Lattice study of transport coefficients in second order dissipative hydrodynamics

Lattice 2010

@ Cagliari, Italy

Jun. 18. 2010

Yasuhiro Kohno[1]

Masayuki Asakawa[1], Masakiyo Kiatazawa[1], Chiho Nonaka[2]

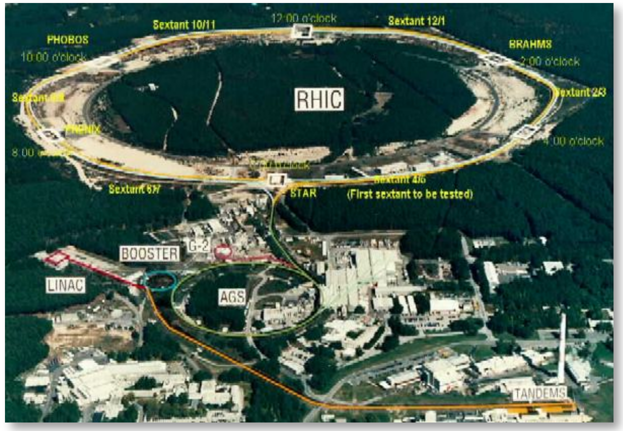
[1]Osaka University, [2]Nagoya University

Background

- Success of ideal hydrodynamic model

> RHIC Scientists Serve Up "Perfect" Liquid
> New state of matter more remarkable than predicted
> -- raising many new questions April 18,2005

New matter（Quark–Gluon Plasma, QGP？）produced @ RHIC behaves like ideal fluid near critical point.



Take account of effects of the viscosities to hydrodynamics for more quantitative discussion of heavy ion collisions.

⇒We adopted second order dissipative (viscous) hydrodynamics by Israel–Stewart.

Relativistic dissipative hydrodynamics

- Basic equations & 2nd law of thermodynamics

$$\partial_\mu T^{\mu\nu} = 0, \quad \partial_\mu N^\mu = 0$$

Conservation of energy–momentum

Conservation of charge

$$\partial_\mu S^\mu \geqq 0$$

2nd law of thermodynamics

5 equations of motion for 14 unknown variables (e, n, etc.).

constraint⇒ 9 eqs. of motion

$T^{\mu\nu} = T^{\mu\nu}_{eq.} + \delta T^{\mu\nu}$: energy-momentum tensor
$N^{\mu} = N^{\mu}_{eq.} + \delta N^{\mu}$: charge current
$S^{\mu} = S^{\mu}_{eq.} + \delta S^{\mu}$: entropy current

Relativistic dissipative hydrodynamics

- Generalized off-equilibrium entropy current

$$S^{\mu} = p\beta^{\mu} \underline{-\alpha N^{\mu} + \beta_{\nu} T^{\mu\nu}} + \underline{Q^{\mu}(\delta N^{\mu}, \delta T^{\mu\nu})}$$

1st order of dissipation

Higher order of dissipation

T : temperature , p : pressure , u^{μ} : 4-current
μ :chemical potential , $\alpha = \mu/T$, $\beta = 1/T$, $\beta_{\mu} = u_{\mu}/T$

Gibbs–Duhem relation & Conservation lows & 2nd low

$$T\partial_{\mu} S^{\mu} = \Pi X - q^{\mu} X_{\mu} + \pi^{\mu\nu} X_{\mu\nu} + T\partial_{\mu} Q^{\mu} \geq 0$$

Π : bulk viscous pressure , q : heat flow
π : shear viscous pressure , X : thermodynamic forces

Relativistic dissipative hydrodynamics

- 1st order theory (by Eckart, Landau–Lifshitz)

$$T\partial_\mu S^\mu = \Pi X - q^\mu X_\mu + \pi^{\mu\nu} X_{\mu\nu} \geq 0$$

the simplest assumption

linear response

$$\Pi = \varsigma X \,,\; q^\mu = \lambda X^\mu \,,\; \pi^{\mu\nu} = 2\eta X^{\mu\nu}$$

positive

ζ : bulk viscosity , λ : heat conductivity , η : shear viscosity

One obtains additive 9 equations of motion.

These equations, however, are known as acausal ones (i.e. violate causality).

C.Eckart, Phys. Rev. **58**, 919(1940)
L.D.Landau and E.M.Lifshitz, *Fluid Mechanics*(Pergamon, New York, 1959)

Relativistic dissipative hydrodynamics

- 2nd order theory (by Israel–Stewart)

$$T\partial_\mu S^\mu = \Pi X - q^\mu X_\mu + \pi^{\mu\nu} X_{\mu\nu} + T\partial_\mu Q^\mu \geq 0$$

$$Q^\mu = -\frac{u^\mu}{2T}\beta_2 \pi_{ij}\pi^{ij} = -\frac{u^\mu}{2T}\frac{\tau_\pi}{2\eta}\pi_{ij}\pi^{ij}$$

only shear part for simplicity

τ_π: relaxation time , η: shear viscosity

$$\beta_2 = \frac{\tau_\pi}{2\eta}$$

Perform the same procedure as in 1st order theory.

Eqs. of motion become causal ones, but new transport coefficients (or phenomenological parameter) appear.

W.Israel and J.M.Stewart, Ann. Phys. (N.Y.) **118**, 341(1979)

Purpose of our study

These new transport coefficients can not be determined by hydrodynamics.
⇒Microscopic theory determine them.

To evaluate the 2nd order transport coefficients by lattice QCD.

$$\beta_0 = \frac{\tau_\Pi}{\zeta}, \ \beta_1 = \frac{\tau_q}{\lambda T}, \ \beta_2 = \frac{\tau_\pi}{2\eta}$$

β_0:bulk viscous part β_1:heat conductive part β_2:shear viscous part

※Focus on only β_2 in following discussion.

See also, Y. Maezawa et. al. (Poster)

Idea

- Boltzmann–Einstein (BE) principle

Probability that a state variable *x* takes a value *A* in equilibrium is given by

$$P(x=A)=\frac{W(A)}{\sum_x W(x)}\propto \exp[S(A)]$$

c.f. $S=\log W$
W : Number of microscopic states

- Applying the entropy of 2nd order theory to BE principle,

$$S=u_\mu S^\mu = S_{eq.} - \frac{V}{2T}\beta_2\pi_{ij}^2$$

$$Q^\mu = -\frac{u^\mu}{2T}\beta_2\pi_{ij}\pi^{ij}$$

$$P(\pi_{ij})\propto W(\pi_{ij})\propto \exp[-\frac{V}{2T}\beta_2\pi_{ij}^2]$$

A. Muronga, Eur. Phys. J. ST 155:107-113(2008)
S. Pratt, Phys. Rev. C**77**, 024910(2008)

Idea

- Gaussian form

$$P(\pi_{ij}) \approx \exp[-\frac{V}{2T}\beta_2 \pi_{ij}^2]$$

β_2 is related with the fluctuation of π_{ij} .

$$<\pi_{ij}^{\ 2}> = \frac{T}{V\beta_2}$$

※$\langle \pi_{ij} \rangle = 0$

- In local rest frame,

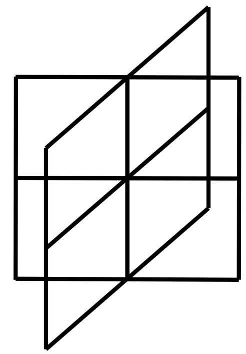
$$\pi_{ij} = \frac{1}{V}\int dV T_{ij}$$

※Measurement of the statistical average $\langle \pi_{ij}^2 \rangle$ on the lattice gives β_2 .

※$\langle \pi_{ij}^2 \rangle$ is UV divergence. We subtract T=0 contribution.

Numerical Setup

- SU(3) pure gauge (Wilson action)
- 1HB + 4OR for each update
- Clover type plaquette for T_{ij}

$T_{ij} \sim$ tr 

- Isotropic lattice
- Lattice size

  β =6.000(a=0.094fm): $16^3 \times 4, 6, 8, 16$

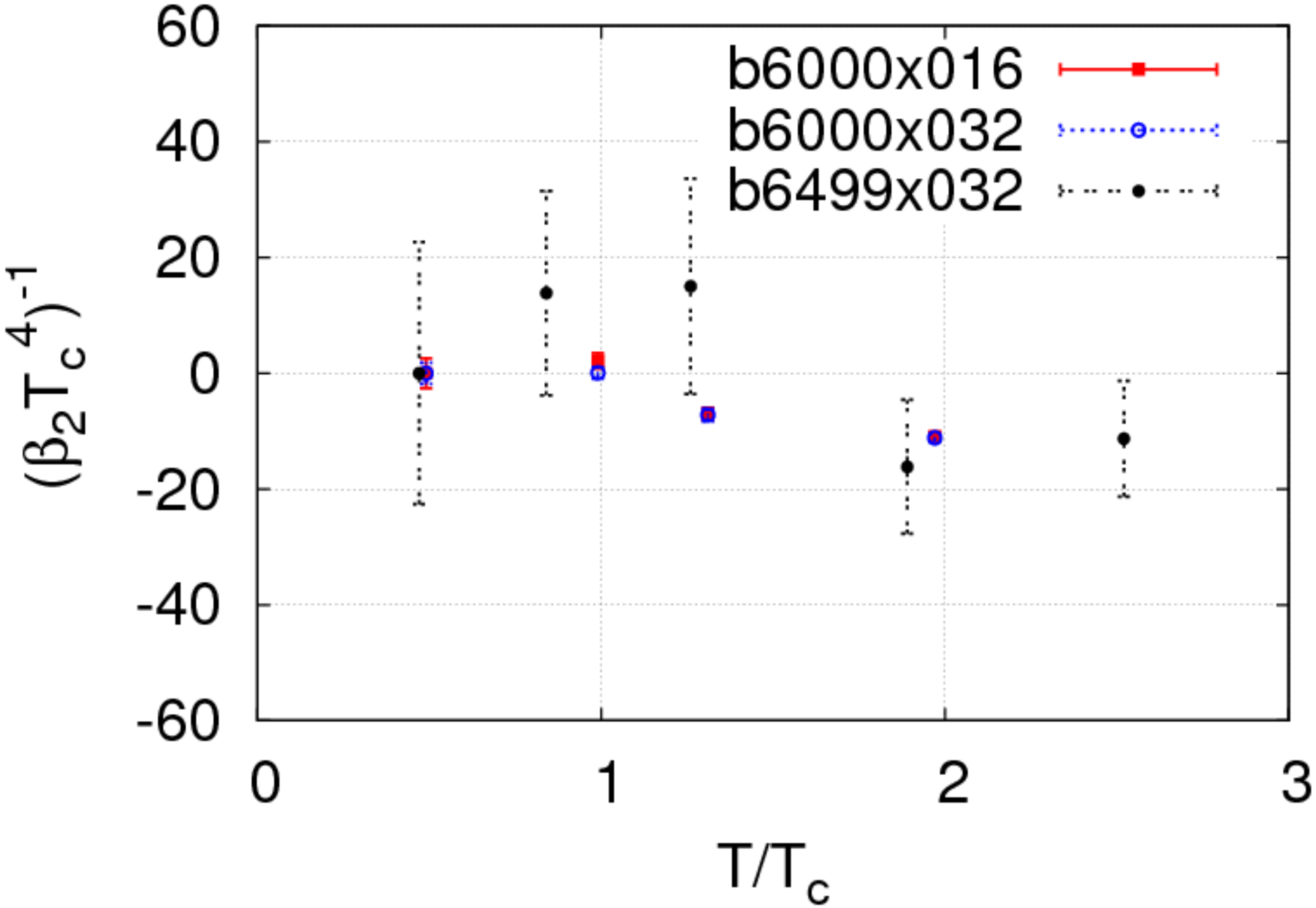
  β =6.000(a=0.094fm): $32^3 \times 4, 6, 8, 16$

  β =6.499(a=0.049fm): $32^3 \times 6, 8, 14, 18, 32$

- $N_{conf.}$=300,000～500,000
- Error estimate by jackknife (binsize=50)

Numerical Results

- $\langle \pi_{ij}^2 \rangle = T/V \beta_2$



- β =6.0 : $\langle \pi_{ij}^2 \rangle$ becomes negative (no V dependence).
- β =6.499 : consistent with 0 within statistics.

Order Estimation of relaxation time

- The lower limit of τ_Π can be estimated from the upper limit of $\langle \pi_{ij}^2 \rangle$

$$\frac{V}{T < \pi_{ij}^2 >^{up}} = \beta_2^{low} = \frac{\tau_\pi^{low}}{2\eta} \Leftrightarrow \tau_\pi^{low} = 2\eta\beta_2^{low}$$

$$= 2(\frac{\eta}{s})(\beta_2 T_C{}^4)^{low} \frac{s}{T_C^4} = 2(\frac{\eta}{s})(\beta_2 T_C{}^4)^{low} \frac{\alpha T^3}{T_C^4}$$

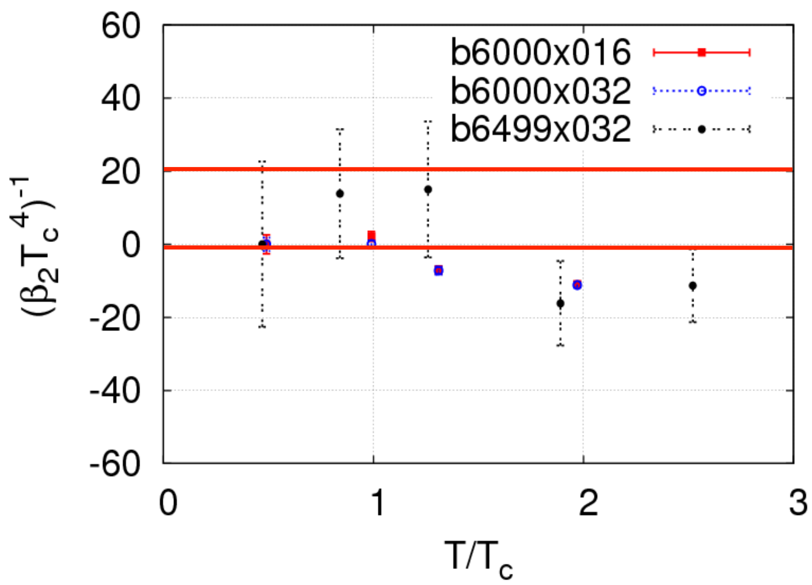
$s = \alpha T^3$

For example,

$\eta / s = 1/4\pi$, $(\beta_2 T_c{}^4)^{-1} = O(1)$,

$\alpha = O(1)$, T=600MeV,

$$\tau_\pi^{low} \cong O(10^{-1})\text{fm}$$



Summary & Future Work

- Summary

1. We evaluated one of the 2nd order transport coefficient i.e. β_2 by lattice QCD with Boltzmann–Einstein principle.
2. The fluctuation becomes negative.
3. For β=6.499, the errorbars of $\langle \pi_{ij}^2 \rangle$ are consistent with τ_π which is considerably shorter than RHIC time scale.

- Future Work

1. Resolve the origin of negative $\langle \pi_{ij}^2 \rangle$.
2. Other transport coefficients β_0 & β_1.

Appendix

Estimation of causality

- Signal velocity in matter is given by

$$v^2 = \frac{1}{2\beta_2(e+p)} = \frac{T_c^4}{2(\beta_2 T_c^4)\cdot \alpha T^4}$$

$(e+p)=\alpha T^4$

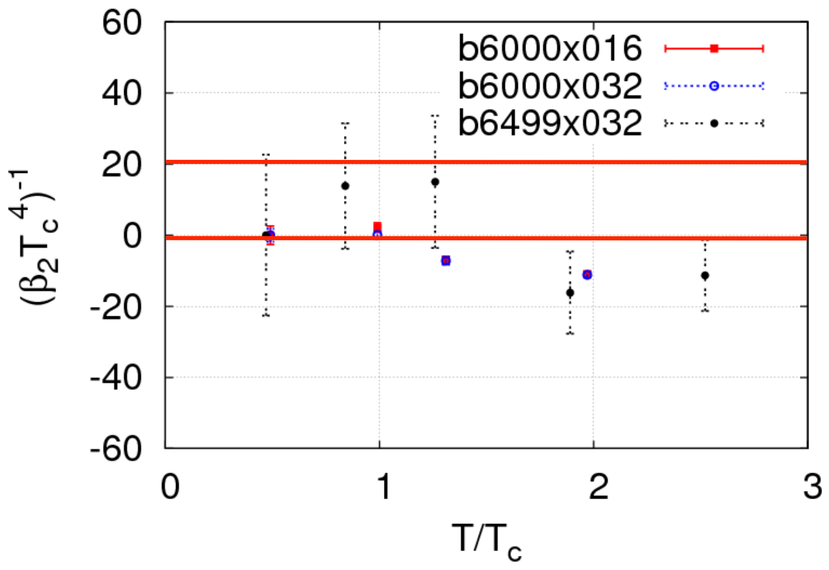
If you choose

$(\beta_2 T_c^4)^{-1}=10$, $\alpha=5$,

T=400MeV,

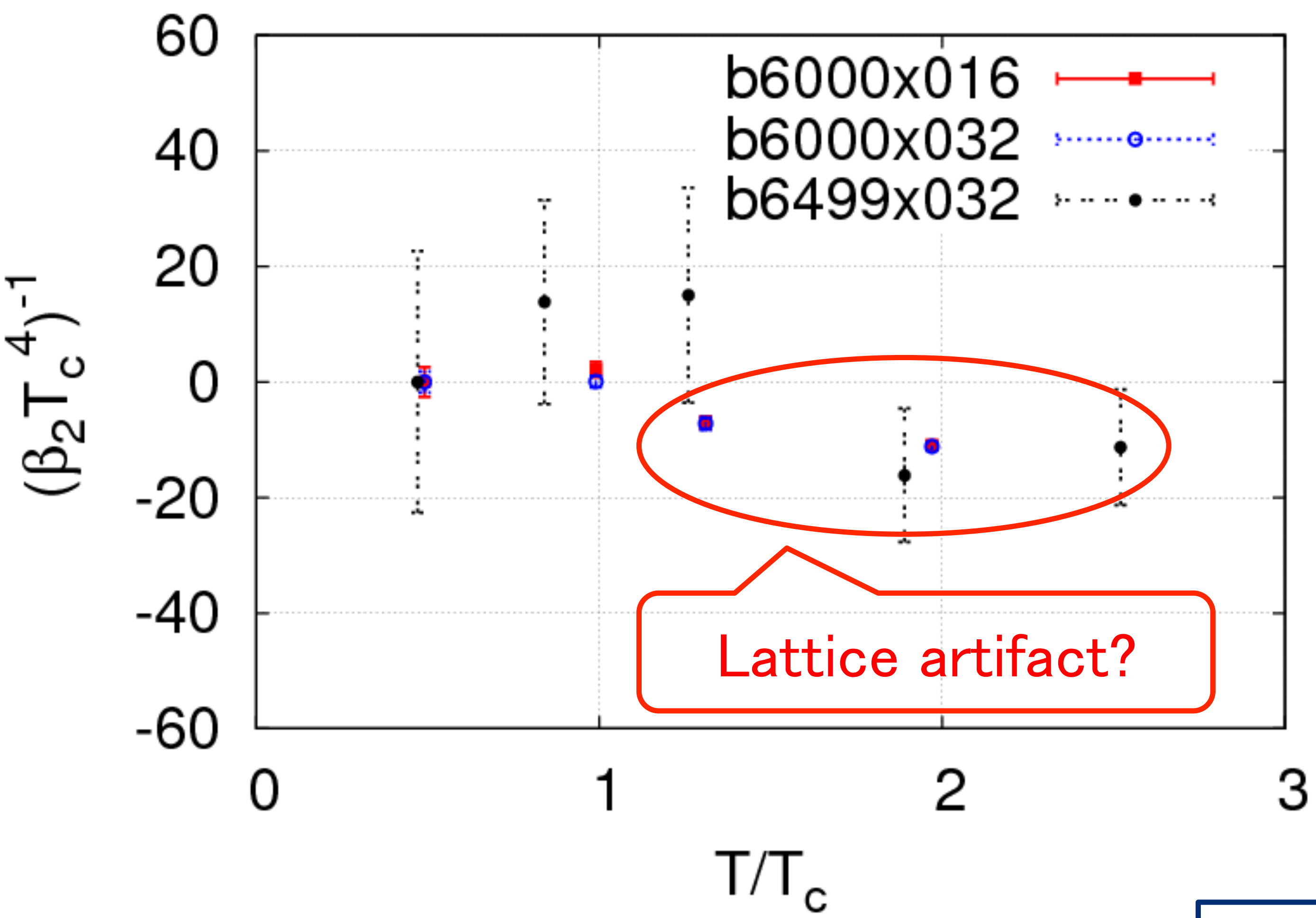
$$v^2 = \frac{81}{256} < c^2$$

Causality OK

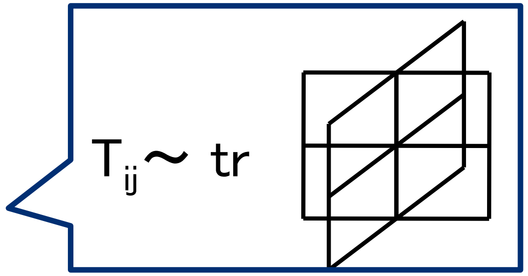


Numerical Results

- $\langle \pi_{ij}^2 \rangle = T/V\,\beta_2$



T_{ij}'s spread on the lattice
so they overlap each other.



Fluctuation of T_{ij}

- Fluctuation of T_{ij}

$$< T_{ij}^2 > - < T_{ij} >^2 = < T_{ij}^2 >$$

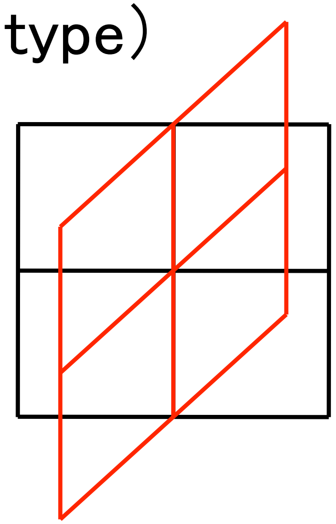
$$T_{ij} \rightarrow \overline{T}_{ij} \equiv \frac{\int dV T_{ij}}{V}$$

$$< T_{ij}^2 > \rightarrow < \overline{T}_{ij}^2 > \equiv \frac{\int dV \int dV' T_{ij}(x) T^{ij}(x')}{VV'}$$

Remove lattice artifact

- T_{ij} on the lattice (clover type)

$T_{ij}^{lat.} \sim$ tr



Operators overlap each other on the lattice

1^{st} order dissipative hydrodynamics

- 1^{st} order theories for dissipative fluid (by Eckart or Landau–Lifshitz) ⇒Introduce dissipative effects linearly

entropy current

dissipative term (q^{μ}: heat flux)

$$S^{\mu} = su^{\mu} + \frac{1}{T} q^{\mu}$$

s : entropy density , u^{μ} : 4-velocity , T : temperature

Transport equation ⇒Causality ×

Transport coefficients : ζ (bulk viscosity),

κ (heat conductivity), η (shear viscosity)

C. Eckart, Phys. Rev. **58**, 919 (1940)
L. D .Landau and E. M. Lifshitz, *Fluid Mechanics* (1959)

2nd order dissipative hydrodynamics

- 2nd order theory for dissipative fluid (by Muller or Israel–Stewart) ⇒ 1st order theory + relaxation times

entropy current

$$S^{\mu} = su^{\mu} + \frac{1}{T}q^{\mu} - \frac{1}{T}(\alpha_0 q^{\mu}\Pi - \alpha_1 q_{\nu}\pi^{\mu\nu}) - \frac{1}{2T}u^{\mu}(\beta_0 \Pi^2 - \beta_1 q^{\nu} q_{\nu} + \beta_2 \pi^{\nu\lambda}\pi_{\nu\lambda})$$

dissipative terms

$$\tau_{\Pi} = \zeta\beta_0,\ \tau_{q} = \kappa T\beta_1,\ \tau_{\pi} = 2\eta\beta_2$$

2nd order theory → 1st order theory ($\tau_i \to 0$,)

Transport equations ⇒ Causality ○

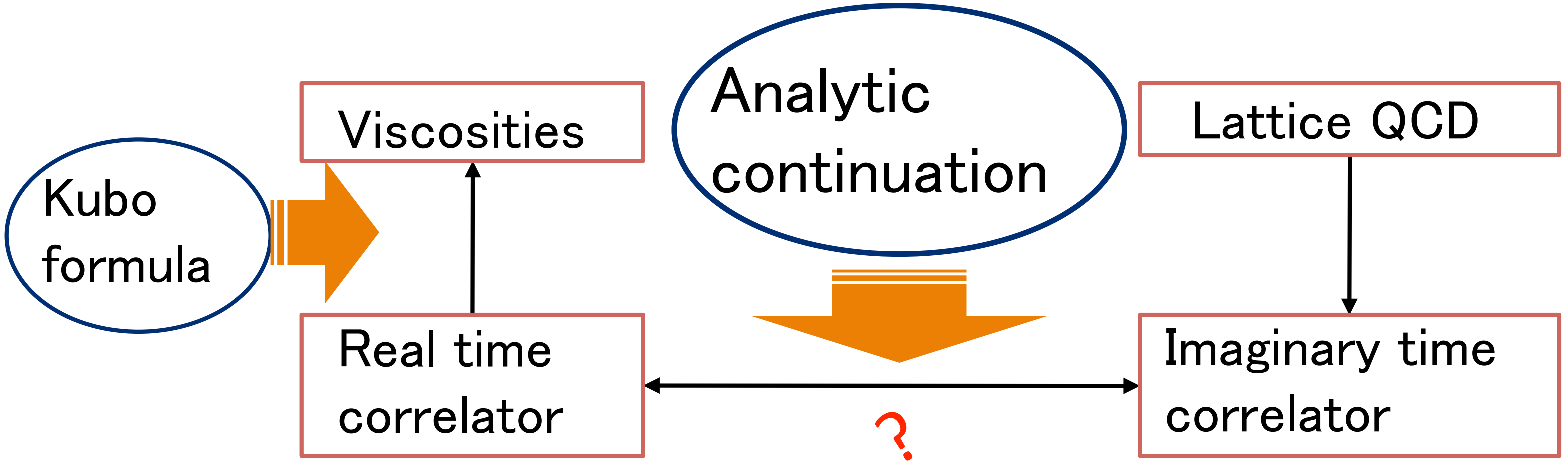
Transport coefficients : ζ, κ, η, α_0, α_1, β_0, β_1, β_2

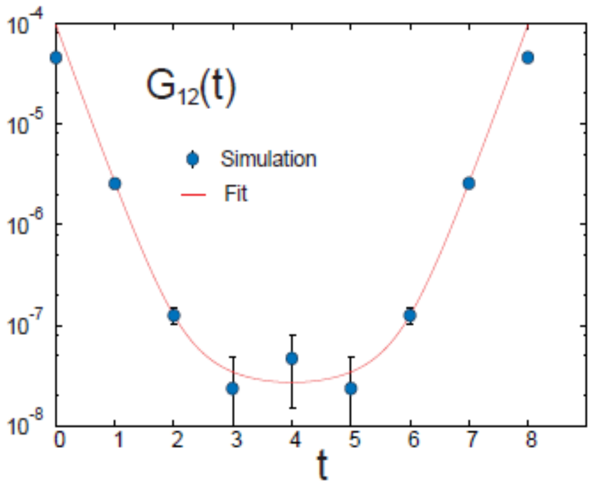
I.Muller, Z. Phys. **198**, 329 (1967)

W.Israel and J.M.Stewart, Ann. Phys. (N.Y.) **118**, 341 (1979)

Previous Work

- Using Kubo formula with ansatz for spectral function. But the validity remains questionable.



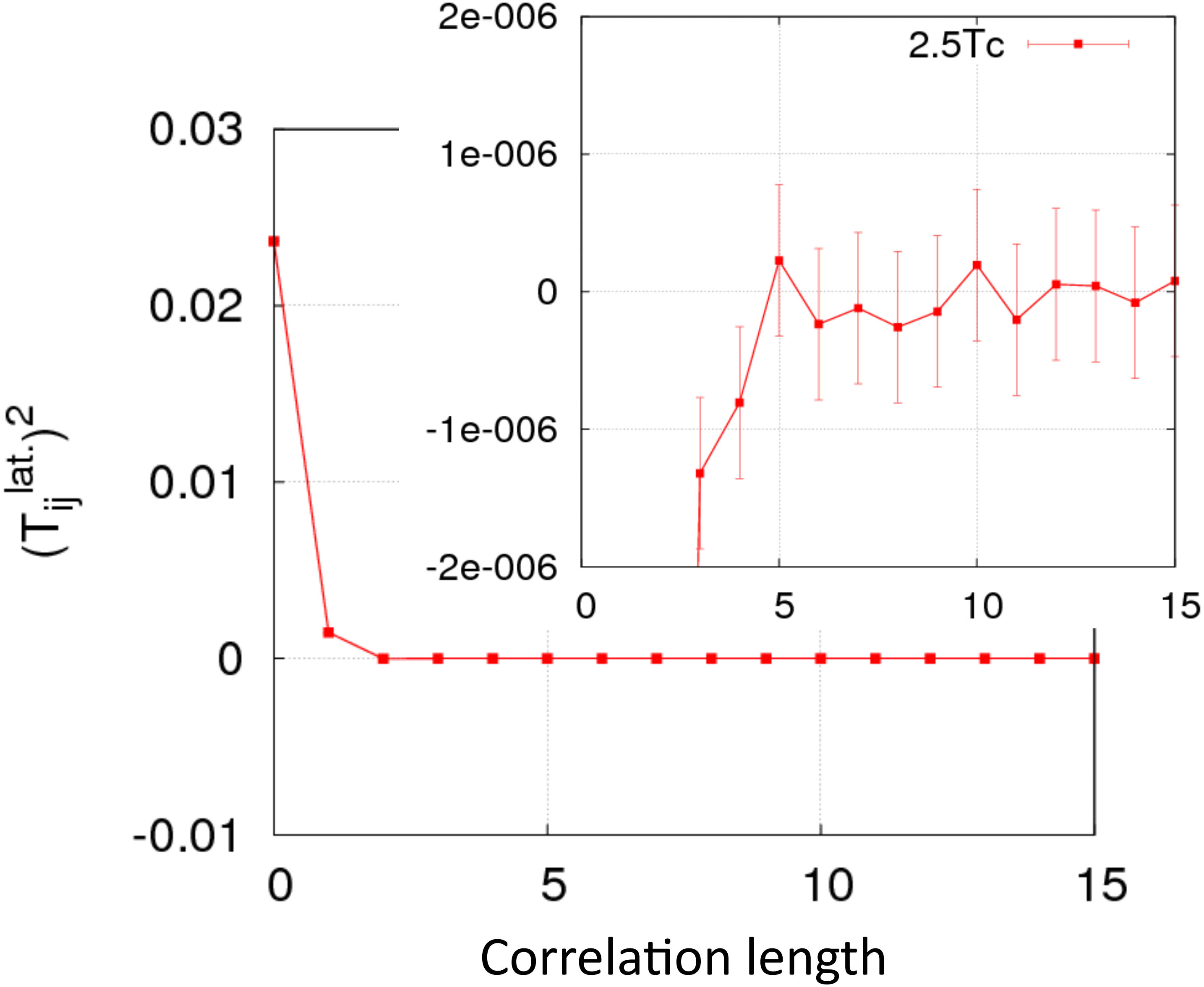


$$\rho(\omega) = \frac{A}{\pi}\left(\frac{\gamma}{(m-\omega)^2+\gamma^2} - \frac{\gamma}{(m+\omega)^2+\gamma^2}\right)$$

F. Karsch and H. W. Wyld, Phys. Rev. D**35**, 2518(1987)
A. Nakamura and S. Sakai, Phys. Rev. Lett. **94**, 072305(2005)
H. B. Meyer, Phys. Rev. D**76**, 101701(2007)

Numerical Results

- Spatial correlation of T_{ij}

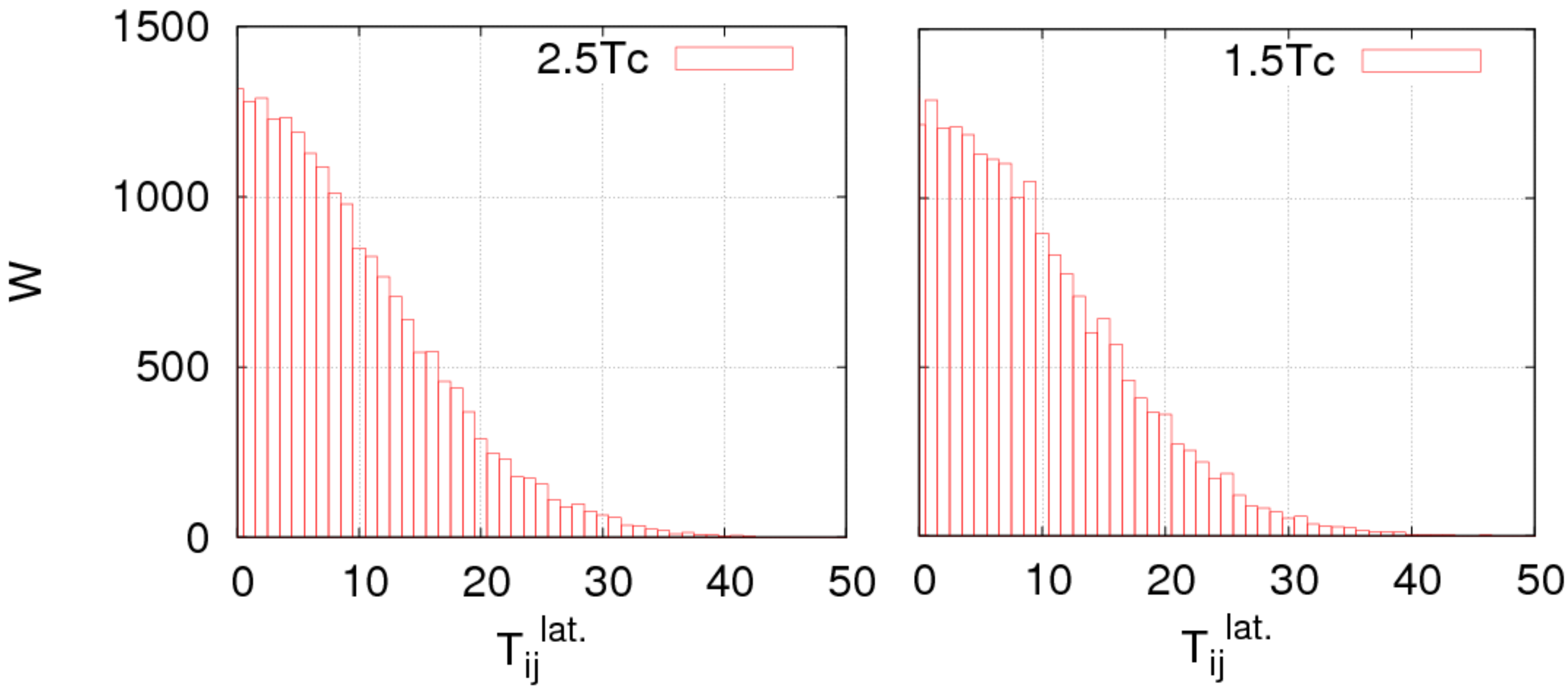


Numerical Results

- *Distribution of T_{ij}*

  ▪SU(3) pure gauge theory ▪Isotropic lattice

  ▪4 x 10,000 configurations for each case



	β	a[fm]	T/T_C	N_τ	N_S
Case1	6.499	0.049	2.5	6	32^3
Case2	6.499	0.049	1.5	10	32^3
Case3	6.499	0.049	1.25	12	32^3

$\beta = 2N_C/g^2$ a: lattice spacing

T_C: critical temperature (~300MeV)

N_τ: number of sites in spatial direction

N_S: number of sites in temporal direction

Numerical Results

- *Temperature dependence of T_{ij}*