SU(3) Deconfinement in 2+1d from Twisted Boundary Conditions and Self-Duality

Nils Strodthoff

Darmstadt University of Technology, Germany

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based on work together with
Sam Edwards and Lorenz von Smekal

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Outline

't Hooft's Twisted Boundary Conditions

Twisted boundary conditions in pure SU(N) gauge theory: ['t Hooft 1979]

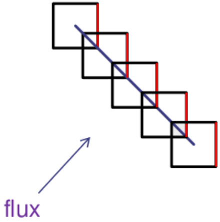
- pure gauge: gauge fields represent the center ($\mathbb{Z}_N$ for SU(N)) trivially
- boundary conditions for the gauge fields in a finite box: fixed up to center elements
- N^3 gauge-inequivalent choices (in 2+1 dimensions)
  $\Rightarrow$'t Hooft's twisted boundary conditions
- at finite T distinguish:
  - temporal twists classified by $\vec{k} \in \mathbb{Z}_N^2$
  - magnetic twists (purely spatial) classified by $m \in \mathbb{Z}_N$
    m is the gauge invariant magnetic flux (set $m = 0$ in the following).

Vortex free energy (per T) F_k:

$$\frac{Z_k(\vec{k})}{Z_k(\vec{0})} \equiv e^{-F_k(\vec{k})}$$

Implementing 't Hooft's twisted boundary conditions on the lattice:

- Multiply one link in each slice by center element z.
- Equivalently, keep periodic boundary conditions but modify action: $\beta \to z\beta$ for a stack of plaquettes perpendicular to plane of twist



$\Rightarrow$ enforce non-vanishing center flux

Electric Flux Partition Functions

- $\mathbb{Z}_N$ Fourier transform of $Z_k(\vec{k})$: electric flux partition function

$$Z_e(\vec{e}) = \frac{1}{N^2} \sum_{\vec{k}} e^{-2\pi i \vec{e}\cdot\vec{k}/N} Z_k(\vec{k})$$

- $\vec{e} \in \mathbb{Z}_N^2$ is gauge invariant electrical flux in $\vec{e}$ direction

Electric flux free energy (per T) F_e:

$$\frac{Z_e(\vec{e})}{Z_e(\vec{0})} \equiv e^{-F_e(\vec{e})}$$

- Interpretation: [de Forcrand, von Smekal 2002]

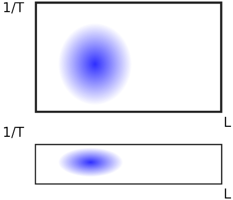
$$\langle \mathcal{P}(\vec{x})\mathcal{P}(\vec{x}+L\vec{e})\rangle_{\text{no-flux}} = e^{-F_e(\vec{e},T,L)}$$

F_e: free energy of fundamental static charge with mirror (anti)charges in the adjacent volume

[Greensite 2003]

Intuitive picture:

- Low T: vortices spread to lower their free energy, disorder temporal Wilson loops
- High T: vortices squeezed



Order parameters for the deconfinement transition:

- center vortex free energy $F_k \sim \tilde{\sigma} L^{d-1}$,
  dual string tension: $\tilde{\sigma} > 0$ for $T > T_c$
- dual order parameter: electric flux free energy $F_e \sim \sigma L/T$,
  string tension: $\sigma > 0$ for $T < T_c$

The Svetitsky-Yaffe Conjecture

Svetitsky, Yaffe 1982

$d+1$ dimensional gauge theory with 2nd order deconfinement transition

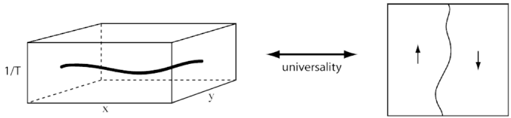
$\Updownarrow$ same universal properties

d dimensional spin model with the same (center) symmetry

The Svetitsky-Yaffe Conjecture II

Correspondence here:

$$\begin{aligned} \text{2+1d SU(3) gauge theory} &\longleftrightarrow \text{2d 3-state Potts model} \\ \text{Polyakov loop correlators} &\longleftrightarrow \text{spin correlators} \\ \text{center vortices} &\longleftrightarrow \text{spin interfaces} \\ \text{free energy } F_k \sim \tilde{\sigma} L &\longleftrightarrow \text{free energy } F_I \sim \sigma_I L \\ \text{dual string tension } \tilde{\sigma} &\longleftrightarrow \text{interface tension } \sigma_I \end{aligned}$$



$\Rightarrow$ well-studied spin models with many exact results available: critical exponents, interface tensions,...

Phase Transitions in pure SU(N) Gauge Theories

Deconfinement phase transitions in $(d+1)$d pure SU(N) gauge theories

	N=2	N=3	N=4
d=3	2nd order	1st order	1st order
d=2	2nd order	2nd order	1st order?

Review: Deconfinement phase transition in 2+1d SU(N) gauge theory [Liddle, Teper 2008]

Phase Transitions in pure SU(N) Gauge Theories

Deconfinement phase transitions in $(d+1)$d pure SU(N) gauge theories

	N=2	N=3	N=4
d=3	2nd order 3d Ising model	1st order -	1st order -
d=2	2nd order 2d Ising model	2nd order 2d 3-state Potts model	1st order? 2d 4-state Potts?

Review: Deconfinement phase transition in 2+1d SU(N) gauge theory [Liddle, Teper 2008]

[Kramers, Wannier 1941] [Savit 1980]

- 2d Ising model $\longleftrightarrow$ 2d Ising model (self-dual)
  with $\beta \sim \tilde{\beta} = -\frac{1}{2} \ln \tanh \beta$, $\beta = J/kT$
- 2d 3-state Potts model $\longleftrightarrow$ 2d 3-state Potts model (self-dual)
  3d Ising model $\longleftrightarrow$ 3d $\mathbb{Z}_2$ gauge theory

Generalization: Finite torus

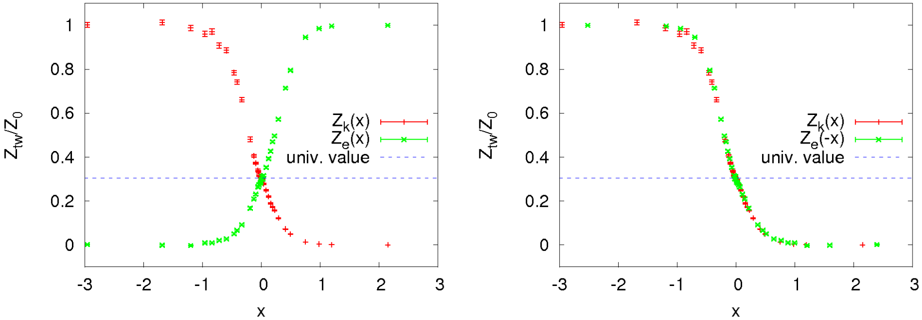
- partition functions with different boundary conditions mix
- e.g. self-duality relation for the Ising model ($N_s \times N_s$, square):

$$\begin{pmatrix} Z_{pp}(\tilde{\beta}) \\ Z_{ap}(\tilde{\beta}) \\ Z_{pa}(\tilde{\beta}) \\ Z_{aa}(\tilde{\beta}) \end{pmatrix} = \frac{1}{2} \sinh(2\beta)^{-{N_s}^2} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix} \begin{pmatrix} Z_{pp}(\beta) \\ Z_{ap}(\beta) \\ Z_{pa}(\beta) \\ Z_{aa}(\beta) \end{pmatrix}$$

- similar relation for all q-state Potts models [Strodthoff, Edwards, von Smekal in prep.]

Checking Self-Duality in SU(3)

- Check self-duality: $\frac{Z_{k,tw}(x)}{Z_{k,0}(x)} \stackrel{?}{=} \frac{Z_{e,tw}(-x)}{Z_{e,0}(-x)}$
  with finite-size scaling (FSS) variable $x = \pm T_c L\,(\pm t)^{\nu} \propto L/\xi^{\pm}$



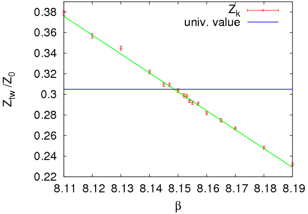
$\Rightarrow$ Numerical evidence for self-duality in SU(3)

1st method: [Edwards, von Smekal 2009]

- Intersect $\frac{Z_{k,tw}}{Z_{k,0}}$ with universal value

  [Park, den Nijs 1988]

  $\Rightarrow$ extract pseudo-critical couplings $\beta_c(N_s, N_t)$

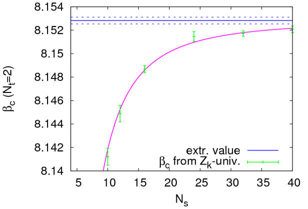


- From the FSS ansatz

$$\frac{Z_{k,tw}}{Z_{k,0}} = Z_{\text{univ}} + b\left(\beta_c(N_s, N_t) - \beta_{c,\infty}(N_t)\right) N_s^{1/\nu} + cN_s^{-\omega} + \ldots$$

get

$$\beta_c(N_s, N_t) = \beta_{c,\infty}(N_t) - d(N_t) N_s^{-(\omega+1/\nu)}$$

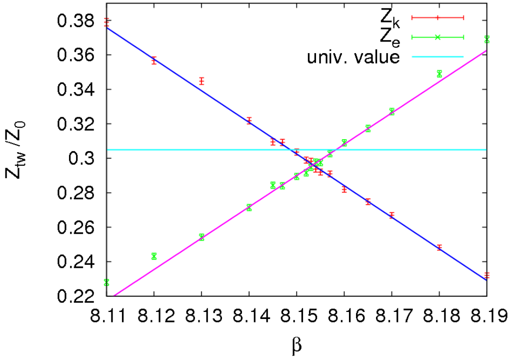


- Determine $\beta_{c,\infty}(N_t)$ by extrapolation

Critical Couplings: Exploiting Self-Duality

2nd method: [Strodthoff, Edwards, von Smekal in prep.]

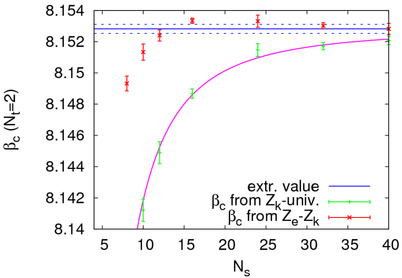
- Determine $\beta_{c,\infty}$ by intersecting Z_e and Z_k

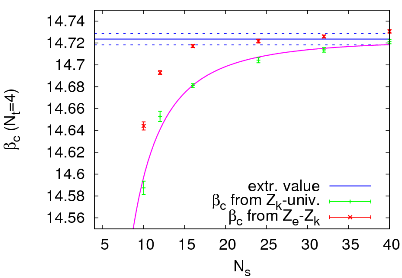


- Advantages:
  - Conceptually: leading corrections to FSS cancel
    $\Rightarrow$ obtain $\beta_{c,\infty}$ in small volumes
  - Numerically: faster convergence

Speed of Convergence and Results

Speed of convergence:





Critical couplings and critical temperature:

N_t	β_c	Lit.
2	8.15309(11)	8.1489(31)†
4	14.7262(9)	14.717(17)†
6	21.357(25)	21.34(4) ‡
8	27.84(12)	-

† [Liddle, Teper 2008] ‡ [Engels et al. 1997]

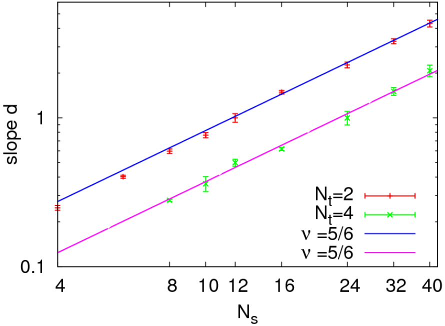
$\Rightarrow \frac{T_c}{{g_3}^2} = 0.5475(3)$; $\frac{T_c}{\sqrt{\sigma}} = 0.9938(9)$
(using literature values for $g^2/\sqrt{\sigma}$)

Literature value: $\frac{T_c}{\sqrt{\sigma}} = 0.9994(40)$

[Liddle, Teper 2008]

Correlation Length Critical Exponent ν from FSS

- For $\beta \sim \beta_c$: $F_k(\beta) = c + d(\beta - \beta_c) + \ldots$
- F_k is a universal function of the FSS variable $x = N_s t^\nu$
- $(\beta - \beta_c) \sim x^{1/\nu} N_s^{-1/\nu} \Rightarrow d(N_s) \sim N_s^{1/\nu}$



- Result e.g. for $N_t = 2$: $\nu = 0.818(25)(20)$
compared to $\nu = 5/6 \approx 0.8333$ for the 2d 3-state Potts model

Motivation: SU(4)

Consider SU(4) in 2+1d:

- SU(4) in 2+1d: order of the phase transition is not entirely clear although there is strong evidence for a first order transition

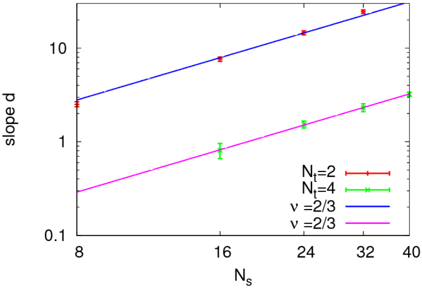
  [de Forcrand, Jahn 2003] [Liddle, Teper 2008] [Holland, Pepe, Wiese 2008]
- If it was 2nd order which of the Ashkin-Teller models? (likely Potts model c.f. [de Forcrand, Jahn 2003])
- Apply our methods to SU(4) to see what happens...

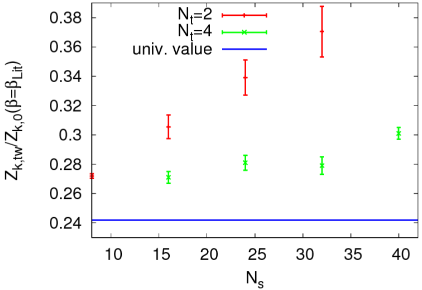
- Critical exponent ($N_t = 4$): $\nu = 0.673(9)$ compared to $\nu = 2/3$ for the 4-state Potts model
- Critical couplings from intersecting with universal value for 4-state Potts model?

N_t	$\beta_{c,\infty}$?	Lit.
2	14.86(2)	14.8403(26)†
4	26.29(2)	26.251(16)‡ 26.228(75)†

† [Liddle, Teper 2008] ‡ [Holland, Pepe, Wiese 2008]

- Does $\frac{Z_{k,tw}}{Z_{k,0}}(\beta_{\text{Lit}})$ converge the universal value?





Summary and Conclusions

- Studied deconfinement transition in pure SU(3) and SU(4) lattice gauge theory in 2+1 dimensions
- Using 't Hooft's twisted boundary conditions measured vortex free energy as an order parameter for the deconfinement transition
- Verified self-duality in SU(3) numerically
- Determined critical couplings $\beta_{c,\infty}$, critical temperature $\frac{T_c}{\sqrt{\sigma}}$ and the critical exponent ν with very high precision
- Presented preliminary results for SU(4)

Thank you for your attention!