features (II)

In 2+1 dim the trace of the energy momentum (EM) tensor is $\frac{\varepsilon - 2p}{T^3} = T\frac{\partial}{\partial T}\left(\frac{p}{T^3}\right)$

We want to determine these quantities from our $N_t \times N_s^2$

lattice with lattice spacing a and Nt point in the (inverse) temperature direction. Define the lattice action

$$S_W(U_\mu(x)) = \beta \sum_P \left(1 - \frac{1}{3}\mathrm{Re}\mathrm{Tr}\, U_P\right) \qquad U_\mu(x), \mu = 0, 1, 2$$

$$Z = \int \prod_{x,\mu} dU_\mu(x)\, e^{-S_W(U_\mu(x))}$$

P denotes one of the $3N_\tau \times N_S^2$ plaquettes

The partition function can not be directly calculated from

The integral method (I)

Monte Carlo methods. We use the integral approach

J. Engels, J. Fingberg, F. Karsch, D. Miller and M. Weber, Phys. Lett. B252(1990)625.

G. Boyd, J. Engels, F. Karsch, E. Laermann, C. Legeland, M. Luetgemeier and B. Petersson, Nucl. Phys. B469(1996)419, [hep-lat/9602007].

we get

$$\frac{d \log Z}{d\beta} = -N_t N_s^2 \langle P_s + 2P_t \rangle$$

at fixed Nt and Ns. The <Ps> and <Pt> are the expectation values of the spatial and temporal plaquettes. This eq. determines p up to a constant with respect to beta so that for some beta0 to be chosen below

$$a^3\, p(\beta, N_\tau, N_S) = -\int_{\beta^0}^{\beta} d\beta' \, \langle P_S + 2P_\tau \rangle_{\beta'} + a^3\, p(\beta^0, N_\tau, N_S).$$

The integral method (II)

where $$\frac{p(\beta, N_\tau, N_S)}{T^3} = N_\tau^3 \int_{\beta^0}^{\beta} d\beta' \Delta S(\beta', N_\tau, N_S),$$

$$\Delta S(\beta, N_\tau, N_S) = 3 \langle P_0 \rangle_\beta - \langle P_S + 2P_\tau \rangle_\beta \,.$$

and <P0> is the expectation value of the plaquette for T=0.

In this work we are interested in the trace of the EM

tensor $$\frac{\varepsilon - 2p}{T^3} = T \frac{\partial}{\partial T} \left(\frac{p}{T^3} \right) = N_t^3 \Delta S(\beta(\frac{T}{T_c}), N_t, N_s) T \frac{d\beta}{dT}$$

where dbeta/dT is estimated through a parametric fit.

Scaling properties with N

It is interesting to study the large N limit. Some dimensionless ratios are constant for large N:

$$\frac{m_0}{\sqrt{\sigma}} = 4.108(20) + \frac{d}{N^2}$$

$$\frac{T_c}{\sqrt{\sigma}} = 0.903(3) + \frac{0.88}{N^2}$$

M. Teper, arXiv:0912.3339 [hep-lat].

B. Lucini and M. Teper, Phys. Rev. D **66** (2002) 097502 [arXiv:hep-lat/0206027].

Moreover

$$\beta = \frac{2N}{ag^2}$$

$$\frac{\sqrt{\sigma}}{g^2 N} = 0.1975 - \frac{0.12}{N^2}$$

$$\sqrt{\sigma} = \frac{0.395 N^2}{a\beta} - \frac{0.24}{a\beta}$$

Now we get the dependence of beta in terms of T:

scaling with N (II)

as $$\frac{T}{T_c} = \frac{T}{\sqrt{\sigma}}\frac{\sqrt{\sigma}}{T_c} = T\frac{a\beta}{(0.395N^2 - 0.24)(0.903 + \frac{0.88}{N^2})}$$

which gives $$\beta = N_\tau \frac{T}{T_c}(0.357N^2 + 0.13 - 0.211/N^2)$$

for N=3 gives 0.34 which coincides with Bialas et al.

P. Bialas, L. Daniel, A. Morel and B. Petersson, Nucl. Phys. B **807** (2009) 547 [arXiv:0807.0855 [hep-lat]].

$$\frac{\beta}{N_\tau} = 3.3\frac{T}{T_c} + \frac{1.5}{N_\tau}.$$

Using the data from J. Liddle and M. Teper, arXiv:0803.2128 [hep-lat].

we get the correction in the limit of large N:

$$\beta = N_\tau \frac{T}{T_c}(0.357N^2 + 0.13 - 0.211/N^2) + (0.22N^2 - 0.5)$$

scaling with N (III)

one can check the validity of the previous formula by plotting the quantity

$$T\frac{\partial\beta}{\partial T}N_\tau^3\Delta S \quad \text{vs} \quad \frac{T}{T_c}$$

t=T/Tc

The result (the trace of the EM-tensor plotted on the scale of critical temperature) it is expected not to be dependent on N and Nt.

Simulations details

Mostly use CHROMA package to measure plaquettes averages

] R. G. Edwards and B. Joo [SciDAC Collaboration and LHPC Collaboration and UKQCD Collaboration], Nucl. Phys. Proc. Suppl. **140** (2005) 832 [arXiv:hep-lat/0409003].

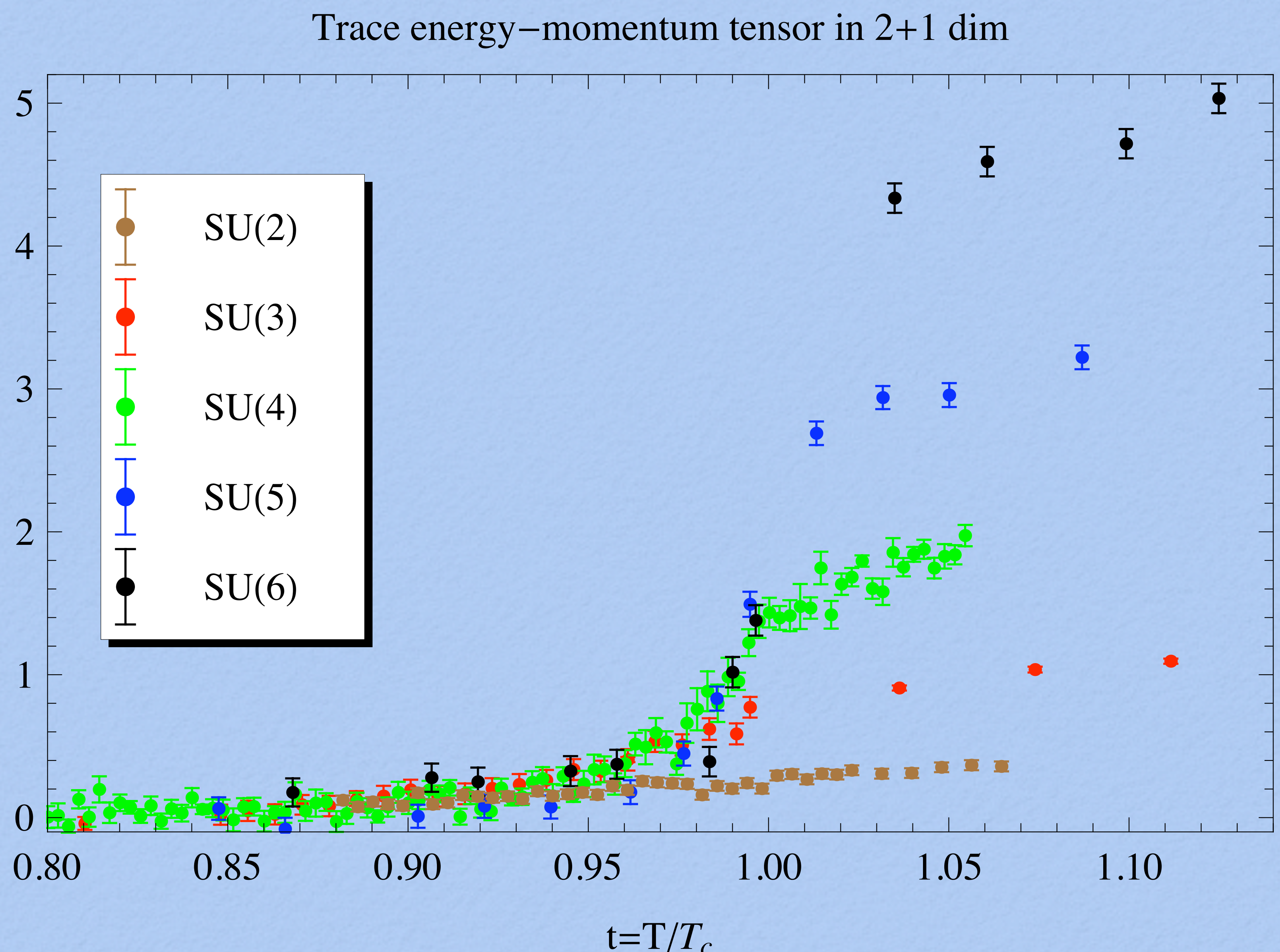
+ own programs (only for SU(2) and SU(4)).

To determine the trace of the EM tensor we compute the averages of spatial and temporal plaquettes in finite lattices with a temporal extent of Nt=6,8 and spatial

volumes Ns^2 such as the aspect ratio Ns/Nt =8. We also

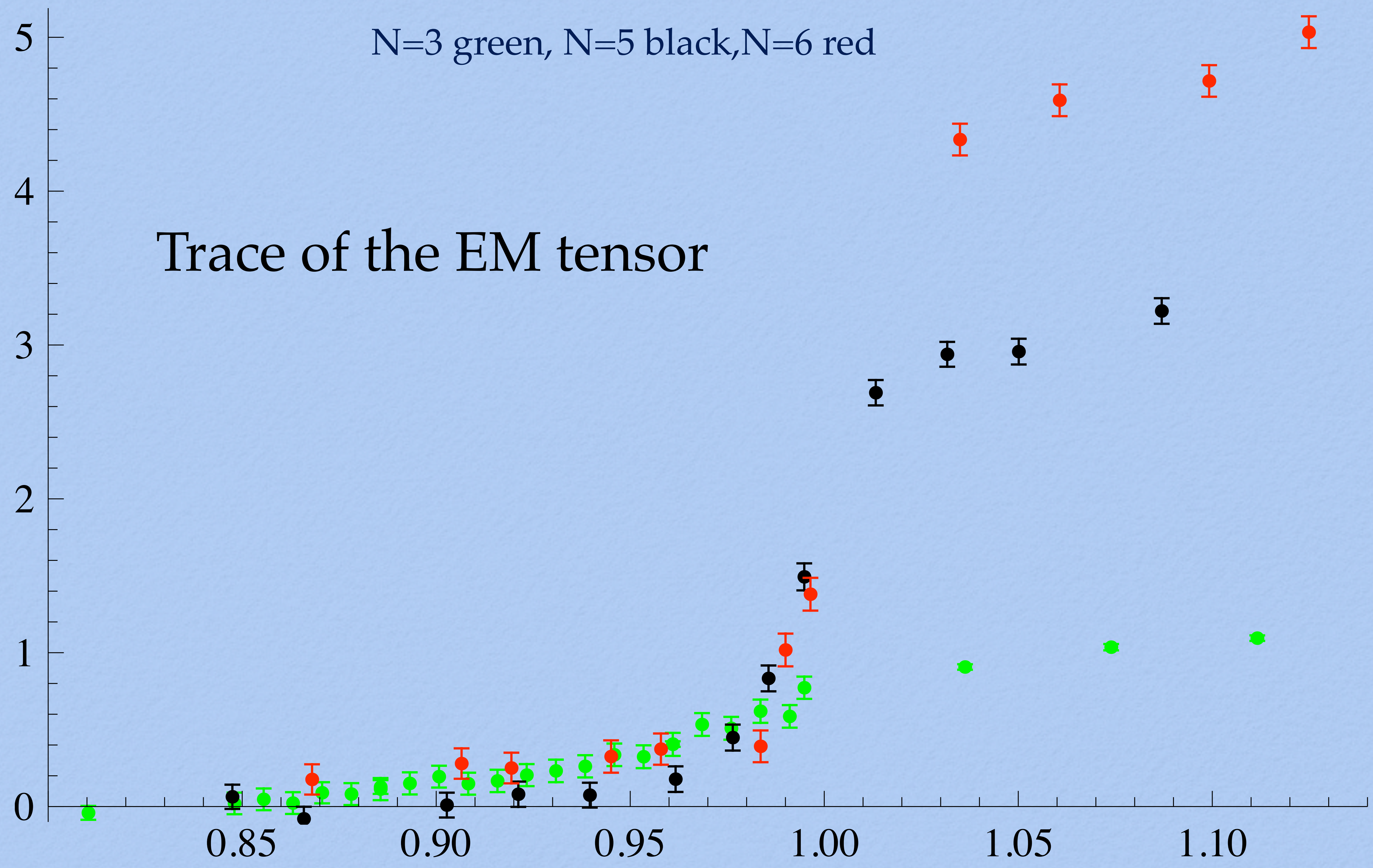
need the value of the plaquette at T=0, <P0> on symmetric lattices Nt = Ns.

scaling Trace of EM tensor

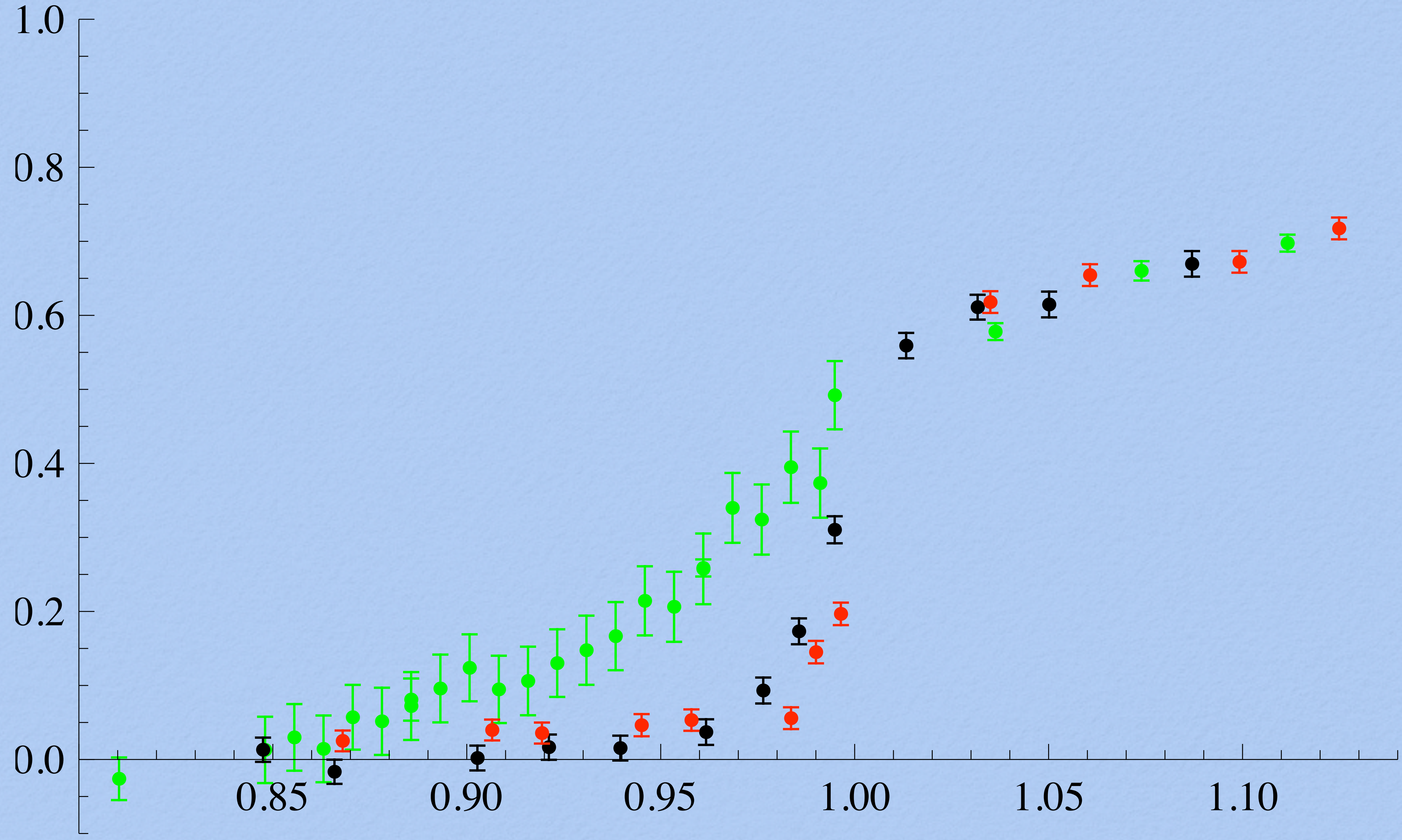


Above Tc the different curves split up and they appear to be ordered according to the high temperature scaling law for the value of N

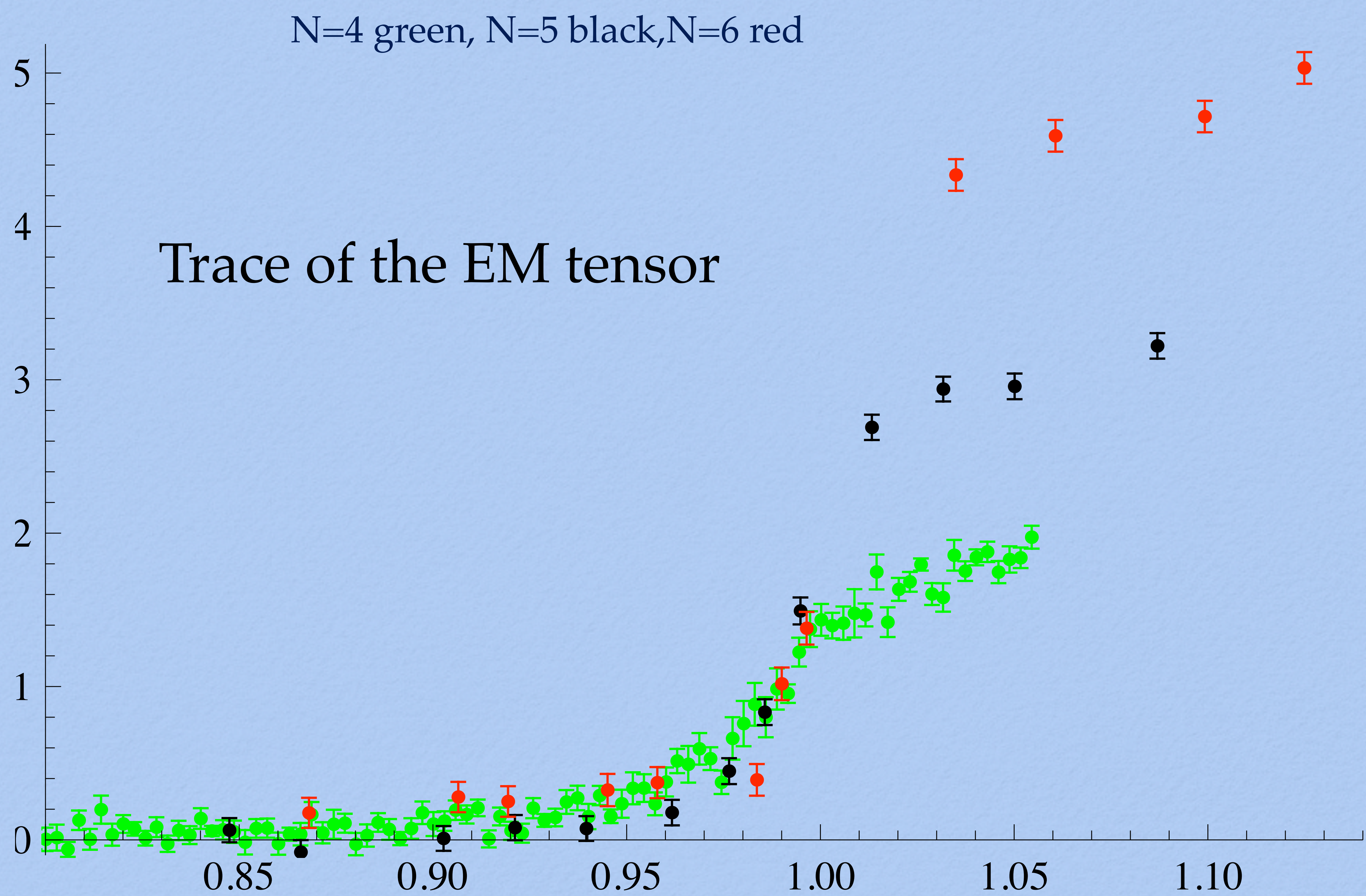
Data for SU(N=3,5,6)



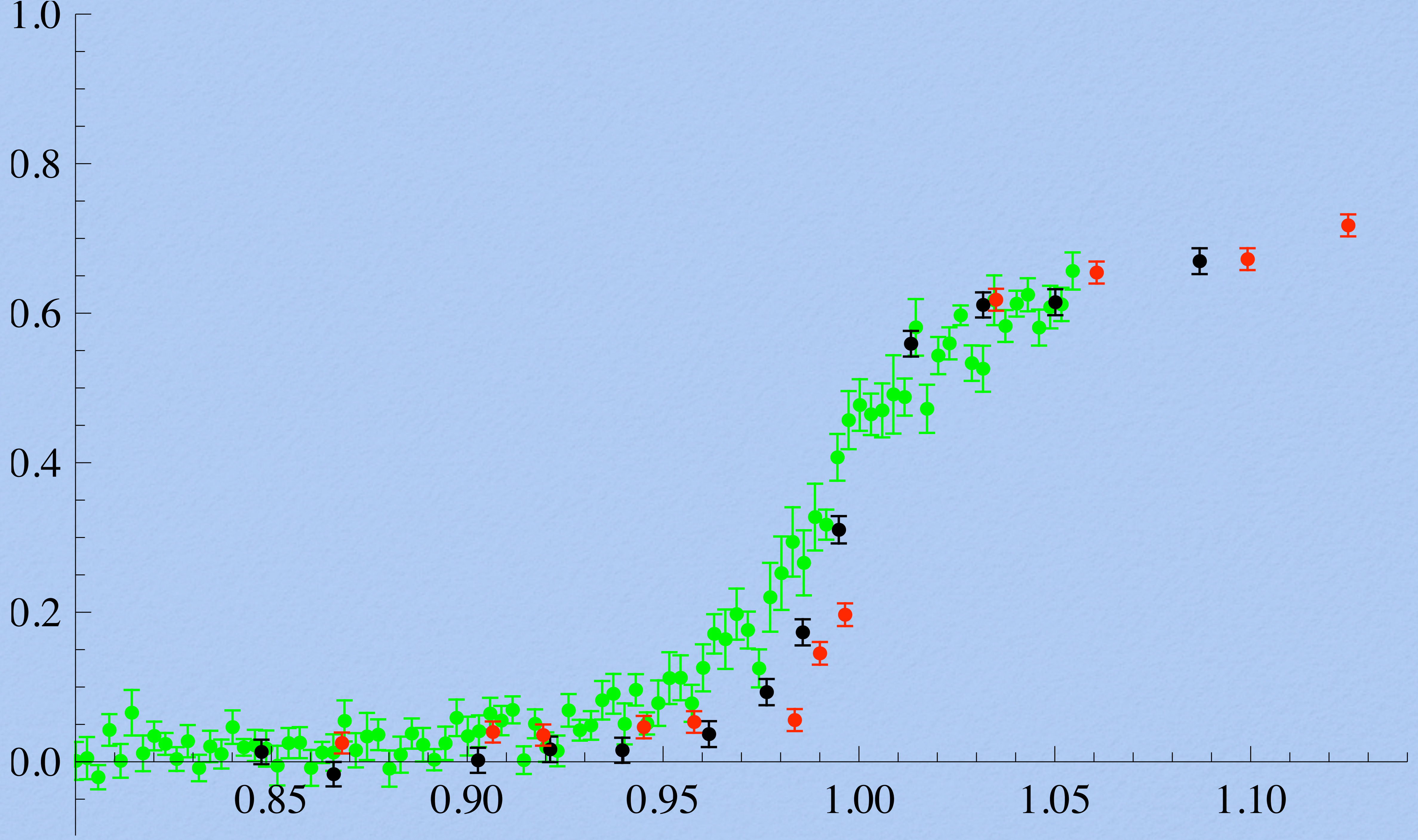
Stefan-Boltzmann norm. SU(N=3,5,6)



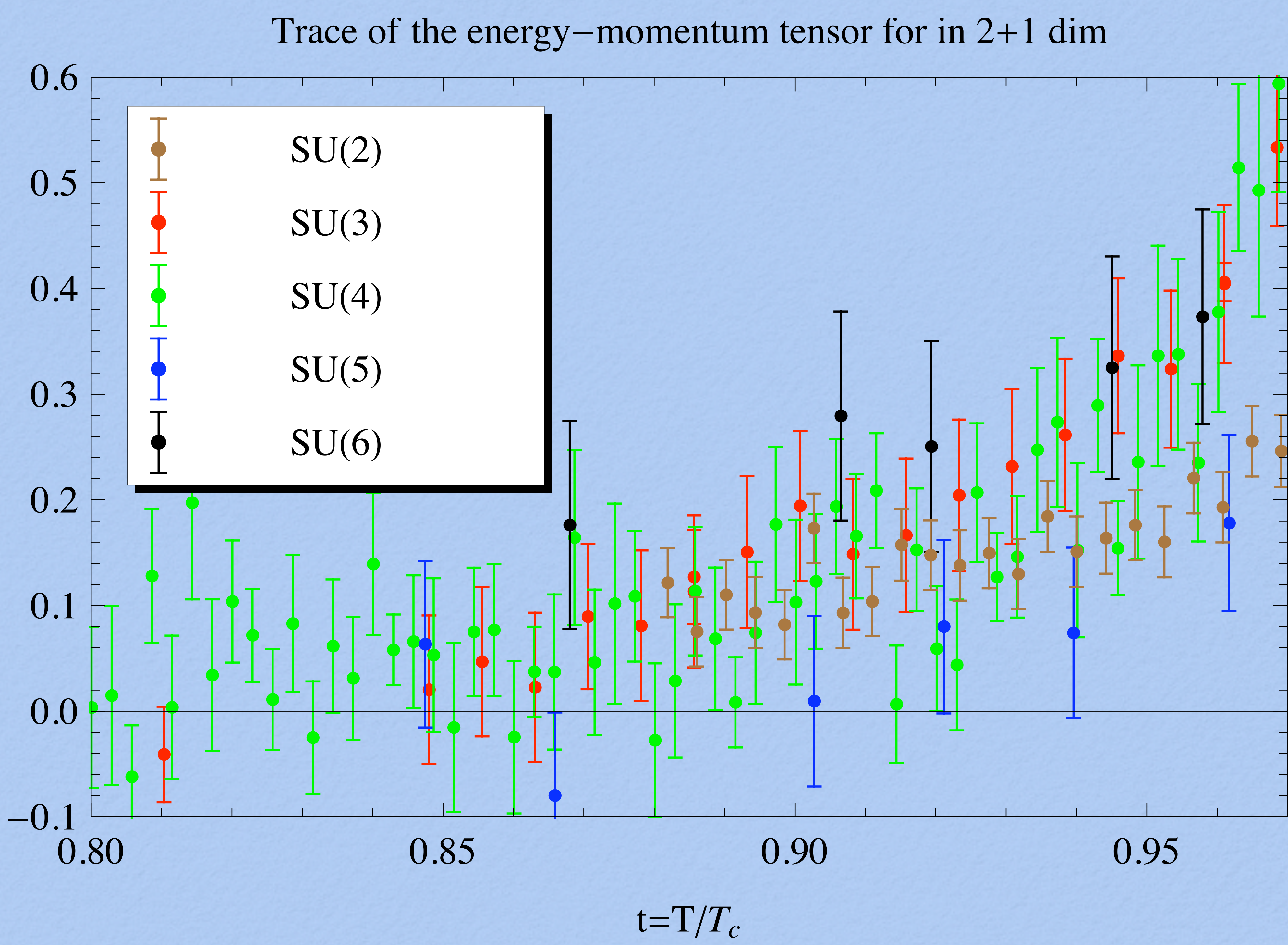
Data for SU(N=4,5,6)



SB norm. SU(N=4,5,6)



Magnified view ...



Gas of Glueballs

We compare our data with the predictions of a glueball gas of non-interactive particles, where we include all the known glueball states below the two-particles threshold.

We use the data for the masses from

M. J. Teper, Phys. Rev. D **59** (1999) 014512 [arXiv:hep-lat/9804008].

to determine $N_t^3 \Delta S$

in terms of the dimensionless quantity m/T:

$$\frac{m}{T} = \frac{m}{\sqrt{\sigma}} \frac{\sqrt{\sigma}}{T} = \frac{m}{\sqrt{\sigma}} (N_t \sqrt{\sigma})$$

Gas of Glueballs (II)

then $$\sqrt{\sigma} = \frac{3.367(50)}{\beta} + \frac{4.1(1.7)}{\beta^2} + \frac{46.5(11.0)}{\beta^3} \qquad \frac{m}{\sqrt{\sigma}} = 4.329(41).$$

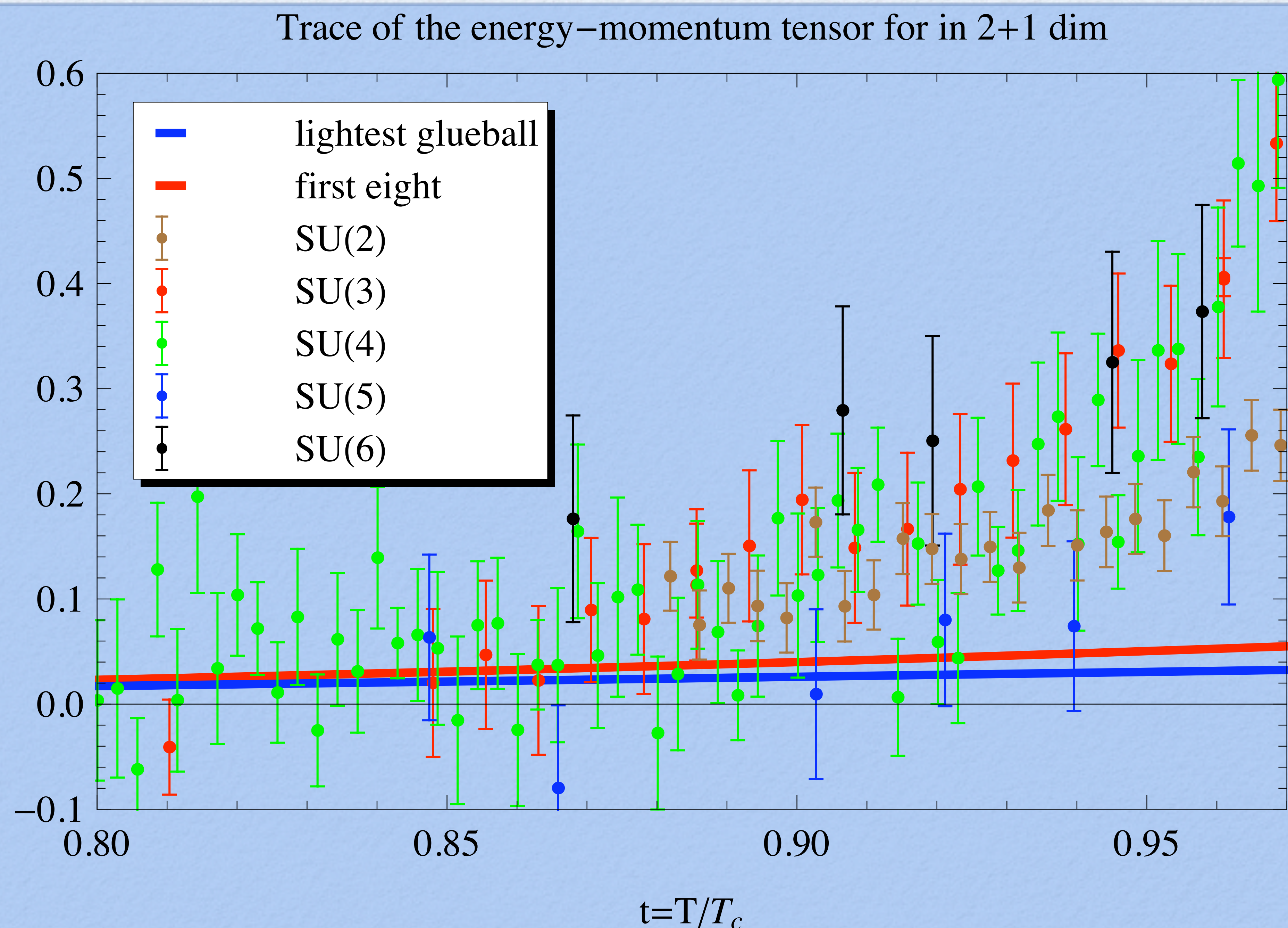
(taking the first mass). We try the functional form:

$$N_t^3 \Delta S = \frac{x^2}{2\pi\beta} \sum \frac{1}{k} e^{-kx}$$

where $$x(\beta) = \frac{m}{T} = 8 * 4.329 * \left(\frac{3.367}{\beta} + \frac{4.1)}{\beta^2} + \frac{46.5)}{\beta^3} \right)$$

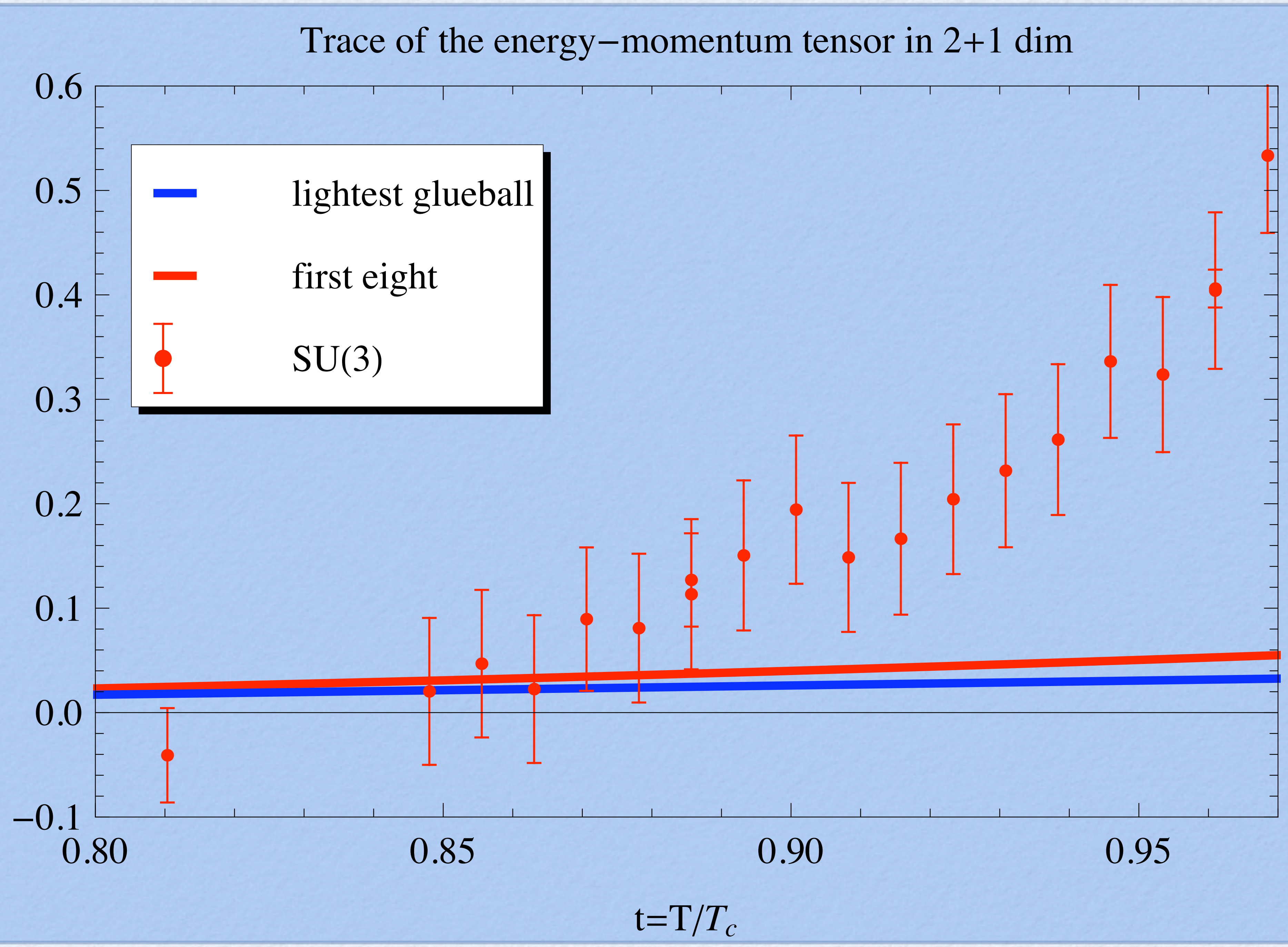
or the eight first masses.

Magnified view

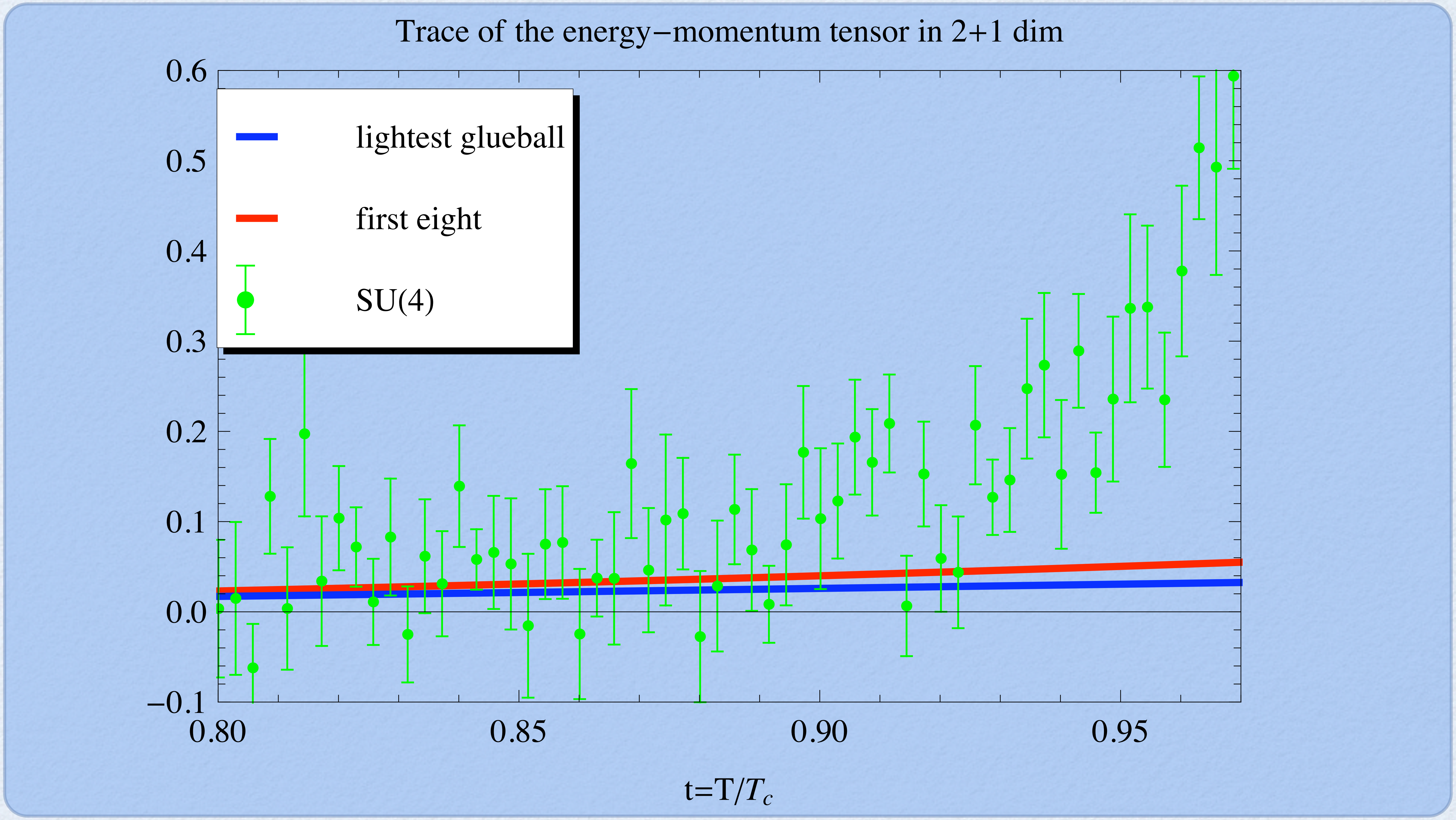


This fits fail to reproduce the data and suggest the necessity of taking into account the full spectrum of glueballs (not only the lowest one)

SU(3)


Trace of the energy−momentum tensor in 2+1 dim
0.6
0.5
0.4
0.3
0.2
0.1
0.0
−0.1
0.80
0.85
0.90
0.95
t=T/T_c
lightest glueball
first eight
SU(3)

SU(4)



Trace of the energy−momentum tensor in 2+1 dim
0.6
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0.3
0.2
0.1
0.0
−0.1
0.80
0.85
0.90
0.95
lightest glueball
first eight
SU(4)
t=T/T_c

Comparison with string

H. B. Meyer, arXiv:0905.4229 [hep-lat].

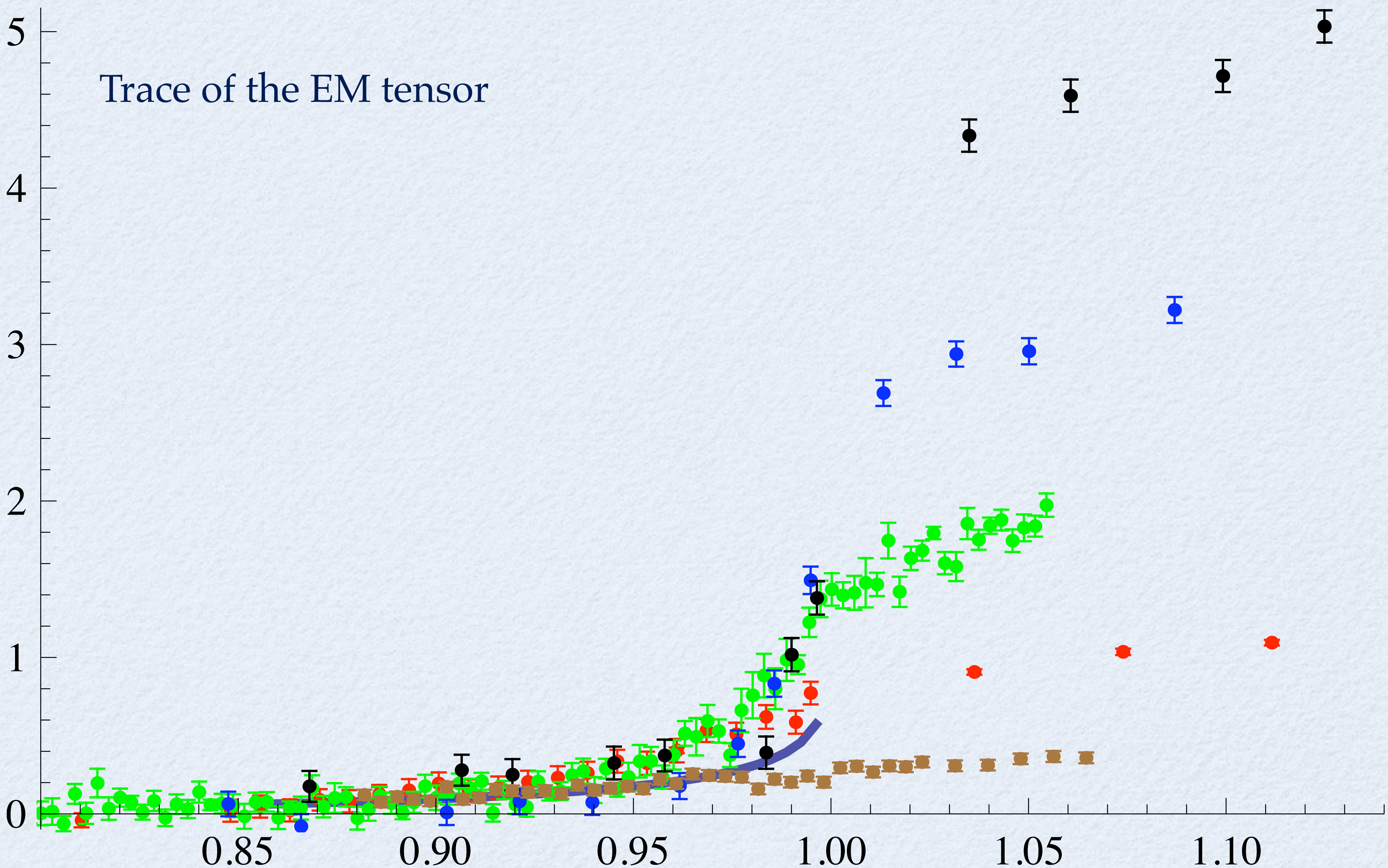
We compared our result with the string prediction for the density of glueballs

Extended to arbitrary dimensions the computation of the density of state of a closed string in D=4 (following [*])

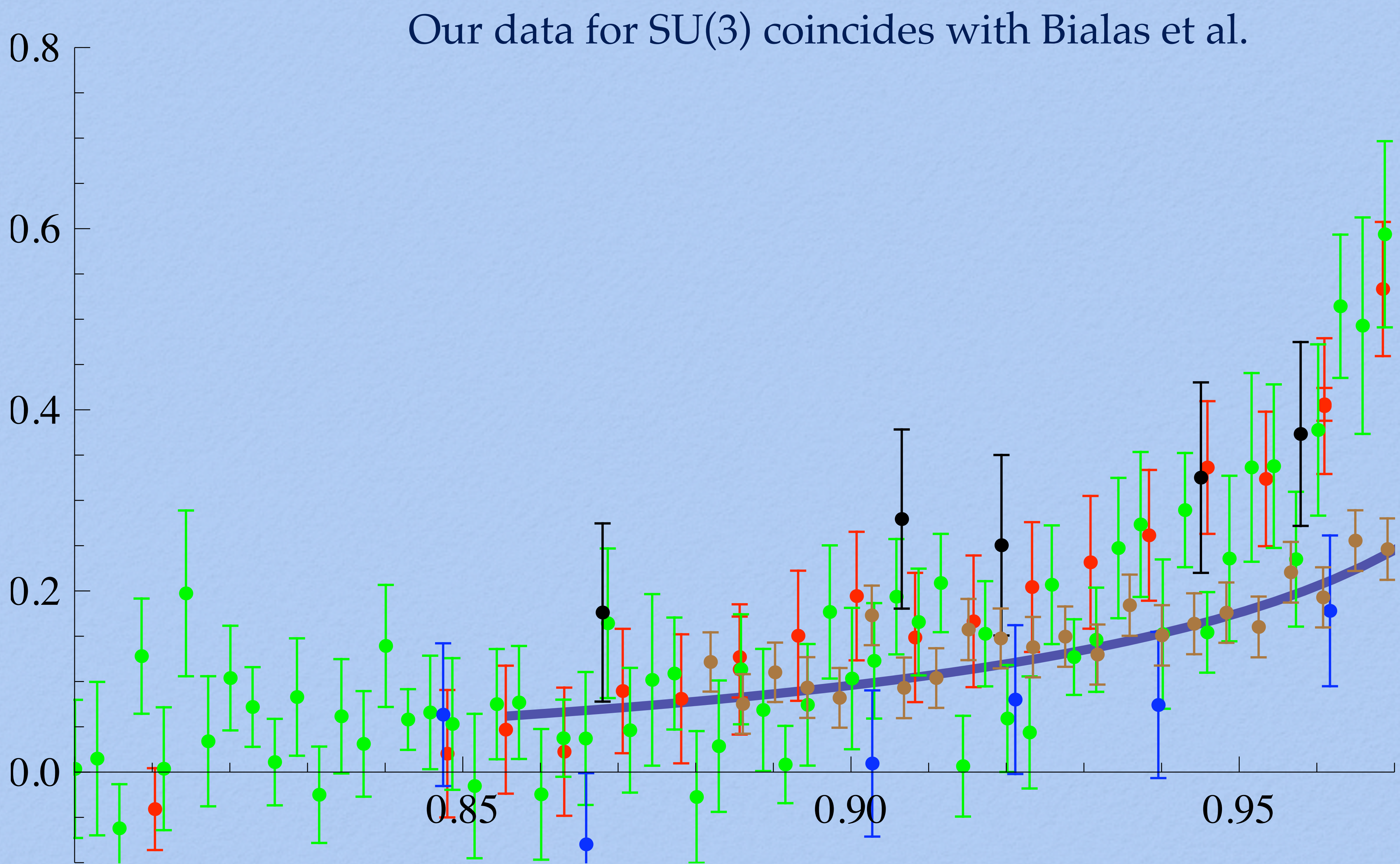
$$\tilde{\rho}_{D-2}(M) = \left(\frac{\pi}{3}\right)^{D-1} \frac{1}{T_H} (D-2)^{\frac{D}{2}-1} \left(\frac{T_H}{M}\right)^{D} e^{M/T_H}$$

[*] B. Zwiebach (2004), A First Course in String Theory, Cambridge, UK: Univ. Pr. 558 p.

Comparison with string (II)



Magnified view...



Fit for the trace EM

We try a functional fit for the trace of the EM tensor in the low temperature region (t < 1):

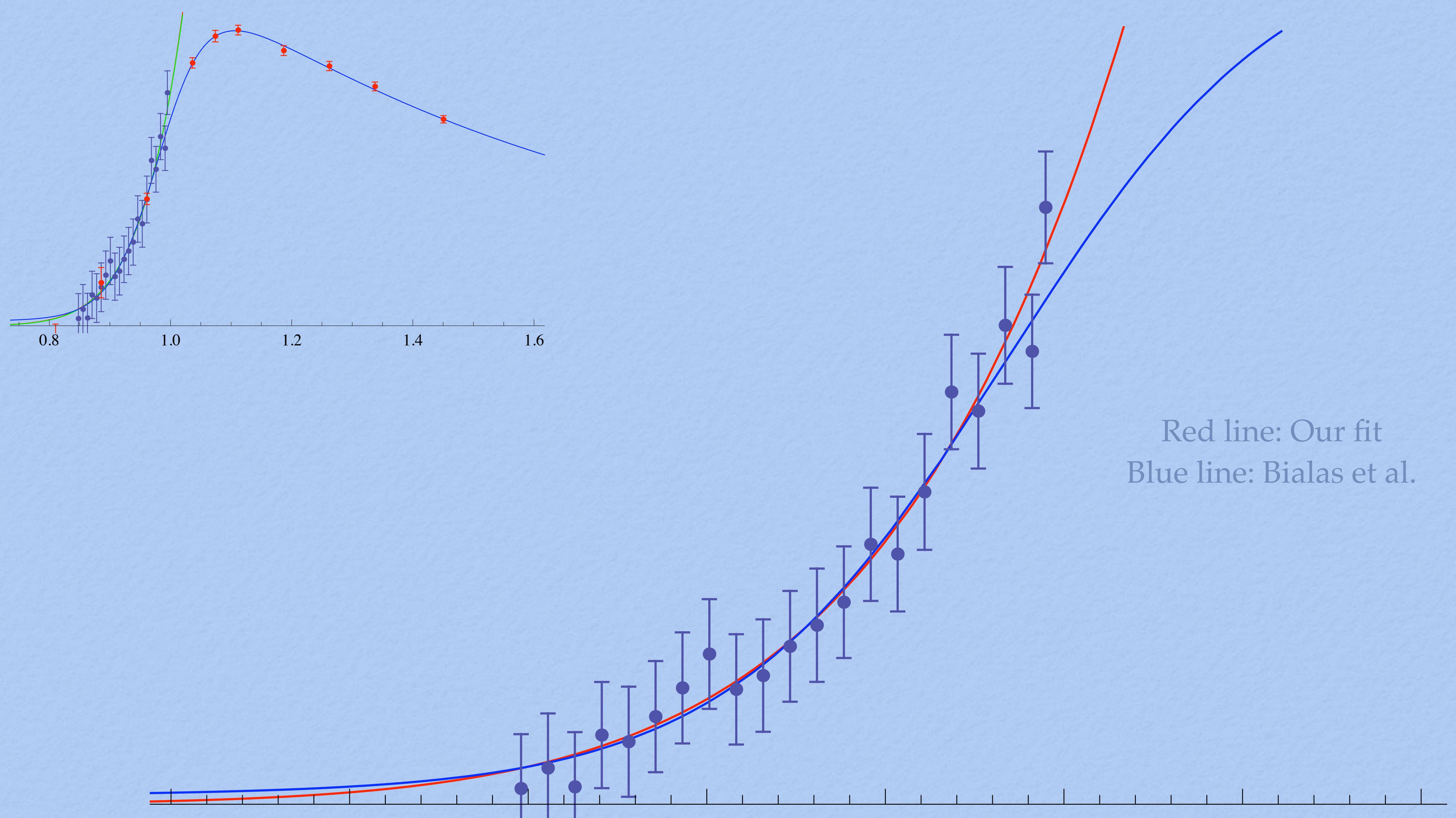
$$e^{p_A - p_C(1-t)/t}$$

to be compared with the fit of Bialas et al. where the interpolating function is to be valid for the whole range of t.

$N_\tau^3 \, \Delta S(\beta(t))$

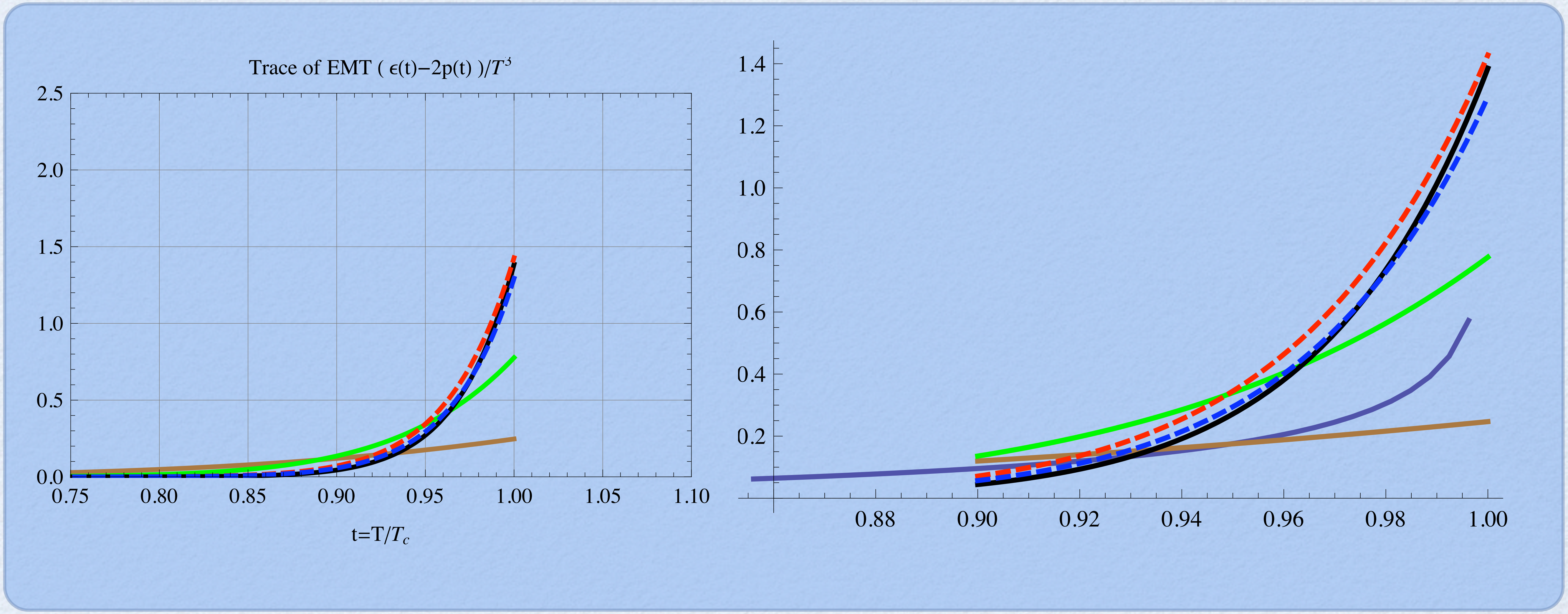
$$f(t) = \frac{a_1 t^2}{1 + a_2 t^4} \frac{1 + a_3\, e^{a_5(t-1)}}{1 + a_4\, e^{a_5(t-1)}}.$$

best Fit (for SU(3))

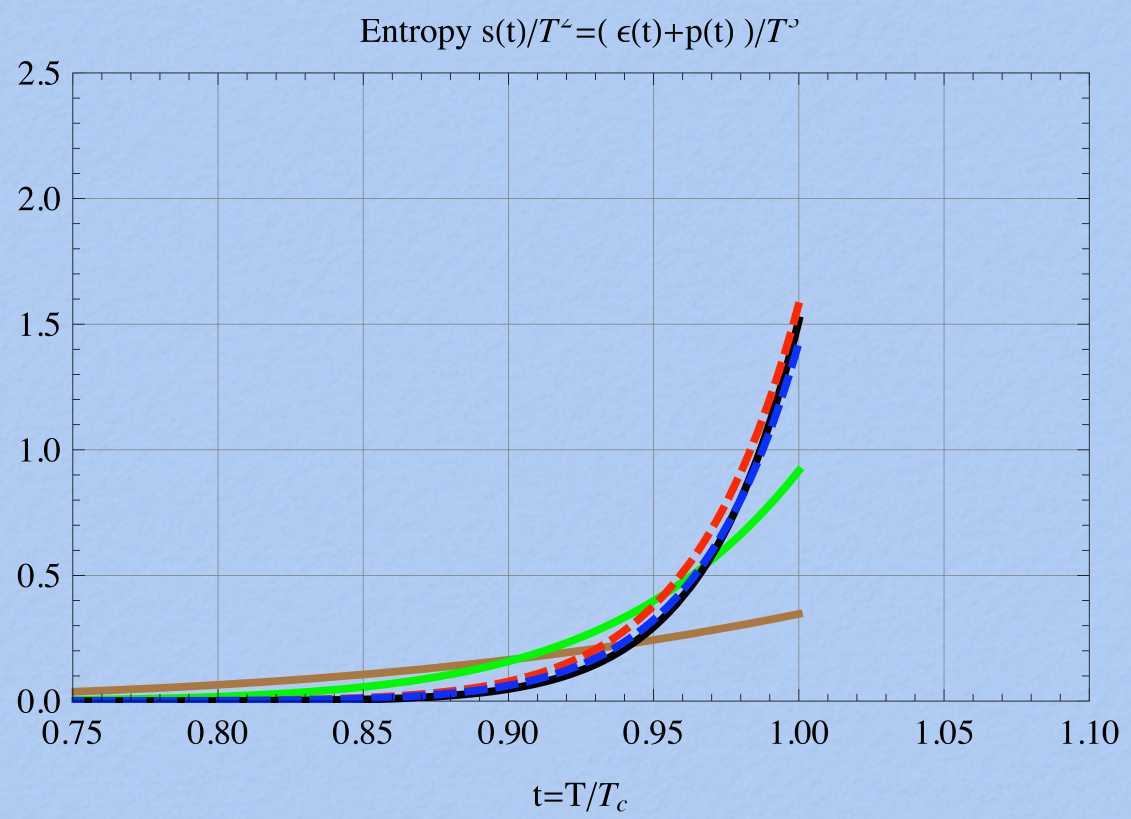


Thermodynamics

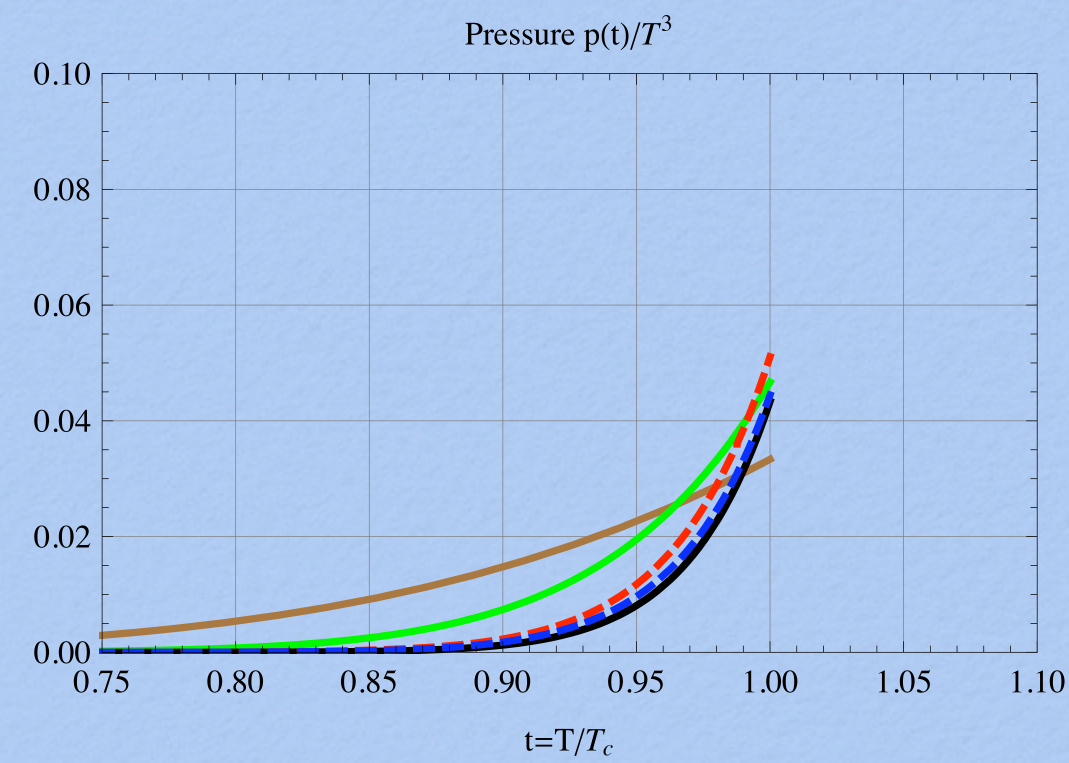
We use the fit to determine the thermodynamics variables :trace of the EM tensor, pressure, Energy density, entropy density...



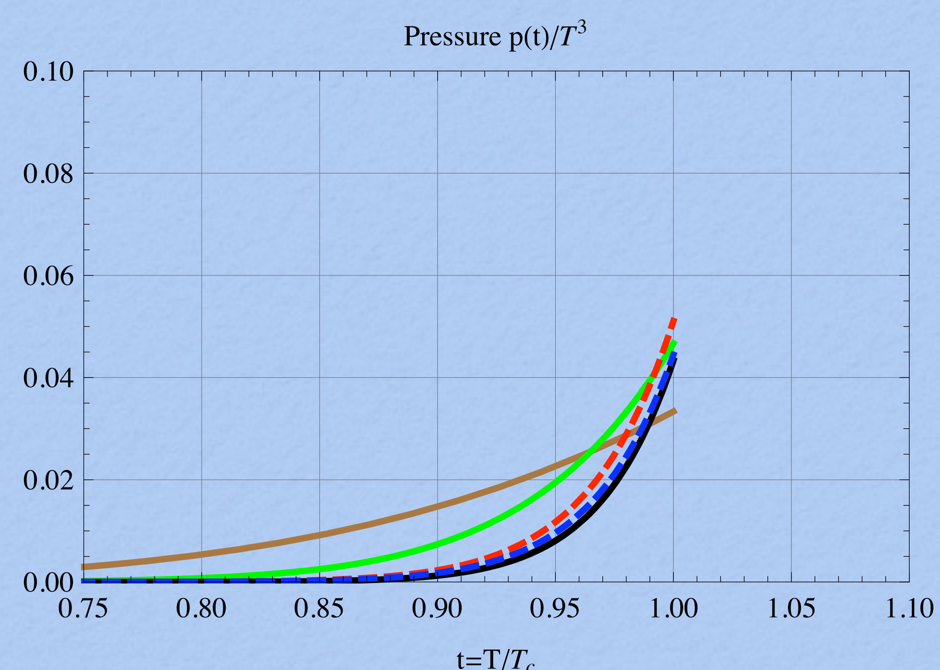
and...



Entropy s(t)/T^2=(ϵ(t)+p(t))/T^3
2.5
2.0
1.5
1.0
0.5
0.0
0.75
0.80
0.85
0.90
0.95
1.00
1.05
1.10
t=T/T_c



Pressure p(t)/T^3
0.10
0.08
0.06
0.04
0.02
0.00
0.75
0.80
0.85
0.90
0.95
1.00
1.05
1.10
t=T/T_c



Pressure p(t)/T^3
0.10
0.08
0.06
0.04
0.02
0.00
0.75
0.80
0.85
0.90
0.95
1.00
1.05
1.10
t=T/T_c

Conclusions

In this work we determine the thermodynamical quantities for SU(N=2,...,6) gauge theories in two spatial dimensions through lattice simulations.

We present results for the computation of the trace of the energy-momentum tensor for (SU(N=2,...,6)) below Tc and show good scaling properties with N.

The description in terms of a gas of Glueballs does not work in reproduce the corresponding data.