Current Status of Improved Fermilab Fermions

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15.June.2010

Outline

- Motivation
- Fermilab Formalism
- Improvement Program
- Action
- Simulation Parameters
- Preliminary Results
- Outlook

Fermilab Formalism

$$\begin{aligned}
\mathcal{L}_{\rm lat} &= m_0 \sum_x \bar\psi(x)\psi(x) + \sum_x \bar\psi(x)\left[\frac{1}{2}(1+\gamma_0)D_0^- - \frac{1}{2}(1-\gamma_0)D_0^+\right]\psi \\
&+ \zeta \sum_x \bar\psi(x)\gamma\cdot\mathbf{D}\psi(x) - \frac{1}{2} a r_s \zeta \sum_x \bar\psi(x)\mathbf{D^2}\psi(x) \\
&- \frac{i}{2} a c_B \zeta \sum_x \bar\psi(x) i\Sigma\cdot\mathbf{B}\psi(x) - \frac{i}{2} a c_E \zeta \sum_x \bar\psi(x) i\alpha\cdot\mathbf{E}\psi(x)
\end{aligned}$$

- Different coefficients for space and time like operators
- Smooth transition to $m_0 a \to 0$ and $m_0 a \to \infty$
- Coefficients depend on $m_0 a, \zeta$ and r_s in a non trivial way.

[A .El-Khadra, A. S. Kronfeld and P. B. Mackenzie,PRD 55, 3993, (1997)].

Improvement Program

- Improvement program starts with considering all the operators of dimesions 6 and 7 of the form $\bar{Q}\Gamma DDDQ$ which respect the symmetries of the continuum QCD ($\mathcal{P}$,$\mathcal{C}$,$\ldots$) and have the power counting λ^3 in HQET and v^6 in NRQCD.

- Space-Time Asymmetry : e.g., $\bar{Q}\gamma_\mu D_\mu^3 Q \to \bar{Q}\gamma_i D_i^3 Q$ and $\bar{Q}\gamma_4 D_4^3 Q$

- Identities : e.g. $\not{D}^2 = D^2 - \frac{i}{2}\sigma_{\mu\nu}F_{\mu\nu}$
$$2\gamma_4 D_4 \boldsymbol{\gamma}\cdot\boldsymbol{D}\gamma_4 D_4 = \{\gamma_4 D_4, \boldsymbol{\alpha}\cdot\boldsymbol{E}\} - \{D_4^2, \boldsymbol{\gamma}\cdot\boldsymbol{D}\}$$
$$2\boldsymbol{\gamma}\cdot\boldsymbol{D}\gamma_4 D_4 \boldsymbol{\gamma}\cdot\boldsymbol{D} = \{\boldsymbol{\gamma}\cdot\boldsymbol{D}, \boldsymbol{\alpha}\cdot\boldsymbol{E}\} - \{\gamma_4 D_4, (\boldsymbol{\gamma}\cdot\boldsymbol{D})^2\}$$

- $\mathcal{L}^{\mathrm{eff}} = \mathcal{L}^{\mathrm{cont.}} + \sum_i c_i a^{s_i - 4}\mathcal{O}_i$ where $s_i = \dim[\mathcal{O}_i]$.

How many are redundant ?

Redundant Directions

Interactions that are induced by the field transformations (equivalently the equations of motion) are "redundant"

- $\bar{Q} \to \bar{Q}\exp[\bar{J}]$ and $Q \to \exp[J]Q$ where

$$\begin{aligned} J &= a\epsilon_1(\gamma_\mu D_\mu + m) + a\delta_1 \boldsymbol{\gamma} \cdot \boldsymbol{D} + a^2\epsilon_2(\gamma_\mu D_\mu + m)^2 - a^2\frac{i}{2}\epsilon_F \sigma_{\mu\nu} F_{\mu\nu} \\ &+ a^2\delta_2(\boldsymbol{\gamma} \cdot \boldsymbol{D})^2 + a^2\delta_B i\boldsymbol{\sigma} \cdot \boldsymbol{B} + a^2\delta_u[\gamma_4 D_4, \boldsymbol{\gamma} \cdot \boldsymbol{D}] \end{aligned}$$

and similarly for A_4 and $\boldsymbol{A}$.

[Sheikholeslami and Wohlert, Nucl. Phys. N259, 572 (1985)].

This process gives

• 19 new operators in addition to original Fermilab Action : 7 dimension six and 12 dimension seven

• In order to determine the coefficients c_i at tree level, a matching calculation needs to be carried out

- Tree-level quark Dispersion Relation up to $\mathcal{O}(p^4)$
- Tree-level chromomagnetic $\mathcal{O}(p^3/m^3)$ and chromoelectric $\mathcal{O}(p^2/m^2)$ interactions (vertices)
- Compton Scattering (lowest order)
- Quark-Quark scattering (lowest order)

Details : [Oktay and Kronfeld, Phys. Rev. D **78** 014504 (2008).]

Feynman Rules

The quark propagator,

$$aS^{-1}(p) = i\gamma_4 \sin(p_4 a) + i\gamma \cdot \boldsymbol{K} + \mu(p) - \cos(p_4 a)$$

where

$$K_i(p) = \sin(p_i a)[-2c_2 \hat{p}^2 a^2 - c_1 p_i^2 a^2]$$
$$\mu(p) = 1 + m_0 a + \hat{p}^2 a^2 \left[\frac{1}{2} r_s \zeta + z_6 \hat{p}^2 a^2\right] + c_4 \sum_i (\hat{p}_i a)^4$$

Temporal single-gluon vertex :

$$\Lambda_4(p', p) = \gamma_4 \cos\left[\frac{1}{2}(p' + p)_4 a\right] - i \sin\left[\frac{1}{2}(p' + p)_4 a\right]$$
$$+ \frac{c_E}{2} \zeta a \cos\left[\frac{1}{2} k_4 a\right] i\boldsymbol{\alpha} \cdot S(\boldsymbol{k})$$
$$+ c_{EE} \gamma \cdot \boldsymbol{S}(k)[S_4(p') - S_4(p)] \cos\left[\frac{1}{2} k_4 a\right] + \ldots$$

Energy

For small momentum

$$E = M_1 + \frac{\mathbf{p}^2}{2M_2} - \frac{w_4}{6}\sum_i p_i^4 - \frac{(\mathbf{p}^2)^2}{8M_4^3}$$

Explicit calculations yield,

$$aM_1 = \log(1 + m_0 a)$$

$$\frac{1}{aM_2} = \frac{2\zeta^2}{m_0 a(2 + m_0 a)} + \frac{r_s\zeta}{1 + m_0 a}$$

$$w_4 = \frac{2\zeta + 6c_2}{m_0 a(2 + m_0 a)} + \frac{r_s\zeta - 24c_4}{4(1 + m_0 a)}$$

$$\frac{1}{M_4^3 a^3} = \frac{8\zeta^4}{[m_0 a(2 + m_0 a)]^3} + \frac{4\zeta^4 + 8r_s\zeta^3(1 + m_0 a)}{[m_0 a(2 + m_0 a)]^2} + \frac{r_s^2\zeta^2}{(1 + m_0 a)^2} + \frac{32\zeta c_1}{m_0 a(2 + m_0 a)} - \frac{8c_3}{1 + m_0 a}$$

Chromoelectric Field

For the interactions with the chromoelectric background field we use the time component of the current J_4

$$J_4^{\text{cont}} = \bar{u}(\xi',0)\left[1 - \frac{\boldsymbol{K}^2 - 2i\boldsymbol{\Sigma}\cdot(\boldsymbol{K}\times\boldsymbol{P})}{8m^2}\right]u(\zeta,0)$$

$$J_4^{\text{lat}} = \bar{u}(\xi',0)\left[1 - \frac{\boldsymbol{K}^2 - 2i\boldsymbol{\Sigma}\cdot(\boldsymbol{K}\times\boldsymbol{P})}{8m_E^2} + \frac{z_E\boldsymbol{K}^2a^2}{1+m_0a}\right]u(\xi,0)$$

where

$$\frac{1}{4m_E^2a^2} = \frac{\zeta^2}{[m_0a(2+m_0a)]^2} + \frac{\zeta^2c_E}{m_0a(2+m_0a)} + \frac{2r_E}{1+m_0a}$$

The correct (tree-level) mathching is achieved if one adjust $z_E = 0$ and (c_E,r_E) such that $m_E = m_2$. Similarly for the chromomagnetic interaction.

New Action

$$
\begin{aligned}
S_0 \quad &= m_0 \sum_x \bar{\psi}(x)\psi(x) + \sum_x \bar{\psi}(x)\gamma_4 D_4 \psi(x) - \frac{1}{2}\sum_x \bar{\psi}(x)\Delta_{4\mathrm{lat}}\psi(x) \\
+ \quad & \zeta \sum_x \bar{\psi}(x)\boldsymbol{\gamma}\cdot\mathbf{D}_{\mathrm{lat}}\psi(x) - \frac{1}{2} r_s \zeta \sum_x \bar{\psi}(x)\Delta^{(3)}_{\mathrm{lat}}\psi(x) \\
- \quad & \frac{1}{2} c_B \zeta \sum_x \bar{\psi}(x) i\boldsymbol{\Sigma}\cdot\mathbf{B}\psi(x) - \frac{1}{2} c_E \zeta \sum_x \bar{\psi}(x) i\boldsymbol{\alpha}\cdot\mathbf{E}\psi(x) \\
+ \quad & c_1 \sum_x \bar{\psi}(x) \sum_i \gamma_i D_i \Delta_{i\mathrm{lat}}\psi(x) + c_2 \sum_x \bar{\psi}(x)\{\boldsymbol{\gamma}\cdot\mathbf{D}, \Delta^{(3)}_{\mathrm{lat}}\}\psi(x) \\
+ \quad & c_3 \sum_x \bar{\psi}(x)\{\boldsymbol{\gamma}\cdot\mathbf{D}, i\boldsymbol{\Sigma}\cdot\mathbf{B}_{\mathrm{lat}}\}\psi(x) + c_{EE} \sum_x \bar{\psi}(x)\{\gamma_4 D_{4\mathrm{lat}}, \boldsymbol{\alpha}\cdot\mathbf{E}_{\mathrm{lat}}\}\psi(x) \\
+ \quad & c_4 \sum_x \bar{\psi}(x) \sum_i \Delta^2_{i\mathrm{lat}}\psi(x) + c_5 \sum_x \bar{\psi}(x) \sum_i \sum_{i \neq j} \{i\Sigma_i B_{i\mathrm{lat}}, \Delta_{j\mathrm{lat}}\}\psi(x)
\end{aligned}
$$

Difference Operators

$$D_{\rho\mathrm{lat}} = (T_\rho - T_{-\rho})/2a$$

$$\Delta_{\rho\mathrm{lat}} = (T_\rho + T_{-\rho} - 2)/2a$$

$$\Delta^{(3)}_{\mathrm{lat}} = \sum_i^3 \Delta_{i\mathrm{lat}}$$

$$F_{\rho\sigma\mathrm{lat}} = \frac{1}{8a^2} \sum_{\bar\rho=\pm\rho} \sum_{\bar\sigma\pm\sigma} \mathrm{sign}\bar\rho\,\mathrm{sign}\bar\sigma\,[T_{\bar\rho} T_{\bar\sigma} T_{-\bar\rho} T_{-\bar\sigma} - T_{\bar\sigma} T_{\bar\rho} T_{-\bar\sigma} T_{-\bar\rho}]$$

$$T_\mu \Psi(x) = U_\mu(x)\Psi(x+\mu) \qquad T_{-\mu}\Psi(x) = U^\dagger_\mu(x-\mu)\Psi(x-\mu)$$

Matching

$$
\begin{aligned}
c_1 &= -\frac{1}{6}\zeta + c_B \frac{m_0 a(2+m_0 a)}{6(1+m_0 a)} \\
c_2 &= \frac{\zeta^3(\zeta^2-1)}{[2m_0 a(2+m_0 a)]^2} - \frac{\zeta^2[\zeta + 2r_s(1+m_0 a) - 3r_s\zeta^2/(1+m_0 a)]}{8m_0 a(2+m_0 a)} \\
&+ \frac{3r_s^2\zeta^3}{16(1+m_0 a)^2} + \frac{m_0 a(2+m_0 a) r_s^2 \zeta}{32(1+m_0 a)^2}\left[\frac{r_s\zeta}{1+m_0 a} - 1\right] \\
c_4 &+ \frac{1}{24} r_s\zeta + \frac{1}{3} c_B\zeta \qquad c_B = r_s \qquad c_3 = c_2 \qquad c_5 = \frac{1}{4} c_B \zeta \\
c_E &= \frac{\zeta^2-1}{m_0 a(2+m_0 a)} + \frac{r_s\zeta}{1+m_0 a} + \frac{r_s^2 m_0 a(2+m_0 a)}{4(1+m_0 a)^2} \\
c_{EE} &= \left[\frac{\zeta(\zeta^2-1)(1+m_0 a)}{[m_0 a(2+m_0 a)]^2} + \frac{c_E\zeta(\zeta^2-1)(1+m_0 a)}{m_0 a(2+m_0 a)}\right. \\
&+ \left.\frac{\zeta(r_s\zeta - 1 - m_0 a)}{2m_0 a(2+m_0 a)} + \frac{1}{2} r_s c_E \zeta^2 - \frac{1}{4} c_E^2 \zeta(1+m_0 a)\right] /(2+m_0 a(2+m_0 a)
\end{aligned}
$$

Action

$$
\begin{aligned}
S &= \sum_x \bar{\psi}_x \psi_x - \kappa_t \sum_x \bar{\psi}_x (1-\gamma_4) T_4 \psi_x - \kappa_t \sum_x \bar{\psi}_x (1+\gamma_4) T_{-4} \psi_x \\
&- \kappa_t \sum_{x,i} \bar{\psi}_x [(r_s \zeta + 8c_4) - \gamma_i(\zeta - 2c_1 - 12c_2)] T_i \psi_x \\
&- \kappa_t \sum_{x,i} \bar{\psi}_x [(r_s \zeta + 8c_4) + \gamma_i(\zeta - 2c_1 - 12c_2)] T_{-i} \psi_x \\
&+ \kappa_t \sum_x \bar{\psi}_x [2c_4 + \gamma_i(2c_2 + c_1)] T_i^2 \psi_x - \kappa_t (c_B \zeta + 16 c_5) \sum_x \bar{\psi}_x i \mathbf{\Sigma} \cdot \mathbf{B} \psi_x \\
&+ \kappa_t \sum_x \bar{\psi}_x [2c_4 - \gamma_i(2c_2 + c_1)] T_{-i}^2 \psi_x - \kappa_t c_E \zeta \sum_x \bar{\psi}_x i \boldsymbol{\alpha} \cdot \mathbf{E} \psi_x \\
&+ \kappa_t c_2 \sum_{x,i \neq j} \bar{\psi}_x \gamma_i \{T_i - T_{-i}, T_j - T_{-j}\} \psi_x + 2\kappa_t c_5 \sum_x \bar{\psi}_x \sum_i \sum_{i \neq j} \{i \Sigma_i B_{i\mathrm{lat}}, (T_j + T_{-j})\} \psi_x \\
&+ 2\kappa_t c_3 \sum_x \bar{\psi}_x \{\boldsymbol{\gamma} \cdot \mathbf{D}_{\mathrm{lat}}, i \mathbf{\Sigma} \cdot \mathbf{B}_{\mathrm{lat}}\} \psi_x + 2\kappa_t c_{EE} \sum_x \bar{\psi}_x \{\gamma_4 D_{4\mathrm{lat}}, \boldsymbol{\alpha} \cdot \mathbf{E}\} \psi_x
\end{aligned}
$$

Coding and Tadpole Improvement

With the form of the action on the previous slide, it is straightforward to code the new action. Our implementation is in QOPQDP.

It is also possible carry out the tadpole improvement :

$$T_{\pm\mu} = u_0\,[T_{\pm\mu}/u_0] = u_0\,\tilde{T}_{\pm\mu}$$

and factor u_0 are absorbed into the couplings. This way, one finds

$$\begin{aligned}
\tilde{\kappa}_t = u_0\kappa_t \quad \tilde{c}_4 &= u_0 c_4 \quad \tilde{c}_2 = u_0 c_2 \\
\tilde{c}_1 + 2\tilde{c}_2 &= c_1 + 2c_2 \\
\tilde{r}\tilde{\zeta} + 8\tilde{c}_4 &= r_s\zeta + 8c_4 \\
\tilde{\zeta} - 2\tilde{c}_1 - 12\tilde{c}_2 &= \zeta - 2c_1 - 12c_2 \\
\tilde{m}_0 a &= \frac{1}{2\tilde{\kappa}_t} - (1 + 3\tilde{r}_s\tilde{\zeta} + 18\tilde{c}_4)
\end{aligned}$$

The terms with no E and B terms are straight forward.

$\bar{\psi}_x\{\boldsymbol{\gamma}\cdot\boldsymbol{D}, i\boldsymbol{\Sigma}\cdot\boldsymbol{B}\}\psi_x$, $\bar{\psi}_x\{\gamma_4 D_4, \boldsymbol{\alpha}\cdot\boldsymbol{E}\}\psi_x$, $\bar{\psi}_x \sum_i \sum_{i\neq j}\{i\Sigma_i B_{i\mathrm{lat}}, \Delta_{j\mathrm{lat}}\}\psi_x$

Simulation Parameters

Test Lattices	
Lattice	N_f = 2+1, $16^3 \times 48$
β	6.600
m_l/m_s	0.029/0.0484 ($\sim$0.6)
u_0	0.8614
a	$\simeq$ 0.15fm

Coefficients at $\kappa = 0.04$	
$\tilde{r}_s,\tilde{\zeta},\tilde{c}_B$	1.0
$\tilde{c}_E$	+0.581
$\tilde{c}_{EE}$	-0.0087
$\tilde{c}_1$	+0.2311
$\tilde{c}_2,\tilde{c}_3$	-0.0955
$\tilde{c}_4$	0.375
$\tilde{c}_5$	0.25

Inconsistency

Since the action is designed to improve $\mathcal{O}(p^4)$ terms, we need to find observables to test these improvements. Binding energies of the kinetic meson mass comes from $\mathcal{O}(p^4)$ term :

$$M_{1\bar{Q}q} = M_{1\bar{Q}} + M_{1q} + B_1 \qquad M_{2\bar{Q}q} = M_{2\bar{Q}} + M_{2q} + B_2$$

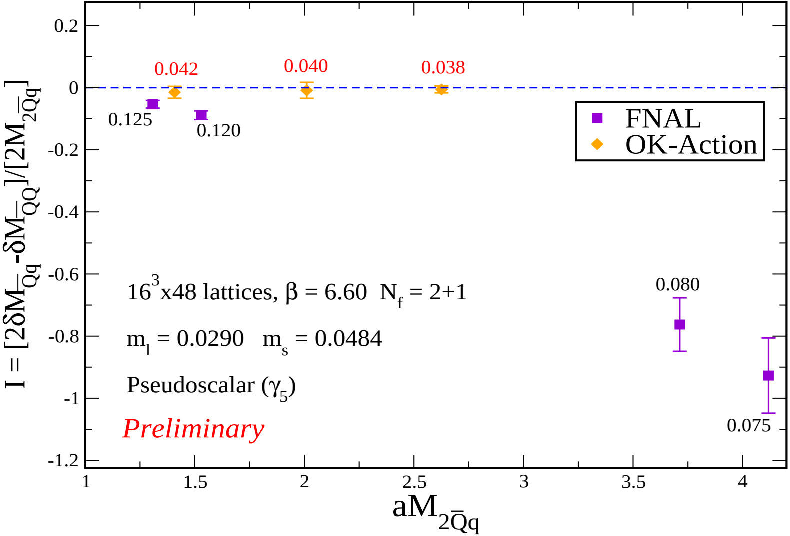
define $\delta M = M_2 - M_1$ and $\delta B = B_2 - B_1$

$$I := \frac{2\delta M_{\bar{Q}q} - (\delta M_{\bar{Q}Q} + \delta M_{\bar{q}q})}{2M_{2\bar{Q}q}} \Rightarrow I = \frac{2\delta B_{\bar{Q}q} - (\delta B_{\bar{Q}Q} + \delta B_{\bar{q}q})}{2M_{2\bar{Q}q}}$$

where B_1 and B_2 are the binding energies. "I" will test the improvement and if the lattice action(s) of the quarks were sufficently accurate, "I" would vanish.

Collins, et.al, Nucl. Phys. B. Proc. Suppl. **47** 445 (1996).
A. S. Kronfeld, Nucl. Phys. Proc. Suppl **53** 401 (1997)

Inconsistency



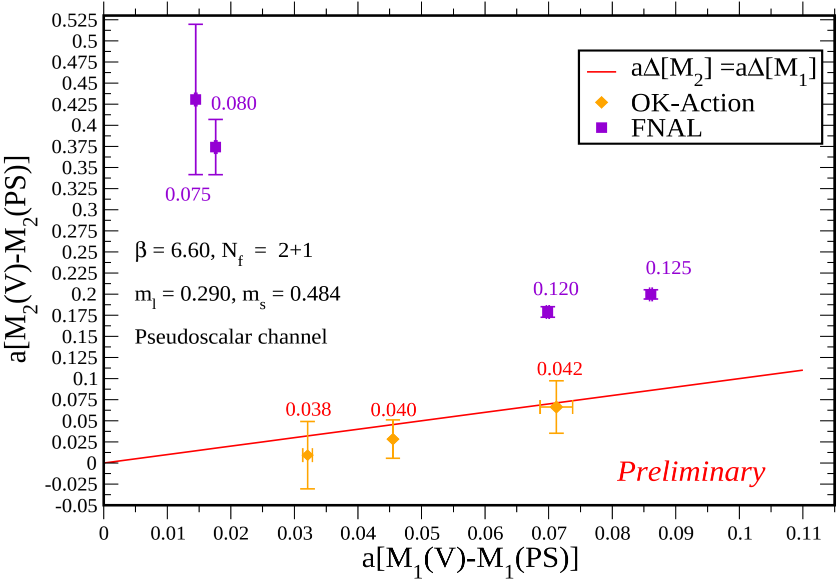
Hyperfine Splittings

Another way to see the effects of improvement is to look at the hyperfine splittings. The rest masses M_1 is order of $\mathcal{O}(p^0)$ and accurate with the Fermilab Formalism but the kinetic masses M_2 will be improved due to higher order corrections e.g., $\bar{\psi}\{\gamma \cdot \boldsymbol{D}, i\Sigma \cdot \boldsymbol{B}\}\psi$. We look at

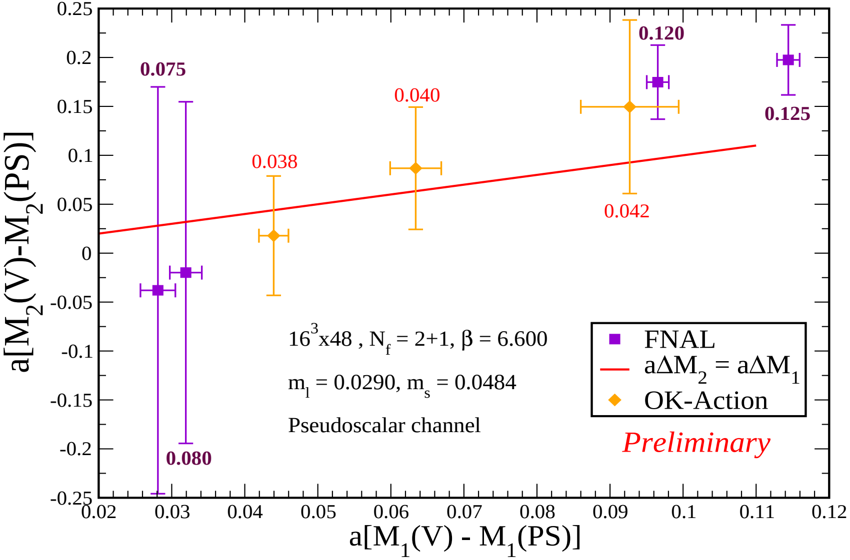
$$[M_2^{\mathrm{V}} - M_2^{\mathrm{PS}}] \stackrel{!}{=} [M_1^{\mathrm{V}} - M_1^{\mathrm{PS}}]$$

and compare it with the FNAL action at fixed light quark mass.

$Q\bar{Q}$ system



$Q\bar{q}$ system



Outlook

- More statistics and data analysis (old data will be revisited).
- Optimization of the code and c_5 operator.
- Implementation in a higher precision study of heavy quark hadrons