Glueballs in J/psi Radiative Decays

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Outline

I. Motivation

II. The widths of J/psi radiatively decaying to glueballs

III. Numerical details

IV. Summary and concluding remarks

I. Introduction

- QCD predicts the existence of glueballs
- Quenched LQCD predicts glueball spectrum
  - Lowest-lying glueballs have masses in the range 1~3GeV
- Experimentally, f0(1370), f0(1500), f0(1710), etc., are glueball candidates, but decisive conclusion cannot be drawn.
- Due to its abundance of gluons, J/psi radiative decay can be the best hunting ground.
- BESIII in Beijing is producing 10^{10} J/psi events

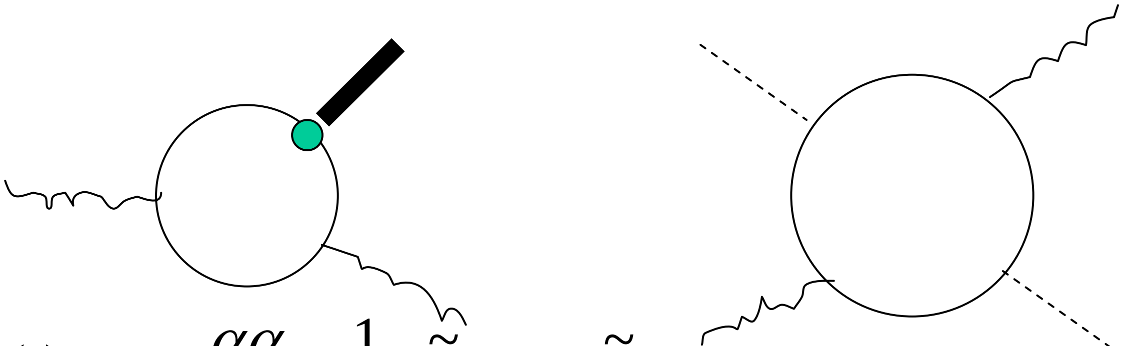
J^{PC}	$r_0 M_G$	M_G (MeV)
0^{++}	4.16(11)(4)	1710(50)(80)
2^{++}	5.83(5)(6)	2390(30)(120)
0^{-+}	6.25(6)(6)	2560(35)(120)
1^{+-}	7.27(4)(7)	2980(30)(140)
2^{-+}	7.42(7)(7)	3040(40)(150)
3^{+-}	8.79(3)(9)	3600(40)(170)
3^{++}	8.94(6)(9)	3670(50)(180)
1^{--}	9.34(4)(9)	3830(40)(190)
2^{--}	9.77(4)(10)	4010(45)(200)
3^{--}	10.25(4))(10)	4200(45)(200)
2^{+-}	10.32(7)(10)	4230(50)(200)
0^{+-}	11.66(7)(12)	4780(60)(230)

Y. Chen et al,
Phys. Rev. D 73, 014516 (2006)

II. The widths of J/psi radiatively decaying to glueballs

(We focus on the scalar glueball in this talk)

- **Method 1:** Effect theory and dispersion relation
  (V.A. Novikov et al. , Nucl. Phys. B 165 (1980) 67.)



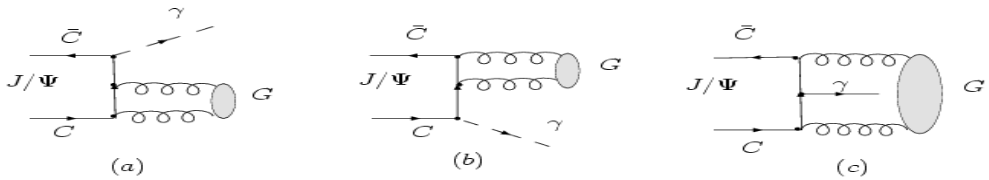
$$L^{(c)}{}_{2\gamma 2g} = \frac{\alpha\alpha_s}{405}\frac{1}{m_c{}^4}\tilde{L}, \qquad \tilde{L} = \{FFGG\}$$

By the use of dispersion relation, we have,

$$\Gamma(J/\psi \to \gamma 0^{++}) = \frac{\alpha^3}{\pi 5^2 3^8 2^3}\frac{m_{J/\psi}^4 \langle 0|\alpha_s G^2|0^{++}\rangle^2}{m_c^4 \Gamma(J/\psi \to e^+e^-)}$$

→ Glueball Matrix Element

- **Method 2: Tree-level perturbative QCD**



$$\langle G_{\lambda_2}\gamma_{\lambda_1}|S|H_\lambda\rangle = (2\pi)^4\delta^4(p_H - p_r - p_G)\frac{e}{(8\omega_\gamma E_G E_H)^{1/2}}T^\lambda_{\lambda_1\lambda_2},$$

$$\begin{aligned} T^\lambda_{\lambda_1,\lambda_2} &= \frac{e_Q g^2}{2\sqrt{3}}\delta_{ab}e^{\lambda_1 *}_\mu(p_\gamma)\int d^4x_1 d^4x_2 \times \\ & Tr\left[\chi_\lambda(x_1,x_2)\gamma^\beta S_F(x_2)\gamma^\mu S_F(-x_1)\gamma^\alpha\right. \\ &+ \chi_\lambda(x_2,0)\gamma_\mu S_F(-x_1)\gamma^\alpha S_F(x_1-x_2)\gamma^\beta \\ &+ \left.\chi_\lambda(0,x_1)\gamma^\alpha S_F(x_1-x_2)\gamma^\beta S_F(x_2)\gamma^\mu\right] G^{ab}_{\alpha\beta}(x_1,x_2)_{\lambda_2}; \end{aligned}$$

J/psi BS wave func.

Glueball BS wave func.

$$\Gamma(J/\psi \to \gamma G) = |\psi(0)|^2 \left|\langle 0|FF|G\rangle\right|^2 K_G(m_c, m_G, m_{J/\psi})$$

Can be calculated explicitly

- **Glueball-to-vacuum matrix elements**

Lattice QCD results (quenched):

Y. Chen et al, (Phys. Rev. D 73, 014516 (2006))

$$s = \langle 0|tr(g^2 G_{\mu\nu}G_{\mu\nu})|0^{++}\rangle = 15.6 \pm 3.2\ (GeV)^3$$
$$p = \langle 0|\varepsilon_{\mu\nu\rho\sigma} tr(g^2 G_{\mu\nu}G_{\rho\sigma})|0^{-+}\rangle = 8.6 \pm 1.3\ (GeV)^3$$
$$t = \varepsilon_{\mu\nu}\langle 0|\frac{1}{2}\Theta_{\mu\nu}|2^{++}\rangle = 0.52 \pm 0.19\ (GeV)^3,$$

H.B. Meyer (arXiv:08083151 [hep-lat])

$$s = 11.5(1.1)(GeV)^3$$
$$t = 0.49(10)(GeV)^3$$

Other phenomenological results:

(a) QCD sum rules with non-perturbative topology effects:
(H. Forkel, hep-ph/0608071) $s = 20(2)(GeV)^3$

(b) Instanton liquid model: $s = 14(GeV)^3$
T. Schaafer and E.V. Shuryak, PRL75(1995)1707

- **Predictions of J/psi radiatively decaying to glueballs**

Method 1: two-photon-two-gluon induced

H.B. Meyer, (arXiv:08083151 [hep-lat])

$$\mathrm{Br}(J/\psi \to G_0\gamma) \approx 0.009.$$

Method 2: Tree-level perturbative QCD results with GME by QLQCD

(Y. Chen, G. Li, and Y.J. Zhang)

m_c	$J/\psi \to \gamma 0^{++}$		$J/\psi \to \gamma 2^{++}$		$J/\psi \to \gamma 0^{-+}$	
(GeV)	Width(GeV)	Br. ratios	Width(GeV)	Br. ratios	Width(GeV)	Br. ratios
1.25	2.2×10^{-4}	2.4	1.5×10^{-7}	1.6×10^{-3}	2.6×10^{-5}	0.28
1.40	2.2×10^{-4}	2.4	9.0×10^{-8}	9.7×10^{-4}	5.5×10^{-6}	0.06
1.50	2.0×10^{-4}	2.2	6.9×10^{-8}	7.3×10^{-4}	2.4×10^{-6}	0.03

TABLE I: The decay widths and branch ratios of $J/\psi \to \gamma G$ with G representing the 0^{++}, 2^{++}, and 0^{-+} glueballs.

- **Method 3:** Direct calculation in LQCD

The decay width of J/psi radiatively decaying to the scalar glueball can be derived from the formular

$$\Gamma(J/\psi \to \gamma G_{0^+}) = \frac{4}{27}\alpha \frac{|p|}{M_{J/\psi}^2}|E_1(0)|^2$$

where E1(0) is the on-shell form factor, which appears in the matrix elements (J.J. Dudek,hep-lat/0601137)

$$\langle S(\vec{p}_S)|j^\mu(0)|V(\vec{p}_V,r)\rangle = \Bigg(E_1(q^2)\left[\varepsilon^\mu(\vec{p}_V,r) - \varepsilon(\vec{p}_V,r)\cdot p_S \frac{p_V^\mu p_V\cdot p_S - m_V^2 p_S^\mu}{\Omega(q^2)}\right] + \frac{C_1(q^2)}{\sqrt{q^2}\Omega(q^2)} m_V \varepsilon(\vec{p}_V,r)\cdot p_S\left[p_V\cdot p_S(p_V+p_S)^\mu - m_S^2 p_V^\mu - m_V^2 p_S^\mu\right]\Bigg)$$

With the vector current insertion $j^\mu = \bar{c}\gamma^\mu c$, these Matrix elements can be calculated through the three point function,

$$\Gamma^{(3)}(\vec{p}_f,\vec{q};t_f,t) = -\sum_{\vec{x},\vec{y}} e^{-i\vec{p}_f\cdot\vec{x}} e^{+i\vec{q}\cdot\vec{y}} \langle O_S(\vec{x},t_f) j^\mu(\vec{y},t) O_V^\dagger(0,0)\rangle$$

$$(t_f \geq t \geq 0)$$

After the intermediate state insertion, the three-point function can be written as

$$\Gamma^{(3),\mu j}(\vec{p}_f, \vec{q}; t_f, t) = \sum_{f,i,r} \frac{e^{-E_f(t_f-t)}e^{-E_i t}}{2E_f(\vec{p}_f)2E_i(\vec{p}_i)} \times \langle 0|O_S(0)|f(\vec{p}_f)\rangle\langle f(\vec{p}_f)|j^\mu(0)|i(\vec{p}_i, r)\rangle\langle i(\vec{p}_i, r)|O_V^{(j)\dagger}|0\rangle$$

$$\langle 0|O_V^\mu|n(\vec{p}, r)\rangle \equiv Z_n \epsilon^\mu(\vec{p}, r)$$

$$\sum_r \epsilon^\mu(\vec{p}, r)\epsilon^{\nu *}(\vec{p}, r) = -g^{\mu\nu} + \frac{p^\mu p^\nu}{m_n^2}$$

$$\vec{p}_i = \vec{p}_f - \vec{q}.$$

Two-point function of vector meson

$$\begin{aligned} C_2^{ij}(\vec{p}, t) &= \sum_{\vec{x}} e^{-i\vec{p}\cdot\vec{x}}\langle 0|O_V^{(i)}(\vec{x}, t)O_V^{(j),\dagger}((\vec{0}), 0)|0\rangle \\ &= \sum_{n,r} \frac{1}{2E_n(\vec{p})}\langle 0|O_V^{(i)}(0)|n(\vec{p}, r)\rangle\langle n(\vec{p}, r)|O_V^{(j),\dagger}(0)|0\rangle e^{-E_n t} \\ &= \sum_n \frac{Z_n Z_n^*}{2E_n(\vec{p})}\left(\delta_{ij} + \frac{p^i p^j}{m_n^2}\right) e^{-E_n t}. \end{aligned}$$

III. Numerical details

1. Lattice and parameters

Anisotropic lattice: $L^3 \times T = 8^3 \times 96 \quad \xi = a_s / a_t = 5$

Strong coupling: $\beta = 2.4 \quad a_s = 0.222(2)\, fm$

2. Actions

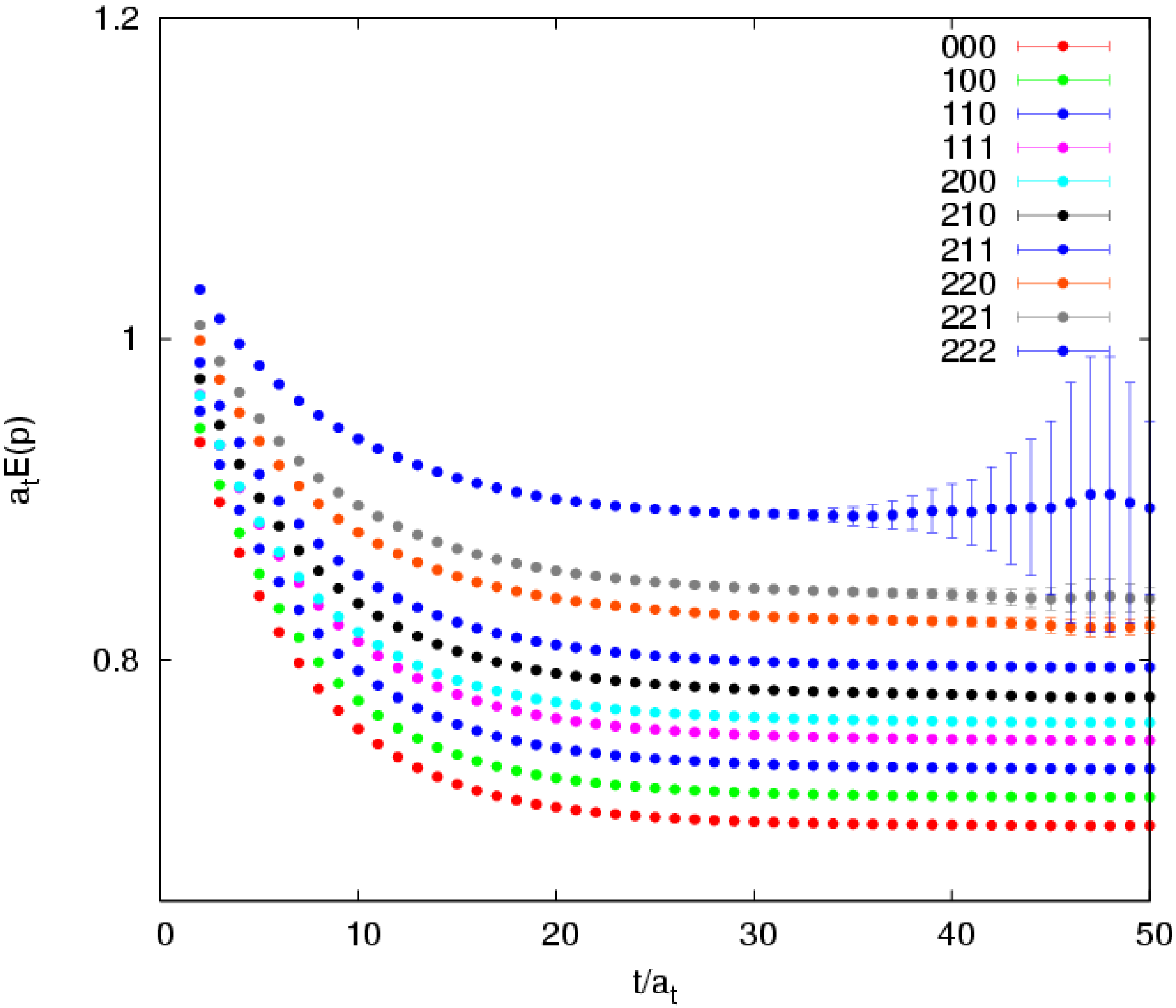
$$S_{IA} = \beta\{\frac{5}{3}\frac{\Omega_{sp}}{\xi u_s^4} + \frac{4}{3}\frac{\xi\Omega_{tp}}{u_t^2 u_s^2} - \frac{1}{12}\frac{\Omega_{sr}}{\xi u_s^6} - \frac{1}{12}\frac{\xi\Omega_{str}}{u_s^4 u_t^2}\}$$

$$\mathcal{A}_{xy} = \delta_{xy}[1/(2\kappa_{max}) + \rho_t \sum_{i=1}^{3} \sigma_{0i}\mathcal{F}_{0i} + \rho_s(\sigma_{12}\mathcal{F}_{12} + \sigma_{23}\mathcal{F}_{23} + \sigma_{31}\mathcal{F}_{31})]$$
$$- \sum_{\mu} \eta_\mu[(1-\gamma_\mu)U_\mu(x)\delta_{x+\mu,y} + (1+\gamma_\mu)U_\mu^+(x-\mu)\delta_{x-\mu,y}]$$

$$\eta_i = \nu/(2u_s), \eta_0 = \xi/2, \sigma = 1/(2\kappa) - 1/(2\kappa_{max}),$$

$$\rho_t = c_{SW}(1+\xi)/(4u_s^2), \rho_s = c_{SW}/(2u_s^4).$$

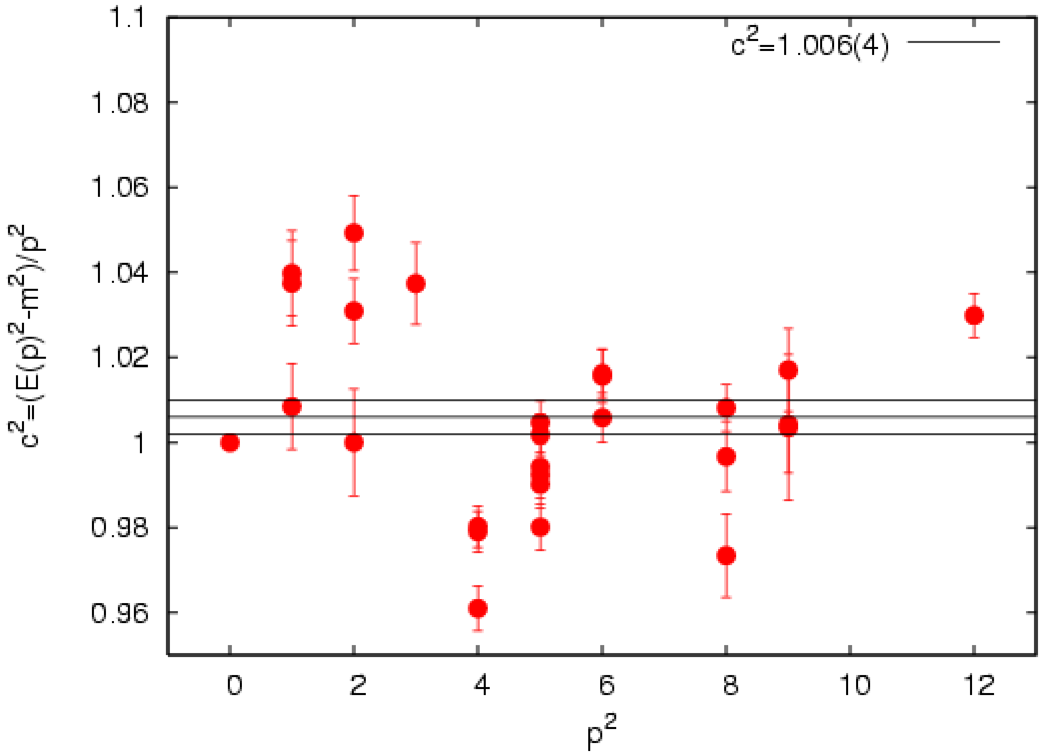
bare speed of light, should be tuned to give the correct dispersion relation



1.2
1
0.8
0
10
20
30
40
50
$a_tE(p)$
t/a_t
000
100
110
111
200
210
211
220
221
222

Self consistent check of the speed of light for J/psi

$$c^2 = \frac{E^2(\vec{p}) - m^2}{p^2}$$



3. Configurations and quark propagators

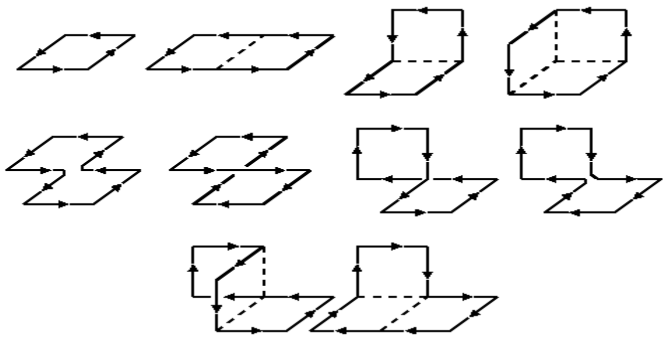
In order to get fair signals of the three point functions, a large enough statistics is required.

- 5000 gauge configurations, separated by 100 HB sweeps
- Charm quark mass is set by the physical mass of J/psi
- On each configuration, 96 charm quark propagators are calculated with point sources on all the 96 time slices. The periodic boundary conditions are used both for the spatial and temporal directions.

$$\Gamma^{(3)\mu i}(\vec{p}_f, \vec{q}; t_f, t) = \frac{1}{T}\sum_{\tau=0}^{T-1}\sum_{\vec{y}} e^{+i\vec{q}\cdot\vec{y}} \left\langle O_G(\vec{p}_f, t_f + \tau) j^{\mu}(\vec{y}, t+\tau) O^{i,+}_{J/\psi}(\tau) \right\rangle$$

4. The glueball operators

Building prototypes
(various Wilson loops)



Smearing: Single link scheme (APE) and double link scheme (fuzzying)

The essence of the VM is to find a set of combinational coefficients

$$\{v_\alpha, \alpha = 1,2,\ldots 24\}$$

such that the operator

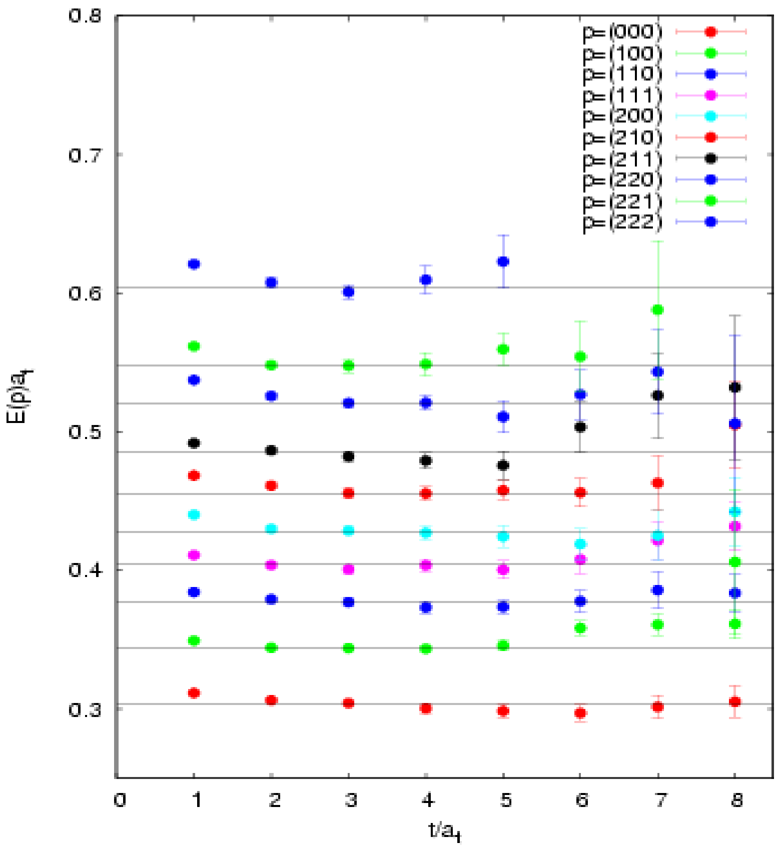
$$\Phi = \sum_\alpha v_\alpha \phi_\alpha$$

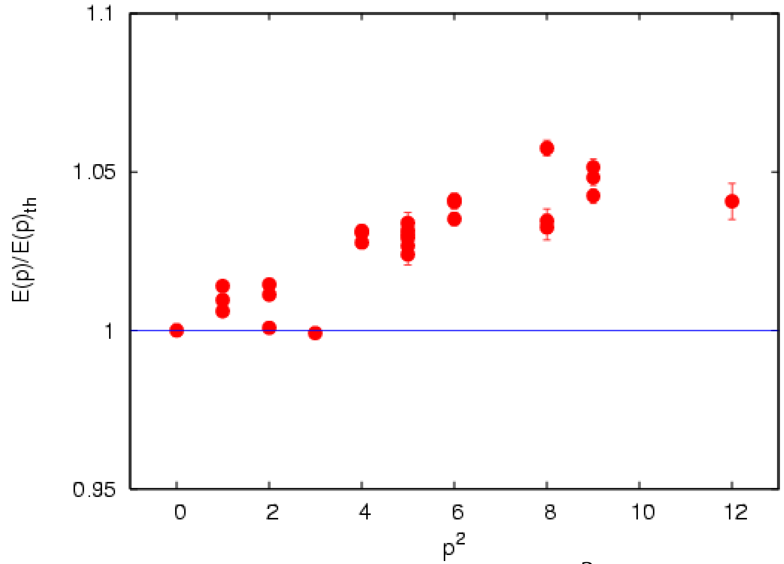
couples mostly to a specific state.

$$\tilde{C}(t_D)\mathbf{v}^{(R)} = e^{-t_D \tilde{m}(t_D)} \tilde{C}(0)\mathbf{v}^{(R)}$$

$$\tilde{C}_{\alpha\beta}(t) = \sum_\tau \langle 0|\phi_\alpha(t+\tau)\phi_\beta(\tau)|0\rangle$$

$$\tilde{m}(t_D) = -\frac{1}{t_D} \ln \frac{\sum_{\alpha\beta} v_\alpha v_\beta \tilde{C}_{\alpha\beta}(t_D)}{\sum_{\alpha\beta} v_\alpha v_\beta \tilde{C}_{\alpha\beta}(0)}$$





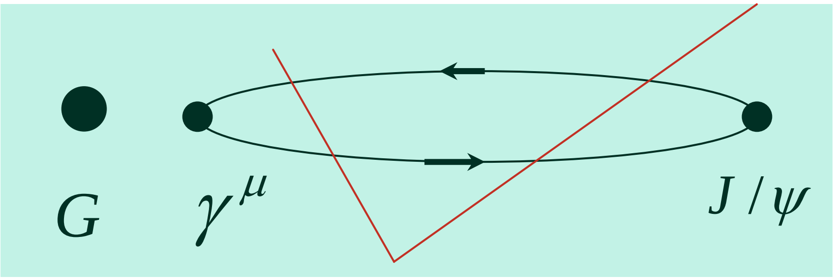
$$a_t^2 E_{th}^2(\vec{p}) = a_t^2 m^2 + \frac{4}{\xi^2} \sum_{i=1}^{3} \sin^2 \frac{p_i}{2},$$

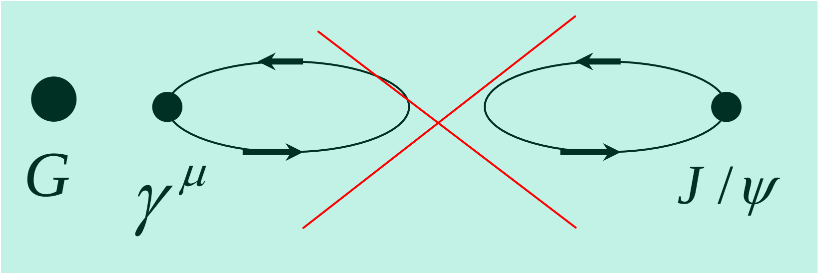
$$\vec{p} = \frac{2\pi}{L}(n_1, n_2, n_3)$$

The energies of 27 momentum modes of scalar glueballs are calculated. Plotted are the plateaus using the optimized operators

The horizontal line is the theoretical prediction with $\xi = 5$.
It is seen that the deviations are less than 5%

5. Three point functions





$$\Gamma^{(3),\mu j}(\vec{p}_f,\vec{q};t_f,t) = \sum_{f,i,r} \frac{e^{-E_f(t_f-t)}e^{-E_i t}}{2E_f(\vec{p}_f)2E_i(\vec{p}_i)} \times \langle 0|O_S(0)|f(\vec{p}_f)\rangle\langle f(\vec{p}_f)|j^\mu(0)|i(\vec{p}_i,r)\rangle\langle i(\vec{p}_i,r)|O_V^{(j)\dagger}|0\rangle$$

$$\vec{p}_i = \vec{p}_f - \vec{q}.$$

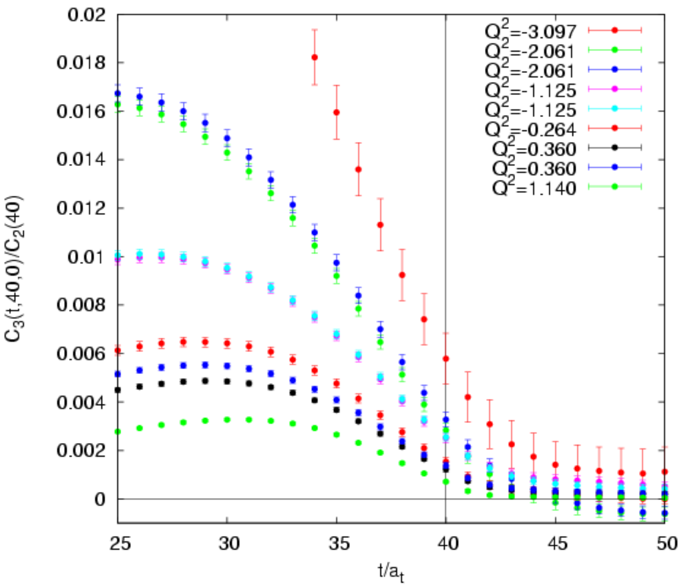
Temporarily, we only analyze the following cases:

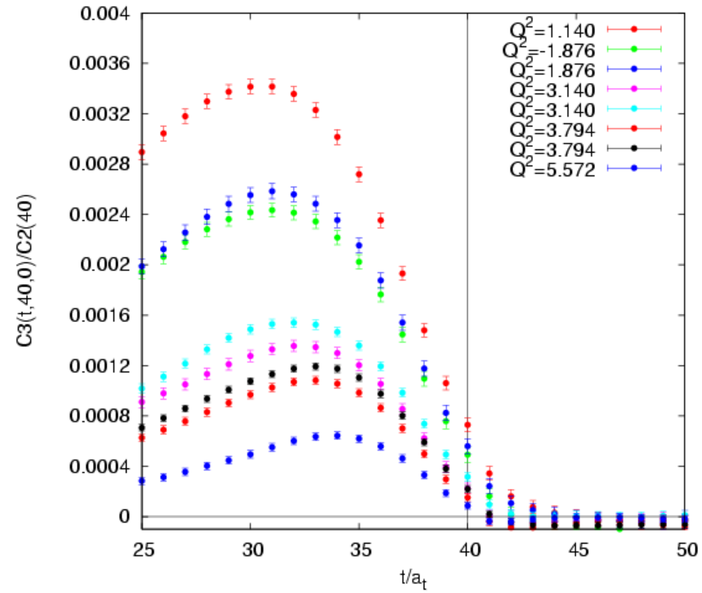
$$\begin{cases} \vec{p}_i = (0,0,0) \\ \mu = i = 1,2,3 \end{cases} \quad \Longleftrightarrow \quad \vec{p}_f = \vec{q} = (000),(100),\cdots,(222)$$

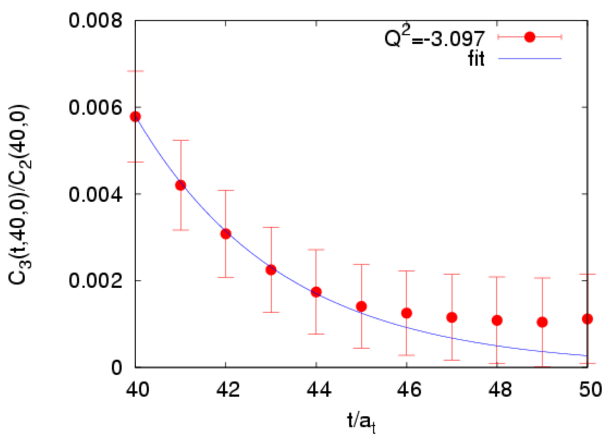
$$Q^2 = -(p_f - p_i)^2 = \vec{p}_f^{\,2} - (M_i - E_f)^2$$

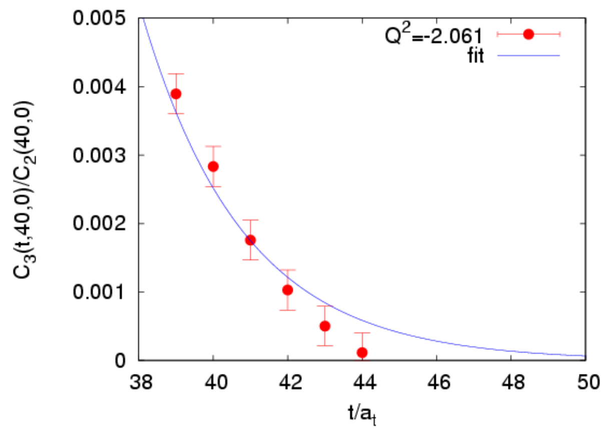
The plots for the ratios

$$\Gamma^{(3)}(t,40,0)/C_2(40)$$

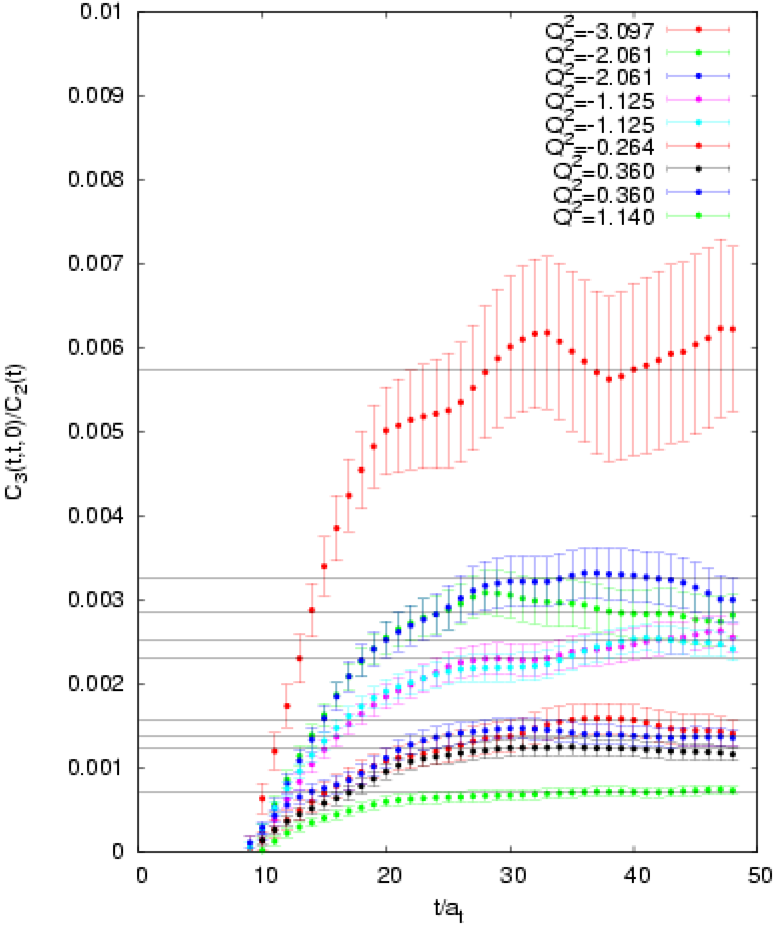


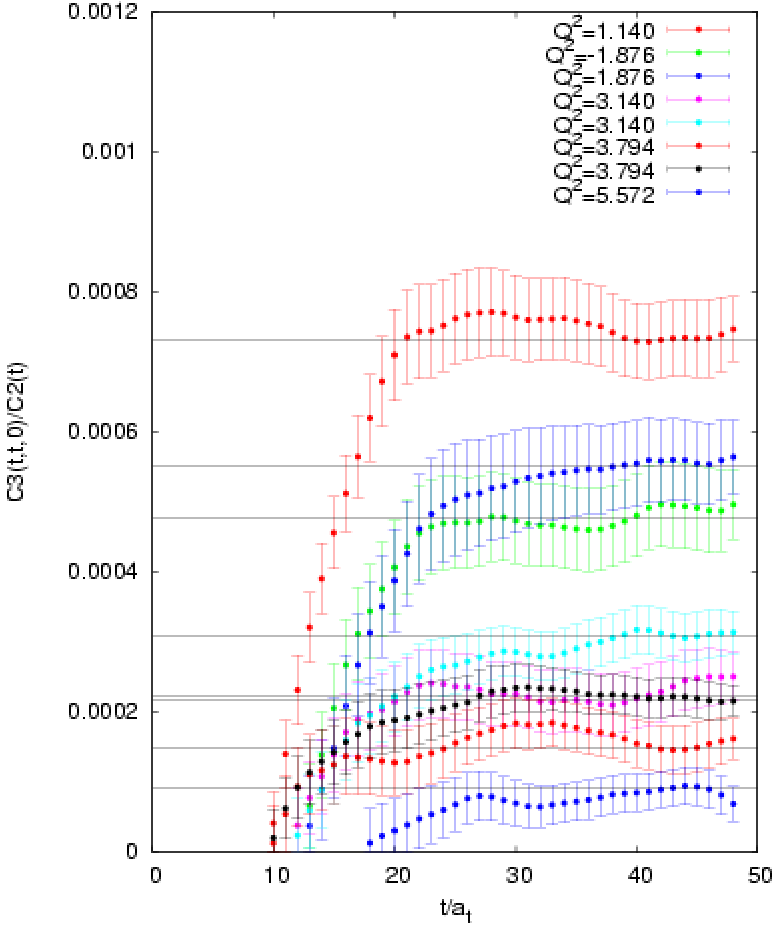






Exponential fit by taking the glueball energies as known parameters

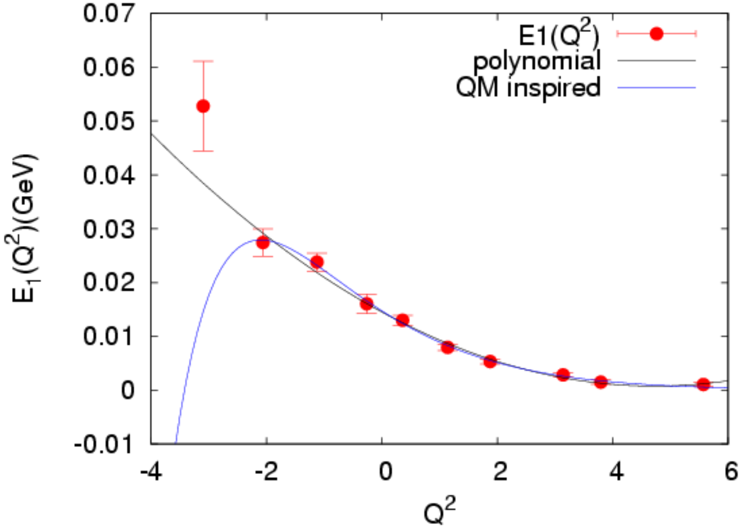




$$\frac{\Gamma^{(3),ii}(\vec{p}_f;t,t)}{C_2^{ii}(\vec{p}_i=0;t)} \equiv \alpha^{ii}(M_f,\vec{p}_f,M_i)E_1(Q^2,t)+\beta^{ii}(M_f,\vec{p}_f,M_i)C_1'(Q^2,t)$$

These are known functions

6. The form factor and the decay width



Polynomial fit:

$$E_1(Q^2) = E_1(0) + aQ^2 + bQ^4$$

$$E_1(0) = 0.0145(13)GeV$$

The branch ratio is

$$\Gamma(J/\psi \to \gamma G_{0^+}) = \frac{4}{27}\alpha \frac{|p|}{M_{J/\psi}^2}|E_1(0)|^2 = 0.030(5)keV$$

$$\frac{\Gamma}{\Gamma_{tot}} = 0.030(5)/93.2 = 3.2(5)\times 10^{-4}$$

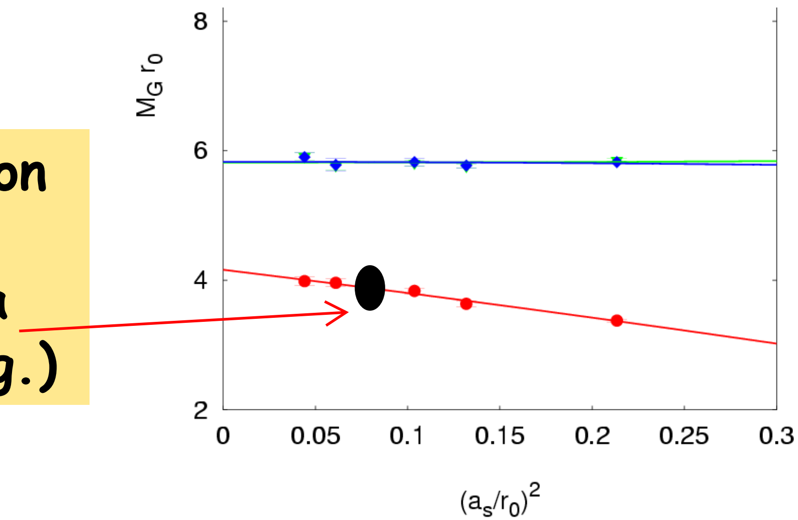
Experimental results for J/psi radiatively decaying to scalars

C. Amsler et al., Phy. Lett. B667, 1 (2008)

Decay modes	**Branch ratio** (Γ_i/Γ)
$J/\psi \to \gamma f_0(1710) \to \gamma K\bar{K}$	$(8.5^{+1.2}_{-0.9}) \times 10^{-4}$
$J/\psi \to \gamma f_0(1710) \to \gamma \pi\pi$	$(4.0 \pm 1.0) \times 10^{-4}$
$J/\psi \to \gamma f_0(1710) \to \gamma \omega\omega$	$(3.1 \pm 1.0) \times 10^{-4}$
$J/\psi \to \gamma f_0(1500)$	$> (5.7 \pm 0.8) \times 10^{-4}$
$J/\psi \to \gamma f_0(1370)$	**N/A**

7. The systematic uncertainties

- The continuum extrapolation has not been carried out. (The same calculation on a finer lattice is undergoing.)



- The lattice vector current has not been renormalized. (We are working on it.)

- The uncertainty owing to the quenched approximation. (Cannot be resolved in the near future.)

IV. Summary and concluding remarks

- For the first time, the form factors relevant to the J/psi radiatively decaying to glueballs are calculated in the quenched lattice QCD.

- With these form factors, the decay widths can be predicted more reliably. We obtain a raw estimation of the branch ratio $\Gamma(J/\psi \to \gamma G_{0^+})/\Gamma_{tot} = 3.2(5)\times 10^{-4}$ We are also working on other channels, such as tensor and pseudoscalar.

- Some of the systematic uncertainties will be addressed in the future work.

Thank You!