# Nucleon structure from RBC/UKQCD 2+1 flavor DWF dynamical ensembles at a nearly physical pion mass

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Talk at Lattice 2010, Sardegna, Italy, June 14-20, 2010

RBC and UKQCD collaborations are generating new dynamical DWF ensembles:

- Iwasaki + dislocation suppressing determinant ratio (DSDR) gauge action, $\beta = 1.75$, and
- Domain-Wall Fermions (DWF) quarks, $L_s = 32$ and $M_5 = 1.8$,
- $a^{-1} \sim 1.368(7)$ GeV: $m_{\rm strange}a = 0.045$, $m_{\rm ud}a = 0.0042$ and 0.001.

Much closer to physical pion mass than the previous set of Iwasaki+DWF ensembles:

- $m_\pi \sim 180$ and 250 MeV, with large volume, $(\sim 4.6\text{fm})^3$ $(32^3 \times 64)$.

Here we report the current status of our nucleon calculations, by

- Meifeng Lin, Yasumichi Aoki, Tom Blum, Chris Dawson, Taku Izubuchi, Chulwoo Jung, SO, Shoichi Sasaki, Takeshi Yamazaki, ...

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Nucleon form factors, measured in elastic scatterings or $\beta$ decay or muon capture:

$$\langle p|V_\mu^+(x)|n\rangle = \bar{u}_p\left[\gamma_\mu F_V(q^2) + \frac{\sigma_{\mu\lambda}q_\lambda}{2m_N}F_T(q^2)\right]u_n e^{iq\cdot x},$$

$$\langle p|A_\mu^+(x)|n\rangle = \bar{u}_p\left[\gamma_\mu\gamma_5 F_A(q^2) + iq_\mu\gamma_5 F_P(q^2)\right]u_n e^{iq\cdot x}.$$

$$F_V = F_1, F_T = F_2; G_E(q^2) = F_1 - \frac{q^2}{4m_N^2}F_2, G_M = F_1 + F_2.$$

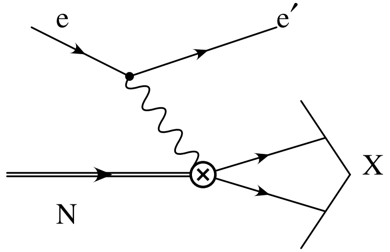
Related to mean-squared charge radius, magnetic moment, $g_V = F_V(0) = G_{\rm Fermi}\cos\theta_{\rm Cabibbo}$, $g_A = F_A(0) = 1.2694(28)g_V$, Goldberger-Treiman relation, $m_N g_A \propto f_\pi g_{\pi NN}$, ... determine much of nuclear physics.

On the lattice, with appropriate nucleon operator, for example, $N = \epsilon_{abc}(u_a^T C\gamma_5 d_b)u_c$, ratio of two- and three-point correlators such as $\dfrac{C_{\rm 3pt}^{\Gamma,O}(t_{\rm sink},t)}{C_{\rm 2pt}(t_{\rm sink})}$ with

$$C_{\rm 2pt}(t_{\rm sink}) = \sum_{\alpha,\beta}\left(\frac{1+\gamma_t}{2}\right)_{\alpha\beta}\langle N_\beta(t_{\rm sink})\bar{N}_\alpha(0)\rangle,$$

$$C_{\rm 3pt}^{\Gamma,O}(t_{\rm sink},t) = \sum_{\alpha,\beta}\Gamma_{\alpha\beta}\langle N_\beta(t_{\rm sink})O(t)\bar{N}_\alpha(0)\rangle,$$

give a plateau in $t$ for a lattice bare value $\langle O\rangle$ for the relevant observable, with appropriate spin ($\Gamma = (1+\gamma_t)/2$ or $(1+\gamma_t)i\gamma_5\gamma_k/2$) or momentum-transfer (if any) projections.

Deep inelastic scatterings 

$: \left|\frac{\mathcal{A}}{4\pi}\right|^2 = \frac{\alpha^2}{Q^4} l^{\mu\nu} W_{\mu\nu},\ W^{\mu\nu} = W^{[\mu\nu]} + W^{\{\mu\nu\}}$

- unpolarized: $W^{\{\mu\nu\}}(x,Q^2) = \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) F_1(x,Q^2) + \left(P^\mu - \frac{\nu}{q^2}q^\mu\right)\left(P^\nu - \frac{\nu}{q^2}q^\nu\right)\frac{F_2(x,Q^2)}{\nu}$,
- polarized: $W^{[\mu\nu]}(x,Q^2) = i\epsilon^{\mu\nu\rho\sigma} q_\rho \left(\frac{S_\sigma}{\nu}(g_1(x,Q^2) + g_2(x,Q^2)) - \frac{q\cdot S P_\sigma}{\nu^2} g_2(x,Q^2)\right)$,

with $\nu = q\cdot P$, $S^2 = -M^2$, $x = Q^2/2\nu$.

Moments of the structure functions are accessible on the lattice:

$$\begin{aligned}
2\int_0^1 dx x^{n-1} F_1(x,Q^2) &= \sum_{q=u,d} c_{1,n}^{(q)}(\mu^2/Q^2, g(\mu))\, \langle x^n\rangle_q(\mu) + \mathcal{O}(1/Q^2),\\
\int_0^1 dx x^{n-2} F_2(x,Q^2) &= \sum_{f=u,d} c_{2,n}^{(q)}(\mu^2/Q^2, g(\mu))\, \langle x^n\rangle_q(\mu) + \mathcal{O}(1/Q^2),\\
2\int_0^1 dx x^{n} g_1(x,Q^2) &= \sum_{q=u,d} e_{1,n}^{(q)}(\mu^2/Q^2, g(\mu))\, \langle x^n\rangle_{\Delta q}(\mu) + \mathcal{O}(1/Q^2),\\
2\int_0^1 dx x^{n} g_2(x,Q^2) &= \frac{1}{2}\frac{n}{n+1}\sum_{q=u,d}\left[e_{2,n}^{q}(\mu^2/Q^2, g(\mu))\, d_n^q(\mu) - 2e_{1,n}^{q}(\mu^2/Q^2, g(\mu))\, \langle x^n\rangle_{\Delta q}(\mu)\right] + \mathcal{O}(1/Q^2)
\end{aligned}$$

- $c_1$, $c_2$, $e_1$, and $e_2$ are the Wilson coefficients (perturbative),
- $\langle x^n\rangle_q(\mu)$, $\langle x^n\rangle_{\Delta q}(\mu)$ and $d_n(\mu)$ are forward nucleon matrix elements of certain local operators,
- so is $\langle 1\rangle_{\delta q}(\mu) = \langle P,S|\bar\psi i\gamma_5\sigma_{\mu\nu}\psi|P,S\rangle$ which may be measured by polarized Drell-Yan and RHIC Spin.

Unpolarized ($F_1/F_2$): on the lattice we can measure: $\langle x\rangle_q$, $\langle x^2\rangle_q$ and $\langle x^3\rangle_q$.

$$\frac{1}{2}\sum_s \langle P,S|\mathcal{O}^q_{\{\mu_1\mu_2\cdots\mu_n\}}|P,S\rangle = 2\langle x^{n-1}\rangle_q(\mu)[P_{\mu_1}P_{\mu_2}\cdots P_{\mu_n} + \cdots - (\text{trace})]$$

$$\mathcal{O}^q_{\mu_1\mu_2\cdots\mu_n} = \bar{q}\left[\left(\frac{i}{2}\right)^{n-1}\gamma_{\mu_1}\overset{\leftrightarrow}{D}_{\mu_2}\cdots\overset{\leftrightarrow}{D}_{\mu_n} - (\text{trace})\right]q$$

Polarized ($g_1/g_2$): on the lattice we can measure: $\langle 1\rangle_{\Delta q}$ ($g_A$), $\langle x\rangle_{\Delta q}$, $\langle x^2\rangle_{\Delta q}$, $d_1$, $d_2$, $\langle 1\rangle_{\delta q}$ and $\langle x\rangle_{\delta q}$.

$$-\langle P,S|\mathcal{O}^{5q}_{\{\sigma\mu_1\mu_2\cdots\mu_n\}}|P,S\rangle = \frac{2}{n+1}\langle x^n\rangle_{\Delta q}(\mu)[S_\sigma P_{\mu_1}P_{\mu_2}\cdots P_{\mu_n} + \cdots - (\text{traces})]$$

$$\mathcal{O}^{5q}_{\sigma\mu_1\mu_2\cdots\mu_n} = \bar{q}\left[\left(\frac{i}{2}\right)^{n}\gamma_5\gamma_\sigma\overset{\leftrightarrow}{D}_{\mu_1}\cdots\overset{\leftrightarrow}{D}_{\mu_n} - (\text{traces})\right]q$$

$$\langle P,S|\mathcal{O}^{[5]q}_{[\sigma\{\mu_1]\mu_2\cdots\mu_n\}}|P,S\rangle = \frac{1}{n+1}d^q_n(\mu)[(S_\sigma P_{\mu_1} - S_{\mu_1}P_\sigma)P_{\mu_2}\cdots P_{\mu_n} + \cdots - (\text{traces})]$$

$$\mathcal{O}^{[5]q}_{[\sigma\mu_1]\mu_2\cdots\mu_n} = \bar{q}\left[\left(\frac{i}{2}\right)^{n}\gamma_5\gamma_{[\sigma}\overset{\leftrightarrow}{D}_{\mu_1]}\cdots\overset{\leftrightarrow}{D}_{\mu_n} - (\text{traces})\right]q$$

and transversity ($h_1$):

$$\langle P,S|\mathcal{O}^{\sigma q}_{\rho\nu\{\mu_1\mu_2\cdots\mu_n\}}|P,S\rangle = \frac{2}{m_N}\langle x^n\rangle_{\delta q}[(S_\rho P_\nu - S_\nu P_\rho)P_{\mu_1}P_{\mu_2}\cdots P_{\mu_n} + \cdots - (\text{traces})]$$

$$\mathcal{O}^{\sigma q}_{\rho\nu\mu_1\mu_2\cdots\mu_n} = \bar{q}[\left(\frac{i}{2}\right)^{n}\gamma_5\sigma_{\rho\nu}\overset{\leftrightarrow}{D}_{\mu_1}\cdots\overset{\leftrightarrow}{D}_{\mu_n} - (\text{traces})]q$$

Higher moment operators mix with lower dimensional ones: Only $\langle x\rangle_q$, $\langle 1\rangle_{\Delta q}$, $\langle x\rangle_{\Delta q}$, $d_1$, and $\langle 1\rangle_{\delta q}$ can be measured with $\vec{P}=0$.

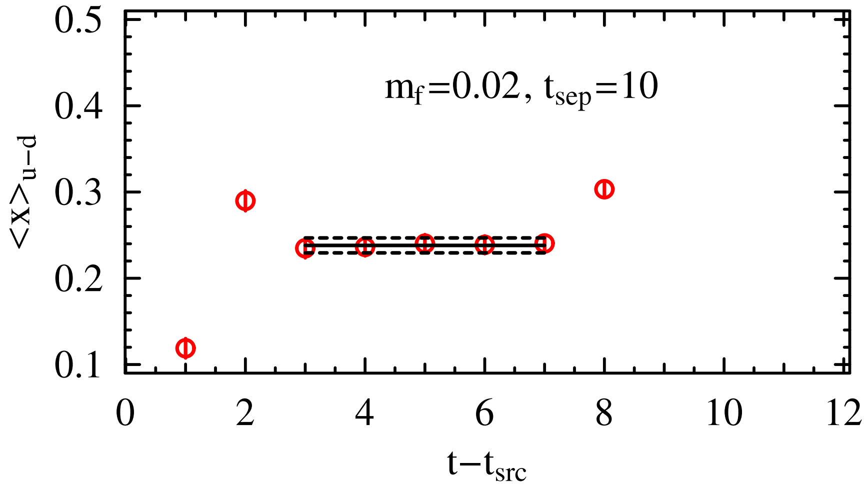
Previous RBC and RBC+UKQCD calculations addressed two important sources of systematics:

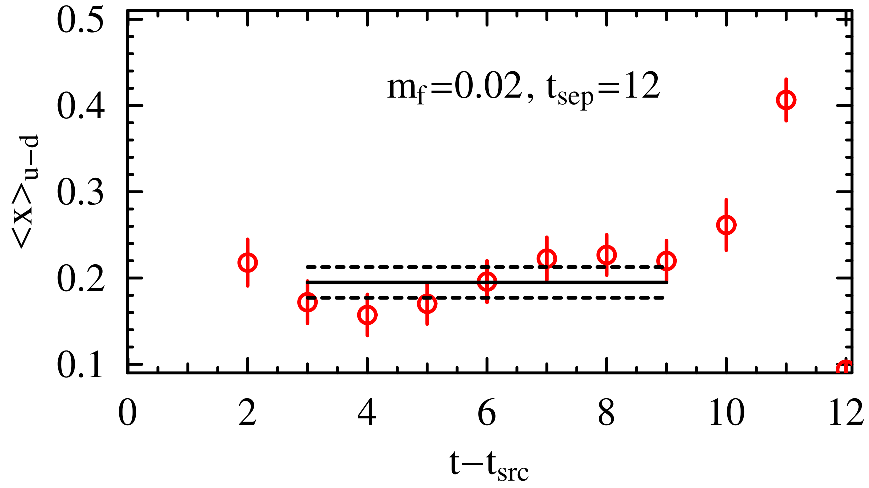
- Time separation between nucleon source and sink,
- Spatial volume.

And though not explicitly addressed yet, a better understanding of quark mass dependence is necessary.

Source/sink time separation:

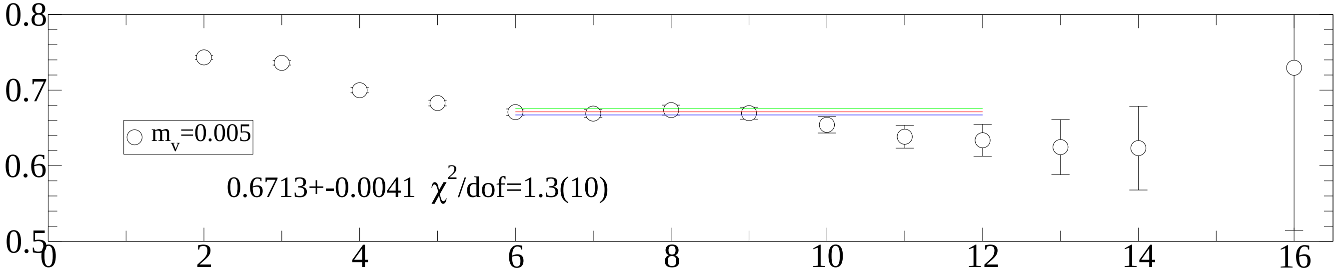
- If too short, too much contamination from excited states, but if too long, the signal is lost.



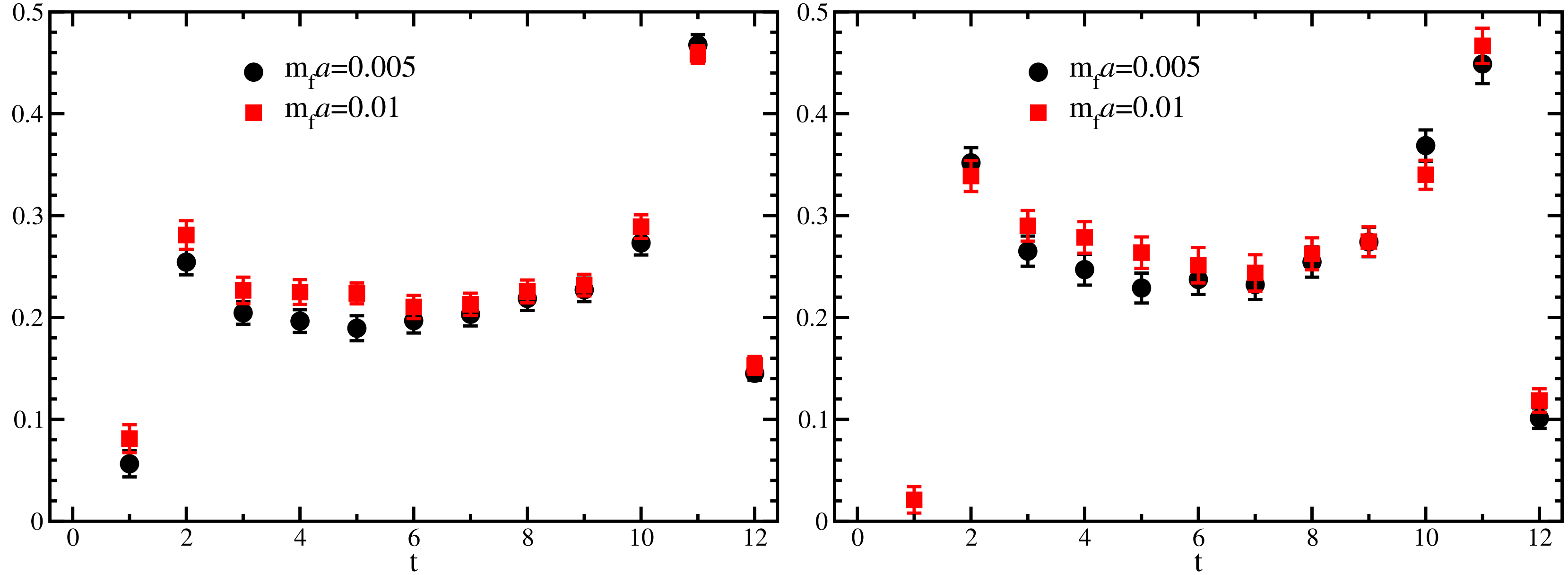




- In an earlier RBC 2-flavor DWF study at $a^{-1} \sim 1.7$ GeV, separation of 10 or 1.1 fm appeared too short.

In the RBC+UKQCD (2+1)-flavor study we choose separation 12 or 13 : ∼1.4 fm:
Mass signal: $m_f = 0.005$

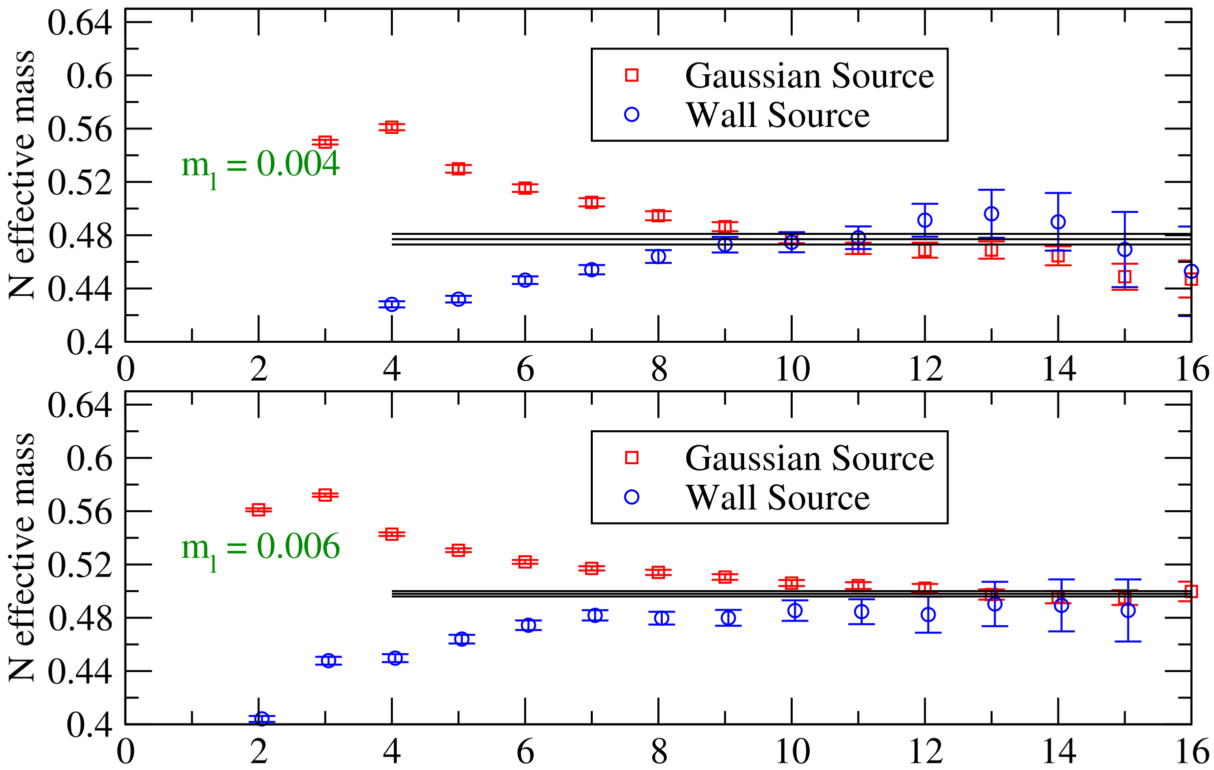



Bare three-point functions: $\langle x\rangle_{u-d}$ (left) and $\langle x\rangle_{\Delta u-\Delta d}$ (right), for $m_f = 0.005$ (red +) and 0.01 (blue ×):




In this study we like to do at least as good, hopefully better: separation of 10 lattice units or longer.

On the other hand, with RBC+UKQCD 2.2-GeV (2+1)-flavor dynamical DWF ensemble:




2-state fits suggest excited-state survives $t_{\rm sink} \geq 9$.

LHP analysis of vector form factors with $t_{\rm sep} = 12$ or 1 fm agree with RBC+UKQCD 1.7-GeV results.

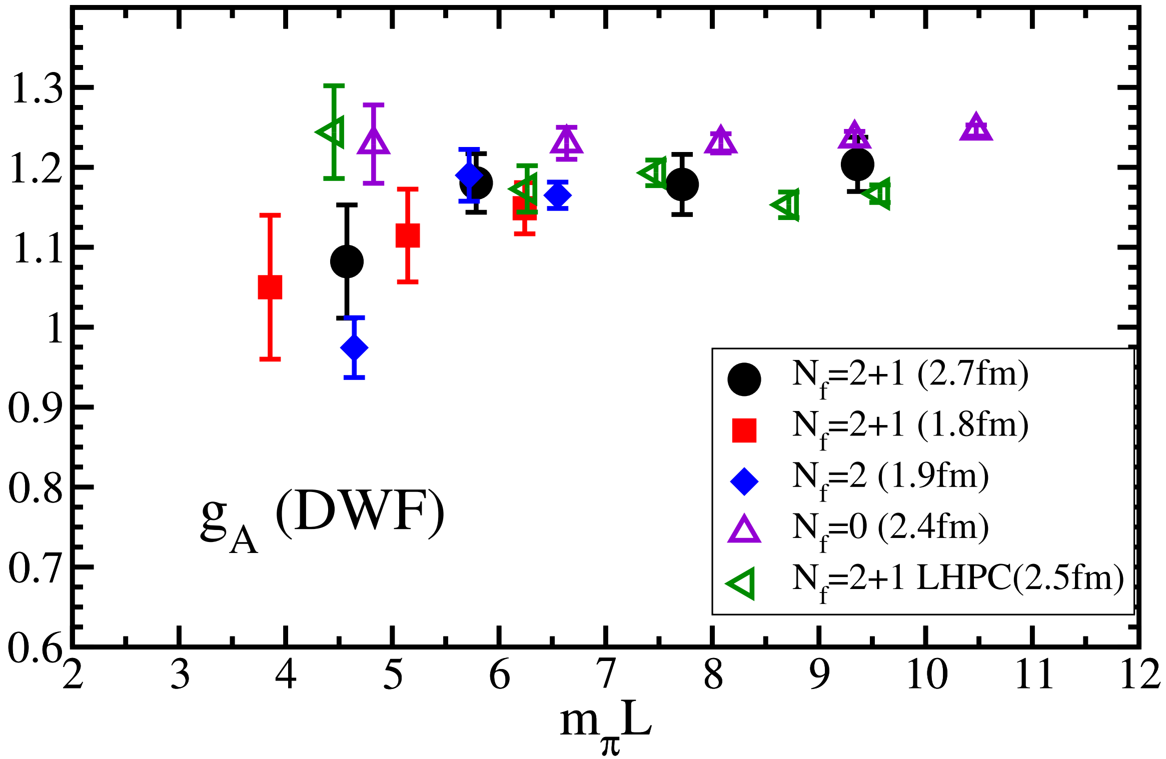
Vector current is less sensitive: conserved charge cannot tell excited-state contamination, for example.

Can we go shorter, ∼1 fm, separation, in spite of our lighter masses?

- Perhaps with better tuned source and sink smearing?
- Would be good as we have to fight growing error, $\sim \exp(-3m_\pi t)$.

Spatial volume. In Lattice 2007 Takeshi Yamazaki reported unexpectedly large finite-size effect:

- in axial charge, $g_A/g_V = 1.2694(28)$, measured in neutron $\beta$ decay, decides neutron life.




Our DWF on quenched and LHPC DWF on MILC calculations are presented for comparison.

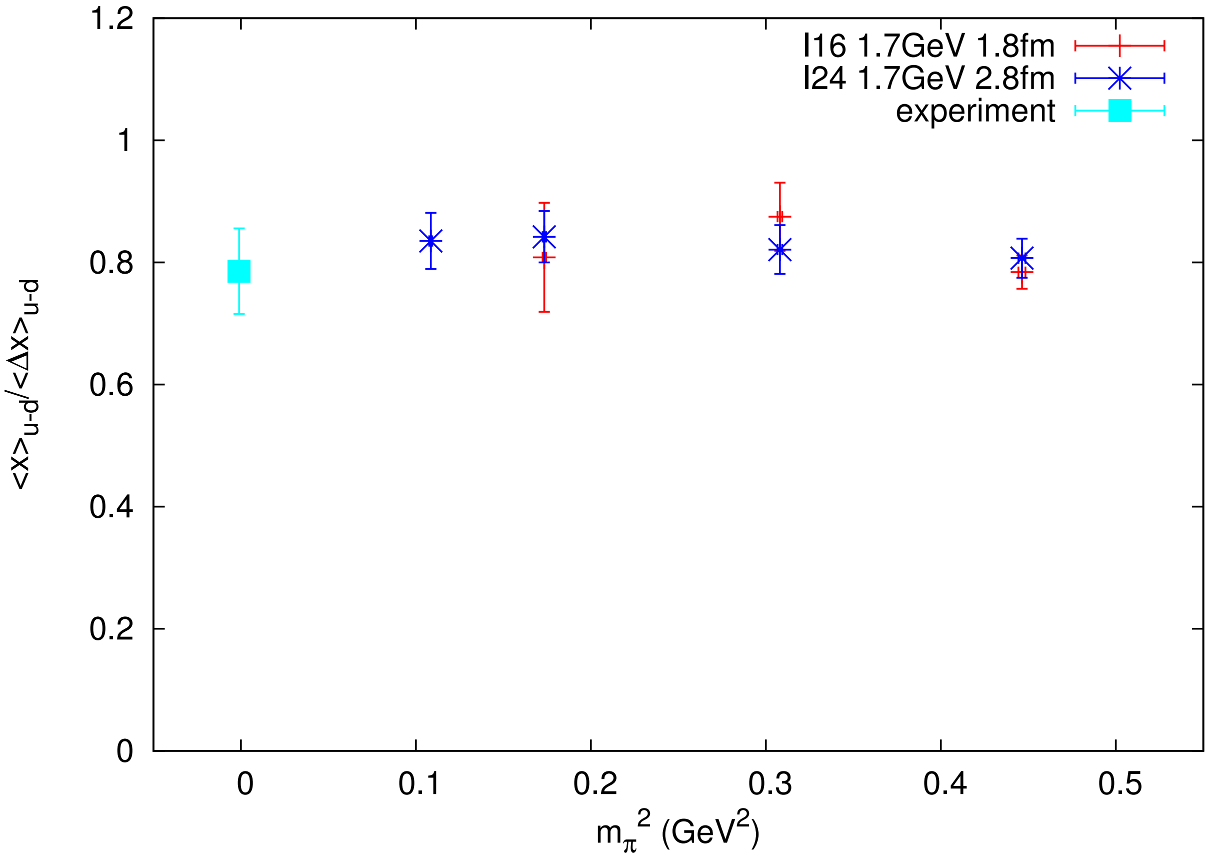
- Heavier quarks: consistent with experiment, no discernible quark-mass dependence.
- Lighter quarks: finite-size sets in as early as $m_\pi L \sim 5$, appear to scale in $m_\pi L$:
  – elastic form factors demand big volumes.

Last year we reported the structure function moments do not seem to suffer so badly, but we need large volume at least for form factors: present $(\sim 4.6\text{fm})^3$ volume is a good start.

RBC/UKQCD (2+1)-flavor, Iwasaki+DWF dynamical, $a^{-1} = 1.73(2)$ GeV, $m_{\rm res} = 0.00315(2)$, $m_{\rm strange} = 0.04$,

- $m_\pi = 0.67$, 0.56, 0.42 and 0.33 GeV; $m_N = 1.55$, 1.39, 1.22 and 1.15 GeV,

Ratio, $\langle x\rangle_{u-d}/\langle x\rangle_{\Delta u-\Delta d}$, of momentum and helicity fractions (naturally renormalized on the lattice),




consistent with experiment, no discernible quark-mass dependence.

No finite-size effect seen, in contrast to $g_A/g_V$ which is also naturally renormalized on the lattice.

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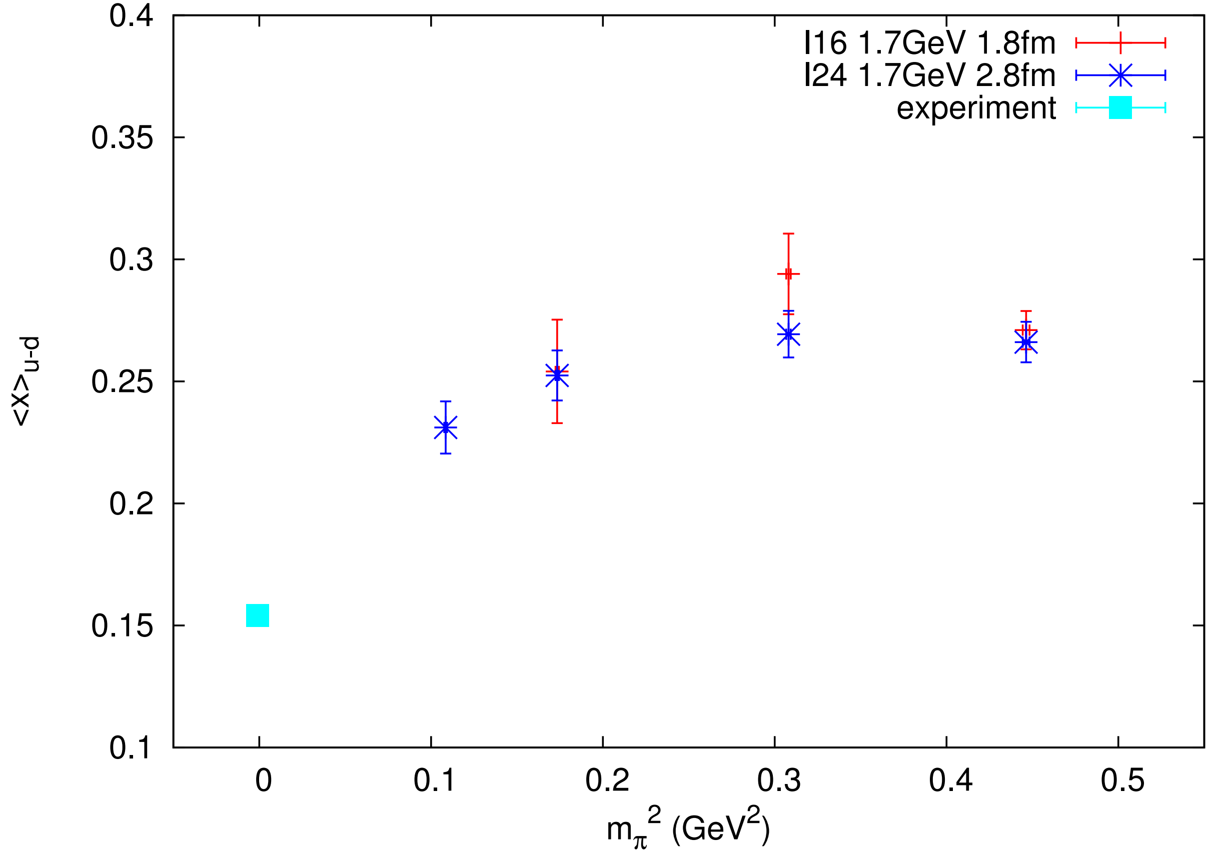
- $m_\pi = 0.67$, 0.56, 0.42 and 0.33 GeV; $m_N = 1.55$, 1.39, 1.22 and 1.15 GeV,

Momentum fraction, $\langle x \rangle_{u-d}$, with NPR, $Z^{\overline{\rm MS}}(2{\rm GeV}) = 1.15(4)$, plotted against $m_\pi^2$,




Absolute values have improved, trending to the experimental values, with NPR, $Z^{\overline{\rm MS}}(2{\rm GeV}) = 1.15(4)$.

No finite size effect seen ($16^3$ (+) and $24^3$ ($\times$) results agree): Likely physical light-quark effect.

A better understanding of quark mass dependence is necessary.

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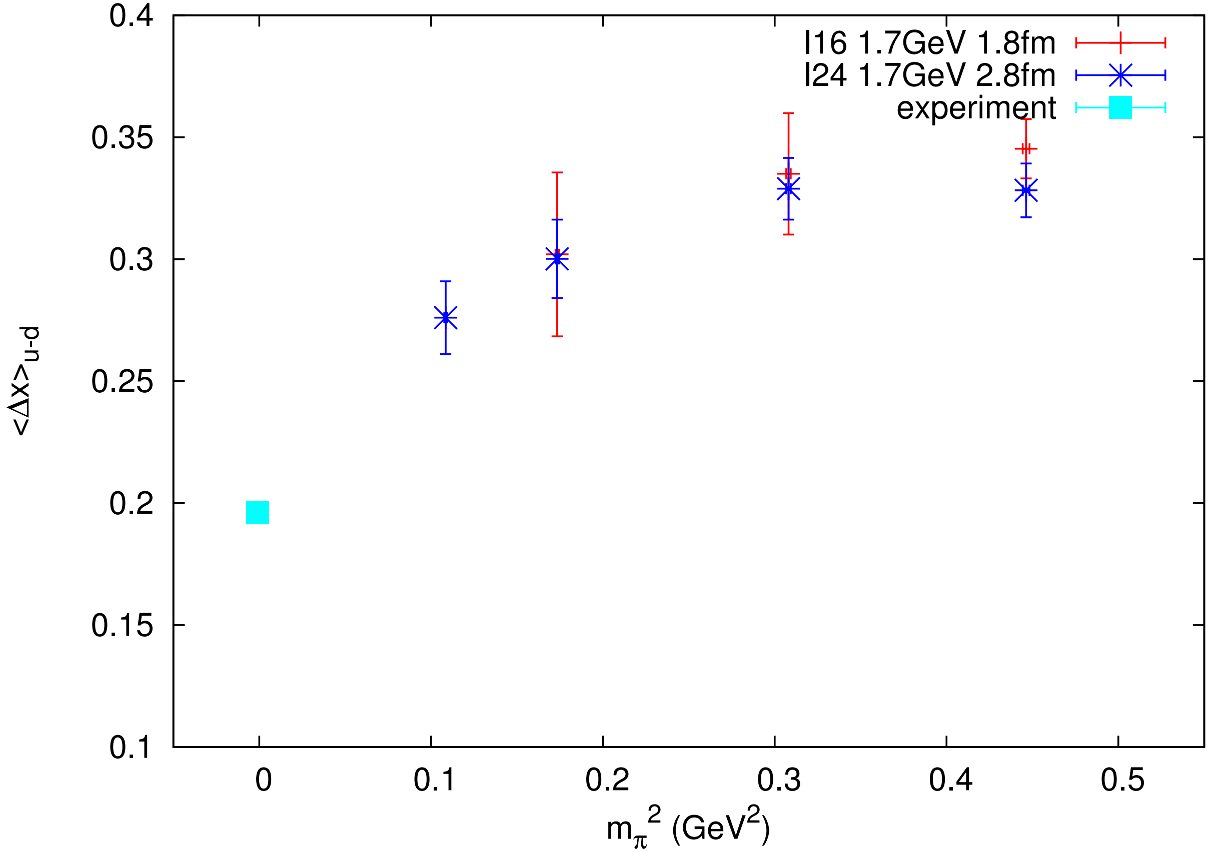
Helicity fraction, $\langle x \rangle_{\Delta u - \Delta d}$, with NPR, $Z^{\overline{\rm MS}(2{\rm GeV})} = 1.15(3)$, plotted against $m_\pi^2$,

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A better understanding of quark mass dependence is necessary.

Now RBC and UKQCD collaborations are jointly generating new (2+1)-flavor DWF ensembles

- with Iwasaki and dislocation-suppressing-determinant-ratio (DSDR) gauge action, $\beta = 1.75$,
- and DWF fermion action, $L_s = 32$ and $M_5 = 1.8$, with $m_{\rm strange} = 0.045$, $m_{\rm ud} = 0.0042$ and 0.001.

We have reasonable topology distribution while maintaining small residual mass:

- lattice scale from $\Omega^-$: $a^{-1} = 1.368(7)$ GeV,
- $m_\pi = 0.1816(8)$ and 0.1267(8), or $\sim$ 250 and 180 MeV.

$32^3 \times 64$ volume is about 4.6 fm across in space, 9.2 fm in time.

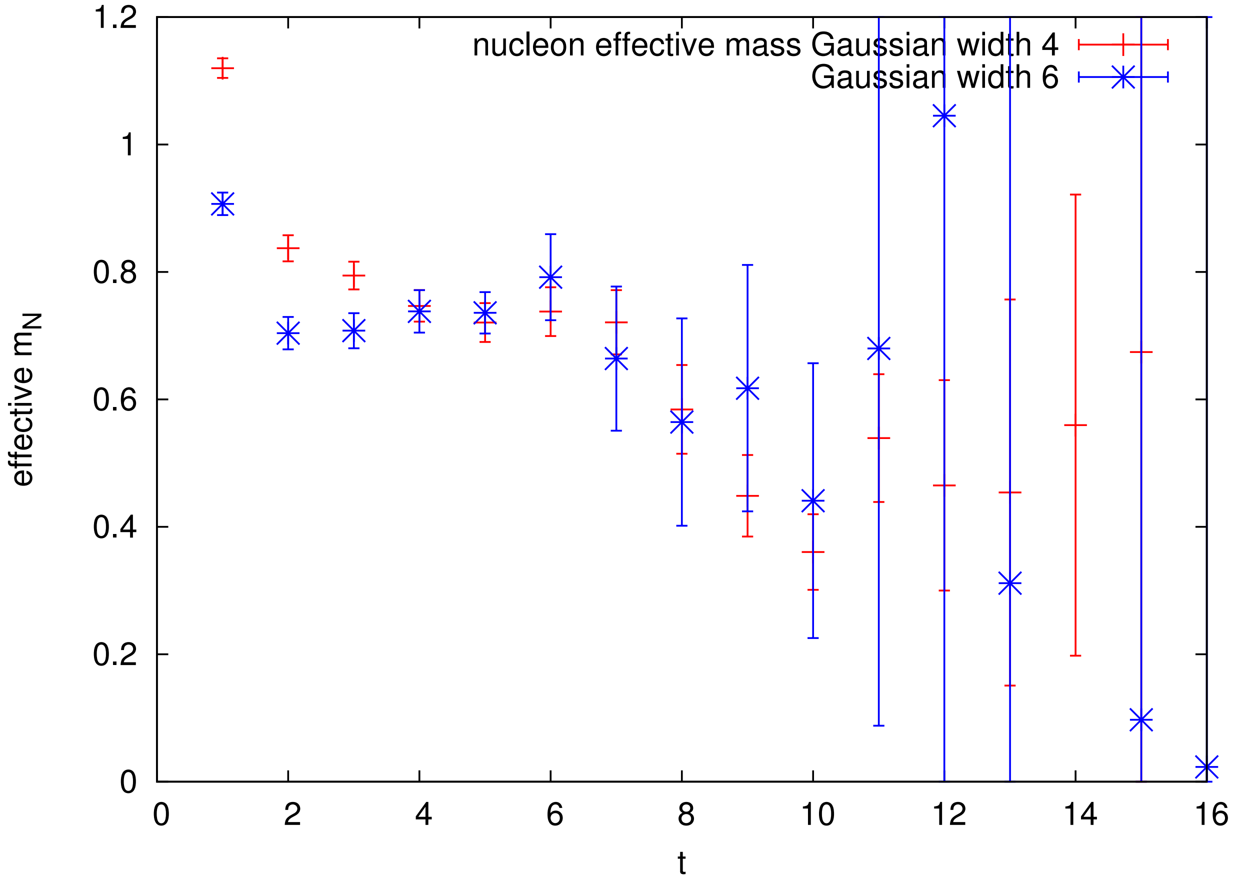
We started nucleon structure calculations using the RICC supercomputing facility at RIKEN, Wako, Japan.

- tuning Gaussian smearing with width 4 and 6,

at this 100-TFlops peak-speed facility.

RBC/UKQCD (2+1)-flavor, ID+DWF dynamical, $a^{-1} = 1.368(7)$ GeV, $m_{\rm strange} = 0.045$,

Nucleon mass signal from the light ($m_{\rm ud} = 0.001$ or $m_\pi = 180$ MeV) ensemble, with ~30 configurations,



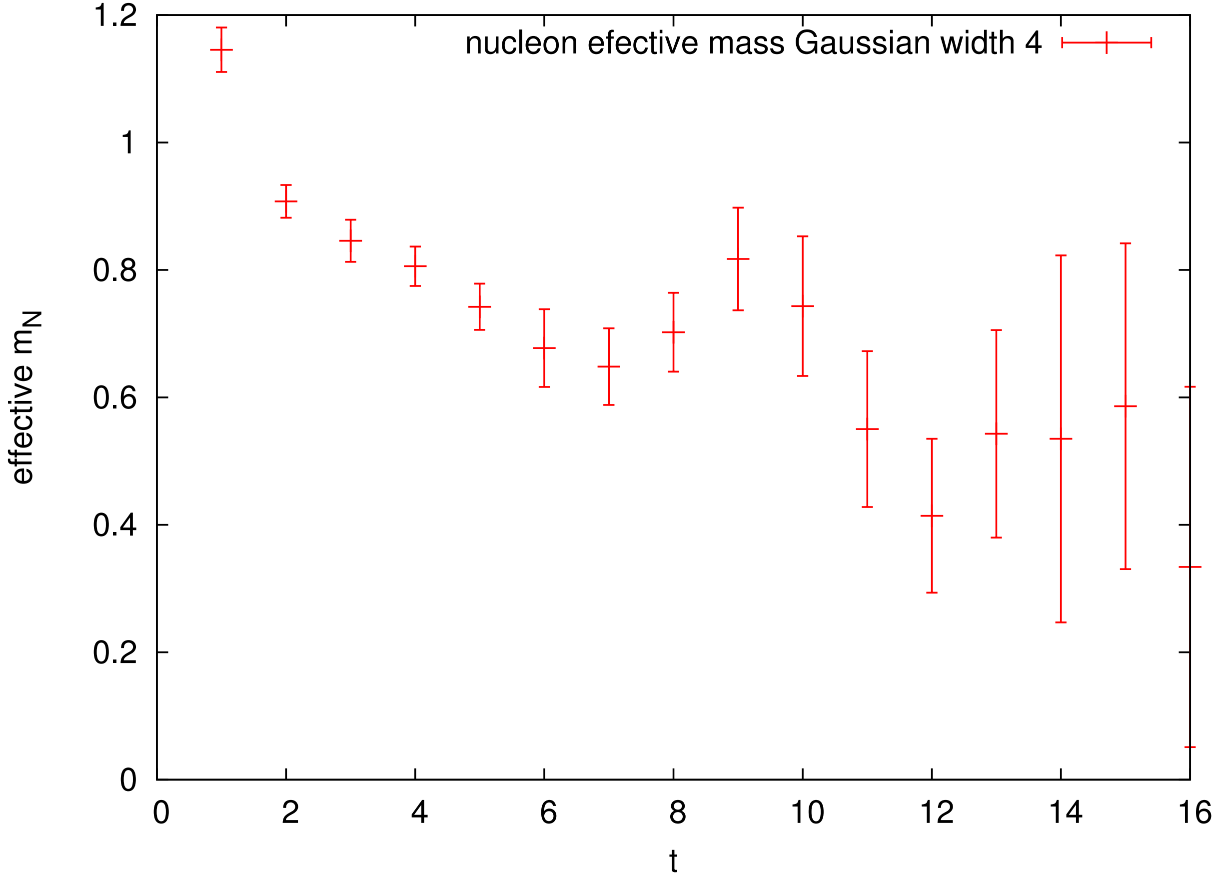

$m_N = 0.721(13)$ or $\sim 0.98$ GeV,

but probably needs a longer plateau for structure calculation to be free of excited-state contamination,

presently increasing the statistics.

RBC/UKQCD (2+1)-flavor, ID+DWF dynamical, $a^{-1} = 1.368(7)$ GeV, $m_{\rm strange} = 0.045$,

Nucleon mass signal from the heavy ($m_{\rm ud} = 0.0042$ or $m_\pi = 250$ MeV) ensemble, with 13 configurations

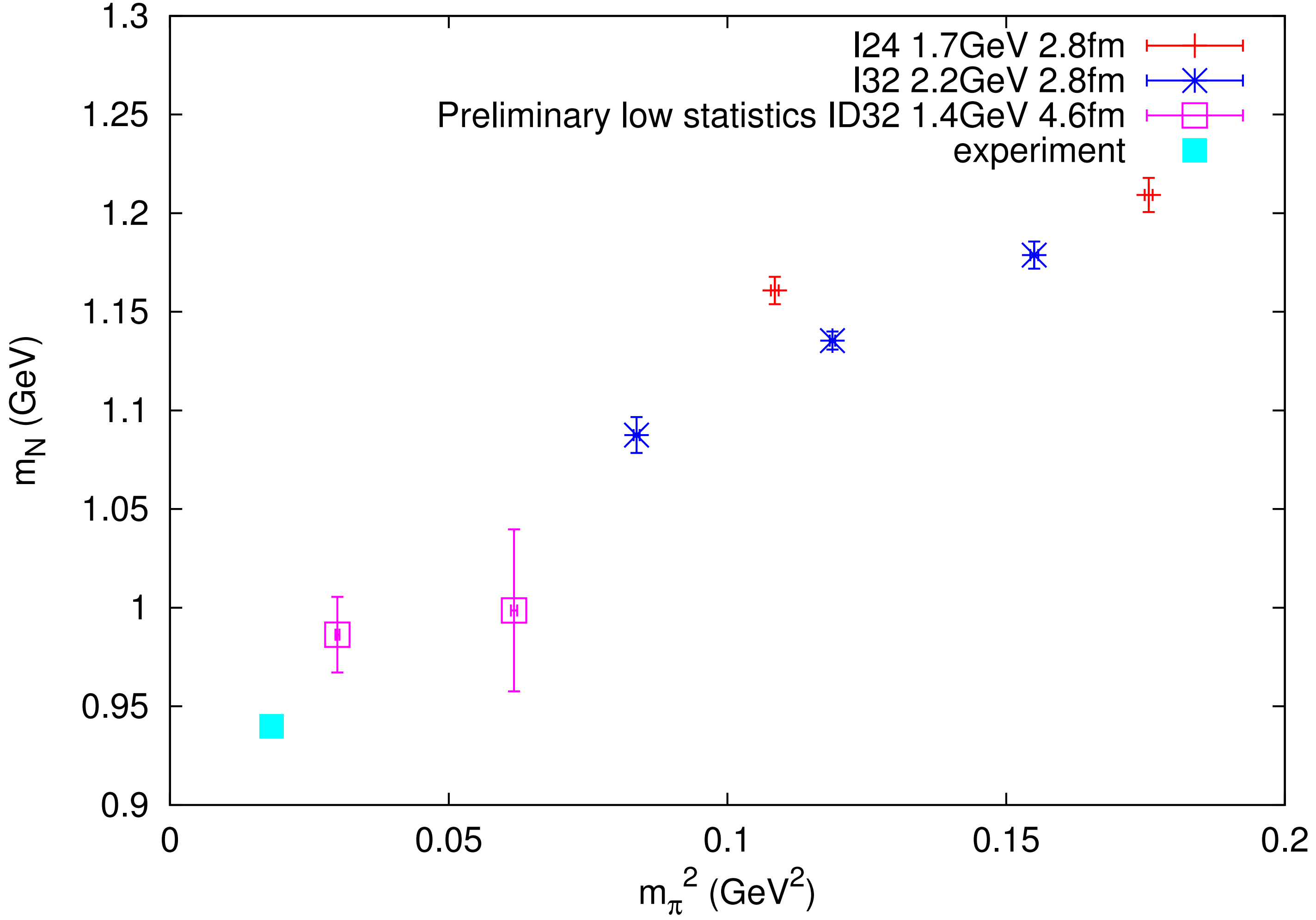



$m_N = 0.73(3)$ or $\sim 1.0$ GeV,

but needs a better plateau for structure calculation to be free of excited-state contamination,

presently increasing the statistics.

Conclusions: RBC/UKQCD (2+1)-flavor, ID+DWF ensembles are being analyzed for nucleon physics.




with $a^{-1} = 1.368(7)$ GeV, $(\sim 4.6\text{fm})^3$ spatial volume.

Closer to physical mass, $m_\pi = 180$ and $250$ MeV, $m_N < 1.0$ GeV,

Isovector form factors and structure function moments will be reported in the near future.