The curvature of the chiral QCD phase transition line in a finite volume

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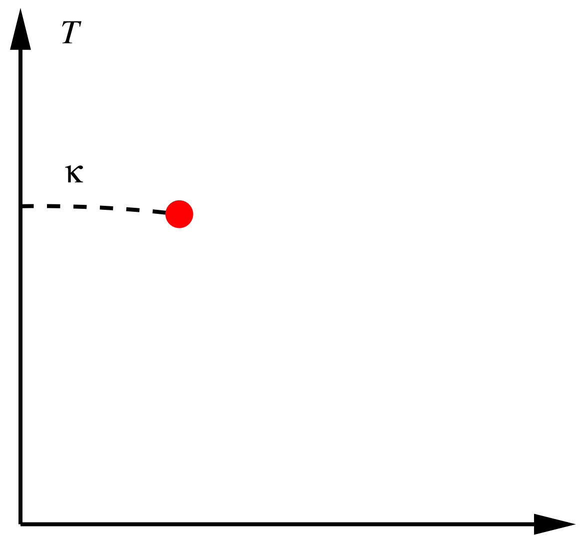
QCD phase diagram

...with only those parts
that I need today...



- second-order phase transition for two flavors in the chiral limit
- crossover at finite quark masses for finite temperature at $\mu = 0$
- expectation: first-order phase transition expected for large chemical potential
- expectation: critical end point of first-order line

Curvature of the phase transition line

Definition of the curvature:

$$\frac{T_\chi(L,m_\pi^*,\mu)}{T_\chi(L,m_\pi^*,\mu\!=\!0)} = 1-\kappa\left(\frac{\mu}{(\pi T_\chi(L,m_\pi^*,0))}\right)^2+\ldots$$

- calculable on the lattice (from μ = 0)
- discretization and volume errors?
- calculable from functional Renormalization Group (FRG) methods

Some results from lattice QCD and functional RG calculations

		N_f	am_c	κ
FRG (condensate)	[1]	1	0	1.13(15)
FRG (critical coupling)	[1]	1	0	0.44(4)
lattice, imaginary μ	[2]	2	0.032	0.500(54)
lattice, imaginary μ	[3]	3	0.026	0.667(6)
lattice, Taylor reweighting	[4]	3	0.005	1.13(45)

[1] J. Braun, Eur. Phys. J. C64, 459 (2009); arXiv:0810.1727 [hep-ph].

[2] P. de Forcrand and O. Philipsen, Nucl. Phys. B642, 290 (2002), hep-lat/0205016. [8^3 x 4]

[3] P. de Forcrand and O. Philipsen, JHEP 01, 077 (2007), hep-lat/0607017. [8^3 x 4]

[4] F. Karsch et al., Nucl. Phys. Proc. Suppl. 129, 614 (2004), hep-lat/0309116. [12^3 x 4, 16^3 x 4]

[Christian Schmidt: talk yesterday, Rosella Falcone: talk later today, Shinji Ejiri: poster]

- Is the result influenced by a volume effect?

Why do we expect finite-volume effects?

- chemical potential: affects quarks (obviously)
- curvature depends on sensitivity of quarks on the chemical potential
- sensitivity of quarks in turn depends on their “constituent quark mass”
- constituent quark mass affected by volume!

Quark-meson model for 2 flavors

- Model for chiral symmetry breaking with 2 quark flavors
- chiral symmetry SU(2) × SU(2) → SU(2) (quark sector) as O(4) → O(3) (meson sector)
- no gauge degrees of freedom

$$\Gamma_\Lambda[\bar{q}, q, \sigma, \vec{\pi}] = \int d^4x \;\; \bar{q}(i\partial\!\!\!/)q + g\bar{q}(\sigma + i\gamma_5\vec{\tau}\cdot\vec{\pi})q$$
$$\frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}(\partial_\mu\vec{\pi})^2 + U_\Lambda(\sigma, \sigma^2 + \vec{\pi}^2)$$

- specify effective action for the model at initial scale Λ
- use functional Renormalization Group (Wetterich equation) to obtain effective action, including fluctuations
[C.Wetterich, Phys. Lett. B 301 (1993) 90.]

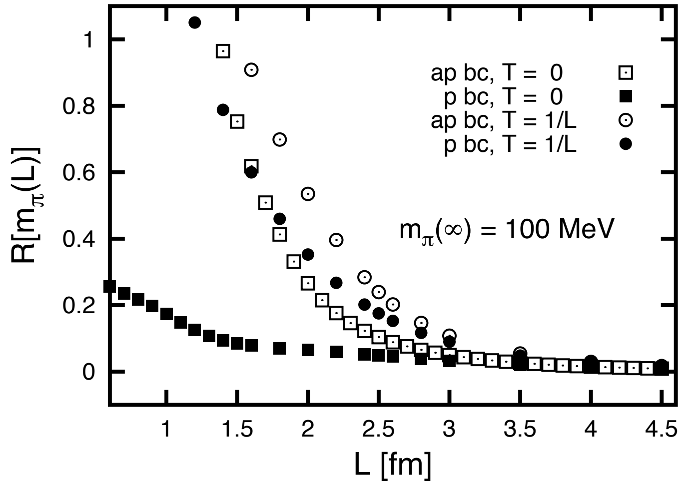
Quark-meson model results in finite volume

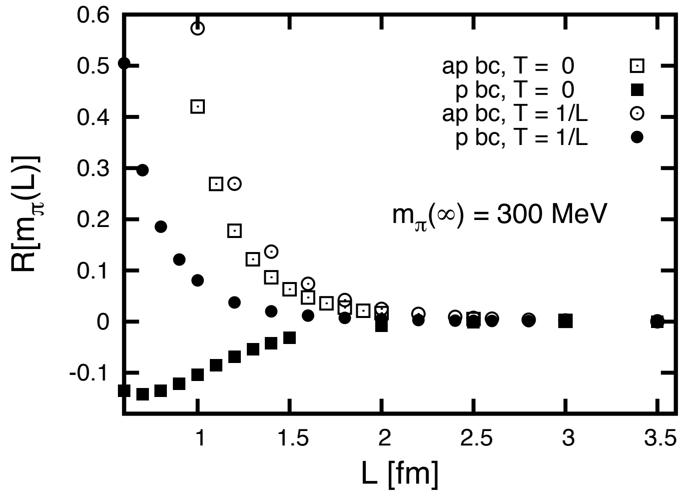
$$m_q \sim \langle \bar{\psi}\psi \rangle$$

$$f_\pi \sim \langle \bar{\psi}\psi \rangle$$

$$m_\pi^2 \sim m_c \frac{\langle \bar{\psi}\psi \rangle}{f_\pi^2} \sim \frac{m_c}{\langle \bar{\psi}\psi \rangle}$$

- condensate vanishes in small volume
- for periodic boundary conditions, it *increases* in intermediate volumes!
- pion mass decreases in intermediate volume

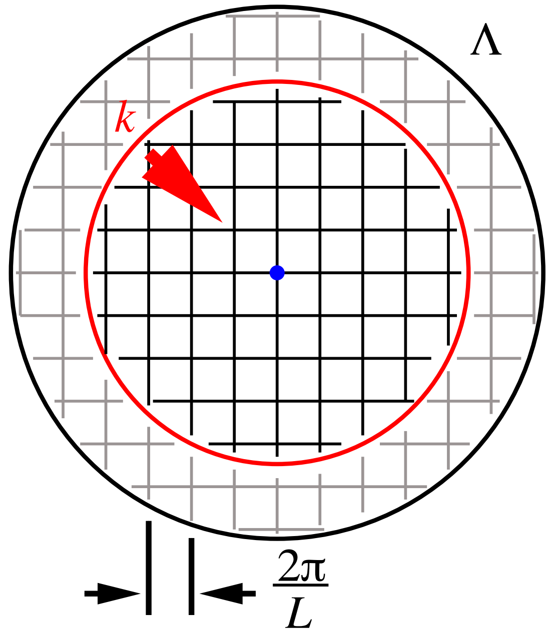




[J. Braun, B. Klein, H.-J. Pirner, Phys. Rev. D72, 034017 (2005).]

Fermionic mode contributions for a finite volume

- fermionic momentum modes contributing to the condensate (and the constituent quark mass) in a large finite volume



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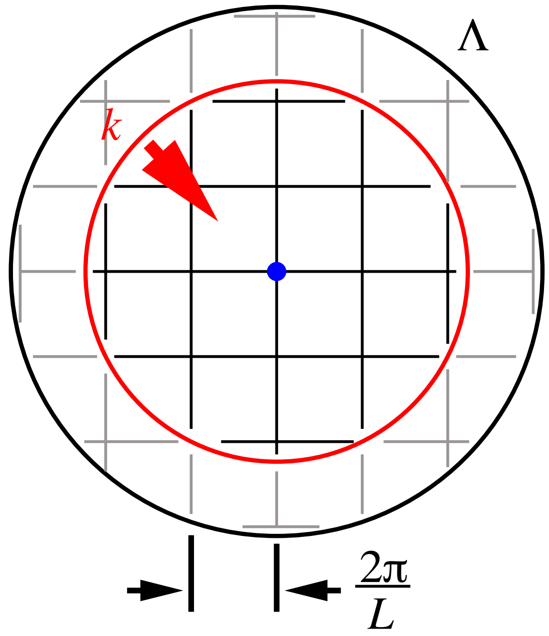


Curvature in infinite volume

m_π [MeV]	100	150	200	250	300
$\kappa(L \to \infty)$	1.391	1.392	1.440	1.463	1.500
$T_\chi(L \to \infty)$	178.1				

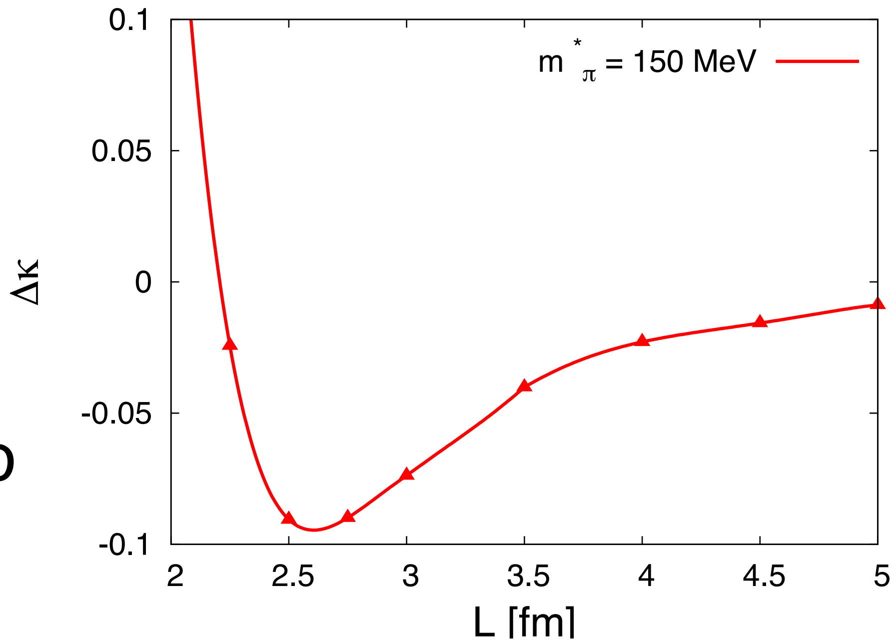
- model: curvature increases with pion mass ($T_c(m_\pi)$!)
- QCD: curvature decreases with pion mass

General observation: NJL-type models for chiral symmetry breaking tend to be more sensitive to the pion mass than QCD

Change in curvature as a function of volume

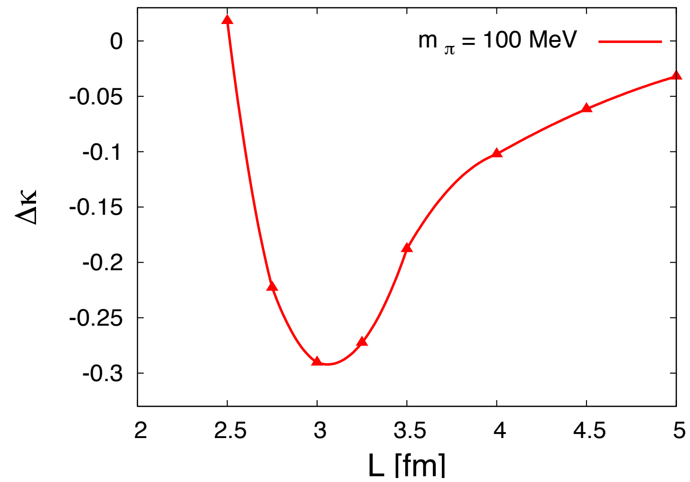
$$\Delta\kappa = \frac{\kappa(L, m_\pi^*) - \kappa(\infty, m_\pi^*)}{\kappa(\infty, m_\pi^*)}$$

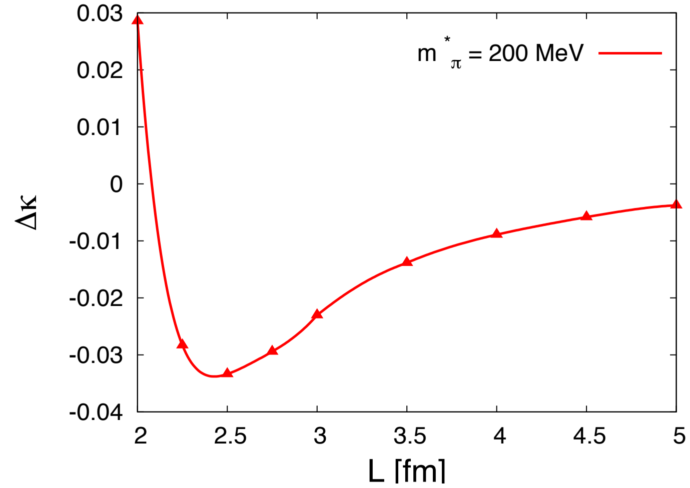
- decreasing curvature in intermediate volume
- corresponds to decreasing pion mass/ increasing constituent quark mass
- decreased sensitivity to chemical potential



Sensitivity of volume dependence on pion mass

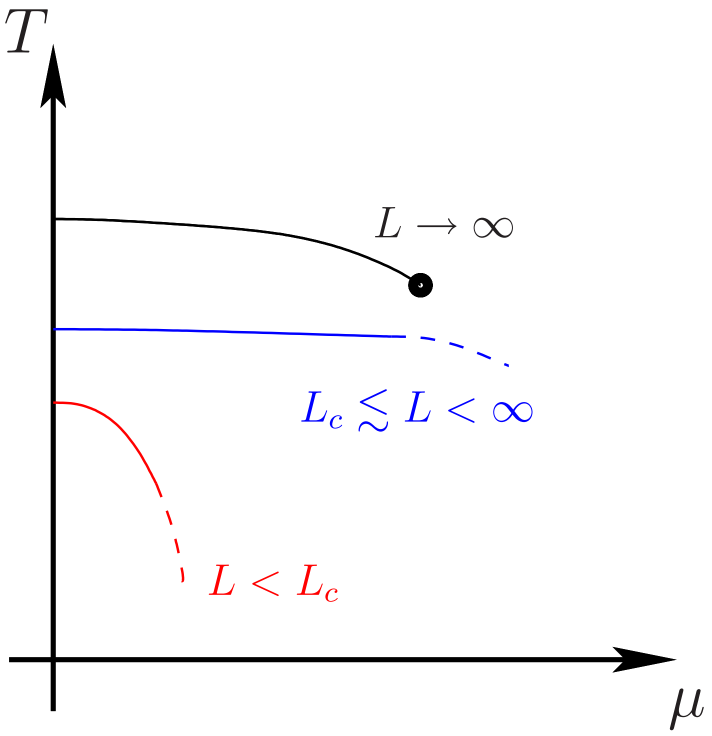
- sensitivity decreases with increasing pion mass
- in agreement with expectations: constituent quark mass rises with pion mass!
- larger constituent quark mass decreases sensitivity





Phase diagram for the QCD models in finite volume - qualitative results

- qualitatively clear effects of finite volume on curvature
- phase transition line tends to *flatten* in an intermediate volume range
- curvature increases dramatically for very small volumes
- consistent with our expectations



Conclusions

- Curvature of finite chemical-potential temperature phase transition line calculated from an NJL-type model *including fermionic and mesonic fluctuations*
- Curvature much larger than in gauge theories
- Finite volume: phase transition line *flattens* in intermediate volume range ➔ curvature smaller!
- possible effects in QCD lattice simulations: expect curvature in small volumes to be smaller