Electromagnetic Effects in Staggered ChiPT

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Motivation: better light quark masses

- The leading uncertainty in the MILC collaboration's calculation of the up to down quark-mass ratio m_u/m_d comes from electromagnetic (EM) effects.
- These also make substantial contributions to individual quark mass errors.

MILC, arXiv:0903.3598 RMP

$$m_u/m_d = 0.42(0)(1)\boxed{(4)}$$

EM error is 10%

$$m_u = 1.9(0)(1)(1)\boxed{(1)}\ \text{MeV}$$
$$m_d = 4.6(0)(2)(2)\boxed{(1)}\ \text{MeV}$$

So...

- We need better control of electromagnetic effects.
  - We use quenched-photon simulations; sea-quark electric charges are set to zero.
  - The use of meson mass splittings, makes this sufficient. Explained below. (Bijnens & Danielsson, hep-lat/0610127)
- See talk by Aaron Torok on quenched photon simulations. (Thursday 14:30, Rm 5 - Hadron spectroscopy)

Origin of the EM Uncertainty

Quark masses are obtained by tuning bare quark masses such that lattice meson masses match experiment. E.g.

$$(M_{K^+}^2)_{\mathrm{QCD}} \equiv M_{K^+}^2 - (M_{K^\pm}^2 - M_{K^0}^2)_{\mathrm{EM}}$$

lattice experiment ≈ electromagnetic effects

Assuming, for now, that EM effects are small for $M_{K^0}^2$.

Origin of the EM Uncertainty

$$(M^2_{K^+})_{\mathrm{QCD}} \equiv M^2_{K^+} - (M^2_{K^\pm} - M^2_{K^0})_{\mathrm{EM}}$$

lattice experiment $\approx$ electromagnetic effects

- Dashen constructed the combination (Phys. Rev. **183**, 1245 (1969))

$$\Delta M^2_D \equiv (M^2_{K^\pm} - M^2_{K^0}) - (M^2_{\pi^+} - M^2_{\pi^0})$$

and pointed out that to leading order, including leading order EM effects, $\Delta M^2_D = 0$.

- The "violation of Dashen's theorem" is often parametrized as

$$(M^2_{K^\pm} - M^2_{K^0})_{\mathrm{EM}} = (1 + \Delta_E)(M^2_{\pi^+} - M^2_{\pi^0})_{\mathrm{EM}}$$

Origin of the EM Uncertainty

$$(M_{K^+}^2)_{\rm QCD} \equiv M_{K^+}^2 - (M_{K^\pm}^2 - M_{K^0}^2)_{\rm EM}$$

lattice — experiment — $\approx$ electromagnetic effects

$$(1+\Delta_E)(M_{\pi^+}^2 - M_{\pi^0}^2)_{\rm EM}$$

lattice — experiment

Partially Quenched Notation

x and y label valence quarks; $\chi_{xy} = B_0(m_x + m_y)$.

sea charge

$$\bar{Q}_2 \equiv \frac{1}{3} \sum_{\sigma \in \text{sea}} q_\sigma^2$$

$q_{x\sigma} = q_x - q_\sigma =$ charge of an $x\sigma$ meson

- Continuum - R. Urech

  Nucl. Phys. B **433**:234 (1995); hep-ph/945341

- Partially Quenched - Bijnens & Danielsson

  PRD **75**:014505 (2007); hep-lat/0610127

The Partially Quenched Result

LEC's

EM

chiral logs

EM

$$
\begin{aligned}
M^2_{xy} \quad = \quad & B_0(m_x + m_y) + q^2_{xy}\Delta_{\rm EM} \\
& + \frac{24}{F_0^2}(2L_6 - L_4)\bar{\chi}_1\chi_{xy} + \frac{8}{F_0^2}(2L_8 - L_5)\chi^2_{xy} \\
& -48e^2 Z_E L_4 q^2_{xy}\bar{\chi}_1 - 16e^2 Z_E L_5 q_{xy}\chi_{xy} \\
& -12e^2(K_1 + K_2 - K_7 - K_8)\bar{Q}_2\chi_{xy} \\
& -4e^2(K_5 + K_6)q^2_p\chi_{xy} + 4e^2(K_9 + K_{10})q^2_p\chi_p + 12e^2 K_8 q^2_{xy}\bar{\chi}_1 \\
& +8e^2(K_{10} + K_{11})q^2_{xy}\chi_{xy} - 4e^2(2K_{18} + K_{19})q_x q_y\chi_{xy} \\
& -\frac{1}{3F_0^2}\bar{A}(\chi_m)\mathcal{R}^m_{n13}\chi_{xy} - \frac{1}{3F_0^2}\bar{A}(\chi_p)\mathcal{R}^p_{q\pi\eta}\chi_{xy} \\
& +e^2\bar{A}(\chi_{xy})q^2_{xy} \\
& -4e^2\bar{B}_1(\chi_\gamma, \chi_{xy}; \chi_{xy})q^2_{xy}\chi_{xy} + 4e^2\bar{B}(\chi_\gamma, \chi_{xy}; \chi_{xy})q^2_{xy}\chi_{xy} \\
& +2e^2 Z_E\bar{A}(\chi_{x\sigma})q_{x\sigma}q_{xy} - 2e^2 Z_E\bar{A}(\chi_{y\sigma})q_{y\sigma}q_{xy}
\end{aligned}
$$

Using quenched photons

- Sea quark charges appear in two places.
  - The analytic term proportional to $\bar{Q}_2$ cancels in mass splittings, or combinations of splittings.
  - Log terms are calculable in chipt, with no unknown LECs.
- Hence, we can use quenched photon simulations (and add the chiral logs back in at the end).

Staggered ChiPT

- Use the replica method: $n_v \to 0 \quad n_s \to 1/4$

- Mesons P_{xy} have taste structure

$$P_{xy} = \sum_{\xi} P_{xy,\xi} \, T^{\xi}$$

$$T^{\xi} \in (\xi_5, \xi_{\mu 5}, \xi_{\mu}, \xi_{\mu\nu} \; \mu < \nu, \xi_I) \quad \text{taste generators}$$

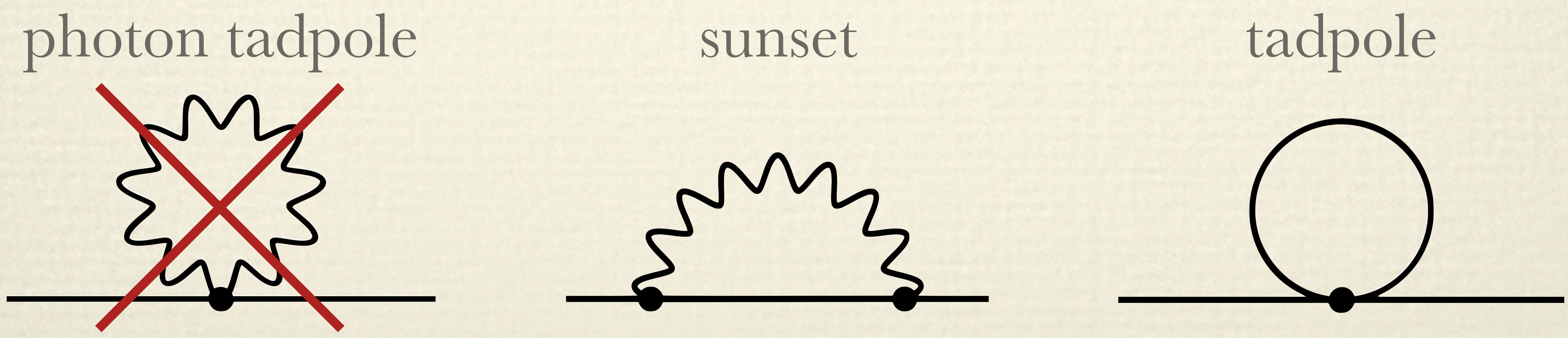
- Taste-breaking potential $a^2 \mathcal{V}$

Staggered Lagrangian

- The (Euclidian) Lagrangian **+ EM**

$$\mathcal{L}_2 = \frac{f^2}{8}\langle d^\mu \Sigma^\dagger d_\mu \Sigma\rangle - \frac{f^2}{8}\langle \chi\Sigma^\dagger + \chi^\dagger\Sigma\rangle - e^2 C\langle Q\Sigma Q\Sigma^\dagger\rangle + \frac{1}{24}m_0^2\langle\phi\rangle^2 + a^2\mathcal{V}$$

$$d_\mu\Sigma \to \partial_\mu\Sigma - iQA_\mu\Sigma + i\Sigma Q A_\mu \qquad Q = \mathrm{diag}(q_x, q_y, q_u, q_d, q_s)$$



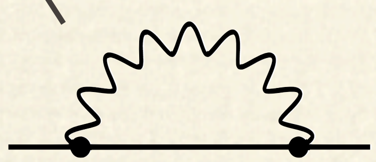
The Calculation

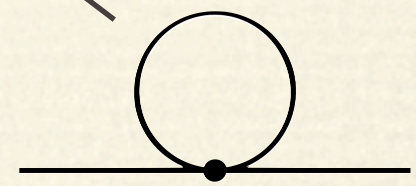
- Calculation closely follows Aubin and Bernard,
  - (PRD **68**: 034014 (2003); hep-lat/0304014).
- Must be careful to use the correct leading-order mass, including EM effects, in the simplifications using $p^2 = -M^2$.
  - This leads to a type of cancellation, noted by Aubin and Bernard, which appears in the EM PQ and EM staggered calculations as well.

The Result: NLO chiral logs

$$\delta M^2_{xy,5} \quad = \quad -\frac{1}{16\pi^2}\, e^2 q^2_{xy}\, M^2_{xy,5}\left[3\ln M^2_{xy,5} - 4\right]$$

$$\frac{-2\Delta_{\rm EM}}{16\pi^2 f^2}\left(\frac{1}{16}\right)\sum_{\xi}\Big[q_{x\sigma}q_{xy}\, I_{x\sigma,\xi} - q_{y\sigma}q_{xy}\, I_{y\sigma,\xi}\Big]$$





with $I_{ij,\xi} \equiv M^2_{ij,\xi}\left(\ln M^2_{ij,\xi} + \ldots\right)$ and $\Delta_{\rm EM} \equiv \dfrac{4e^2 C}{f^2}$

The Result: NLO analytical terms

Examples

$$\mathcal{O}^P_{u4} \equiv \langle \Sigma Q \Sigma^\dagger \xi_5 Q \xi_5 \rangle + \langle \Sigma \xi_5 Q \xi_5 \Sigma^\dagger Q \rangle \propto a^2 q^2_{xy}$$

$$\mathcal{O}^T_{u14} \equiv \langle \Sigma Q \Sigma^\dagger Q \rangle \langle \Sigma \xi_{\mu\nu} \Sigma^\dagger \xi_{\nu\mu} \rangle \propto a^2 q^2_{xy}$$

$$\mathcal{O}^V_{m8} \equiv \langle \Sigma Q \Sigma^\dagger \xi_\mu \Sigma^\dagger Q \Sigma \xi_\mu \rangle \propto a^2 (q^2_x + q^2_y)$$

All reduce to

$\propto a^2 q^2_{xy}$ and $a^2(q^2_x + q^2_y)$

at tree level.

No $a^2 \bar{Q}_2$ term.

$M_{\pi^0}^2$ vs. $M_{\pi'}^2$

- The true π^0 is a $\bar{u}u - \bar{d}d$ state.
  - There are EM disconnected-diagram contributions to its mass even for $m_u = m_d$.
  - This is difficult to simulate.
- Instead, we simulate a meson created from distinct, mass-degenerate, light quarks and anti-quarks with opposite electric charges.
  - We call this neutral meson the π'.

$M^2_{\pi^0}$ vs. $M^2_{\pi'}$

- We make the replacement

$$M^2_{\pi^\pm} - M^2_{\pi^0} \quad \rightarrow \quad M^2_{\pi^\pm} - M^2_{\pi'} \,.$$

- The error in this approximation is

$$\delta^{\rm EM} M^2_{\pi^0} = \frac{2\Delta_{\rm EM}}{16\pi^2 f^2} M^2_\pi \left(\ln M^2_\pi + 1\right) + e^2 M^2_\pi ({\rm LEC's})\ ,$$

which is small, at the physical point, compared to the EM correction to the $\pi^\pm$, dominated by the kaon mass

$$\delta^{\rm EM} M^2_{\pi^\pm} \approx \frac{2\Delta_{\rm EM}}{16\pi^2 f^2} M^2_K \ln M^2_K \,.$$

$M^2_{\pi^0}$ vs. $M^2_{\pi'}$

- We make the replacement

$$M^2_{\pi^\pm} - M^2_{\pi^0} \quad \to \quad M^2_{\pi^\pm} - M^2_{\pi'} \, .$$

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Summary

- Improved physics needs EM (lattice) ChiPT
- Mass splittings, or combinations like Dashen's, can be determined with quenched QED.
- The staggered calculation is done.
- Points to note:
  - $p^2 = -M^2$ yields substantial cancellations;
  - only $a^2 q_{xy}^2$ and $a^2(q_x^2 + q_y^2)$ analytic terms;
  - π^0 vs π'.