Hadronic structure from the lattice

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QCDSF Collaboration

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Lattice 2010
14 June 2010

Outline

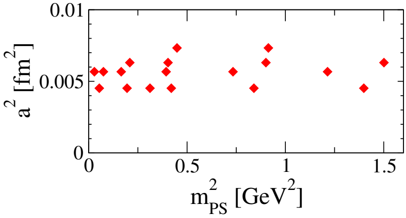
- Status of QCDSF $N_{\mathrm{f}} = 2$ simulations
- Lowest moments of nucleon twist 2 PDFs
  - n=1: $\langle 1 \rangle_{\Delta q} = g_{\mathrm{A}}$, $\langle 1 \rangle_{\delta q} = g_{\mathrm{T}}$
  - n=2: $\langle x \rangle_q$, $\langle x \rangle_{\Delta q}$
- Nucleon form factors
  - Electro-magnetic form factors

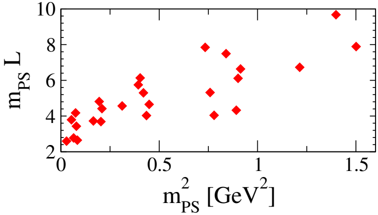
For 2-point function results $\rightarrow$ Talk by G. Schierholz

Credits to QCDSF colleagues

S. Collins, M. Göckeler, P. Hägler, R. Horsley, Y. Nakamura, A. Nobile, H. Perlt, P.E.L. Rakow, A. Schäfer, G. Schierholz, E. Scholz, A. Sternbeck, H. Stüben, F. Winter, J.M. Zanotti

$N_f = 2$ NP-Clover Simulation Status





- Goal: Explore region where $m_\pi^{\text{phys}} \leq m_{\text{PS}} \lesssim 200\,\text{MeV}$ on large lattices
- Rational for quantities of interest:
  - Discretisation errors tend to be small
  - Finite size effects can be significant
  - Chiral extrapolations are difficult to control
- Dedicated finite size runs where $m_{\text{PS}} L \lesssim 3$

Workhorse for Generating Gauge Fields

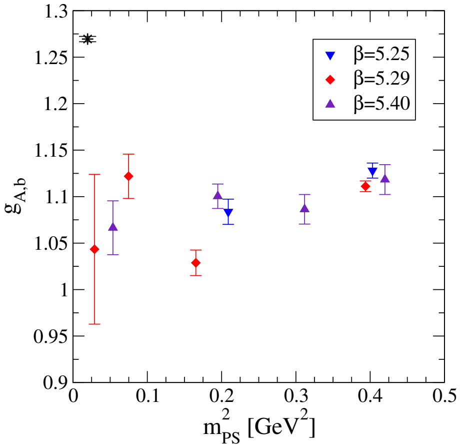
- Aggregate peak performance: 200 TFlops (double precision)
- Stable production mode since end 2009
- For details on performance
  $\rightarrow$ Talk by A. Nobile
- Program used: BQCD
  $\rightarrow$ Poster by Y. Nakamura and H. Stüben

Calculating Observables

- Calculation of bare matrix elements
  - Standard sequential sources methods
  - (Smeared) point sources
  - PC-cluster type of machines running Chroma
- Non-perturbative renomalisation
  - Mostly RI'-MOM used $\rightarrow$ Talk by M. Göckeler
  - For Z_V we use $F_1^{(v)}(0) = 1$
  - Continuum PT for conversion to $\overline{\mathrm{MS}}$

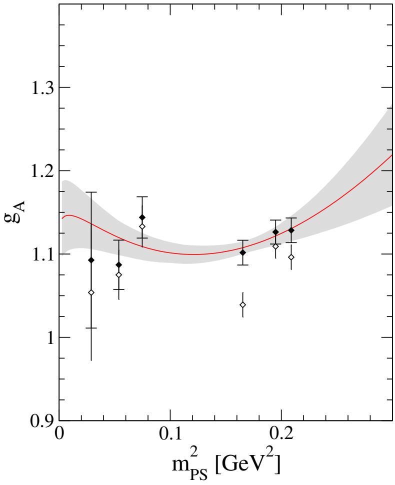
    4 loop β function, 2-3 loop anomalous dimension
- Consistent choice of scale: $r_0 = 0.467\,\mathrm{fm}$

n=1 Moment of Polarized PDFs: g_A



- Data relatively constant over large quark mass range
- Discretization effects seem to be small
- Lattice data significantly smaller than experimental value

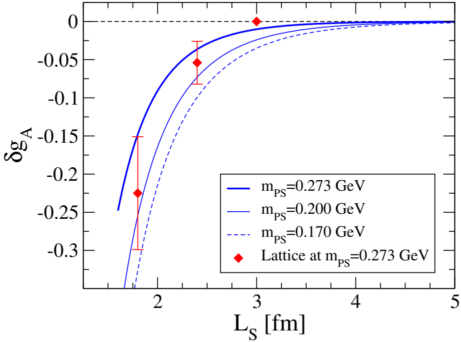
Chiral Extrapolation



- Calculation based on the SSE scheme of ChEFT in infinite and finite volumes gives guidance on
  - quark mass dependence
  - finite volume effects
- Fit details:
  - Fit restricted to $m_{PS} \lesssim 0.45\,\text{GeV}$
  - Discretization effects ignored
  - Finite size effect corrections enabled
  - Various parameters fixed to experimental/phenomenological value
- Result O(10%) below experiment

Finite Size Effects

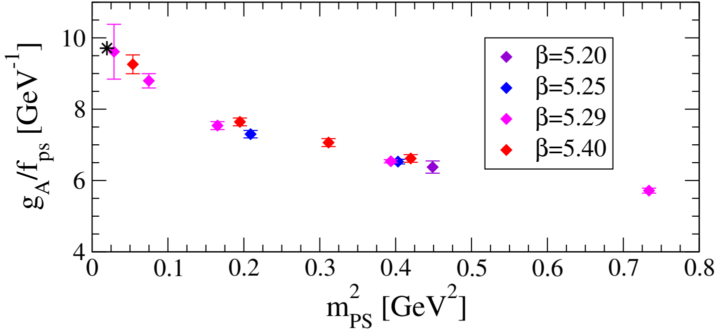
- Relative shift: $\delta_{g_\mathrm{A}}(L) = \frac{g_\mathrm{A}(L) - g_\mathrm{A}(\infty)}{g_\mathrm{A}(\infty)}$



- Comparison with data not included in fit
- Lattice data: Use largest lattice as reference
- Fit (slightly) underestimates finite size corrections

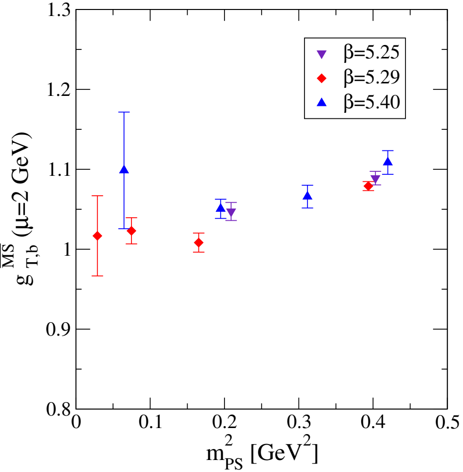
- Finite size effects do not seem to explain discrepancy with experimental value

g_A in Units of f_{PS}^{-1}



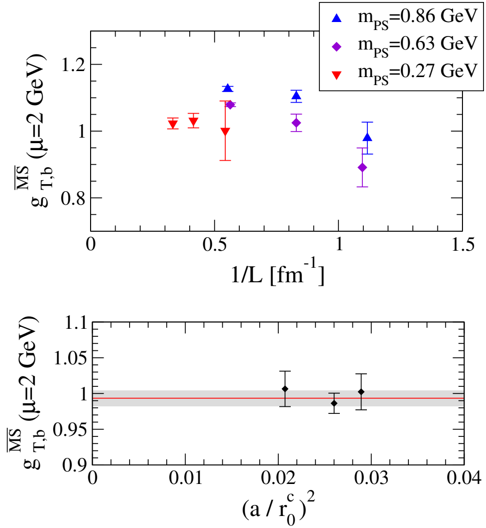
- Find good agreement with experiment
- Non-perturbative value for Z_A too small?

n=1 Moment of Tensor PDFs: g_T



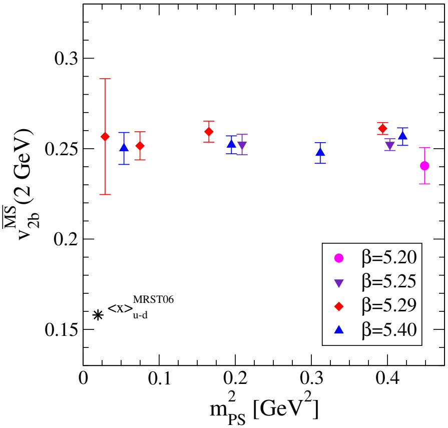
- Mild quark mass dependence
- Discretization effects seem to be small
- No experimental number
  - Models seem to support $\delta u > \Delta u$ and $\delta d \simeq \Delta d$ at (much smaller) model scale

g_{T}: Finite Size and Discretization Effects



- Finite size effects seem to be absent for $L \gtrsim 3\,\mathrm{fm}$
- Performed linear extrapolation to chiral limit
- Continuum extrapolation show no significant discretization effects

n=2 Moment of Unpolarised PDFs: $\langle x \rangle_q^{(v)}$

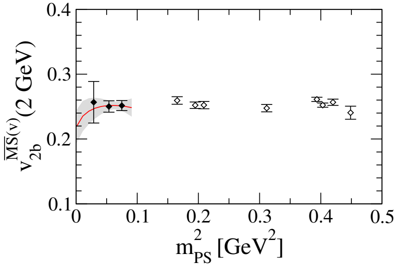


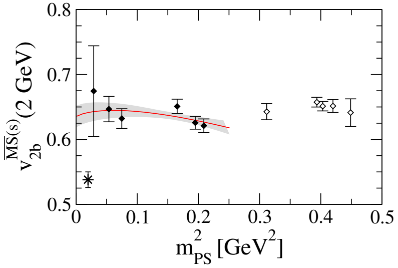
- For $O(a)$-improvement of the operator

$$\mathcal{O}^{\gamma}_{\mu\nu} \to (1 + am_{\text{q}}c_0)\ \mathcal{O}^{\gamma}_{\mu\nu}$$

  with perturbative value for c_0.
- Data reveals little quark mass, lattice scale and volume dependence
- Results are significantly larger then pheno-menological value(s)

$\langle x \rangle_q^{(v)}$ and $\langle x \rangle_q^{(s)}$: Chiral Extrapolation

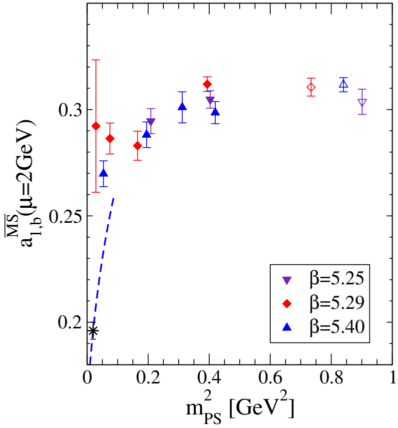




- Calculations based on ChEFT predict “bending down”
  - Used here: BChPT calculations from Dorati et al.
- Fit details:
  - Fit restricted to
    - $m_{PS} \lesssim 0.3$ GeV (iso-vector)
    - $m_{PS} \lesssim 0.5$ GeV (iso-scalar)
  - Discretization effects ignored
  - Various parameters fixed to experimental/phenomenological value ☞ 2 free parameters
- Iso-scalar case lacks disconnected contributions

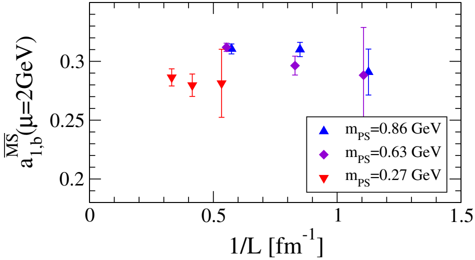
  → Poster by S. Collins

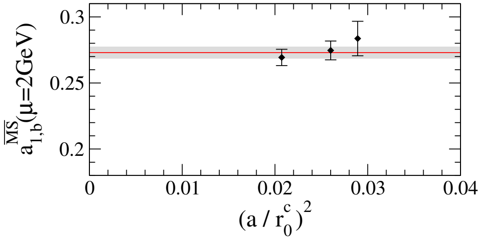
n=2 Moment of Polarised PDFs: $\langle x \rangle_{\Delta q}$



- Discretization effects seem to be absent in data
- Slight bending down for $m_{\text{PS}} \lesssim 0.5\,\text{GeV}$
- Results for $m_{\text{PS}} \simeq 0.2\,\text{GeV}$ significantly larger then expected from HBChPT
    - HBChPT parameters chosen to match phenomenological value

$\langle x \rangle_{\Delta q}$: Finite Size and Discretization Effects





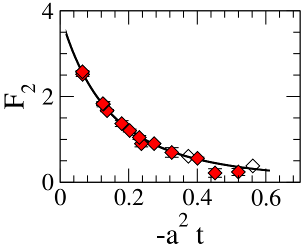
- Comparison of results from dedicated finite size runs confirm absense of finite size effects
- Continuum extrapolation
  - Based on linear extrapolation of results with $m_{\text{PS}} \lesssim 0.7\,\text{GeV}$
  - Discretization effects small compared to statistical errors

Electro-magnetic Form Factors

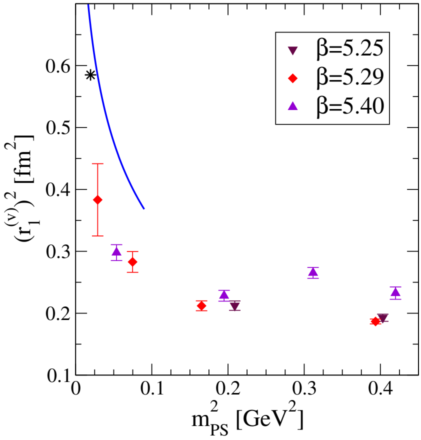
- Calculation of form factor radii and magnetic moment
- Parametrization of form factors required:
  - Popular choice "p-pole"

$$F_i(Q^2) = \frac{F_i(0)}{\left[1 + \frac{Q^2}{p m_i^2}\right]^p}$$

- Our choice:
  - F1: di-pole fit
  - F2: tri-pole fit
- Data not sufficiently precise to guide this decision

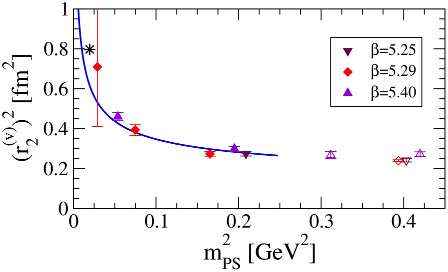


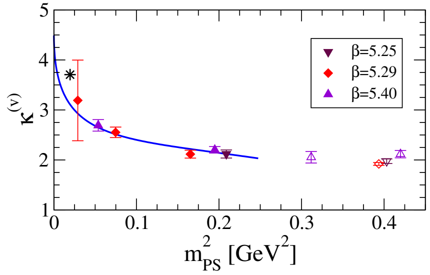
Iso-vector Dirac Form Factor



- Comparison of Pauli radius with predictions from the SSE scheme of ChEFT based calculations
  - No fit to lattice data
- "Bending up" starts to emerge in lattice data
- Values still significantly below phenomenological value(s)

Iso-vector Pauli Form Factor

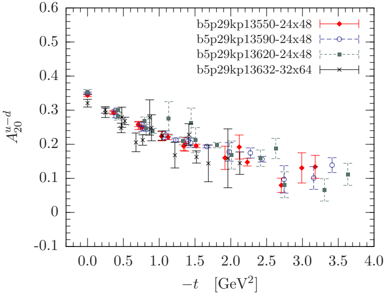


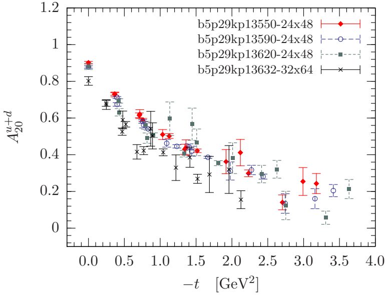


- Combined fit to ChEFT results
  - Required to determine unknown/little known parameters
  - Stable fit requires $m_{\text{PS}} \lesssim 0.5\,\text{GeV}$
    - probably out of scope where ChEFT is applicable
- Data seems to suppport “bending up”
- Statistical errors too large to check consistency

Generalized Form Factor A_{20}

[A. Sternbeck]





- Here: no conversion to $\overline{\text{MS}}$
- Iso-vector channel: Little quark mass dependence in data
- Stronger curvature in iso-scalar channel at lighter quark mass

Summary and Conclusions

- We have presented an update on QCDSF's results for the
  - Lowest moments of the nucleon twist 2 PDFs
  - Nucleon form factors
- Current data confirm the importance of performing calculations at $m_{\mathrm{PS}} \lesssim 200\,\mathrm{MeV}$
  - Statistical errors still rather large
  - Observation: For some observables current data does not support predictions from ChEFT
- For most quantities finite volume effects seem to be small
- Discretization errors seem to be absent in data