Non-perturbative renormalization of quark mass in Nf=2+1 QCD with the Schrödinger functional scheme

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Center for Computational Physics, University of Tsukuba

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Purpose of this project

- Determine the fundamental parameters of QCD
  - Fundamental parameters of $N_f = 2 + 1$ QCD

$$\mathcal{L} = -\frac{1}{4g_s^2} F^a_{\mu\nu} F^a_{\mu\nu} + \overline{\psi}_i \left(\gamma_\mu D_\mu + m_i\right) \psi_i$$

    - Strong coupling: g_s (PACS-CS 2009)
    - Light quark masses: m_{ud}, m_s (This year)
  - Determine m_{ud}, m_s with inputs of low energy observable on the lattice.
    - m_π, m_K, m_Ω (PACS-CS), m_π, m_ρ, m_K (m_ϕ) (CP-PACS)
  - Convert them in RGI mass or in the $\overline{\mathrm{MS}}$ scheme

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How to derive the RGI quark mass

- Quark mass is given by the PCAC mass.
- Definition of the RGI mass

$$M = \overline{m}(\mu)\left(2b_0\overline{g}^2(\mu)\right)^{-\frac{d_0}{2b_0}} \exp\left(-\int_0^{\overline{g}(\mu)} dg\left(\frac{\tau(g)}{\beta(g)} - \frac{d_0}{b_0 g}\right)\right)$$

- Window problem: $1/L \ll \Lambda_{QCD} \ll \mu \ll 1/a$
- Procedure is given as follows (ALPHA)

$$m^{(\mathrm{bare})}_{\mathrm{PCAC}}(g_0)\underbrace{\left.\frac{Z_A(g_0)}{Z_P(g_0, a/L_{max})}\right|_{a\to 0}}_{\mathrm{NPR}}\underbrace{\frac{\overline{m}(1/L_n)}{\overline{m}(1/L_{max})}}_{\mathrm{NP\ running}}\underbrace{\frac{M}{\overline{m}(1/L_n)}}_{\mathrm{PT\ running}}$$

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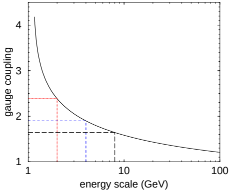
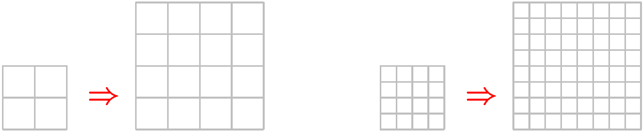
Step Scaling Function

- Discrete renormalization group flow: $\overline{\mathbf{m}}(\mathbf{L}) \to \overline{\mathbf{m}}(\mathbf{2L})$



Follow the RG flow
in discretized way.

- Take continuum limit for every step of RG flow.



- Covers $\Lambda_{\rm QCD} < \mu < 2^{10}\Lambda_{\rm QCD}$ with **10** independent SSF's.
  - keeping $\mathbf{a}/\mathbf{L} \ll \mathbf{1}$ at each step.

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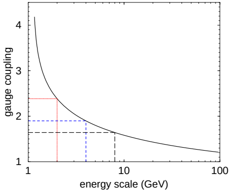

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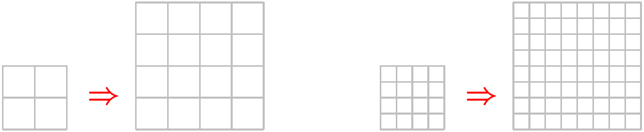

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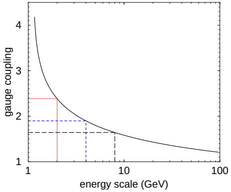

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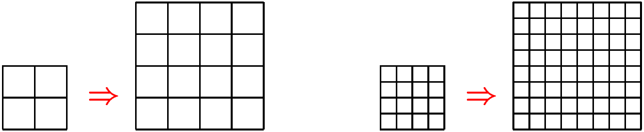

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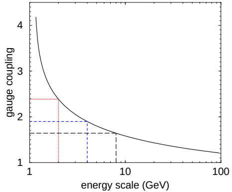

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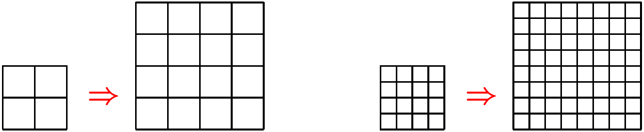

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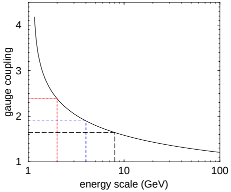

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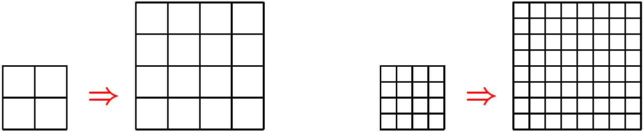

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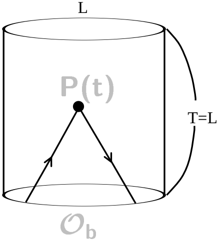
Pseudo scalar density SSF

- Renormalization factor of pseudo scalar density



$$Z_P = \frac{\langle P(t) \cdot \mathcal{O}_b \rangle_{tree}}{\langle P(t) \cdot \mathcal{O}_b \rangle}$$

- Step Scaling Function of pseudo scalar density

$$\Sigma_P\left(u, \frac{a}{L}\right) = \left.\frac{Z_P(g_0, 2L)}{Z_P(g_0, L)}\right|_{\bar{g}^2(L)=u, m=0}$$

- Iwasaki gauge action + NP imp. clover fermion.

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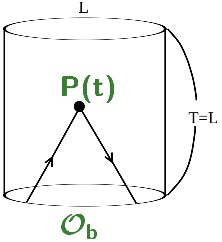

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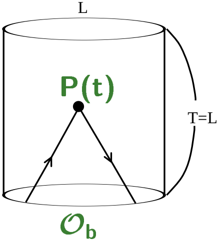

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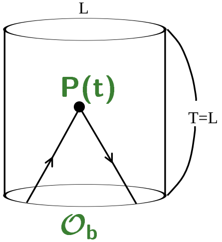

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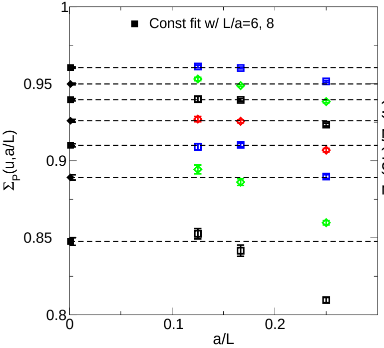
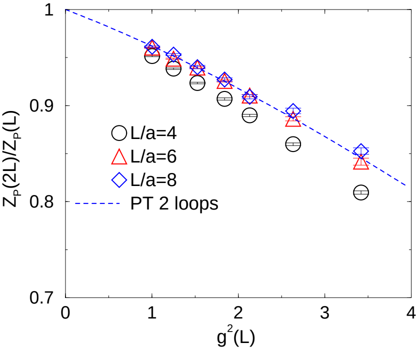
Scaling behavior





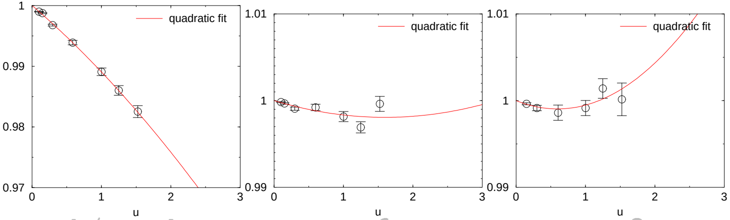
- Not very good.
  - Need improvement.

Perturbative improvement of the Step Scaling Function

- We need deviation from the continuum SSF

$$\frac{\Sigma_P(u, a/L) - \sigma_{\mathrm{PT}}(u)}{\sigma_{\mathrm{PT}}(u)}$$

  - PT result not available for Iwasaki action
  - Instead of the analytic evaluation
  - We try numerical evaluation at very weak coupling region.
  - Quadratic fit of the numerical data.



L/a = 4 6 8

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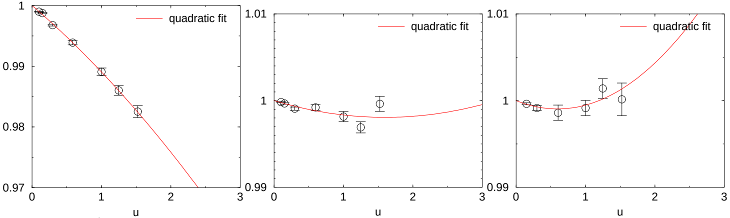

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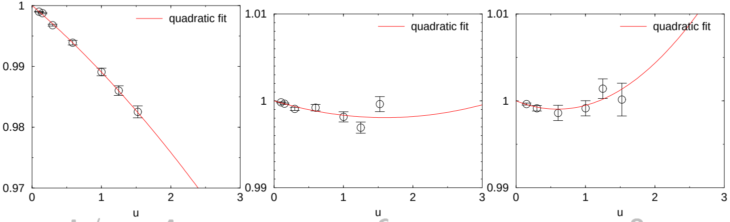

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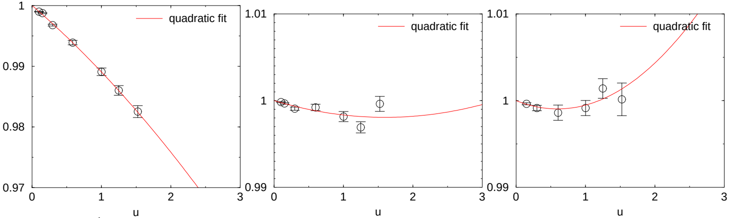

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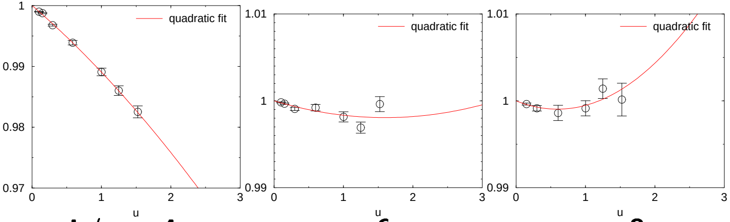

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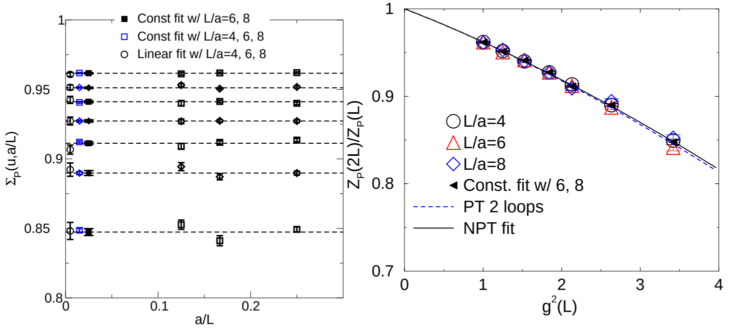

- Scaling behavior becomes much better.
- Consistency between three continuum extrapolations
  - Constant extrapolation with two/three data
  - Linear extrapolation

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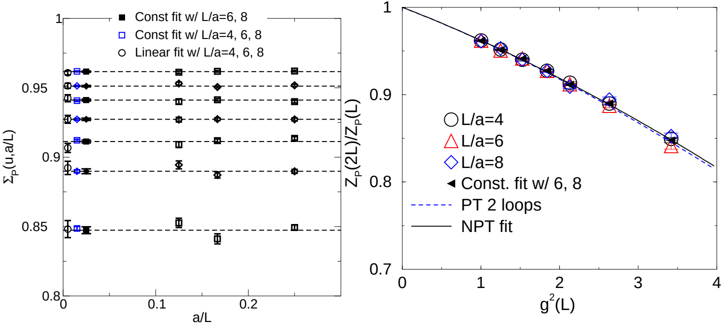

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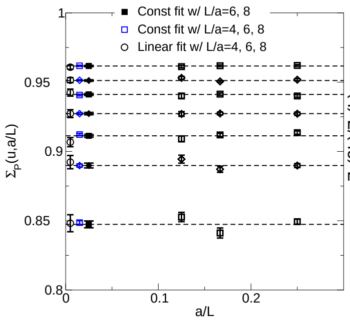

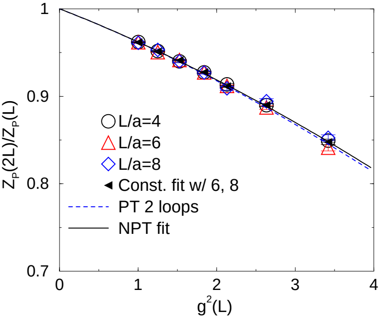

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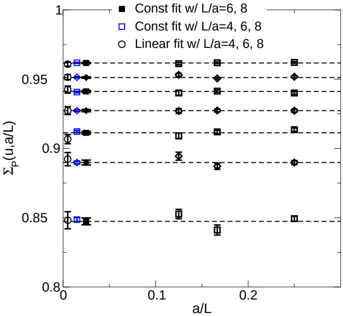

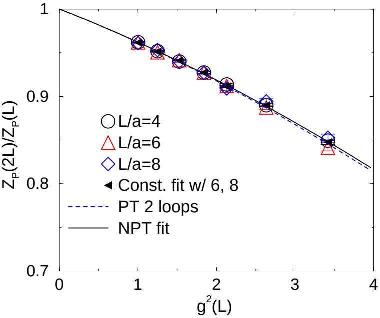

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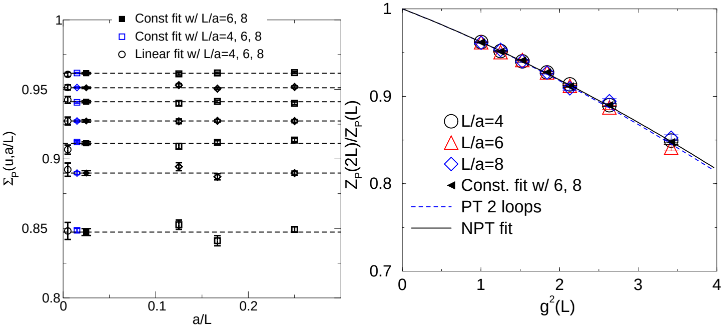

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- Consistency between three continuum extrapolations
    - Constant extrapolation with two/three data
    - Linear extrapolation

Non-perturbative running

- Step Scaling Function of pseudo scalar density

$$\sigma_P(u) = \lim_{a\to 0} \frac{Z_P(g_0, 2L)}{Z_P(g_0, L)}\bigg|_{\bar{g}^2(L)=u, m=0}$$

- Non-perturbative running factor

$$\frac{\overline{m}(1/L_n)}{\overline{m}(1/L_{max})} = \prod_{i=1}^{n} \sigma_P(u_i), \quad L_n = 2^{-n} L_{max}$$

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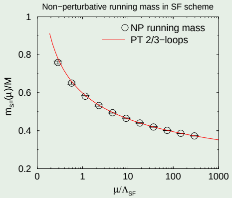
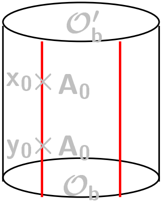
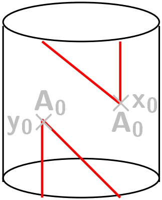


Renormalization factor of axial vector current

$$m_{PCAC}^{(bare)}(\beta) \left. \frac{Z_A(\beta)}{Z_P(\beta, a/L_{max})} \right|_{a\to 0} \left. \frac{\overline{m}(1/L_n)}{\overline{m}(1/L_{max})} \right|_{NP} \left. \frac{M}{\overline{m}(1/L_n)} \right|_{PT}$$

- Z_A at three $\beta = 1.83$, $\beta = 1.90$, $\beta = 2.05$
- From WT identity: $$Z_A^2 = \frac{\langle \mathcal{O}'_b \mathcal{O}_b \rangle}{\langle \mathcal{O}'_b \cdot A_0(x_0) A_0(y_0) \cdot \mathcal{O}_b \rangle}$$
  - Connected
  - Disconnected





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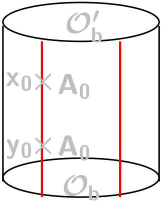

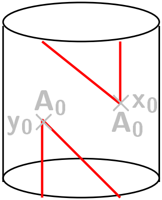

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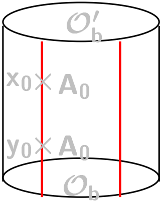

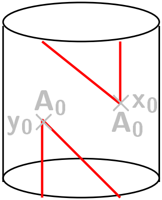

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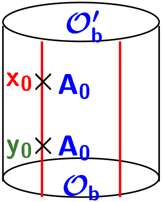

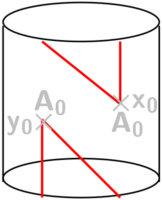

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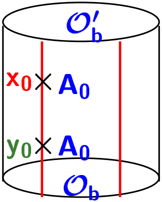

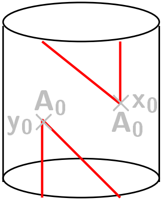

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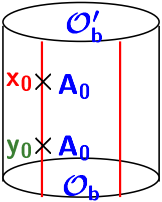

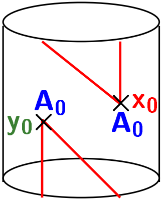

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$\Rightarrow 0$
$(m \to 0,$
$a \to 0)$

Renormalization factor of axial vector current

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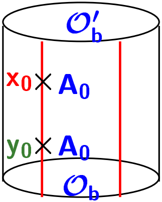
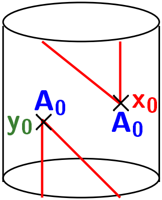
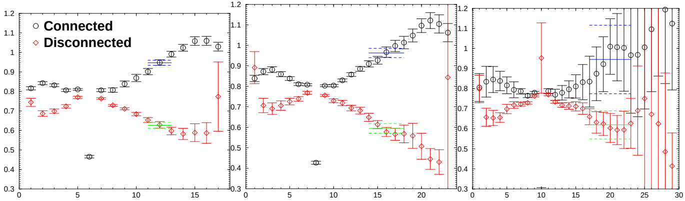


Should be flat in x_0 by WT id.

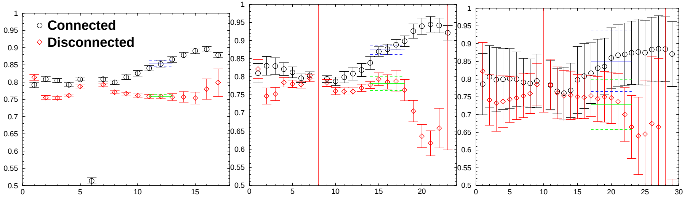


$\Rightarrow 0$ $(m \to 0,\ a \to 0)$

- Plateau as an estimate of the $O((a/L)^2)$ artifact
- beta = 1.83 (a = 0.117 fm)



- β = 1.90 (a = 0.090 fm)



L/a = 8 10 12

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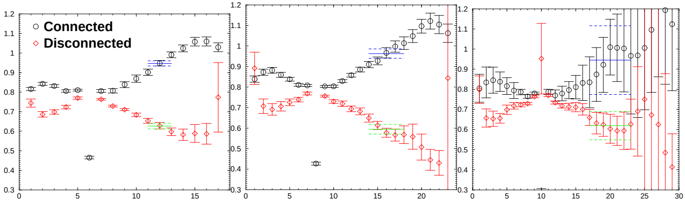

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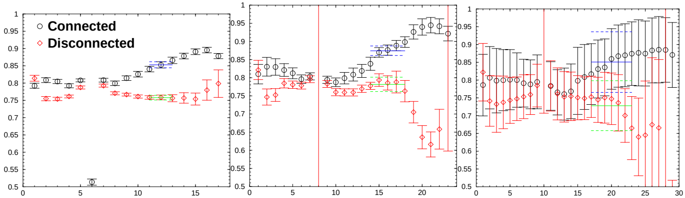

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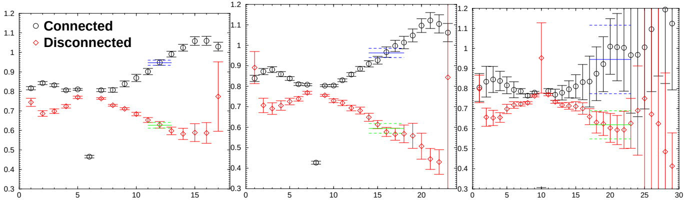

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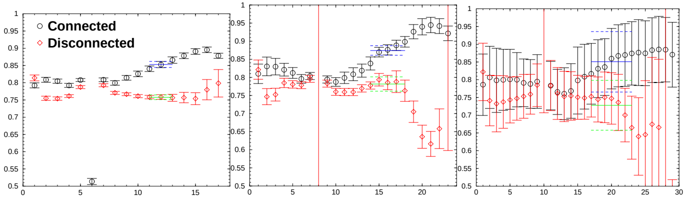
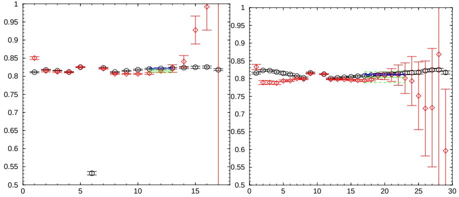


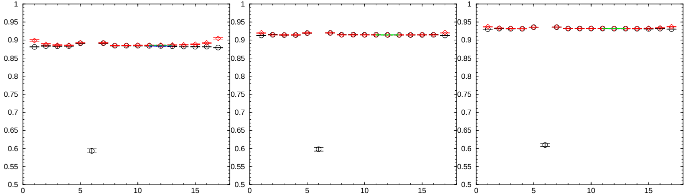
$L/a = 8$ 10 12

- $\beta = 2.05$ (a = 0.070 fm)



L/a = 8 12

- $\beta = 3.0$ $\beta = 4.0$ $\beta = 5.0$



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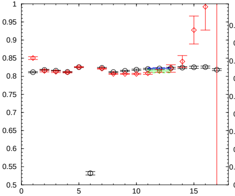

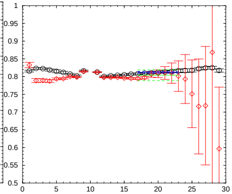

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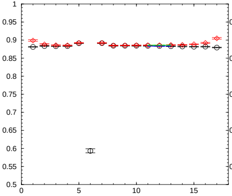

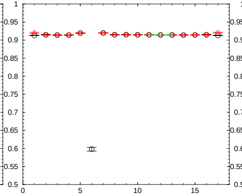

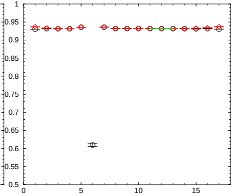

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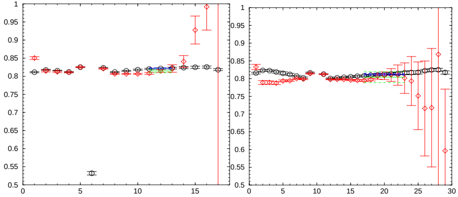

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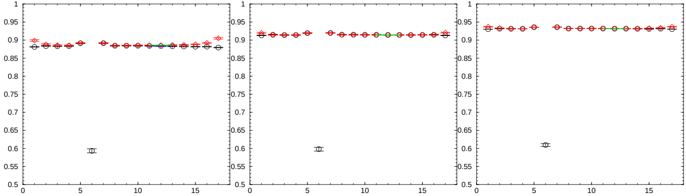

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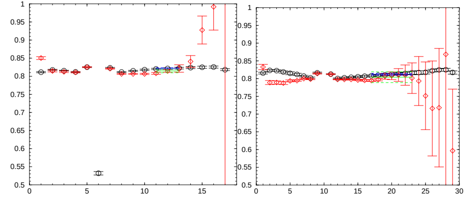

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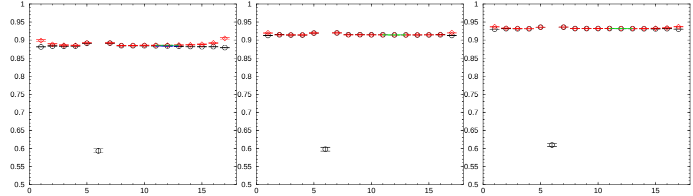
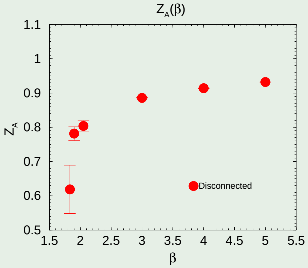


$L/a = 8$ 8 8

Recommended choice for Z_A

- With disconnected diagram.
- Larger box size: 12^3 ($\beta = 1.83$), 10^3 ($\beta = 1.90$)
- $\beta \geq 2.05$ is recommended for small lattice artifact.

β dependence of Z_A



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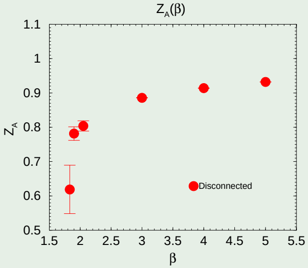

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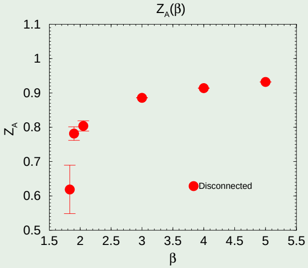

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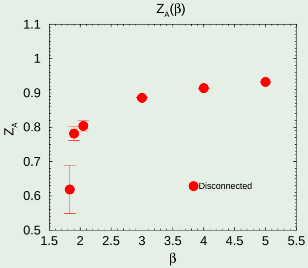

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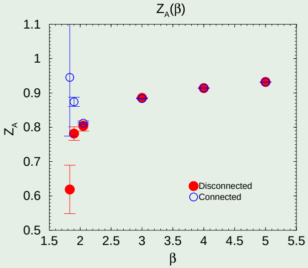

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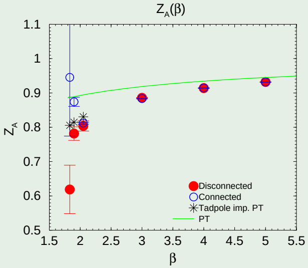
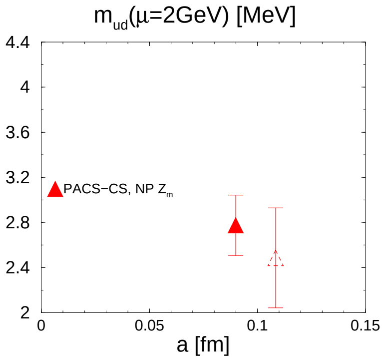


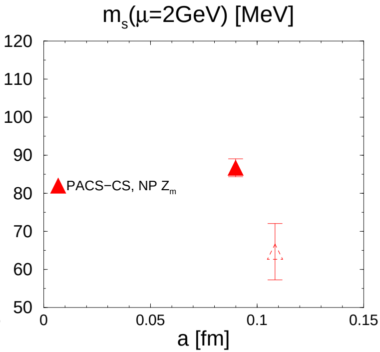
Light quark masses

△ PACS-CS: $\mathbf{m_\pi \sim 155}$ MeV

- On physical point with reweighting

○ CP-PACS/JLQCD: $\mathbf{m_\pi \sim 500}$ MeV





$\mathbf{m_{ud}^{\overline{MS}}(2\ GeV) = 2.78(27)}$, $\mathbf{m_{s}^{\overline{MS}} = 86.7(2.3)}$ MeV

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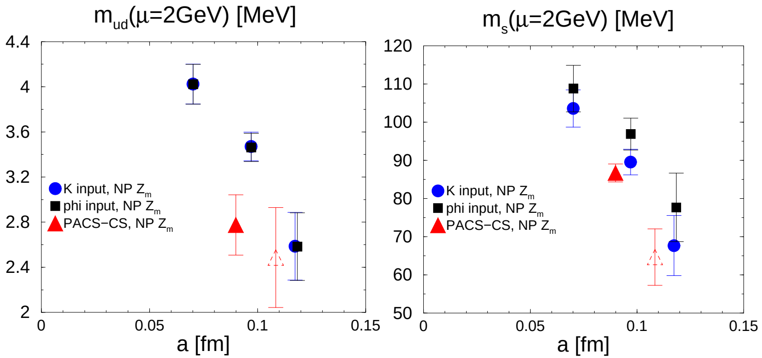

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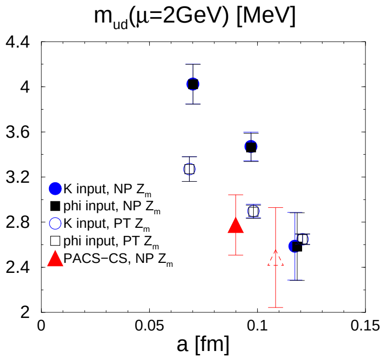
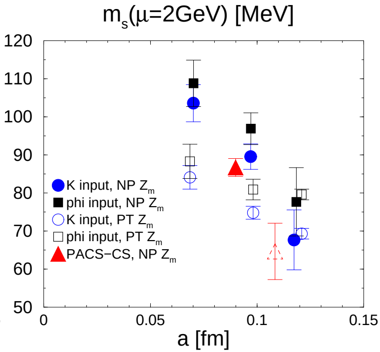


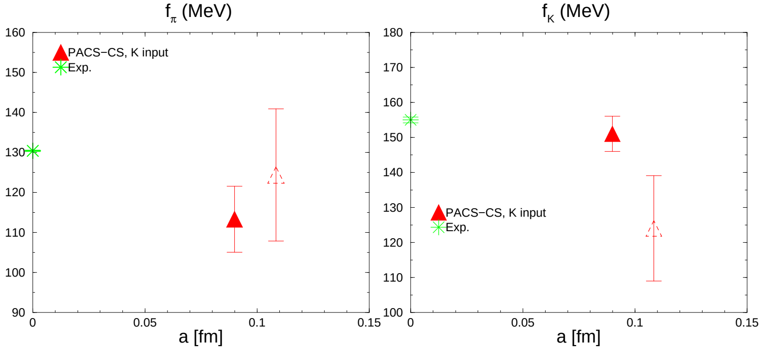


$\mathbf{m_{ud}^{\overline{MS}}(2\ GeV) = 2.78(27)}$, $\mathbf{m_s^{\overline{MS}} = 86.7(2.3)}$ MeV

Decay constants

△ PACS-CS: $m_\pi \sim 155$ MeV

◯ CP-PACS/JLQCD: $m_\pi \sim 500$ MeV



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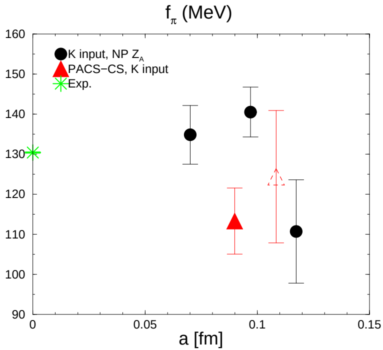

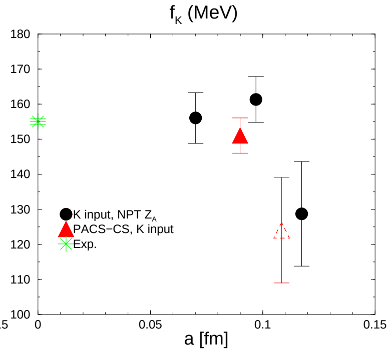

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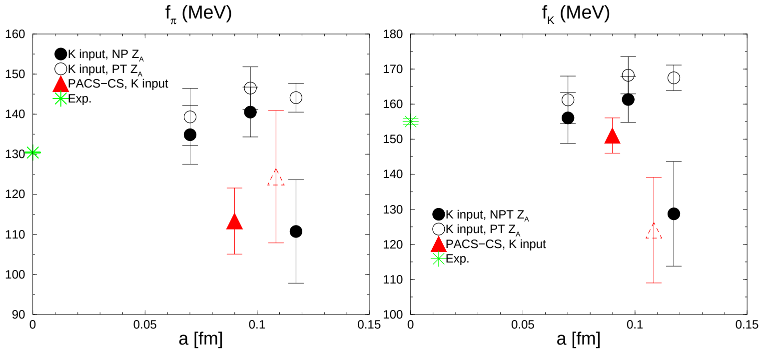
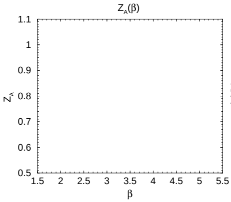
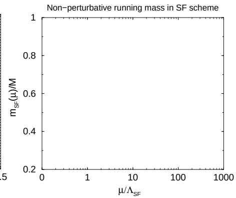


Conclusion

$$\underbrace{m_{\mathrm{PCAC}}^{(\mathrm{bare})}(g_0)}_{\mathrm{PACS-CS}} \underbrace{\left.\frac{Z_A(g_0)}{Z_P(g_0, a/L_{max})}\right|_{a\to 0}}_{\mathrm{NPR}} \underbrace{\frac{\overline{m}(1/L_n)}{\overline{m}(1/L_{max})}}_{\mathrm{NP\ running}} \underbrace{\frac{M}{\overline{m}(1/L_n)}}_{\mathrm{PT\ running}}$$

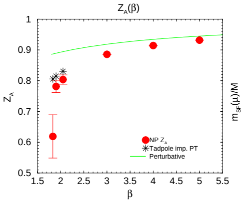


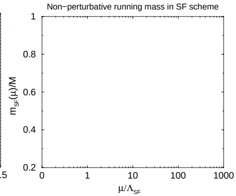


- $Z_m^{\overline{\mathrm{MS}}}(\beta = 1.90) = 1.347(36)$ (cf. $Z_m^{\mathrm{PT}} = 1.11322$)
- Applied to PACS-CS result
  - $m_{ud}^{\overline{\mathrm{MS}}}(2\ \mathrm{GeV}) = 2.78(27)$, $m_s^{\overline{\mathrm{MS}}} = 86.7(2.3)$ MeV
- We need result at $\beta = 2.05$.

Conclusion

$$\underbrace{m_{PCAC}^{(bare)}(g_0)}_{PACS-CS} \underbrace{\left.\frac{Z_A(g_0)}{Z_P(g_0, a/L_{max})}\right|_{a\to 0}}_{NPR} \underbrace{\frac{\overline{m}(1/L_n)}{\overline{m}(1/L_{max})}}_{NP\ running} \underbrace{\frac{M}{\overline{m}(1/L_n)}}_{PT\ running}$$





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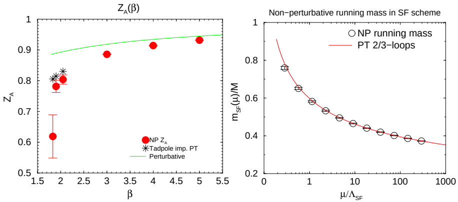

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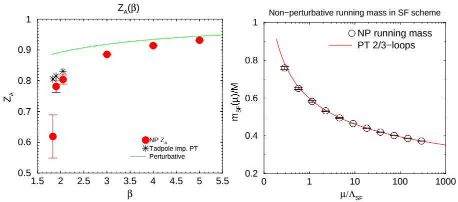

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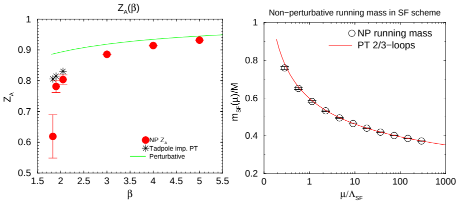

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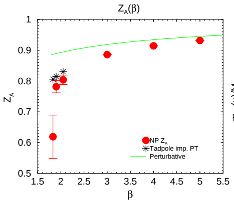

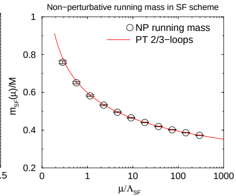

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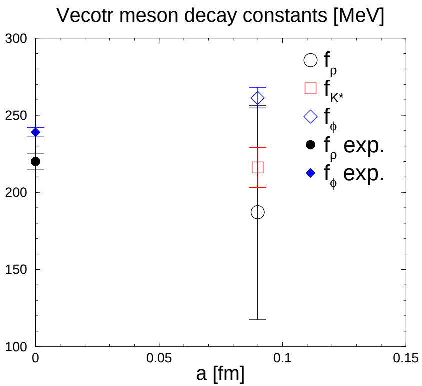
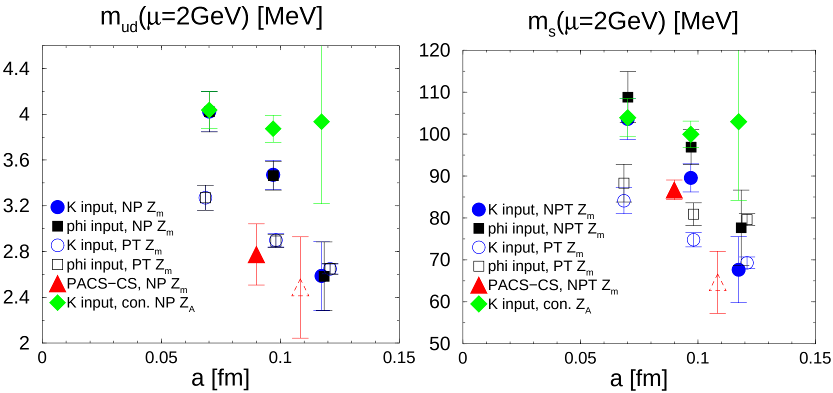
Vector meson decay constants



NP renormalization without disconnected diagram



NP renormalization without disconnected diagram