Lattice QCD study of baryon-baryon interactions in the (S,I)=(-2,0) system using the coupled-channel formalism

Kenji Sasaki (*University of Tsukuba*)

for HAL QCD collaboration

HAL(Hadrons to Atomic nuclei from Lattice) QCD Collaboration

S. Aoki (*Univ. of Tsukuba*)

T. Doi (*Univ. of Tsukuba*)

T. Hatsuda (*Univ. of Tokyo*)

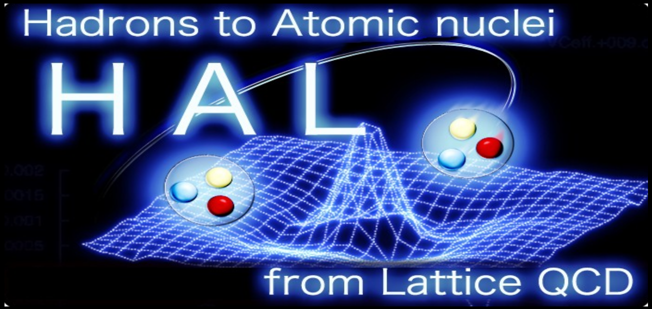
Y. Ikeda (*RIKEN*)

T. Inoue (*Nihon Univ.*)

N. Ishii (*Univ. of Tokyo*)

K. Murano (*KEK*)

H. Nemura (*Tohoku Univ.*)



Introduction

Strangeness in nuclei opened the new frontier of nuclear physics.

Quest for the hyperon-nucleon (YN) and hyperon-hyperon (YY) interactions

One of the most important subject in the (hyper-) nuclear physics

Their informations enable us to deeper understand the baryon-baryon interactions.

This work :

Baryon-baryon interactions in strangeness = -2 and isospin = 0 system

We want to reveal the Λ-Λ interaction from Lattice QCD simulation.
The SU(3) invariant or breaking effects in the BB interaction.

The SU(3) singlet state is involved in this system.

Strength of ΛΛ interaction

Conclusions of the "NAGARA Event"

The clear double- hypernuclear event has been confirmed.
Lower limit of H mass : $m_H \geq 2m_\Lambda$ - 7.3MeV
The result demonstrates that the Λ-Λ interaction is weakly attractive.

Channel coupling

Baryon-baryon system with strangeness = -2 and isospin = 0

This state consistes of the ΛΛ, NΞ and ΣΣ component

$m_{\Lambda\Lambda}$ = 1115 + 1115 = 2230 MeV

$m_{N\Xi}$ = 940 + 1320 = 2260 MeV

$m_{\Sigma\Sigma}$ = 1190 + 1190 = 2380 MeV

Δm=30MeV

Δm=120MeV

This is serious problem especially in S=-2 system.

BS wave function

$$W(t-t_0,\vec{r}) = \sum_{\vec{x}} \sum_i \langle 0 | B_\alpha(\vec{x}+\vec{r}) B_\beta(\vec{x}) | m_i \rangle e^{-E_i(t-t_0)} \langle m_i | \bar{B}_\gamma \bar{B}_\delta | 0 \rangle$$

Source operator can hit some states (m_1, m_2, …) with the same quantum number.

Small energy difference becomes the origin of contaminations

We need a large t_{sat} to extract the ground state.

The ordinary HAL's procedure would not work well if the energy levels are close.

To remove contaminations from excited states

Variational method to separate the states

Dioagonalization of the source operator.

Valiational method ← Powerful tool for analysis of excited states

Linear combination of several independent operators

$$\mathcal{I}(t) = \sum_{\alpha} v_{\alpha} \mathcal{J}^{\alpha}(t)$$

Forming the effective mass of this operator

$$m(T_D) = -\frac{1}{T_D} \ln\left[\frac{\langle \mathcal{I}(T+T_D)\mathcal{I}^{\dagger}(0)\rangle}{\langle \mathcal{I}(T)\mathcal{I}^{\dagger}(0)\rangle}\right] = -\frac{1}{T_D} \ln\left[\frac{\sum_{\alpha\beta} v_{\alpha} v_{\beta} C_{\alpha\beta}(T+T_D)}{\sum_{\alpha\beta} v_{\alpha} v_{\beta} C_{\alpha\beta}(T)}\right]$$

with the correlation matrix defined as

$$C^{IJ}(T) \equiv \langle \mathcal{J}^{I}_{\mathrm{snk}}(T) \mathcal{J}^{I\dagger}_{\mathrm{src}}(0)\rangle$$

Flat wall sink was considered
to symmetrize the correlation matrix

Search the stationary point of m against *v*

$$\frac{\partial m(T_D)}{\partial v_{\gamma}} = 0 \quad \longrightarrow \quad C(T+T_D)\vec{v} = e^{-m(T_D)T_D} C(T)\vec{v}$$

Diagonalization

We can obtain the source operators I_0, I_1, …

which strongly couples to the ground state, 1^{st} excited state, … .

BS wave functions with diagonalized sources.

We prepare three operators for (S,I)=(-2,0) system.

$J_{\Lambda\Lambda}(\vec{x})=\Lambda(\vec{x})\Lambda(0)$ $J_{N\Xi}(\vec{x})=N(\vec{x})\Xi(0)$ $J_{\Sigma\Sigma}(\vec{x})=\Sigma(\vec{x})\Sigma(0)$

After diagonalizing source operators, we can select three states with E_0, E_1 and E_2.

Using diagonalized source operators...

BS wave function in (S,I) = (-2,0) channel

Ground state	1st excited state	2nd excited state
$\Psi_{\Lambda\Lambda}(x,E_0)\equiv\langle 0\vert J_{\Lambda\Lambda}(x)\vert E_0\rangle$	$\Psi_{\Lambda\Lambda}(x,E_1)\equiv\langle 0\vert J_{\Lambda\Lambda}(x)\vert E_1\rangle$	$\Psi_{\Lambda\Lambda}(x,E_2)\equiv\langle 0\vert J_{\Lambda\Lambda}(x)\vert E_2\rangle$
$\Psi_{N\Xi}(x,E_0)\equiv\langle 0\vert J_{N\Xi}(x)\vert E_0\rangle$	$\Psi_{N\Xi}(x,E_1)\equiv\langle 0\vert J_{N\Xi}(x)\vert E_1\rangle$	$\Psi_{N\Xi}(x,E_2)\equiv\langle 0\vert J_{N\Xi}(x)\vert E_2\rangle$
$\Psi_{\Sigma\Sigma}(x,E_0)\equiv\langle 0\vert J_{\Sigma\Sigma}(x)\vert E_0\rangle$	$\Psi_{\Sigma\Sigma}(x,E_1)\equiv\langle 0\vert J_{\Sigma\Sigma}(x)\vert E_1\rangle$	$\Psi_{\Sigma\Sigma}(x,E_2)\equiv\langle 0\vert J_{\Sigma\Sigma}(x)\vert E_2\rangle$

We can obtain wave functions, $\Psi_{\Lambda\Lambda}$ $\Psi_{N\Xi}$ $\Psi_{\Sigma\Sigma}$ for each energy.

3 diagonalized sources ; 3 independent sink operators
→ 9 energy independent potentials

We should consider the coupled channel Schrödinger equarion.

Coupled channel Schrödinger equation

Asymptotic region ($x \to \infty$)

$$\left(E - M_\alpha - \frac{p_\alpha^2}{2\mu_\alpha}\right)\Psi_\alpha(\vec{x}, E) = 0$$

p_α : asymptotic momentum of channel α.

We can determine the asymptotic momentum p_α.

The region where the interaction is active

Assuming that potentials are *energy independent*,

Coupled channel Schrödinger equation.

$$\left(\frac{p_\alpha^2}{2\mu_\alpha} - \frac{\nabla^2}{2\mu_\alpha}\right)\Psi_\alpha(\vec{x}, E) = V_{\alpha\alpha}(\vec{x})\Psi_\alpha(\vec{x}, E) + V_{\alpha\beta}(\vec{x})\Psi_\beta(\vec{x}, E) + V_{\alpha\gamma}(\vec{x})\Psi_\gamma(\vec{x}, E)$$

$$\begin{pmatrix} V^{\Lambda\Lambda,\Lambda\Lambda}(\vec{x}) \\ V^{\Lambda\Lambda,N\Xi}(\vec{x}) \\ V^{\Lambda\Lambda,\Sigma\Sigma}(\vec{x}) \end{pmatrix} = \frac{1}{2\mu_{\Lambda\Lambda}} \begin{pmatrix} \Psi_0^{\Lambda\Lambda} & \Psi_0^{N\Xi} & \Psi_0^{\Sigma\Sigma} \\ \Psi_1^{\Lambda\Lambda} & \Psi_1^{N\Xi} & \Psi_1^{\Sigma\Sigma} \\ \Psi_2^{\Lambda\Lambda} & \Psi_2^{N\Xi} & \Psi_2^{\Sigma\Sigma} \end{pmatrix}^{-1} \begin{pmatrix} (\nabla^2 + p_0^2)\,\Psi_0^{\Lambda\Lambda}(\vec{x}) \\ (\nabla^2 + p_1^2)\,\Psi_1^{\Lambda\Lambda}(\vec{x}) \\ (\nabla^2 + p_2^2)\,\Psi_2^{\Lambda\Lambda}(\vec{x}) \end{pmatrix}$$

$$\begin{pmatrix} V^{N\Xi,\Lambda\Lambda}(\vec{x}) \\ V^{N\Xi,N\Xi}(\vec{x}) \\ V^{N\Xi,\Sigma\Sigma}(\vec{x}) \end{pmatrix} = \frac{1}{2\mu_{N\Xi}} \begin{pmatrix} \Psi_0^{\Lambda\Lambda} & \Psi_0^{N\Xi} & \Psi_0^{\Sigma\Sigma} \\ \Psi_1^{\Lambda\Lambda} & \Psi_1^{N\Xi} & \Psi_1^{\Sigma\Sigma} \\ \Psi_2^{\Lambda\Lambda} & \Psi_2^{N\Xi} & \Psi_2^{\Sigma\Sigma} \end{pmatrix}^{-1} \begin{pmatrix} (\nabla^2 + q_0^2)\,\Psi_0^{N\Xi}(\vec{x}) \\ (\nabla^2 + q_1^2)\,\Psi_1^{N\Xi}(\vec{x}) \\ (\nabla^2 + q_2^2)\,\Psi_2^{N\Xi}(\vec{x}) \end{pmatrix}$$

$$\begin{pmatrix} V^{\Lambda\Lambda,\Lambda\Lambda}(\vec{x}) \\ V^{\Lambda\Lambda,N\Xi}(\vec{x}) \\ V^{\Lambda\Lambda,\Sigma\Sigma}(\vec{x}) \end{pmatrix} = \frac{1}{2\mu_{\Sigma\Sigma}} \begin{pmatrix} \Psi_0^{\Lambda\Lambda} & \Psi_0^{N\Xi} & \Psi_0^{\Sigma\Sigma} \\ \Psi_1^{\Lambda\Lambda} & \Psi_1^{N\Xi} & \Psi_1^{\Sigma\Sigma} \\ \Psi_2^{\Lambda\Lambda} & \Psi_2^{N\Xi} & \Psi_2^{\Sigma\Sigma} \end{pmatrix}^{-1} \begin{pmatrix} (\nabla^2 + k_0^2)\,\Psi_0^{\Sigma\Sigma}(\vec{x}) \\ (\nabla^2 + k_1^2)\,\Psi_1^{\Sigma\Sigma}(\vec{x}) \\ (\nabla^2 + k_2^2)\,\Psi_2^{\Sigma\Sigma}(\vec{x}) \end{pmatrix}$$

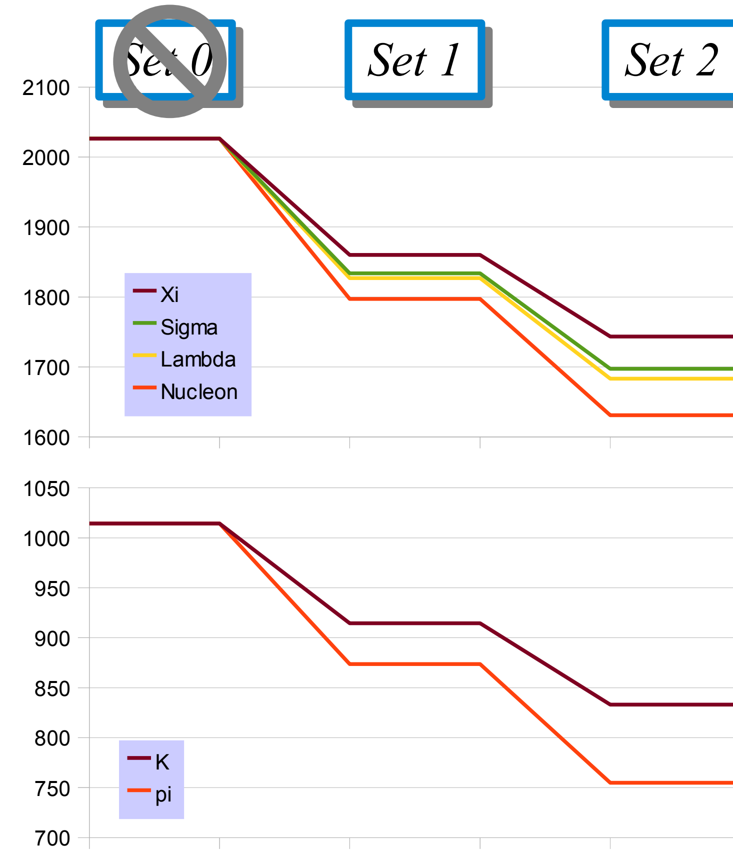
We can obtain the potential matrix.

Hermiticity can be checked as well.

Numerical set up

- 2+1 flavor gauge configurations by CP-PACS/JLQCD.
  - RG improved gauge & O(a) improved clover quark
  - 16^3x32 lattice, a = 0.1209 [fm], L = 1.934 [fm].
- Flat wall source is considered to produce S-wave B-B state.
- The supercomputer system Blue Gene/L at KEK had been used.
- Following three configurations are used,

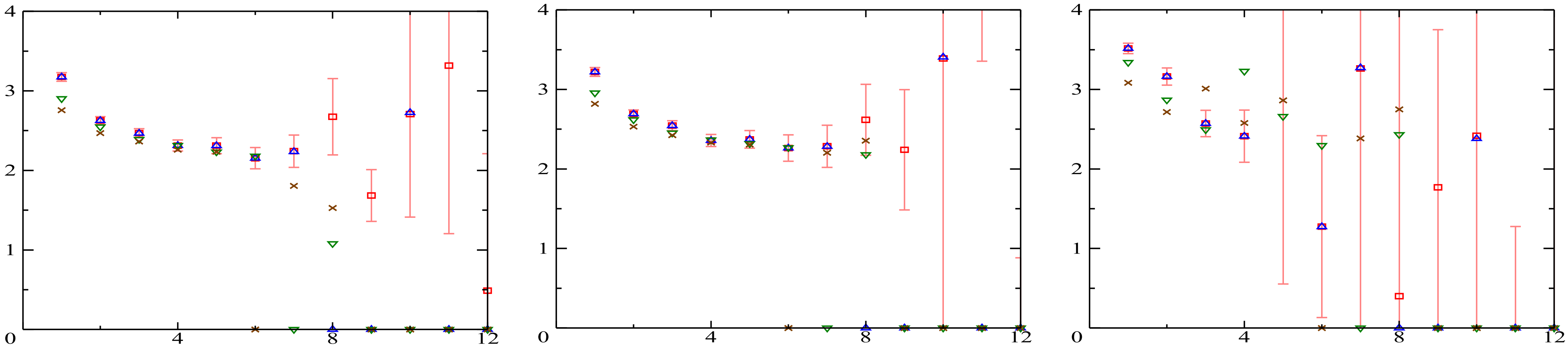
	Set 0	Set 1	Set 2
	κ_s=0.13710		
	κ_{ud}=0.13710	κ_{ud}=0.13760	κ_{ud}=0.13800
π	1014.2±1.1	873.7±2.7	754.9±5.3
K	1014.2±1.1	914.5±2.6	833.1±5.1
m_π/m_K	1	0.956	0.906
N	2026.2±3.4	1797.1±9.2	1631.1±17.9
Λ	2026.2±3.4	1827.0±8.9	1683.4±16.0
Σ	2026.2±3.4	1833.8±8.9	1697.3±17.0
Ξ	2026.2±3.4	1860.0±8.5	1743.3±14.8



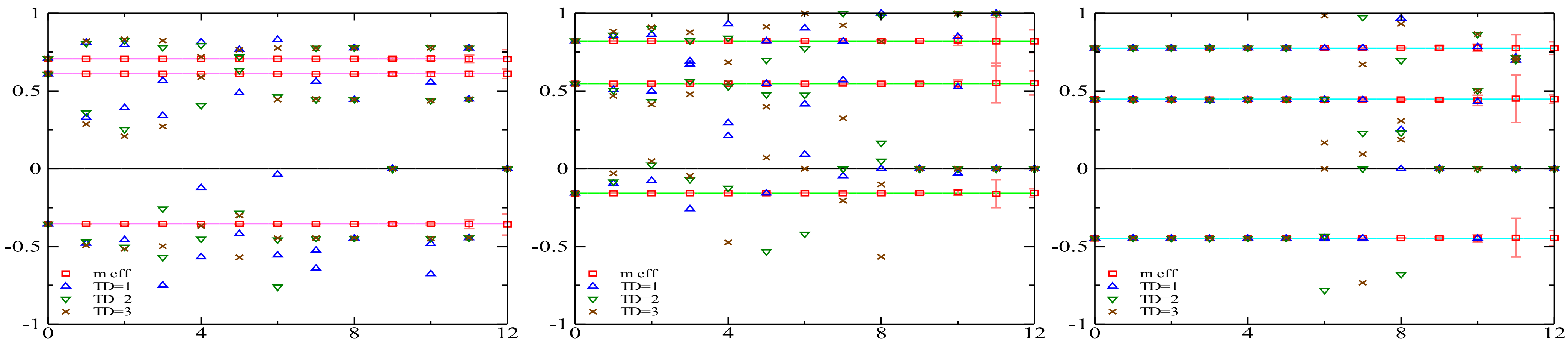
Effective masses and eigen vectors

Set 1

Eigen values (effective mass) of diagonalized correlator matrix



Eigen vectors of diagonalized correlator matrix



We choose the set of eigen vectors at T=5 with TD=1.

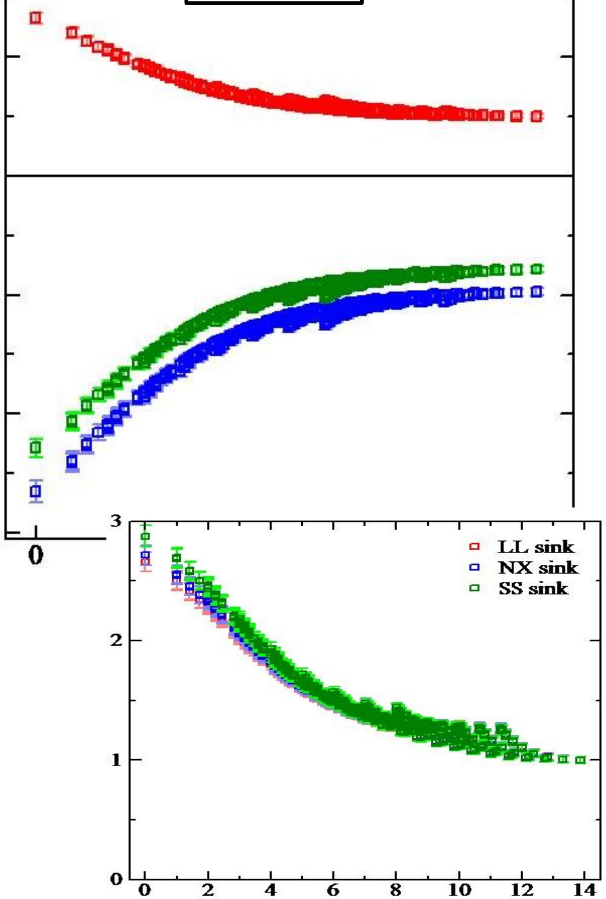
BS wave functions for (S,I)=(-2,0) channel.

Set 1

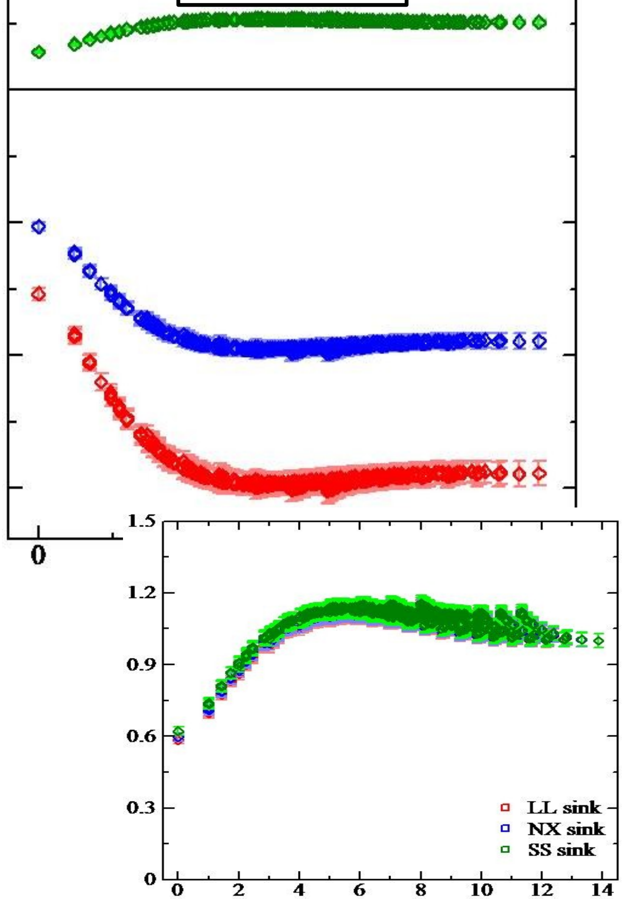
Wave functions with diagonalized souces

ΛΛ, NΞ, ΣΣ

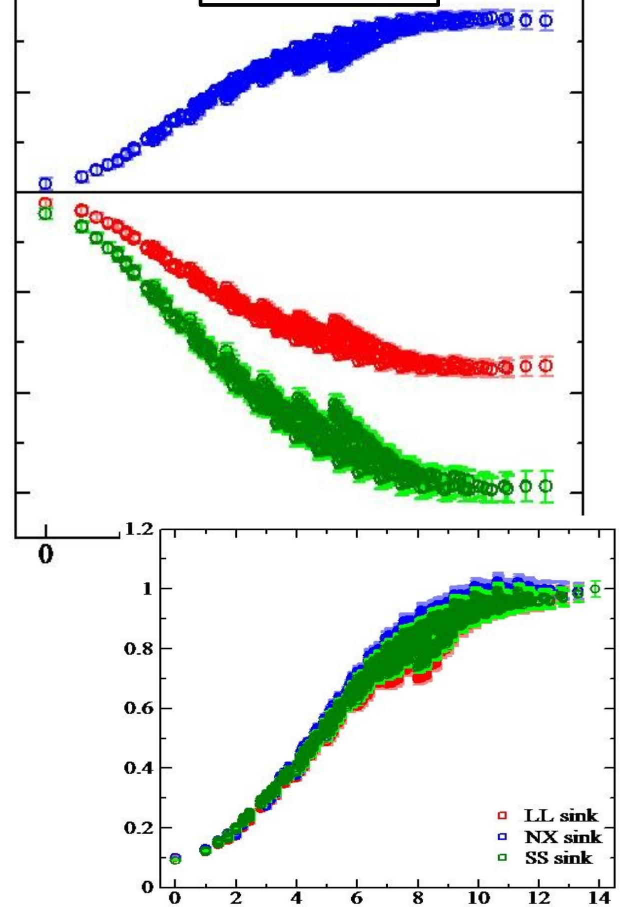
Lowest



1st excited



2nd excited



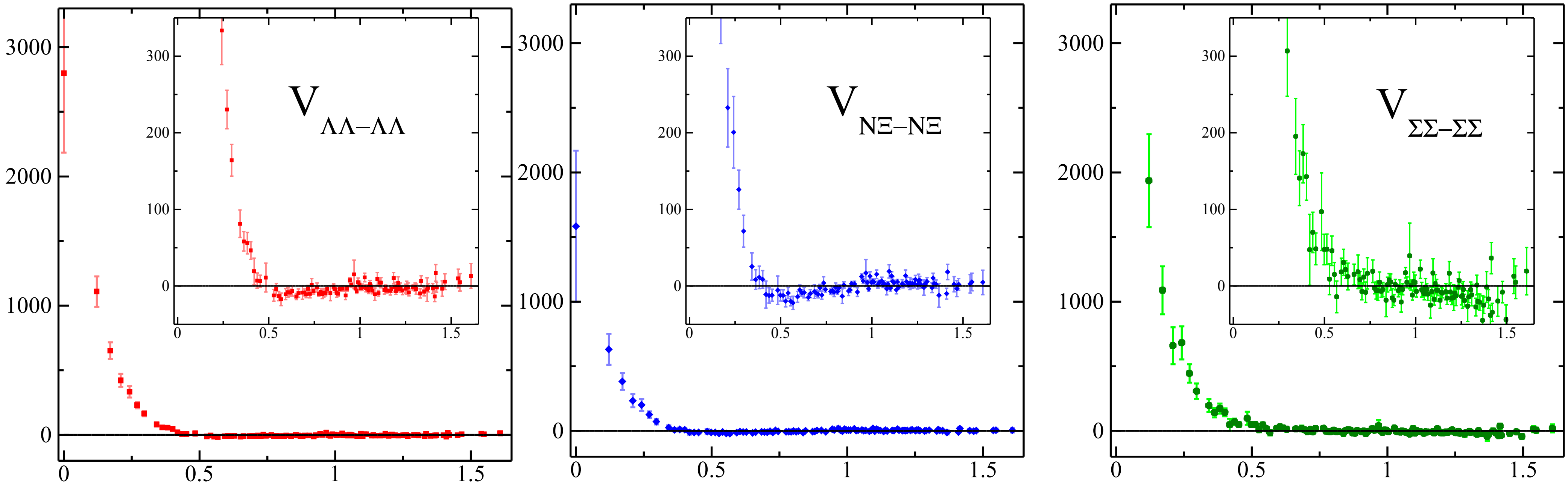
The qualitative behavior of wave functions are same as the result of $SU(3)_f$ limit.
We can see a dependence on the sink operator due to the $SU(3)_f$ breaking effect.

Diagonal part of potential matrix with (S,I)=(-2,0).

Set 1

Potential matrix

$$\begin{pmatrix} V_{\Lambda\Lambda-\Lambda\Lambda} & V_{\Lambda\Lambda-N\Xi} & V_{\Lambda\Lambda-\Sigma\Sigma} \\ V_{N\Xi-\Lambda\Lambda} & V_{N\Xi-N\Xi} & V_{N\Xi-\Sigma\Sigma} \\ V_{\Sigma\Sigma-\Lambda\Lambda} & V_{\Sigma\Sigma-N\Xi} & V_{\Sigma\Sigma-\Sigma\Sigma} \end{pmatrix}$$



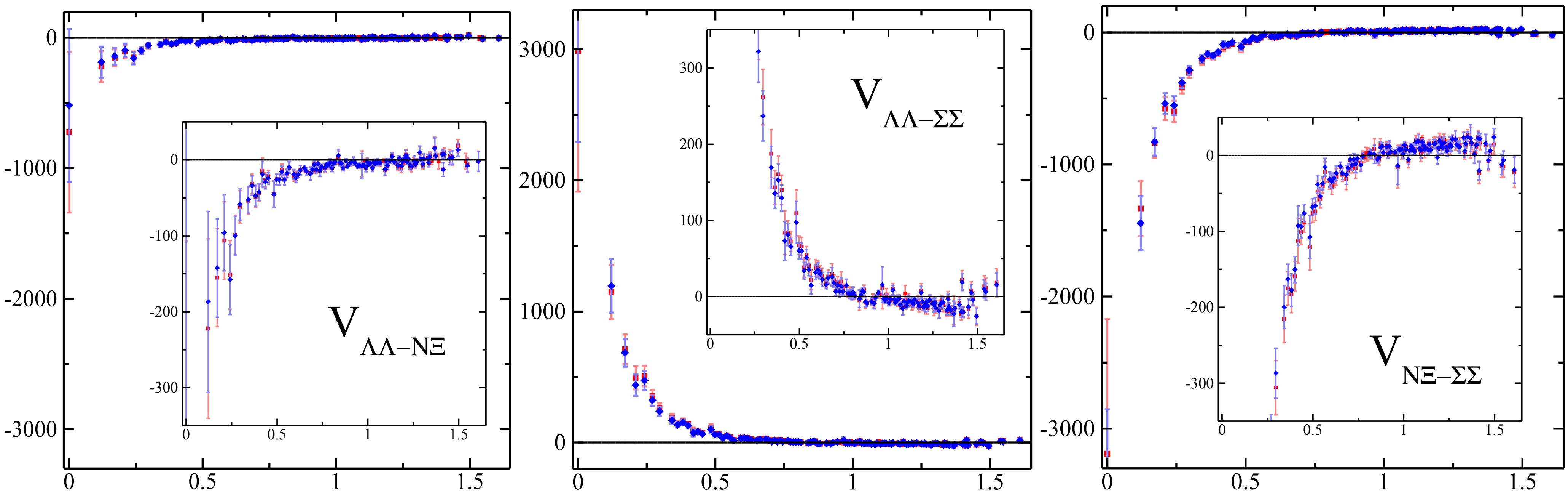
- The potential matrix is calculated using wave functions at $t\text{-}t_0=10$.
  - We can see an attraxtive pocket in the ΛΛ and NΞ potentials.
  - No attractive pocket can be seen in the ΣΣ potential.
- All potentials are repulsive at short range region.
  - We can see the flavor dependence of the height of repulsive core.
-

Non-diagonal part of potential matrix with (S,I)=(-2,0)

Set 1

Potential matrix

$$\begin{pmatrix} V_{\Lambda\Lambda-\Lambda\Lambda} & V_{\Lambda\Lambda-N\Xi} & V_{\Lambda\Lambda-\Sigma\Sigma} \\ V_{N\Xi-\Lambda\Lambda} & V_{N\Xi-N\Xi} & V_{N\Xi-\Sigma\Sigma} \\ V_{\Sigma\Sigma-\Lambda\Lambda} & V_{\Sigma\Sigma-N\Xi} & V_{\Sigma\Sigma-\Sigma\Sigma} \end{pmatrix}$$



- The potential matrix is calculated using wave functions at $t\text{-}t_0=10$.
  - The sign of transition potential is meaningless. (relative sign of wave functgions)
  - The strengths of $V_{\Lambda\Lambda-\Sigma\Sigma}$ and $V_{N\Xi-\Sigma\Sigma}$ are similar.
  - The ΛΛ−NΞ transition potential is much weaker than the others.
- Hermiticity is proper within the error bar.
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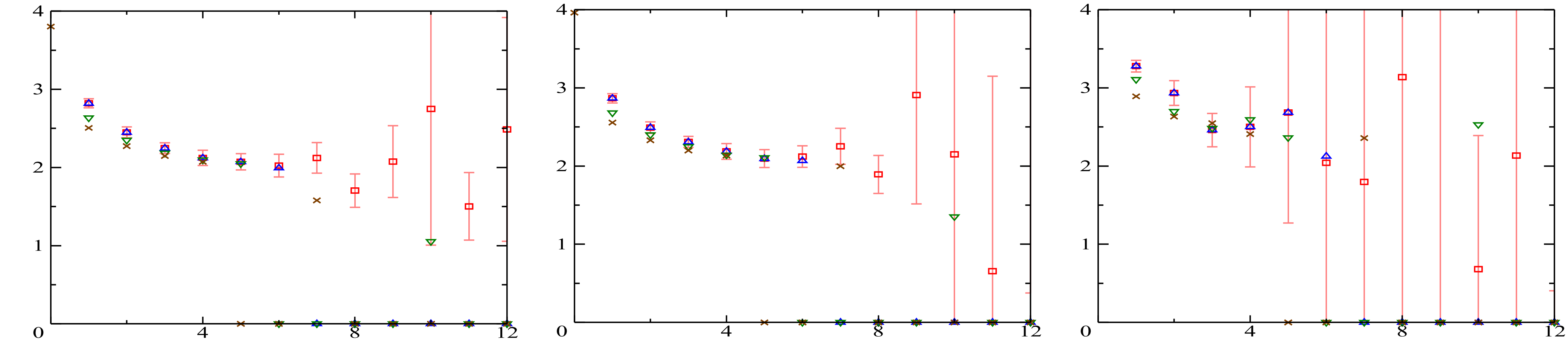
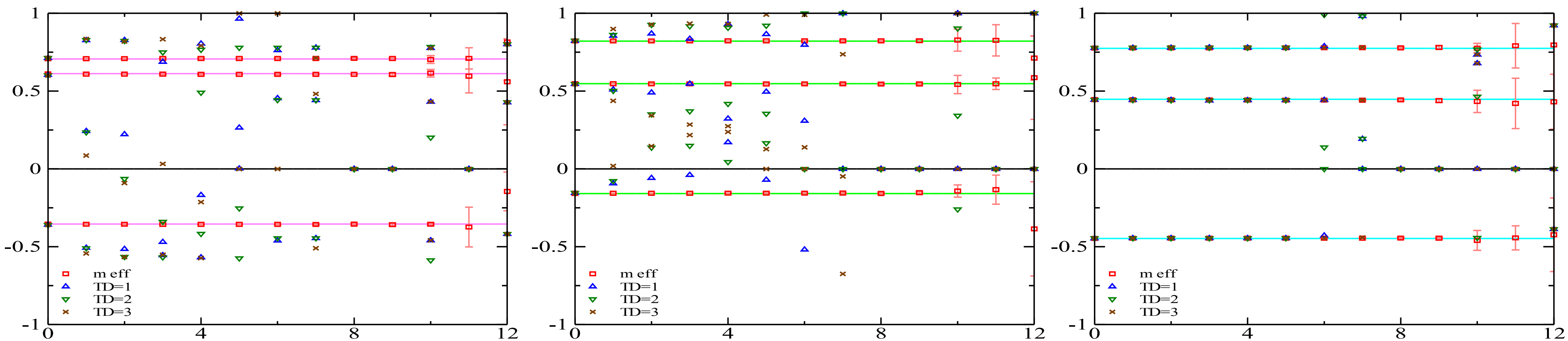
Effective masses and eigen vectors

Set 2

Eigen values (effective mass) of diagonalized correlator matrix



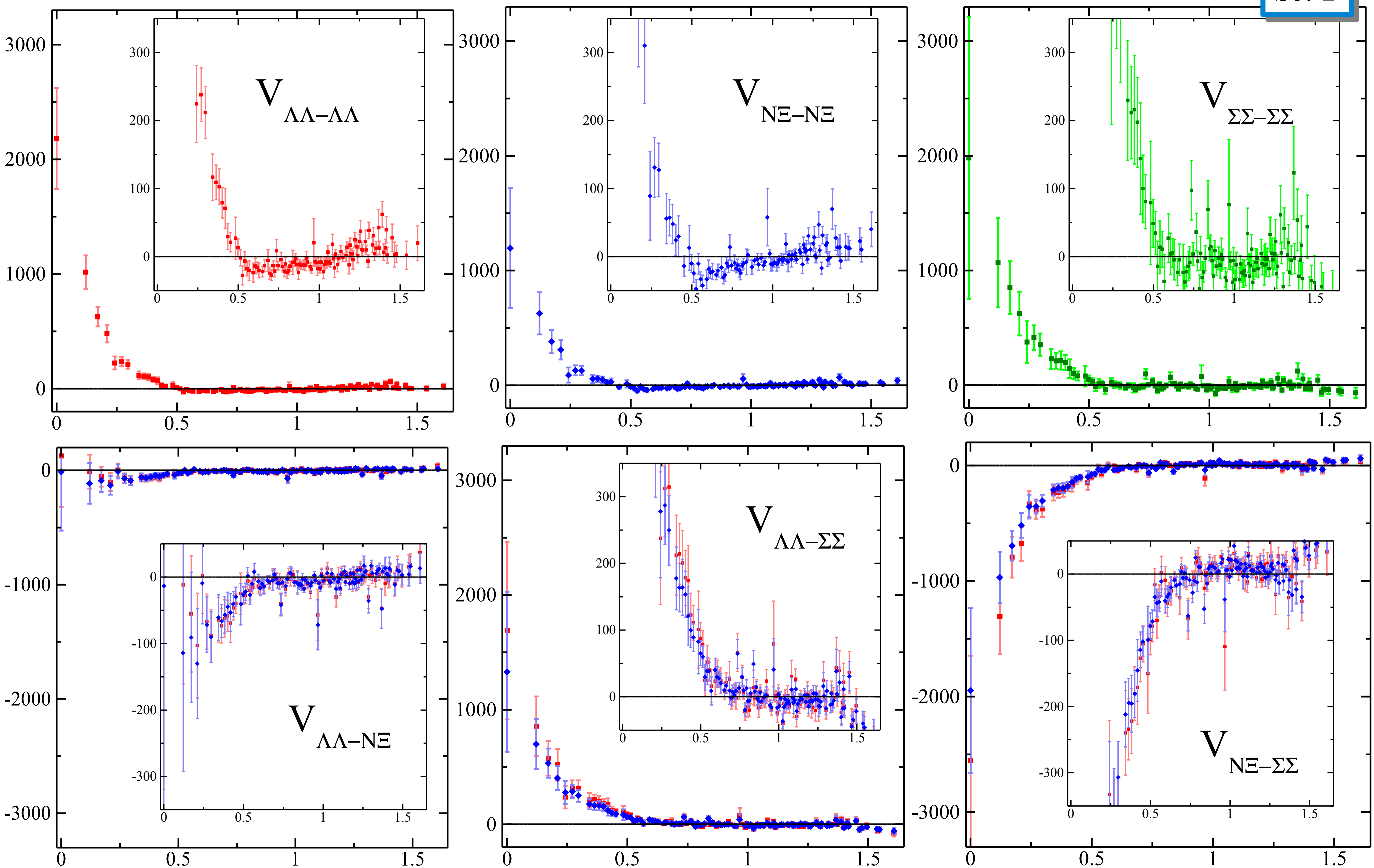
Eigen vectors of diagonalized correlator matrix



We choose the set of eigen vectors at T=4 with TD=2.

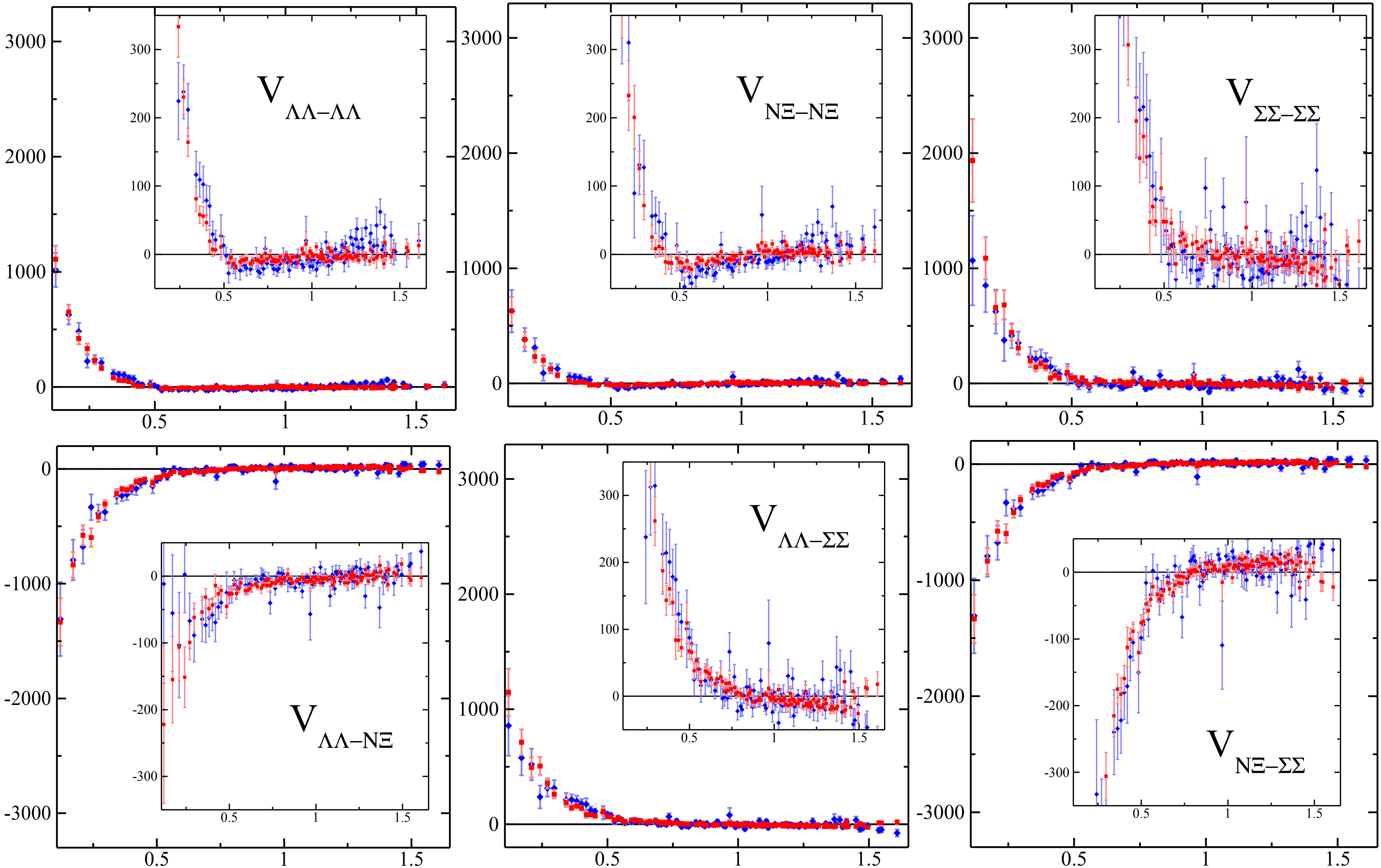
Potential matrix

Set 2



Comparison of potential matrices

Set 1
Set 2



Summaries and Outlooks

- We have investigated the YY (and YN) interactions from lattice QCD.
- In order to deal with a variety of interactions, we devised the efficient way to treat it.
  - Utilize the diagonalized source operators which enable us to obtain the wave functions with each energy.
  - Assuming energy independence to the potential which allows us to construct the coupled channel equation for the considering system.
  - Using this technique we can obtain the potential matrix for the coupled channel Schrödinger equation.

- We have considered the $SU(3)_f$ breaking effects of the YY (and YN) interactions by changing the light-flavored quark mass.
  - We need more statistics to make a clear conclusion.

- We confirmed that this technique working well.
- Coupled channel technique is powerful and widely applicable
  - We will challenge to reveal all baryon-baryon interactions with S=0, -1, … -6 below the pion production threshold.

Backup slides

HAL QCD's strategy

- Calculate Bethe-Salpeter (BS) wave function on any gauge configuration.

$$W(t-t_0,\vec{r}) = \sum_{\vec{x}} \langle 0|B_i(t,\vec{x}+\vec{r})B_j(t,\vec{x})\,\bar{B}_k(t_0)\bar{B}_l(t_0)|0\rangle$$

- Define the non-relativistic Schrödinger equation (general form)

$$\left(E-\frac{\nabla^2}{2\mu}\right)\Psi(\vec{x}) = \int U(\vec{x}-\vec{y})\Psi(\vec{y})d^3y$$

- Performing the derivative expansion for the interaction kernel

$$U(\vec{x}-\vec{y}) \simeq V_0(\vec{x})\delta(\vec{x}-\vec{y}) + V_1(\vec{x},\nabla)\delta(\vec{x}-\vec{y}) \cdots$$

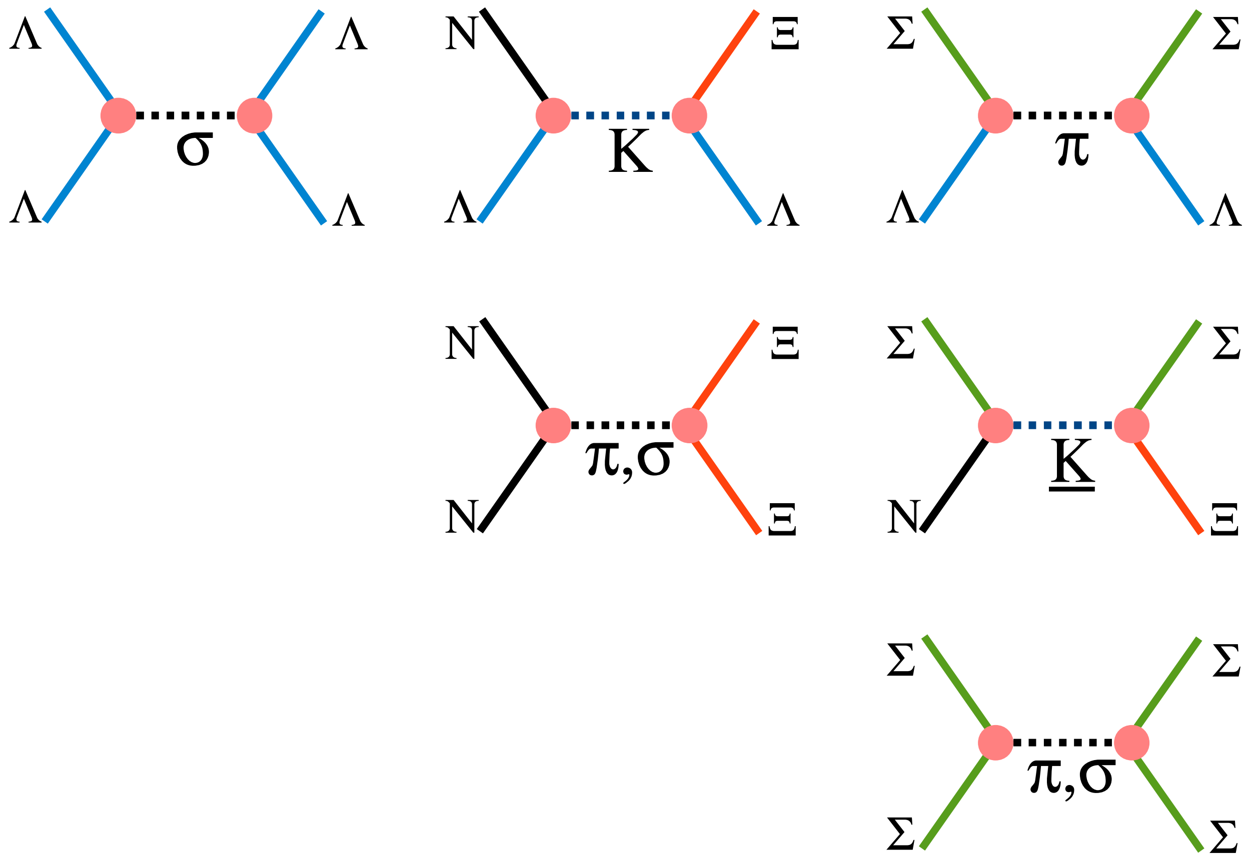
- The potential is given as

$$V(\vec{x}) = E - \frac{1}{2\mu}\frac{\nabla^2\Psi(\vec{x})}{\Psi(\vec{x})}$$

- This technique is widely applicable for hadronic systems

Extention to the YN and YY systems

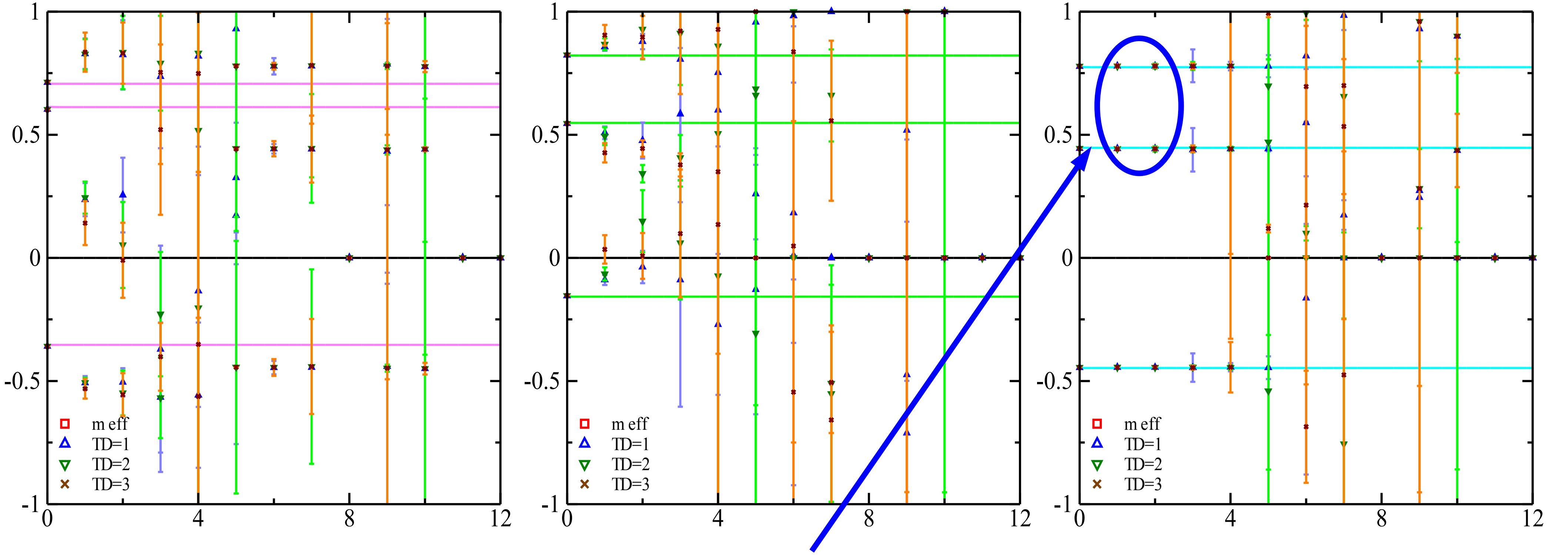
Potentials



Effective masses and eigen vectors

Set 2

Eigen vectors of diagonalized correlator matrix



Error bar is basically given this much

Need more efficient way to calculate error bars.