

# The Decay Constants $f_{D_s}$ , $f_{D^+}$ , $f_{B_s}$ and $f_B$ from Lattice QCD

## **Fermilab Lattice and MILC Collaborations:**

J.A. Bailey, A. Bazavov, C. Bernard, C. Bouchard, C. DeTar,  
A.X. El-Khadra, E.D. Freeland, E. Gámiz, Steven Gottlieb  
U.M. Heller, J.E. Hetrick, A.S. Kronfeld, J. Laiho, L. Levkova,  
P.B. Mackenzie, M.B. Oktay, J.N. Simone, R.L. Sugar,  
D. Toussaint, and R.S. Van de Water

June 14, 2010

# Introduction

This talk presents summary, though preliminary, results for the  $D$  and  $B$  decay constants from the now concluded initial phase of the FNAL/MILC flavor physics program.

## Project roadmap

phase	years	sea	spacings [fm]	valence light	valence heavy
I	now	Asqtad	0.09, 0.125, 0.15	Asqtad	clover
II	+1	Asqtad	above + 0.06	Asqtad	clover
IIb	+2	Asqtad	above + 0.045	Asqtad	clover
III	+5	HISQ	similar	HISQ	HISQ charm

## Features

- I → II Re-run with refined inputs and  $4\times$  the statistics.
- II and IIb Additional finer lattice spacings.
- III Four-flavor HISQ. Include run at physical quark masses.

# MILC three-flavor ensembles – three lattice spacings

This calculation was performed on the eleven MILC Asqtad ensembles listed here:

$a$ [fm]	$am_h$	$am_l$	$\beta$	$r_1/a$	configs
0.09	0.031	0.0031	7.08	3.69	906
		0.0062	7.09	3.70	557
		0.0124	7.11	3.72	518
0.125	0.05	0.005	6.76	2.64	678
		0.007	6.76	2.63	833
		0.01	6.76	2.62	592
		0.02	6.79	2.65	460
		0.03	6.81	2.66	549
0.15	0.0484	0.0097	6.572	2.13	631
		0.0194	6.586	2.13	631
		0.029	6.600	2.13	576

Table: MILC three-flavor lattice parameters.

The full collection of MILC Asqtad ensembles is freely available and is published on the ILDG.

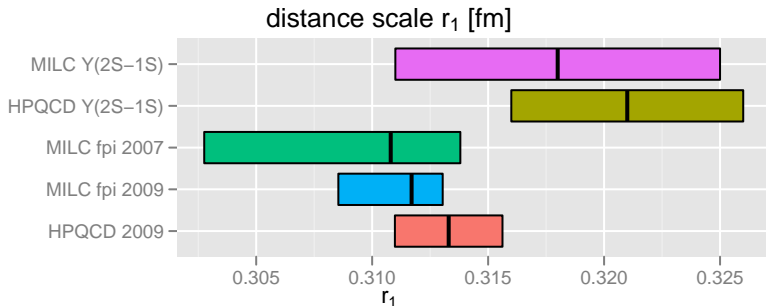
# Highlights of the calculation

- Chiral fits use NLO expression for  $\phi = f\sqrt{M}$  from partially-quenched staggered chiral perturbation theory [Aubin and Bernard].
- Add NNLO analytic (quadratic in quark mass) terms.
- Model both light- and heavy-quark discretization effects in the fits.
- Input the distance scale  $r_1$ , quark masses  $m_s$ ,  $m_d$  and  $m_u$  and  $O(a^2)$  LECs from MILC light meson fits.
- Bulk of HL current renormalizations are nonperturbative ( $Z_V^{hh}$  and  $Z_V^{ll}$ ), the remainder ( $\rho_{A_4}$ ) is known to one loop.

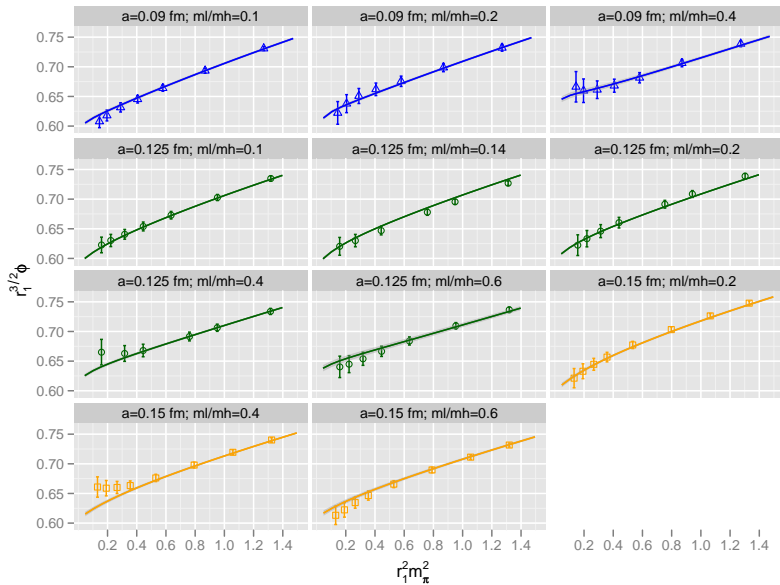
# Distance scale $r_1$

As reported at LAT09, we have adopted the  $r_1$  value from the MILC  $f_\pi$  determination as an input.

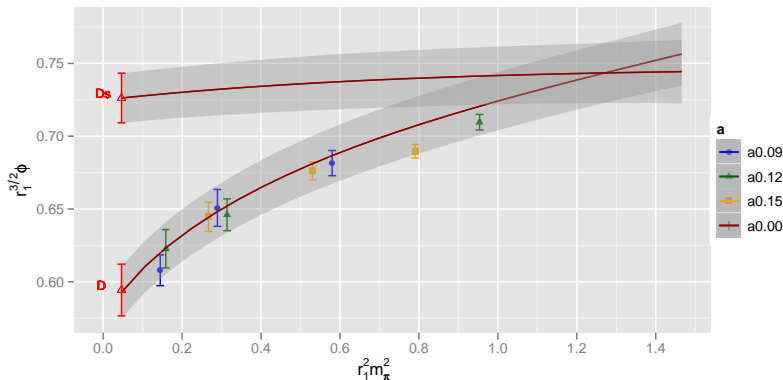
The recent MILC values agree well with HPQCD's recent  $r_1$  value determined from a combination of several quantities.



# The $D$ -meson fit at finite lattice spacing



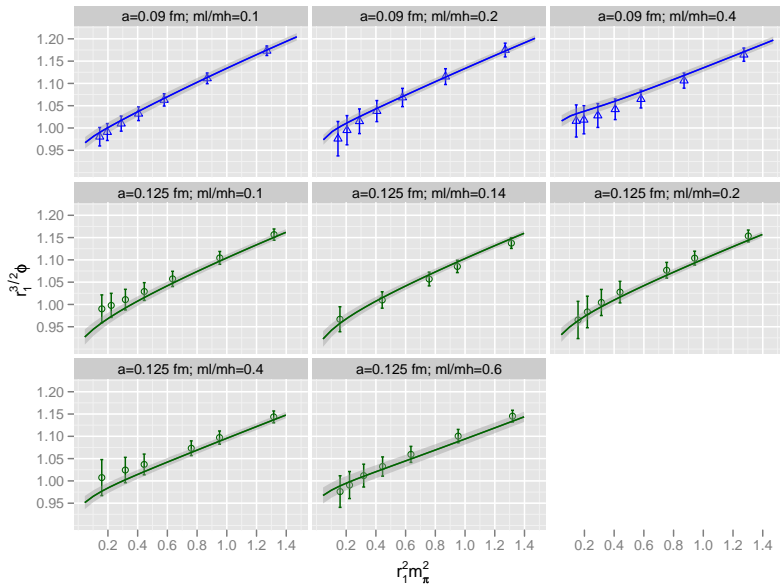
# The $f_{D_s}$ and $f_{D^+}$ extrapolation at zero lattice spacing



Two projections of the extrapolated fit surface are shown. The  $D_s$  curve has the valence mass fixed:  $m_q = m_s$ , while the valence and sea mass vary together for the  $D$  curve.

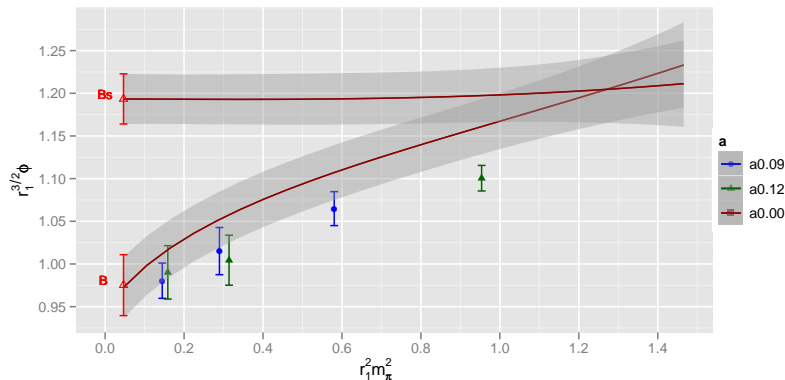
The points having  $m_q$  near  $m_s$ , visible in the previous slide, do not lie exactly on the  $D_s$  projection.

# The $B$ meson system fit at finite lattice spacing





# The $f_{B_s}$ and $f_B$ extrapolation at zero lattice spacing



# Results

## D-meson system

$$f_{D_s} = 261 \pm 8 \pm 5 \text{ MeV}$$

$$f_{D^+} = 220 \pm 8 \pm 5 \text{ MeV}$$

$$f_{D_s}/f_{D^+} = 1.19 \pm 0.01 \pm 0.02$$

## B-meson system

$$f_{B_s} = 256 \pm 6 \pm 6 \text{ MeV}$$

$$f_B = 212 \pm 6 \pm 6 \text{ MeV}$$

$$f_{B_s}/f_B = 1.21 \pm 0.01 \pm 0.02$$

Total error is about 3 to 4 percent for the decay constants.

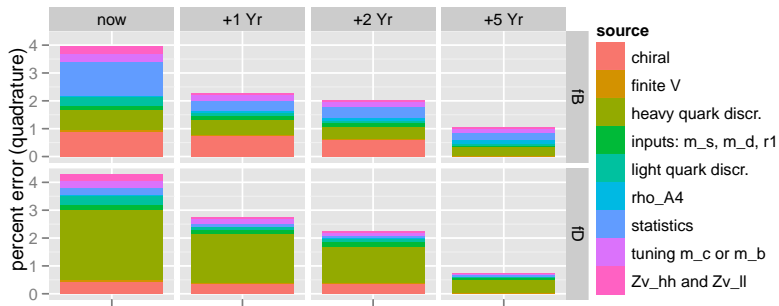
Statistical as well as systematic errors tend to cancel in the two ratios yielding errors around 2 percent.

# Error budget

source	$f_{D_s}$	$f_{D^+}$	$f_{D_s}/f_{D^+}$	$f_{B_s}$	$f_B$	$f_{B_s}/f_B$
statistics and discretization effects	2.9	3.6	1.1	2.3	2.9	1.1
chiral extrapolation	0.8	1.4	1.2	1.3	1.9	1.2
inputs $r_1$ , $m_s$ , $m_d$ and $m_u$	0.7	0.8	0.1	0.7	0.8	0.1
input $m_c$ or $m_b$	1.2	1.0	0.2	1.1	1.1	0.1
$Z_V^{hh}$ and $Z_V^{qq}$	1.0	1.0	0	1.0	1.0	0
higher-order $\rho_{A_4}$	0.3	0.3	0.2	0.4	0.4	0.1
finite volume	0.2	0.4	0.4	0.2	0.4	0.4
total	3.5	4.2	1.7	3.1	3.9	1.7

**Table:** Uncertainties as a percentage of the decay constants and their ratio. The total combines all of the errors in quadrature.

# Projected $f_B$ and $f_{D^+}$ error budgets

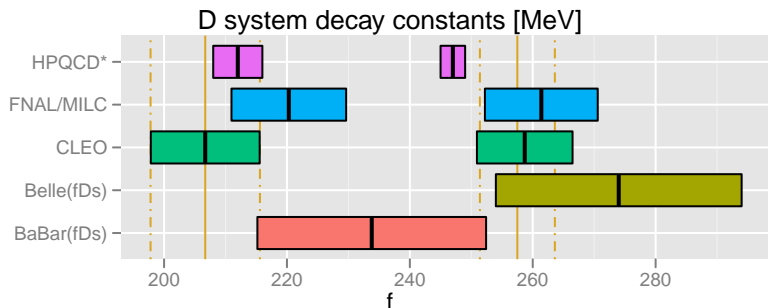


**H+L Discr.:** finer lattices; HISQ (light and charm).

**Statistics:** more configurations and get more info per config.

**Chiral:** run nearer physical  $m_u, m_d$ . Vary sea  $m_h$ .

# Compare $f_{D^+}$ and $f_{D_s}$

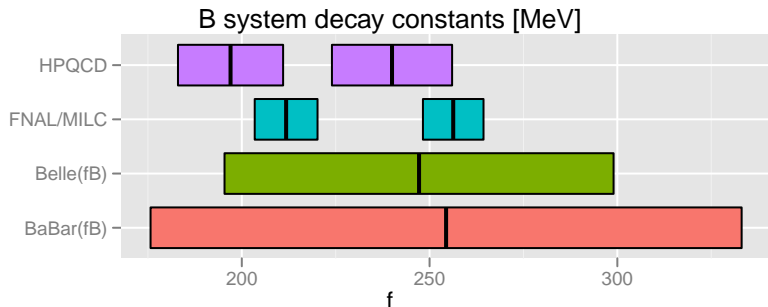


HPQCD  $f_{D^+}$  value has been adjusted upwards (about one sigma) to reflect their latest  $r_1$ . An official update is anticipated.

Experimental averages and errors are indicated in gold.

Rosner and Stone review [[arXiv:1002.1655](https://arxiv.org/abs/1002.1655)]

# Compare $f_B$ and $f_{B_s}$



Assumed  $|V_{ub}| = 3.97 \times 10^{-3}$  (avg. exclusive and inclusive)

Rosner and Stone review [[arXiv:1002.1655](https://arxiv.org/abs/1002.1655)]

# Summary

The first (nearing publication) phase of the FNAL/MILC has yielded 3 to 4% errors for the decay constants.

The second phase of the program (underway), including refined inputs and finer lattices, aims to reduce errors to the 2% level.

The program using HISQ lattices and valence quarks has begun and aims for errors at the percent level for the decay constants.