Low-energy parameters from $N_f = 2$ clover fermions at physical quark masses

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– QCDSF Collaboration –

Special mention:

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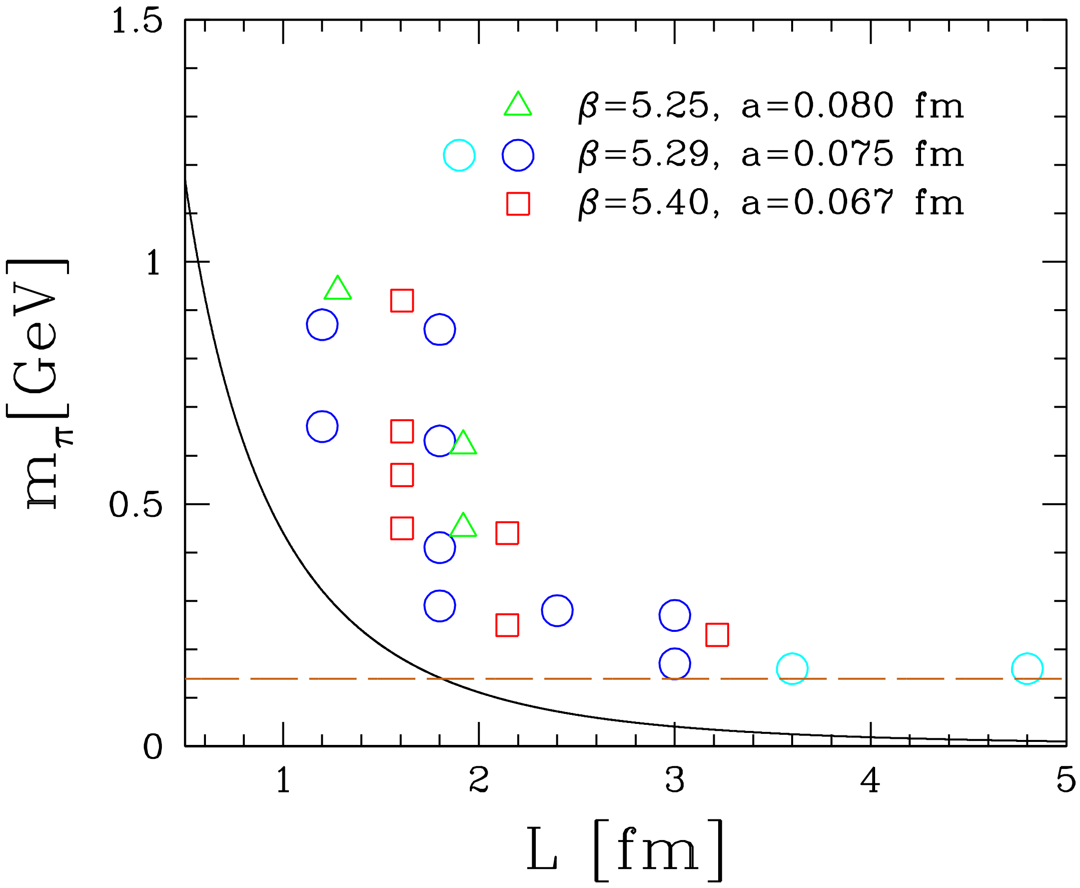
D. Pleiter, P.E.L. Rakow, T. Streuer and J. Zanotti

Landscape



β=5.25, a=0.080 fm
β=5.29, a=0.075 fm
β=5.40, a=0.067 fm
m_π [GeV]
L [fm]
1.5
1
0.5
0
1
2
3
4
5

Action

$$N_f = 2$$

$$S = S_G + S_F$$

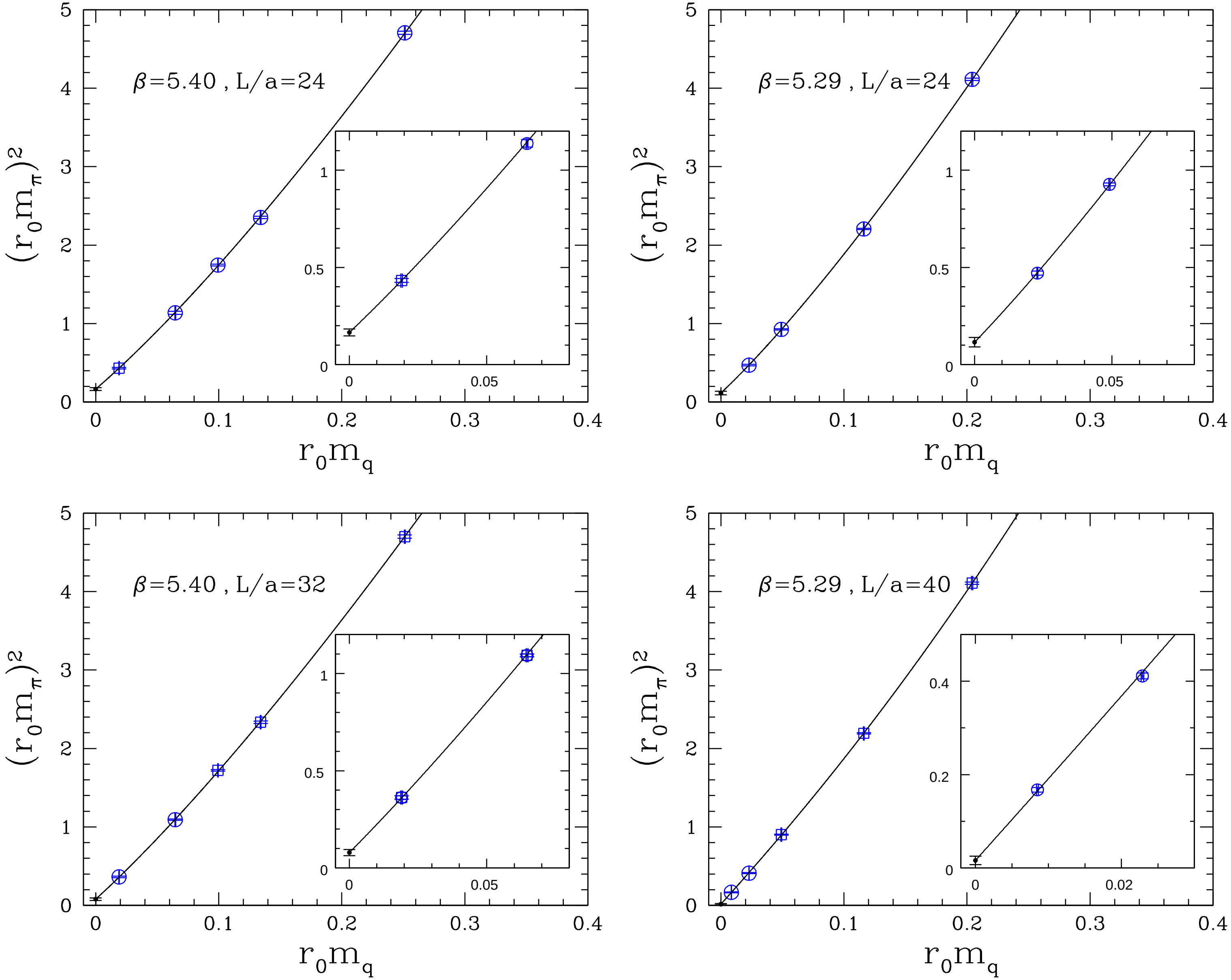
$$S_G = \beta \sum_{x,\mu<\nu} \left(1 - \frac{1}{3}\mathrm{Re\,Tr}\, U_{\mu\nu}(x)\right)$$

$$S_F = \sum_x \Big\{ \bar{\psi}(x)\psi(x) - \kappa\, \bar{\psi}(x) U_\mu^\dagger(x-\hat{\mu})[1+\gamma_\mu]\psi(x-\hat{\mu}) - \kappa\, \bar{\psi}(x) U_\mu(x)[1-\gamma_\mu]\psi(x+\hat{\mu}) - \frac{1}{2}\kappa\, c_{SW}\, g\, \bar{\psi}(x)\sigma_{\mu\nu}F_{\mu\nu}(x)\psi(x) \Big\}$$

Clover Fermions

Pion & Kaon

Pion in a Box


β=5.40 , L/a=24
β=5.29 , L/a=24
β=5.40 , L/a=32
β=5.29 , L/a=40
$(r_0 m_\pi)^2$
$r_0 m_q$
0
0.1
0.2
0.3
0.4
1
2
3
4
5
0.5
0.05
0.02

δ Regime

$m_\pi L \ll 1\,,\ L \ll T$

$$m_\pi^{\mathrm{res}} = \frac{3}{2 f_\pi^2 L^3 (1+\Delta)}$$

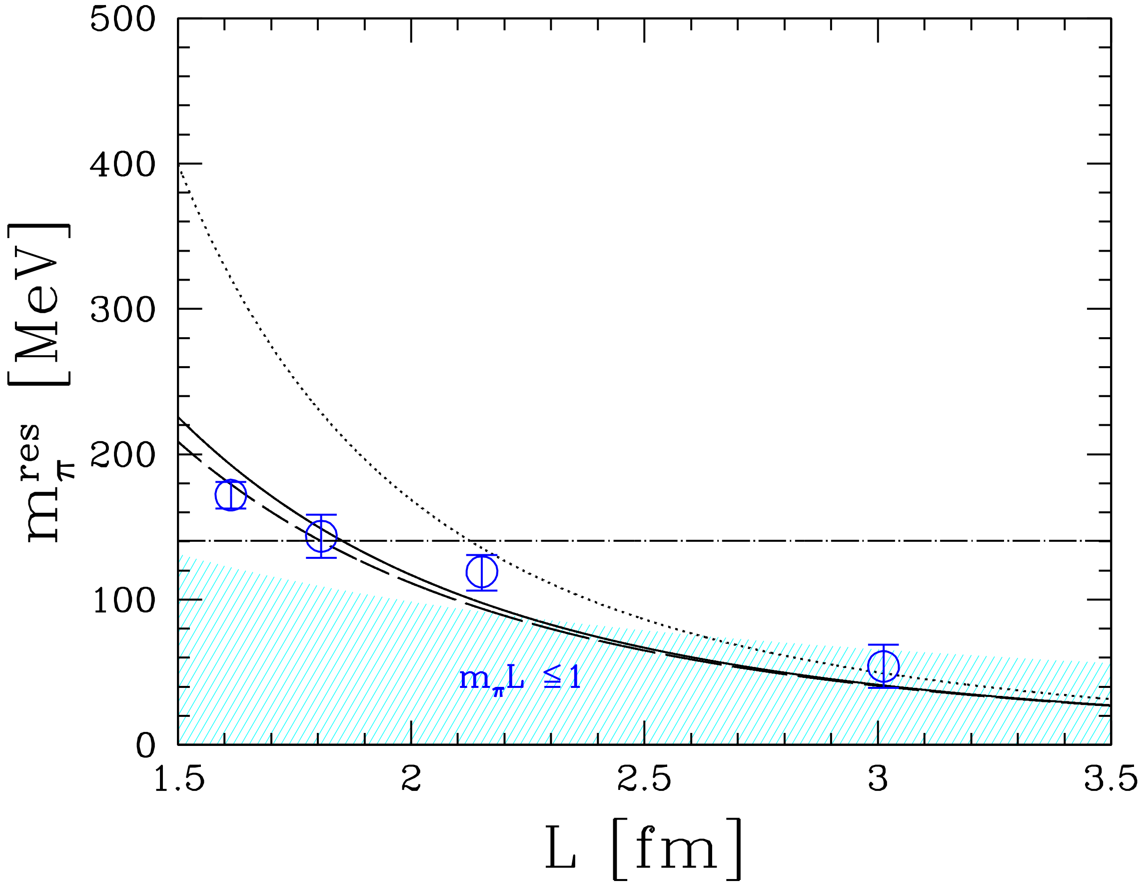
with

$$\Delta = \frac{2}{f_\pi^2 L^2}\, 0.2257849591 + \frac{1}{f_\pi^4 L^4} \left[0.088431628 - \frac{0.8375369106}{3\pi^2} \left(\frac{1}{4} \ln\left(\Lambda_1 L\right)^2 + \ln\left(\Lambda_2 L\right)^2 \right) \right]$$

$$\ln\left(\Lambda_i / m_\pi^{\mathrm{phys}}\right)^2 = \bar{l}_i$$

Leutwyler, Niedermayer & Hasenfratz

Mass Gap

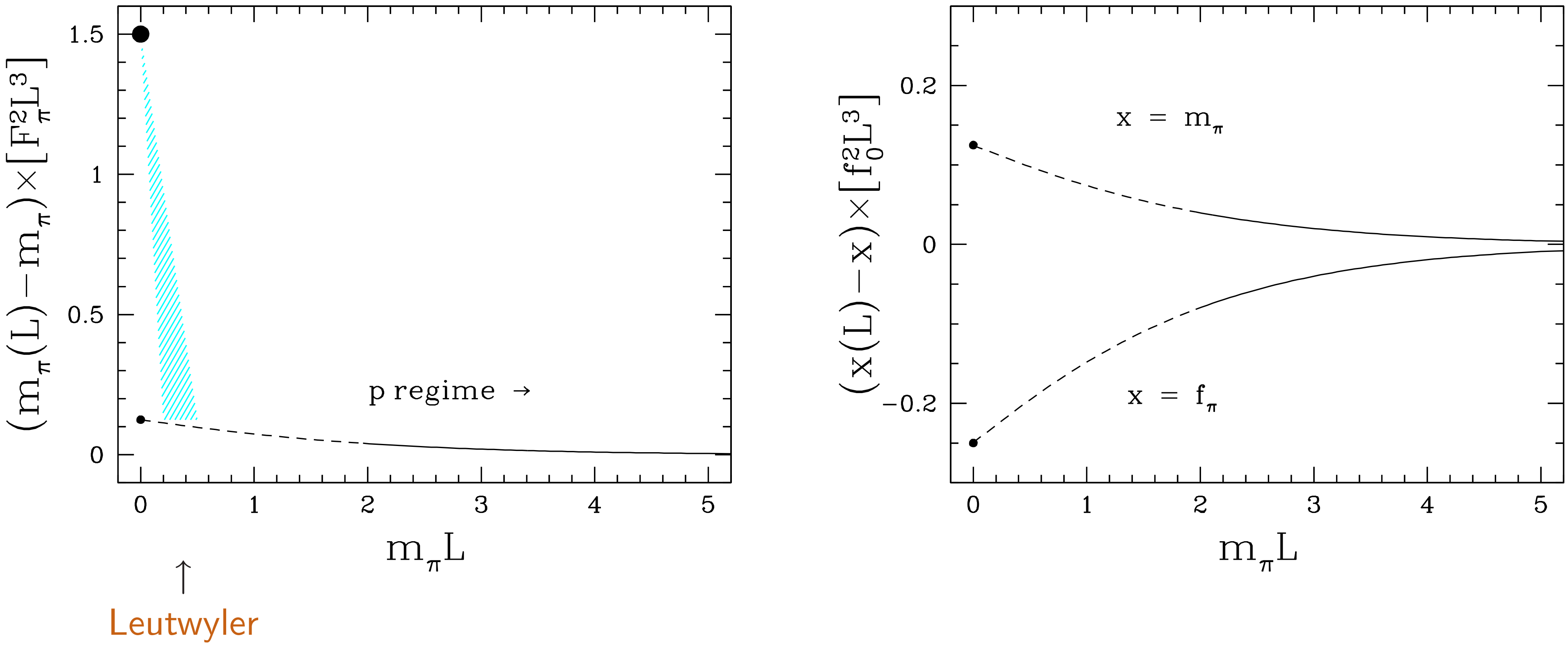


$$f_0 = f_\pi|_{m_\pi=0} = 78^{+14}_{-10}\ \text{MeV}$$

$r_0 = 0.47(1)$

from m_N

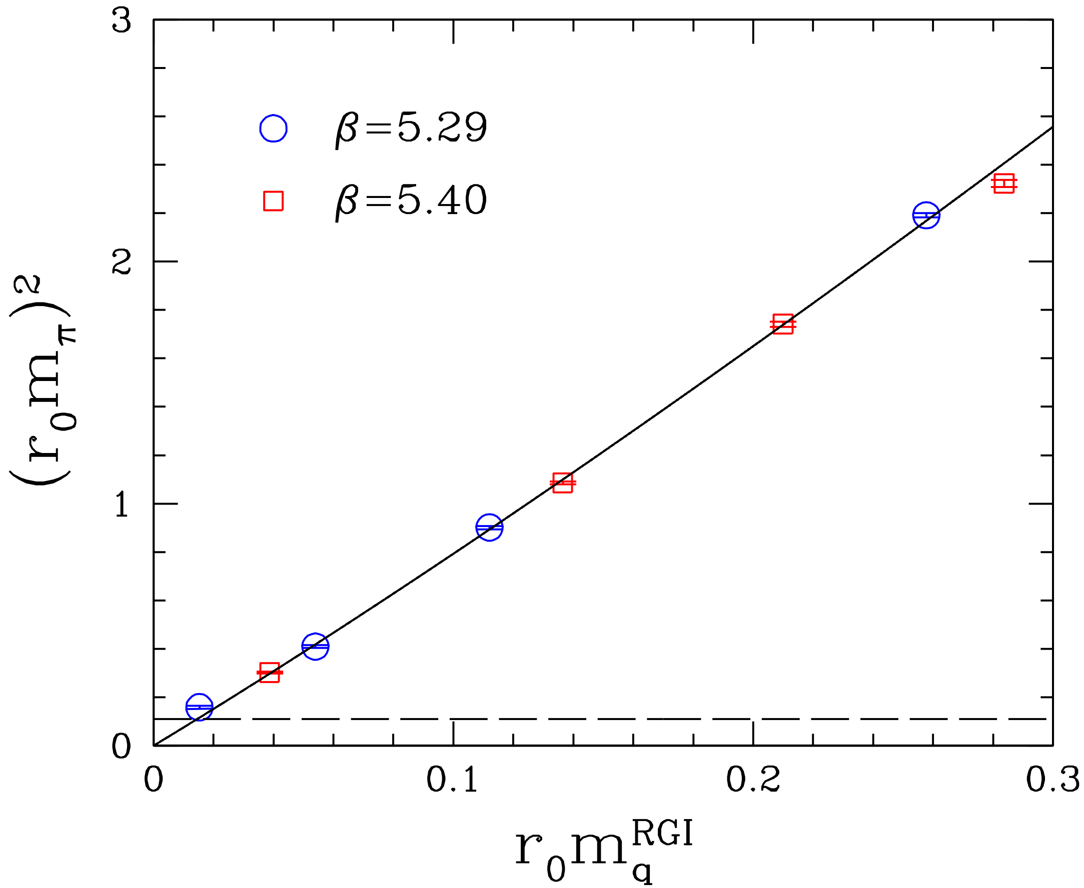
Finite Size



$$m_\pi(L) - m_\pi \overset{\text{LO}}{=} \frac{m_\pi^3}{16\pi^2 f_\pi^2} \sum_{\vec{n} \neq 0} \frac{K_1(m_\pi |\vec{n}| L)}{m_\pi |\vec{n}| L} \overset{m_\pi L \ll 1}{\simeq} \frac{m_\pi^3}{16\pi^2 f_\pi^2} \int_0^\infty d\nu \, 4\pi \nu^2 \, \frac{K_1(m_\pi \nu L)}{m_\pi \nu L}$$

$$= \frac{1}{8 f_\pi^2 L^3}$$

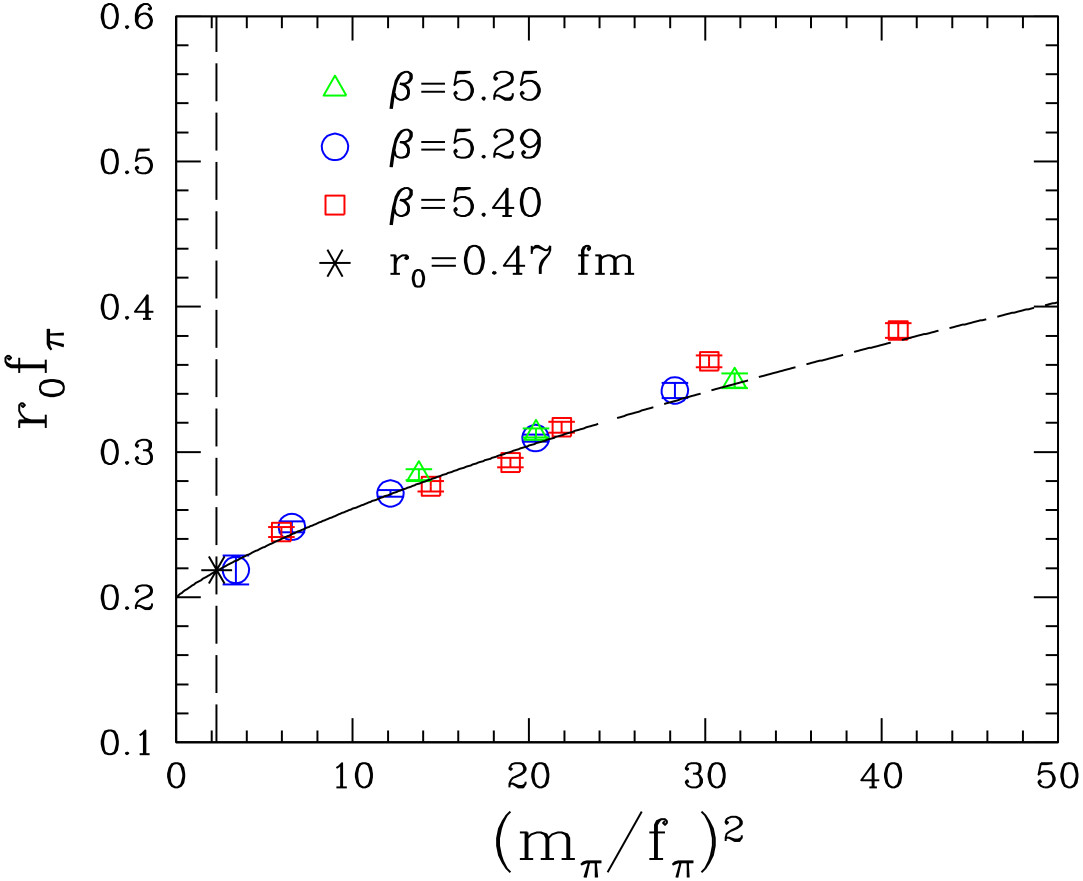
Infinite Volume



$$m_\pi^2 = 2B_0 m_q \left[1 - \frac{1}{2}\xi\, l_3 + O(\xi^2)\right] , \quad \xi = \frac{2B_0 m_q}{16\pi^2 f_0^2}$$

$$\bar{l}_3 = 5.0(3)$$

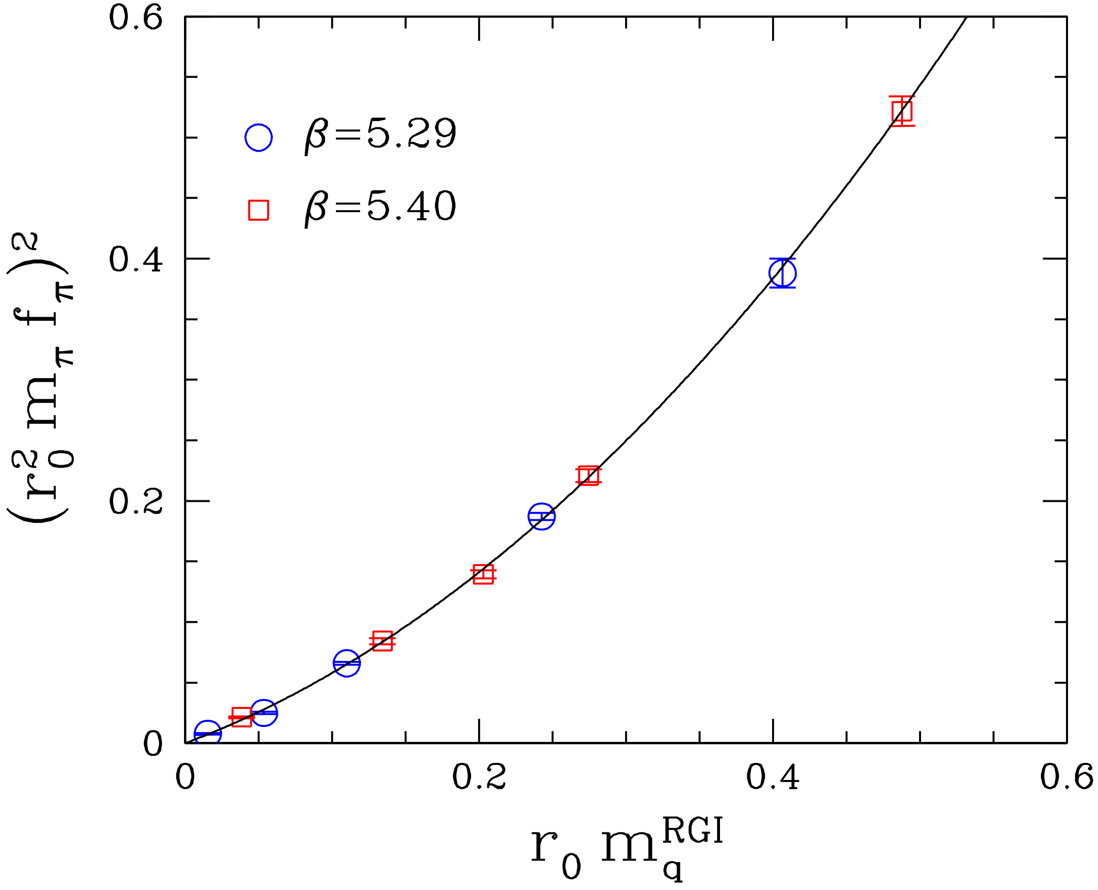
Pion Decay Constant



$$f_\pi^2 = 2B_0 m_q \left[1 + \xi\, l_4 + O(\xi^2)\right]\,, \quad \xi = \frac{2B_0 m_q}{16\pi^2 f_0^2}$$

$\bar{l}_4 = 4.58(12)$

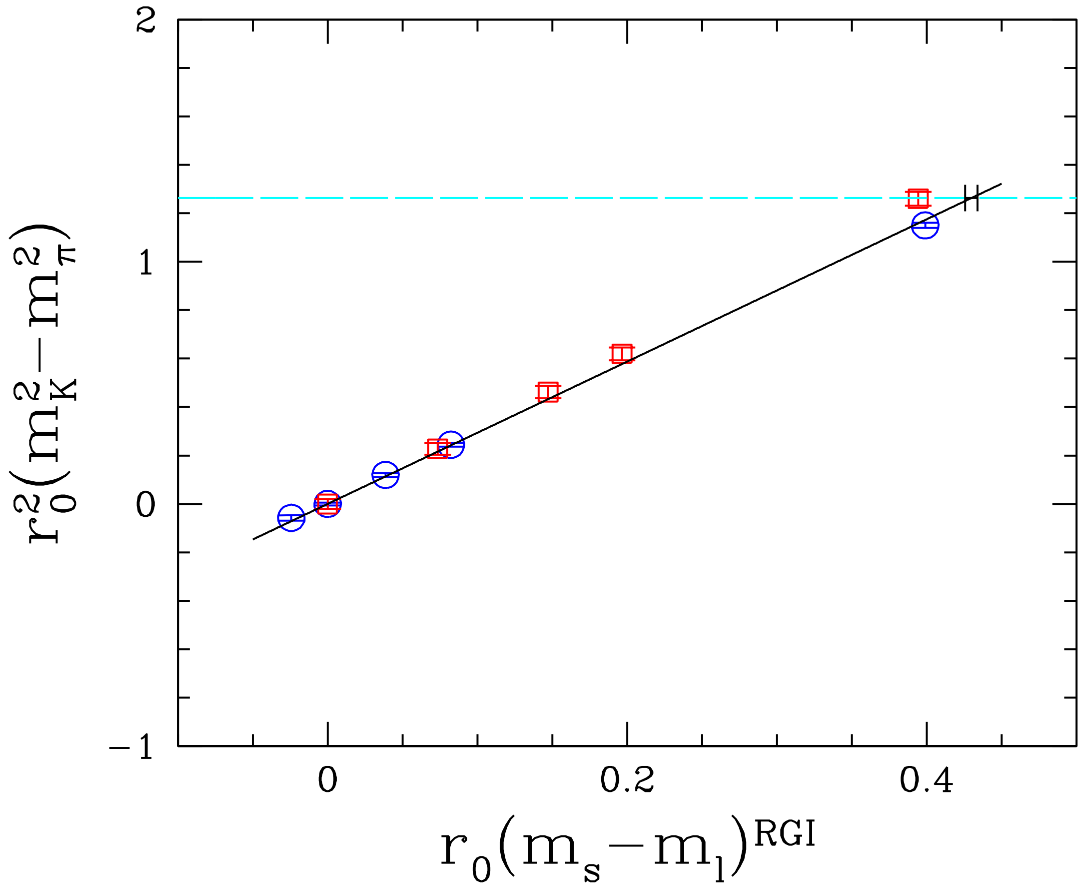
Chiral Condensate



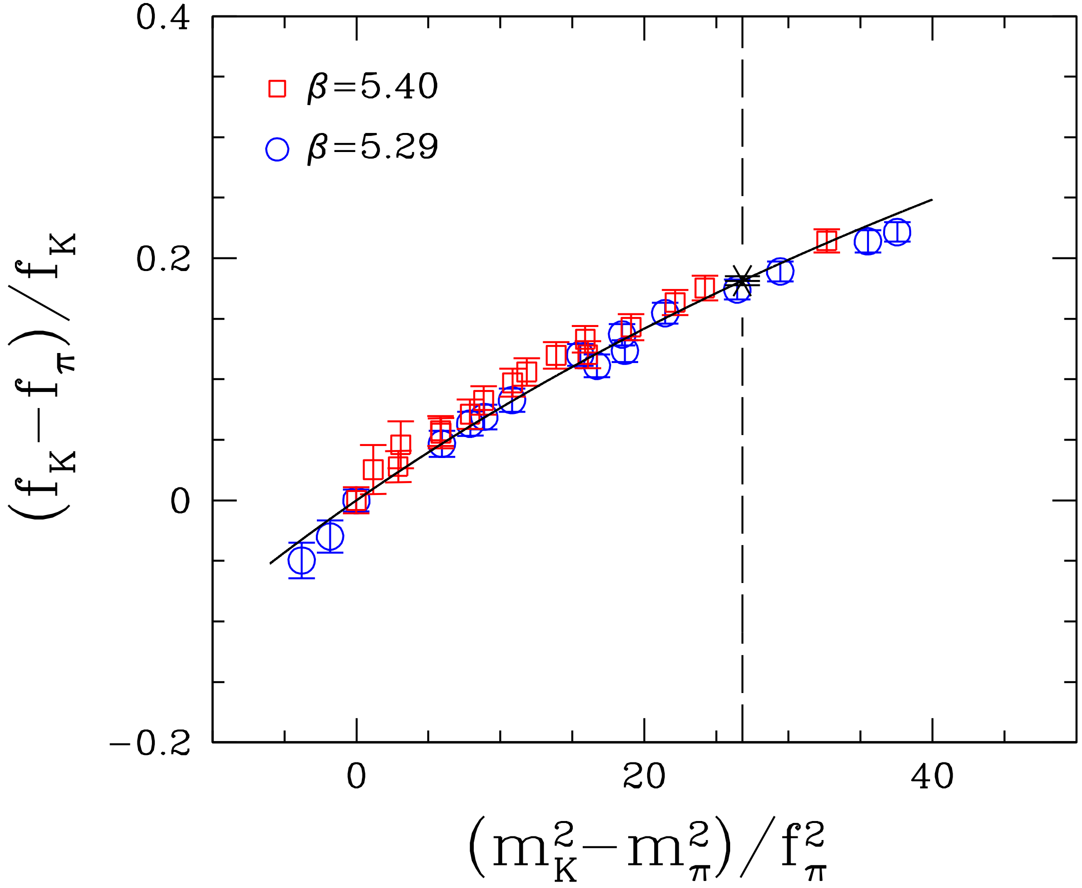
GOR: $m_\pi^2\, f_\pi^2 = 2\,\Sigma\, m_q$

$\Sigma^{\overline{MS}}(2\ \text{GeV}) = (289(5)\ \text{MeV})^3$

Kaon Mass



Kaon Decay Constant

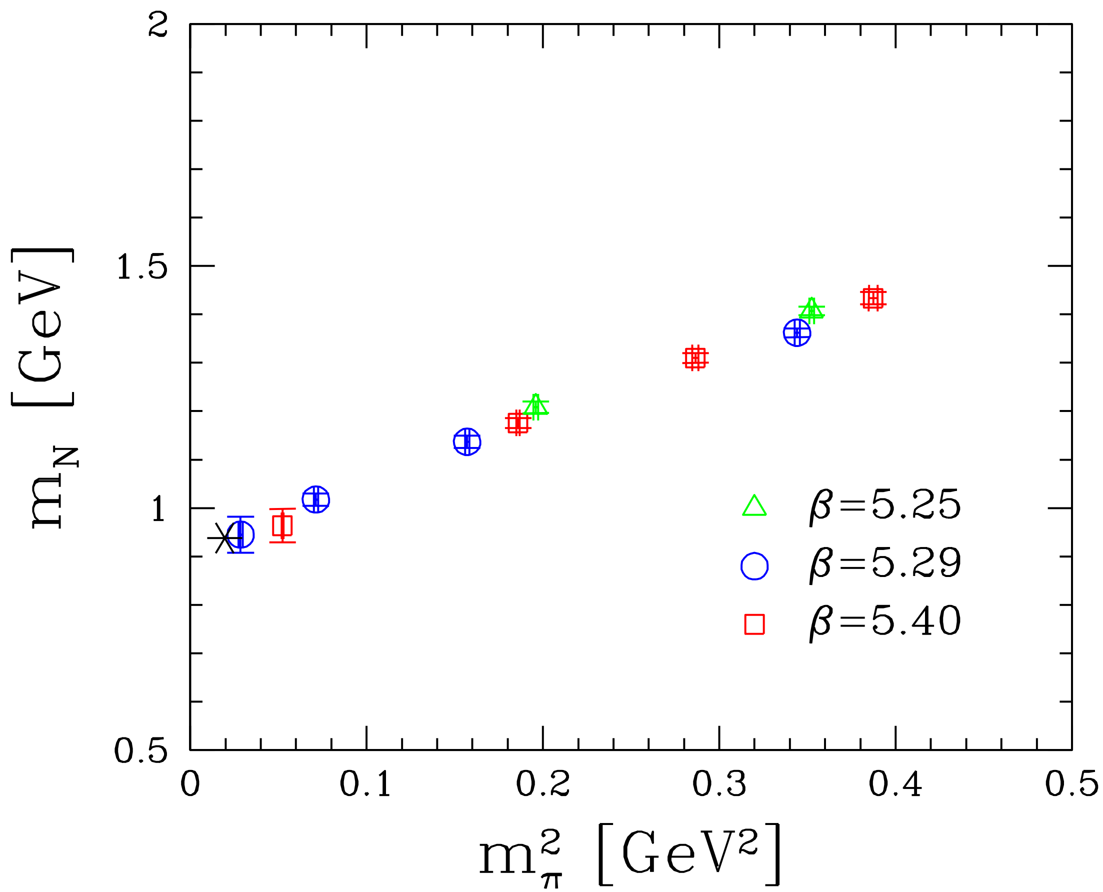


$$\frac{f_K - f_\pi}{f_K} = \frac{4L_5(m_K^2 - m_\pi^2)/f_\pi^2}{1 + 4L_5(m_K^2 - m_\pi^2)/f_\pi^2} + \text{logs}$$

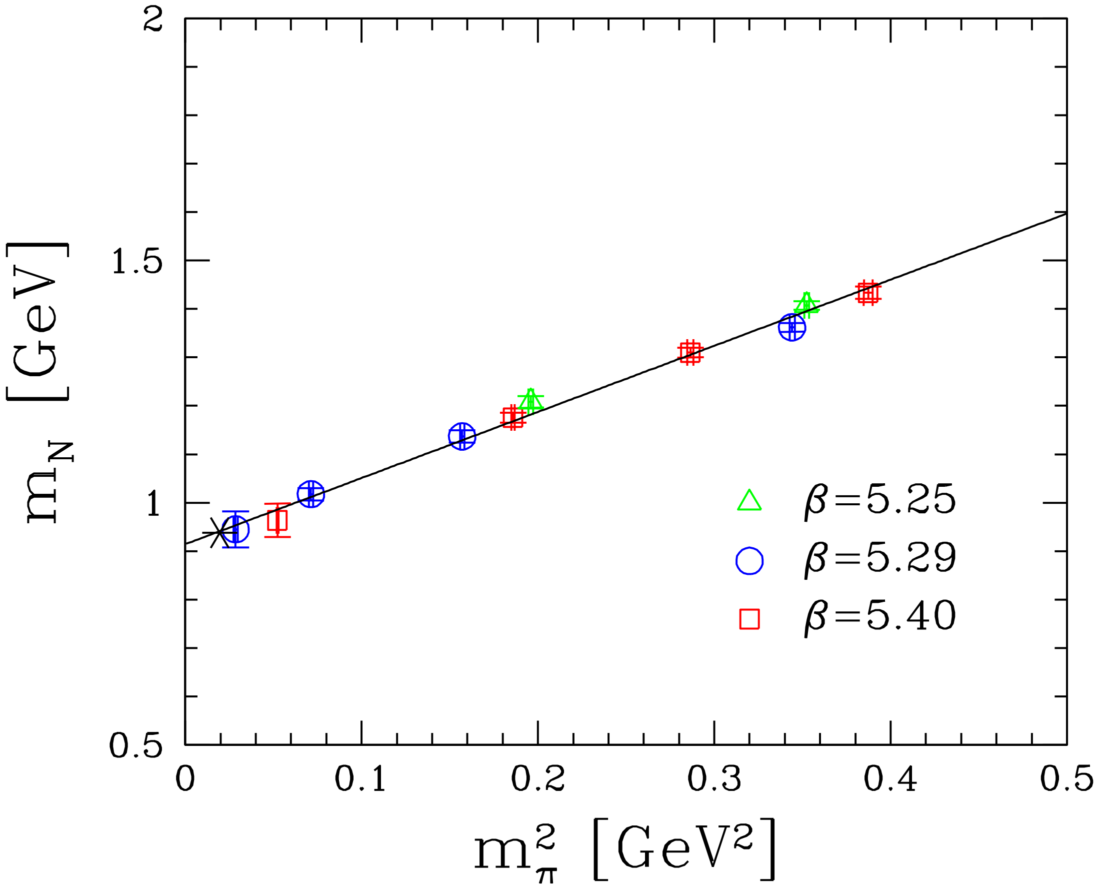
$$f_K/f_\pi = 1.222(6)\,, \quad L_5 = 0.00207(3)$$

Nucleon

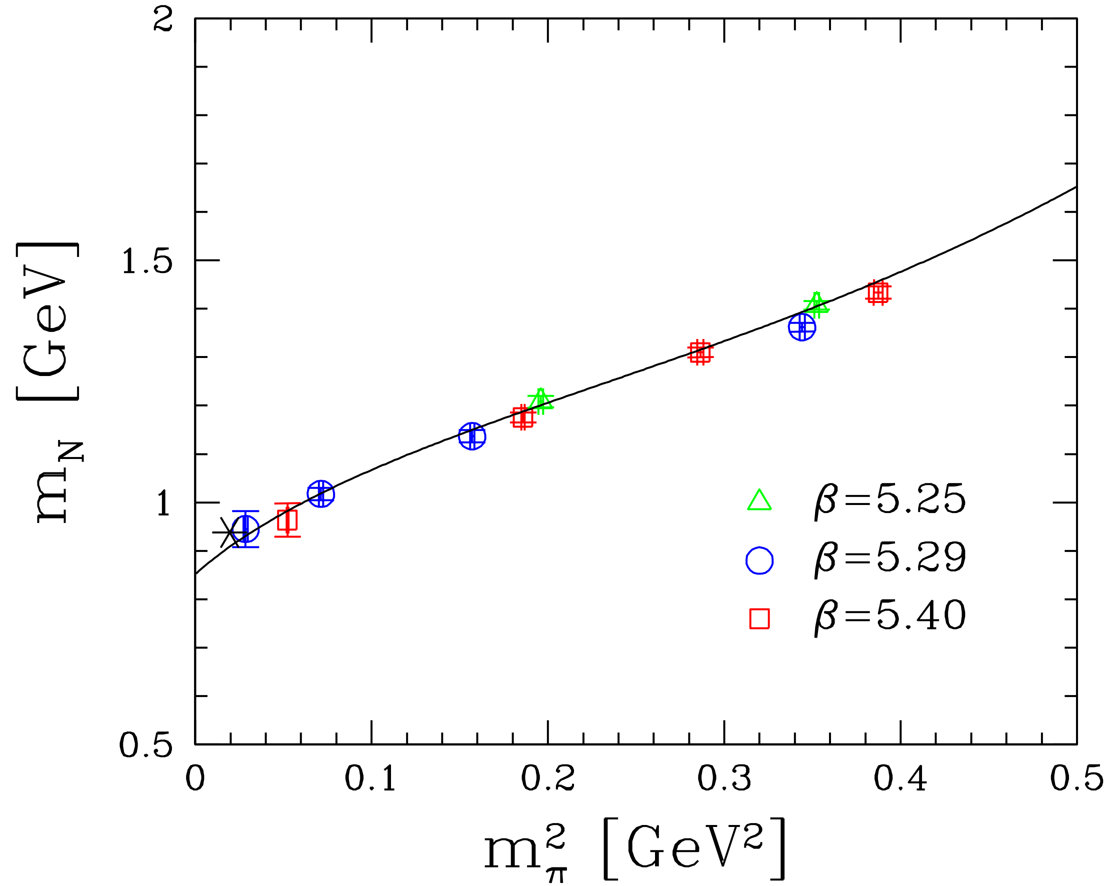
Nucleon Mass



Linear Fit

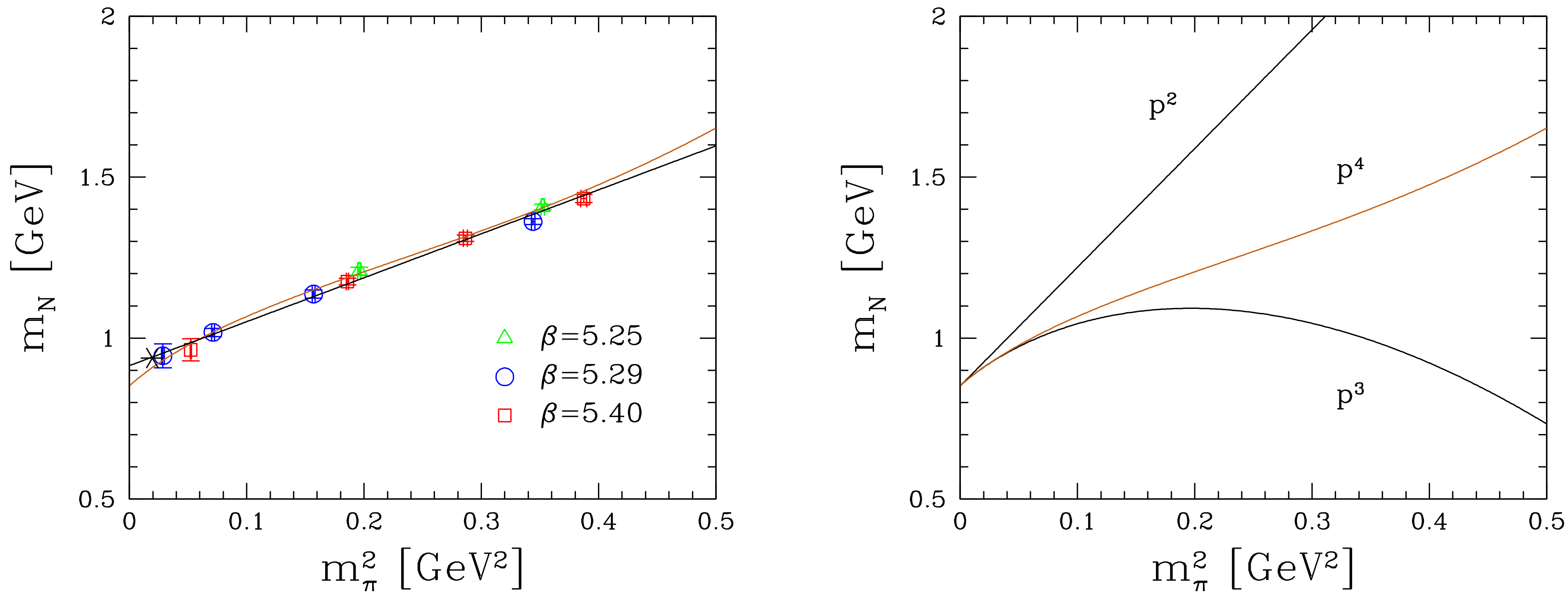


HBChPT



c_2, c_3 fixed to phenomenological values

Use of HBChPT ?



$$m_N = m_0 - 4c_1 m_\pi^2 - \frac{3 g_A^{0\,2}}{32\pi f_0^2} m_\pi^3 + \left[e_1(\mu) - \frac{3}{64\pi^2 f_0^2}\left(\frac{g_A^{0\,2}}{m_0} - \frac{c_2}{2}\right) - \frac{3 g_A^{0\,2}}{32\pi^2 f_0^2}\left(\frac{g_A^{0\,2}}{m_0} - 8c_1 + c_2 + 4c_3\right) \ln\frac{m_\pi}{\mu}\right] m_\pi^4 + \frac{3 g_A^{0\,2}}{256\pi f_0^2 m_0^2} m_\pi^5 + O(m_\pi^6)$$

Procura et al.

$$m_N - m_N(L) = -\frac{3 g_A^{0\,2} m_0\, m_\pi^2}{16\pi^2 f_0^2} \sum_{|\vec{n}|\neq 0} \int_0^\infty dz K_0\left(\sqrt{m_0^2 z^2/m_\pi^2 + (1-z)}\,\lambda\right) - \frac{3 m_\pi^4}{4\pi^2 f_0^2} \sum_{|\vec{n}|\neq 0} \left[(2c_1 - c_3)\frac{K_1(\lambda)}{\lambda} + c_2 \frac{K_2(\lambda)}{\lambda^2}\right] + O(m_\pi^5)$$

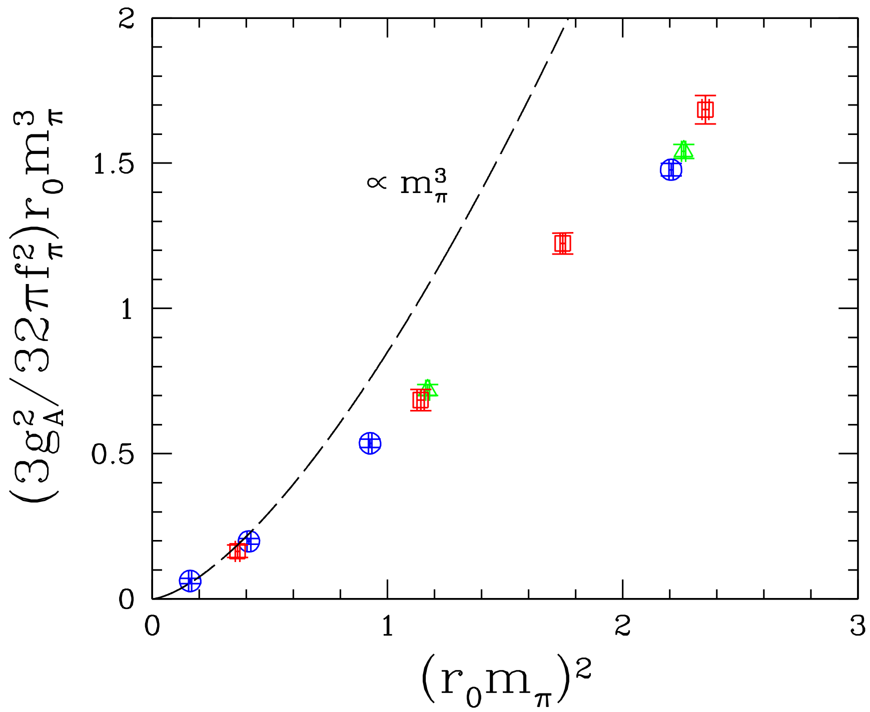
$$d_{00}^+ = -\frac{m_\pi^2}{f_\pi^2}\,(2c_1 - c_3) + O(m_\pi^3) \qquad \lambda = m_\pi |\vec{n}| L$$

$$d_{10}^+ = \frac{2}{f_\pi^2}\, c_2 + O(m_\pi)$$

QCDSF

Compatible ?

$$\frac{g_A^{0\,2}}{f_0^2} \longrightarrow \frac{g_A^2}{f_\pi^2} \; : \qquad \frac{g_A^2}{f_\pi^2}\, m_\pi^3 \;\propto\; m_\pi^2$$

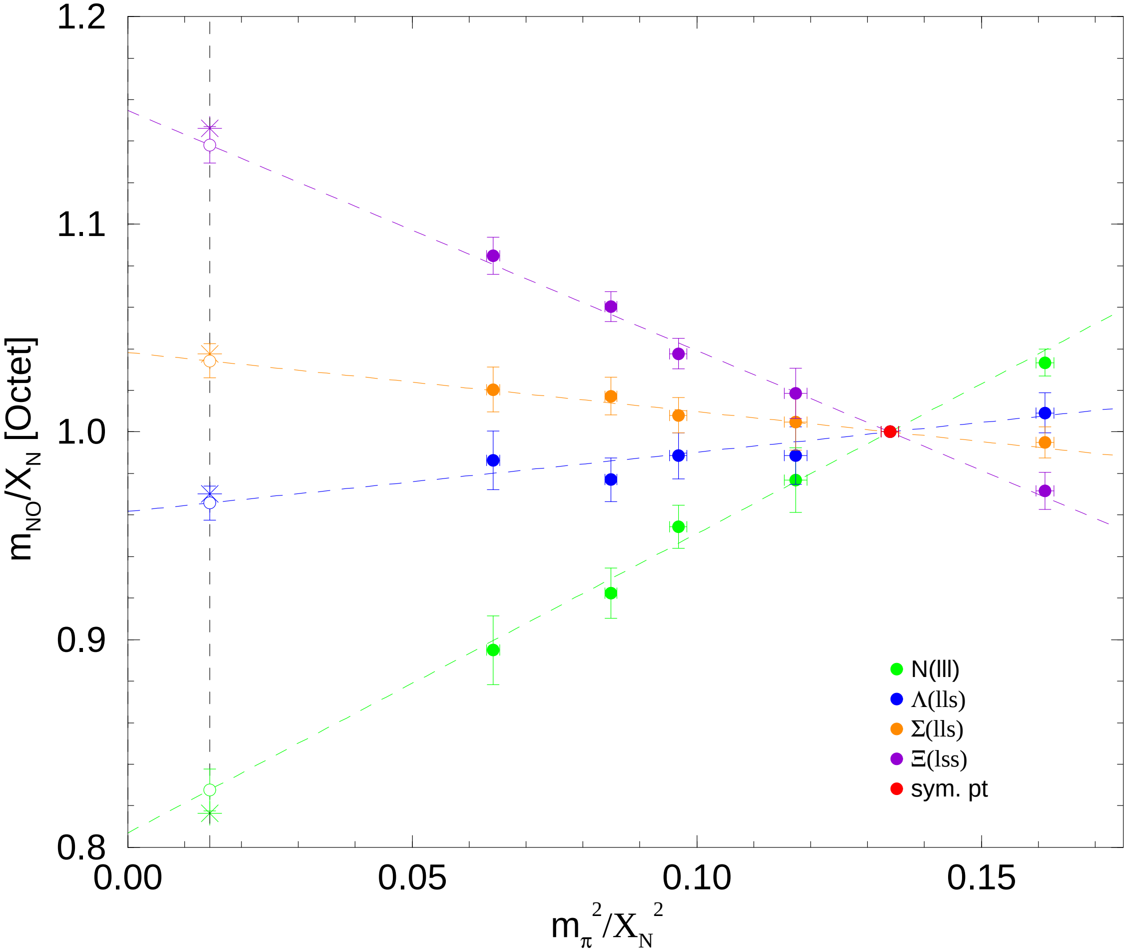


$$\frac{g_A^2}{m_0} = \frac{c_2}{2}$$ √

$$\frac{g_A^2}{m_0} - 8\,c_1 + c_2 + 4\,c_3 = -8\,c_1 + \frac{3}{2}c_2 + 4\,c_3 = 0$$ √

Reduced to finite volume corrections ?

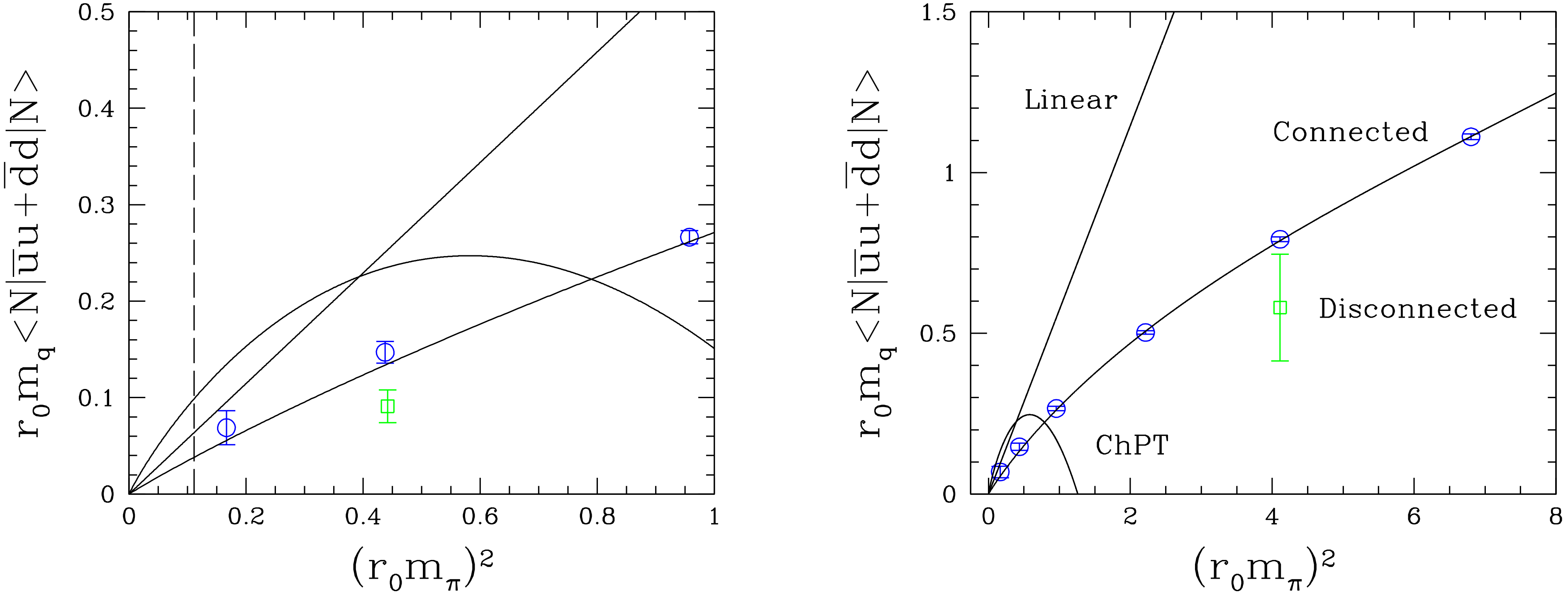
$N_f = 2 + 1$



Roger Horsley's talk

Nucleon Sigma Term

$$\sigma_N = 2m_q \langle N|\bar{q}q|N\rangle \Big|_{m_q=m_q^{\rm phys}} \qquad 2m_q \langle N|\bar{q}q|N\rangle = m_q \frac{d\,m_N(m_q)}{d\,m_q} \stackrel{!}{=} m_\pi^2 \frac{d\,m_N(m_\pi)}{d\,m_\pi^2}$$



Collins

Linear (LO) $\sigma_N = 26.8 \pm 0.3 \pm 0.6$ MeV

HBChPT $\sigma_N = 50.5 \pm 1.4 \pm 1.1$ MeV

Connected $\sigma_N = 27.4 \pm 0.3 \pm 0.6$ MeV

Summary

- Meson sector in broad agreement with the predictions of ChPT, including finite volume effects

- Use of HBChPT dodgy, though observed finite volume effects tend to be well reproduced. Further tests are needed

r_0	$=$	0.47(1) fm	
$\bar{l}_3$	$=$	5.0(3)	
$\bar{l}_4$	$=$	4.58(12)	
L_5	$=$	0.00207(3)	
f_K/f_π	$=$	1.222(6)	
$\Sigma^{\overline{MS}}(2\,\text{GeV})$	$=$	$(289 \pm 5 \pm 6\,\text{MeV})^3$	
σ_N	$=$	$26.8 \pm 0.3 \pm 0.6\,\text{MeV}$	From linear fit
σ_N	$=$	$50.5 \pm 1.4 \pm 1.1\,\text{MeV}$	From chiral fit
σ_N	$=$	$27.4 \pm 0.3 \pm 0.6\,\text{MeV}$	Connected contribution $2m_q\langle N\|\bar{q}q\|N\rangle$
c_2	$\approx$	g_A^2/m_0	
c_3	$\approx$	$2c_1 - 3/8\,c_2$	