

Exploring center symmetry with electrically charged quarks

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OUTLINE

- Center symmetry
 - glue
 - dynamical quarks
- Including **electromagnetism**
- Our model
 - disordering
 - preliminary results



CENTER SYMMETRY

- Symmetry of **pure** $SU(N)$ gauge theory

$$SU(N)/\mathbb{Z}_N, \quad z = e^{in2\pi/N} \in \mathbb{Z}$$

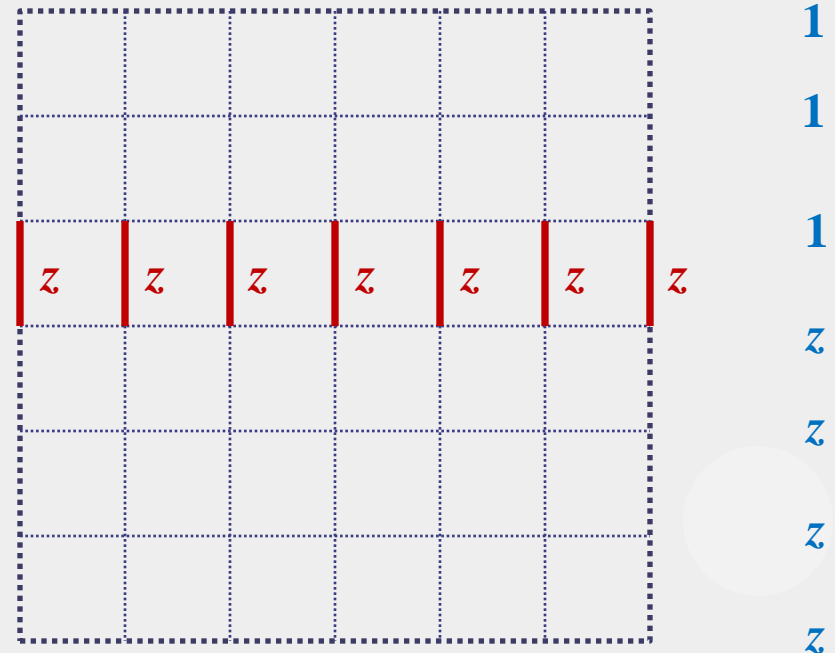
- **Center transformation** - gauge trans. periodic only up to center phase in time direction

- action invariant

$$\square \longrightarrow z^* z \square = \square$$

- Polyakov loop picks up phase

$$P = \text{Tr} \uparrow \longrightarrow z \text{Tr} \uparrow$$

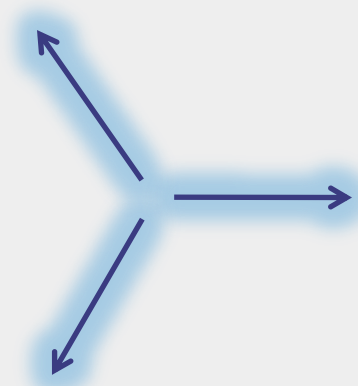


CENTER SYMMETRY - CONFINEMENT

- VEV determines whether center symmetry is realized
- low T , confined, center symmetric phase

$$\langle P \rangle = 0$$

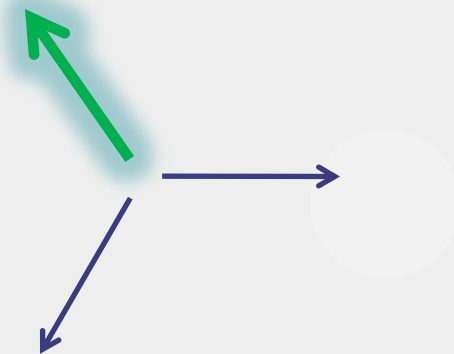
average to zero
- disordered by **center vortices**



- high T , deconfined, center broken phase

$$\langle P \rangle \neq 0$$

sector spontaneously chosen



CENTER SYMMETRY - CONFINEMENT

- deconfinement for pure $SU(N)$ is a **center symmetry breaking** phase transition

- VEV of Polyakov loop is an **order parameter**
 - (see also vortex free energy)

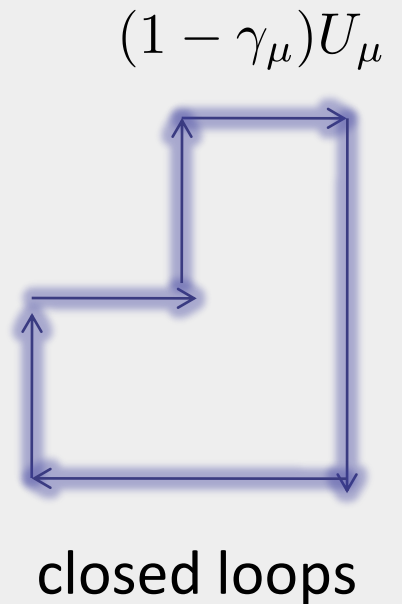
- **infinitely heavy quarks**, i.e. static



DYNAMICAL QUARKS (WILSON)

- explicitly break center symmetry
- Hopping expansion

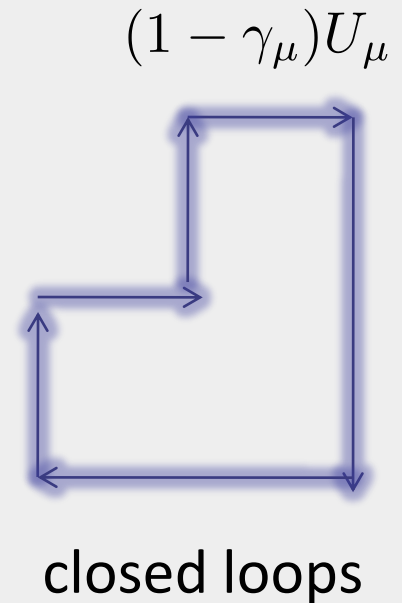
$$\det M = \exp\left(-\sum_j \frac{\kappa^j}{j} \text{Tr} H^j\right), \quad \kappa = \frac{1}{2am + 8}$$



DYNAMICAL QUARKS (WILSON)

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- $N_t = 4$ – plaquettes & Polyakov loops

$$\det M = \exp\left(8\kappa^4 \sum_{\square} 2\text{Re Tr } \square + 32\kappa^4 \sum_{\vec{x}} 2N_c \text{Re } P(\vec{x}) + \dots\right)$$

DYNAMICAL QUARKS – $N_T = 4$

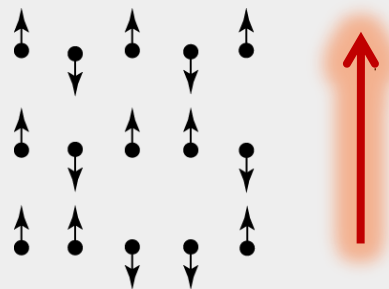
↓ in partition $f^n e^{-S}$

$$S_{eff} = -\frac{\tilde{\beta}}{N_c} \sum_{\square} \text{Re Tr } \square - 32\kappa^4 \sum_{\vec{x}} 2N_c \text{Re } P(\vec{x}) + \dots$$

modify gauge coupling

ordering effect (S_{eff} minimized for $P=1$)

c.f. spin system in magnetic field

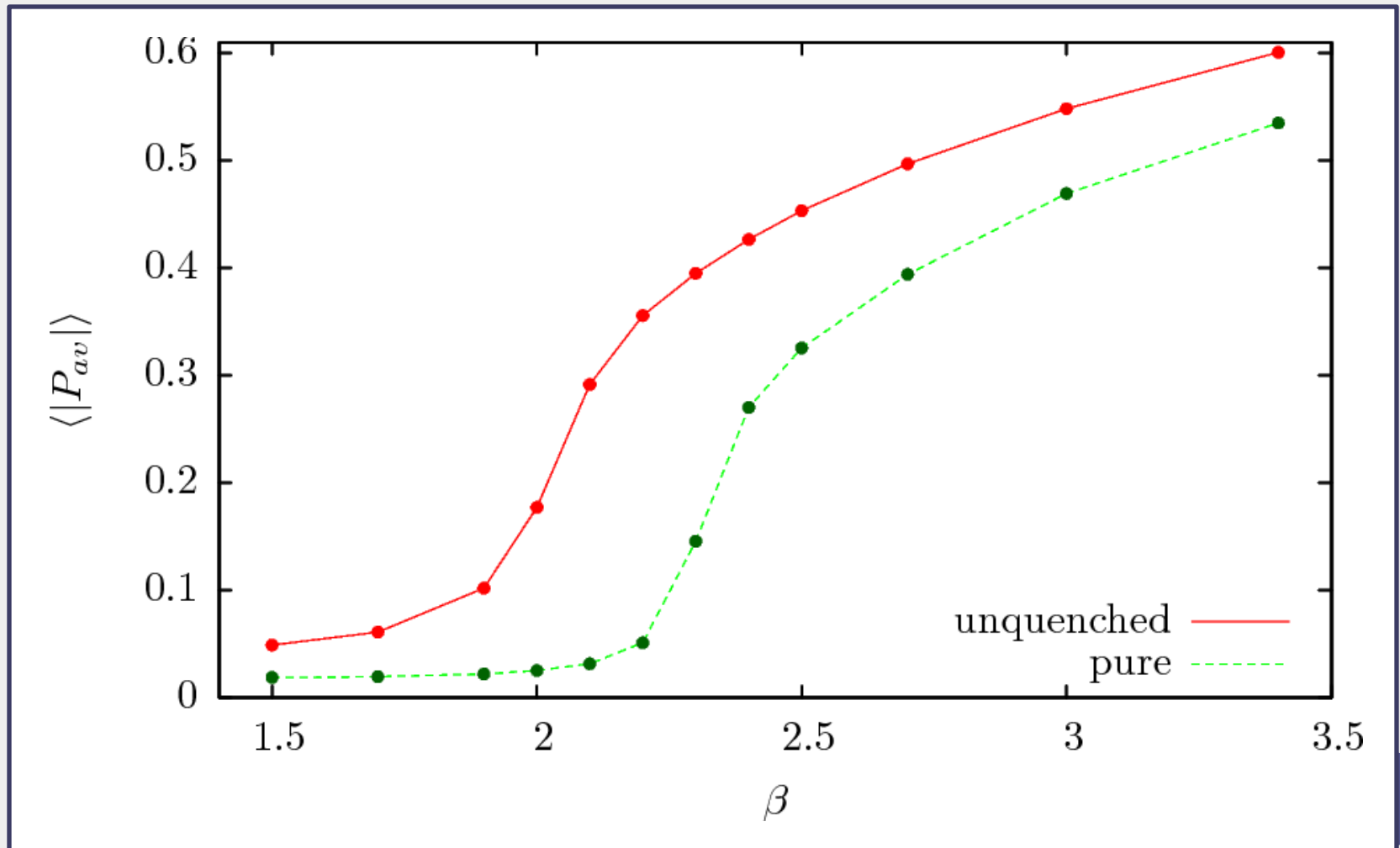


Effect of fermions - ordering external field



$$P_{av} = \frac{1}{V} \sum_{\vec{x}} P$$

DYNAMICAL FERMIONS – SU(2)



$8^3 \times 4, \kappa = 0.15$

...BUT QUARKS HAVE ELECTRIC CHARGE

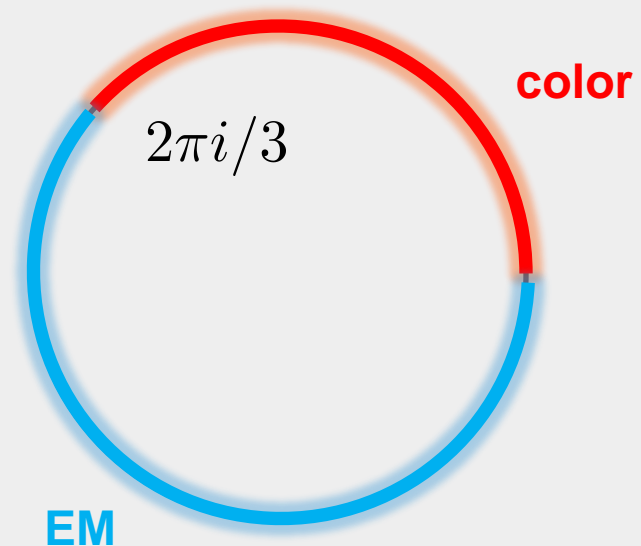
- What if we include **electromagnetism**?

$$q_u = +\frac{2}{3}e, \quad q_d = -\frac{1}{3}e$$

- Exactly compensate color center phase by U(1) phase

- **Gauge group**

$$SU(3) \times U(1)_{em} / \mathbb{Z}_3$$



HIDDEN SYMMETRY

- True Standard Model symmetry group

$$SU(3) \times SU(2) \times U(1)/\mathbb{Z}_6$$

before electroweak
trans.

- Importance

- ▶ unification, e.g. $SU(5)$, $SO(10)$ GUT
- ▶ topological objects - color-EM monopoles/vortices

- Existence of a **global gauge symmetry** that may be spontaneously broken

- relevant to **confinement?**

electroweak trans. –
Zubhov, Veselov,
Bakker



TOY LATTICE MODEL

- 2 flavors of dynamical Wilson fermions, gauge group

$$SU(2) \times U(1)_{em}/\mathbb{Z}_2$$

- u/d quarks with $\pm \frac{1}{2}$ charge relative to $U(1)_{em}$ gauge action

$$S = - \sum_{\square} \left(\frac{\beta_{col}}{2} \text{Re Tr } \square_{SU(2)} + \beta_{em} \cos \square_{\theta} \right) + S_{f,W}$$

- implement via HMC



$$U_{\mu} \exp i \frac{\theta}{2}$$

parallel transporters give both color and electromagnetic contribution to quarks – no net phase for $-1 \times -1 = 1$



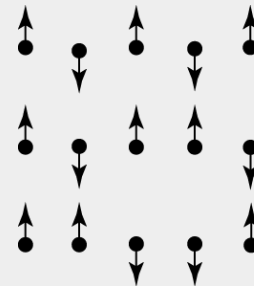
EFFECT OF THE U(1)

- Expect $U(1)_{em}$ to have a **disordering effect**
- Recall Polyakov loop term from Hopping expansion

$$\propto -\text{Re Tr} \begin{array}{c} \uparrow \\ \text{color} \end{array} \cdot \text{Re} \begin{array}{c} \uparrow \\ \text{EM} \end{array} \quad \text{transporter for quarks}$$

- If U(1) loop random over space \rightarrow c.f. spin model in a **random external field**

$$\mathcal{H} = -J \sum_{\langle i,j \rangle} s_i s_j - h \sum_i h_i s_i$$

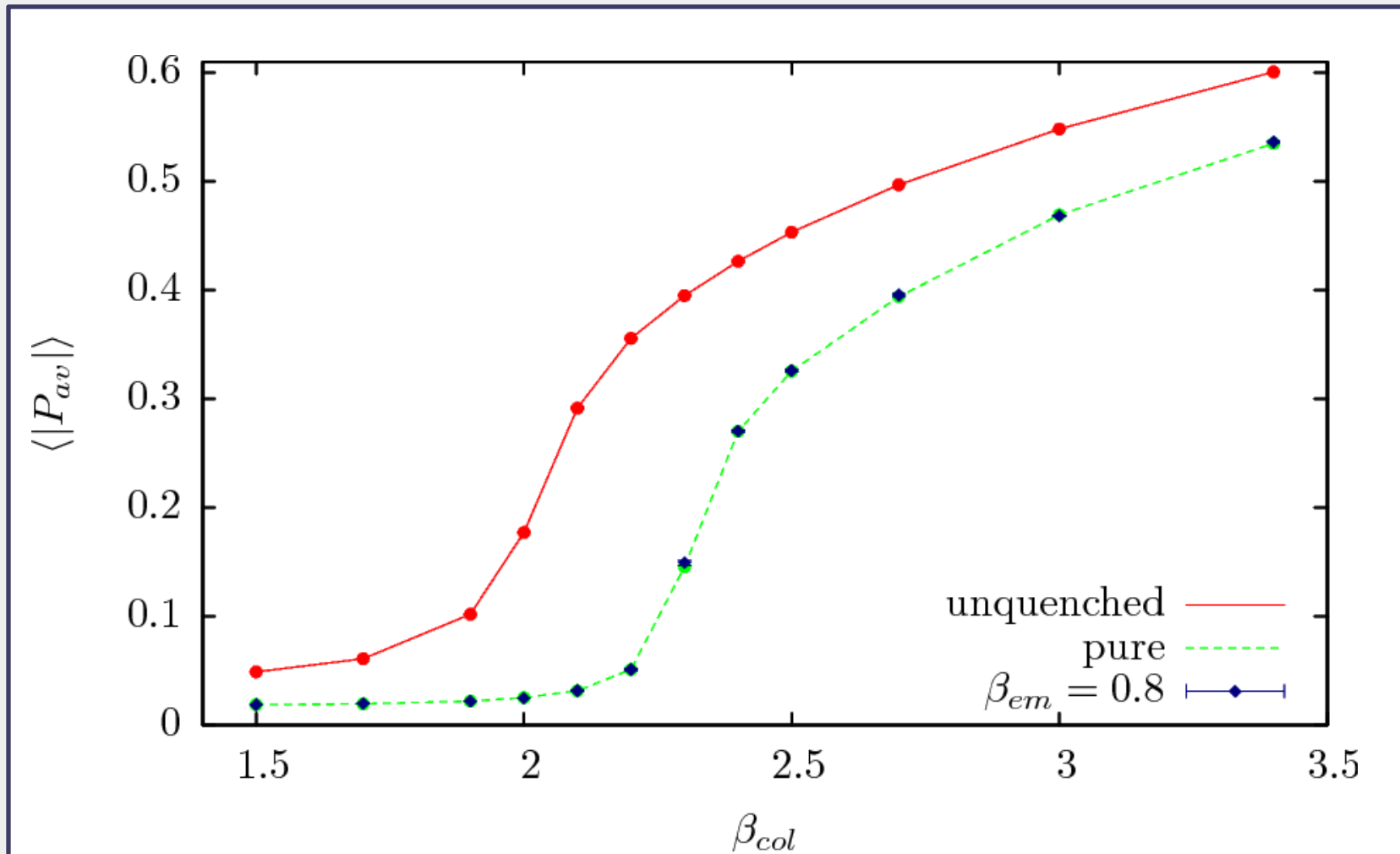


A WORD ON (PURE) COMPACT QED

- Confining at small values of the lattice coupling (i.e. **strong** coupling)
 - i.e. **disordered phase** for EM Polyakov loop
- Phase transition to Coulomb phase at $\beta \approx 1.01$
- Expect U(1) to have large effect for small lattice coupling



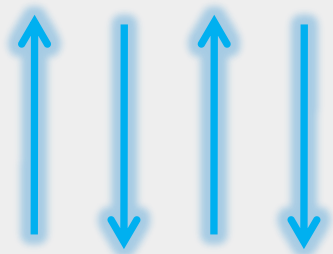
RESTORATION OF QUENCHED BEHAVIOR FOR $P_{SU(2)}$ $SU(2) \times U(1)/Z_2$



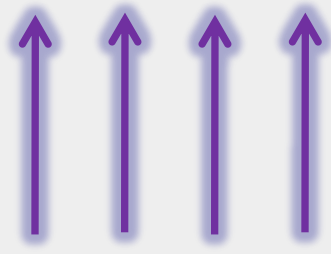
$8^3 \times 4$, $\kappa = 0.15$ (heavy quarks!)

IMPORTANCE OF QUARKS' FRACTIONAL CHARGE

- U(1) angles in gauge action are **twice** those seen by quarks
 - phases differing by π for quarks are not distinguished by the U(1) action
- As we cross $\beta_{em} \approx 1$, particles of unit electric charge should be deconfined - i.e. Polyakov loop of 'electron' becomes finite
- **BUT there is still room for Z_2 disorder in the links as seen by quarks**



for quarks

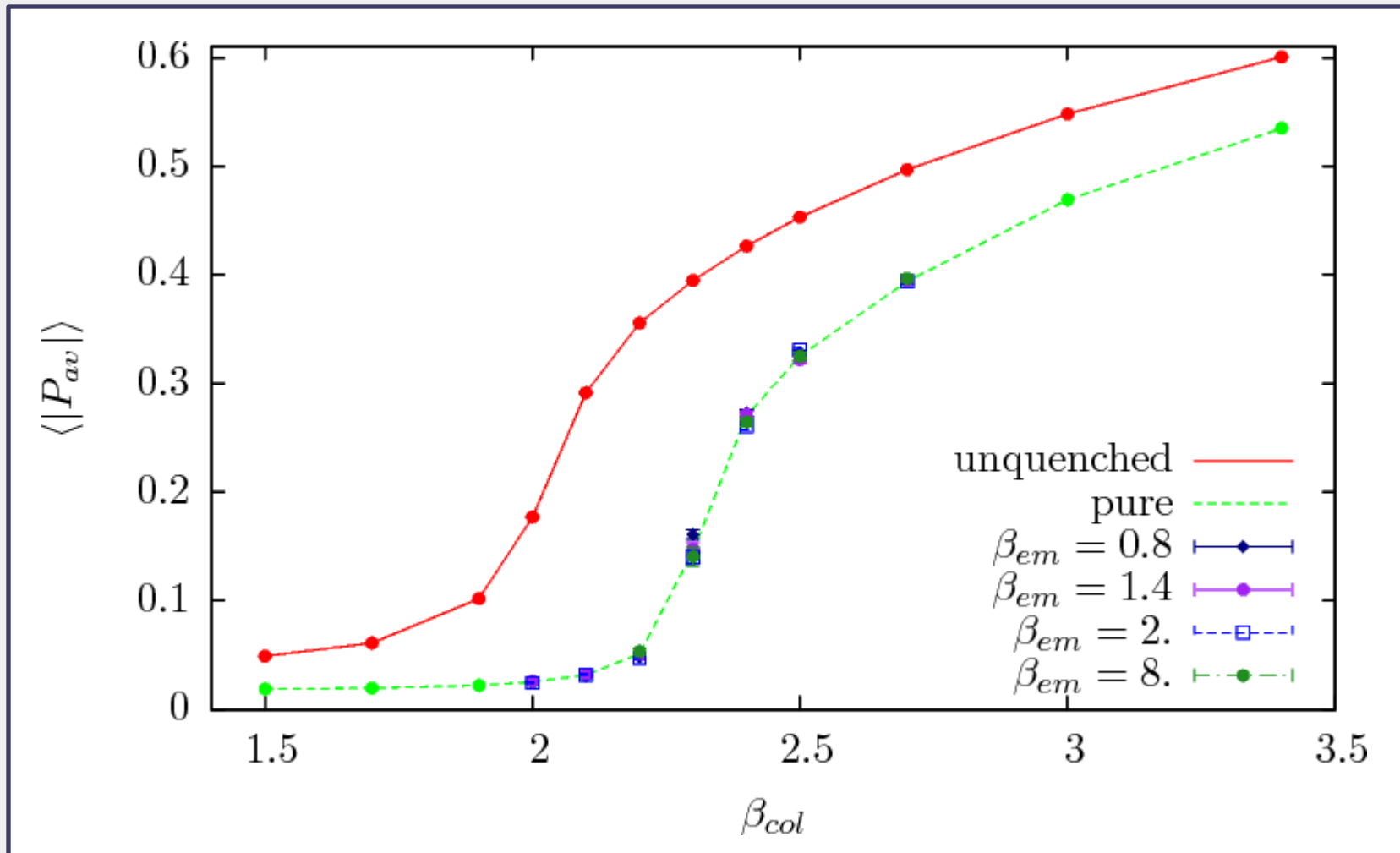


for electrons /
gauge action

Disorder SU(2) Polyakov
loop in the Coulomb
phase for unit electric
charges?



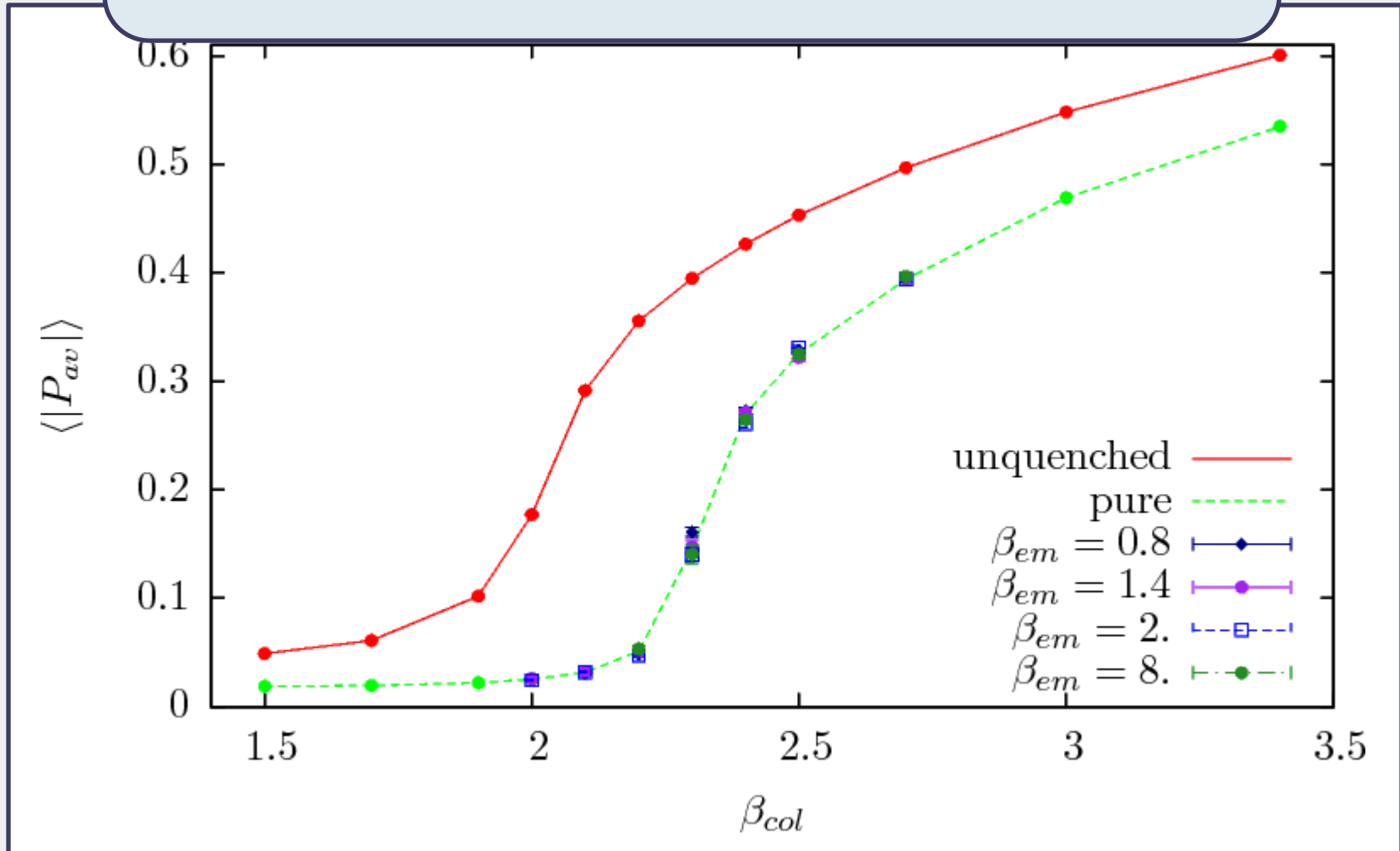
HOT START - $P_{SU(2)}$



$8^3 \times 4$, $\kappa = 0.15$, ~ 6000 traj.

U(1) gauge action is not able to remove the disorder for quarks, even deep in the Coulomb phase for unit charges

Ho



$8^3 \times 4$, $\kappa = 0.15$, ~ 6000 traj.

MUCH TO BE UNDERSTOOD...

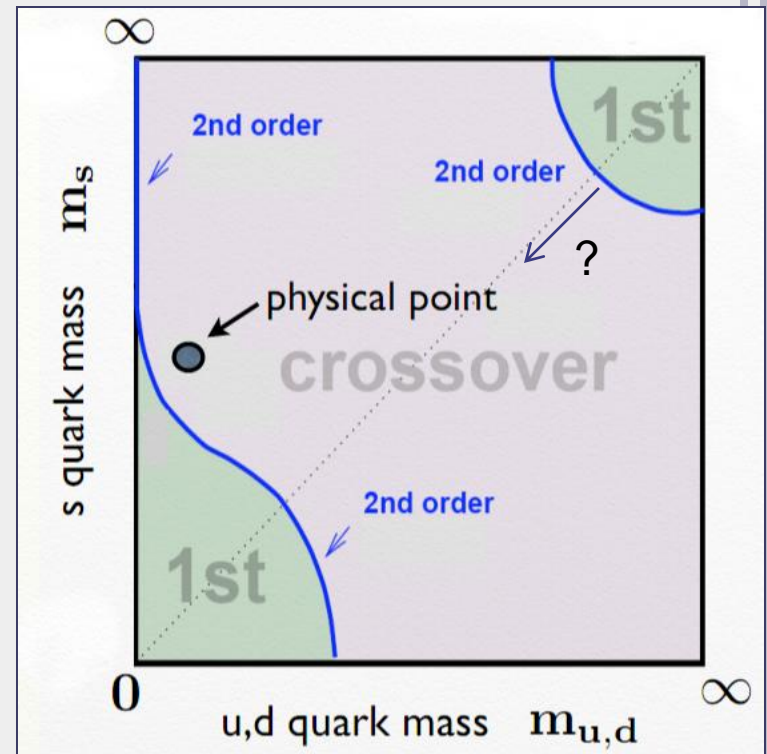
- Cold starts – disorder for quarks only persists for a window beyond $\beta_{em} \approx 1$ – algorithmic issue only?
- Suppression of -1 transporters for light quarks? **Competition** between plaquette like terms

- **Speculation** for $SU(3) \times U(1) / Z_3$

first order transition persists for lighter quarks?

?

sharpen crossover if it doesn't reach physical quark masses?



SUMMARY

- **Center symmetry** recovered when $U(1)$ is added to QCD with dynamical quarks
- **Disordering** effect of $U(1)$
 - How much can **electromagnetism** influence **color** dynamics?
- First steps in a toy $SU(2) \times U(1) / Z_2$ model



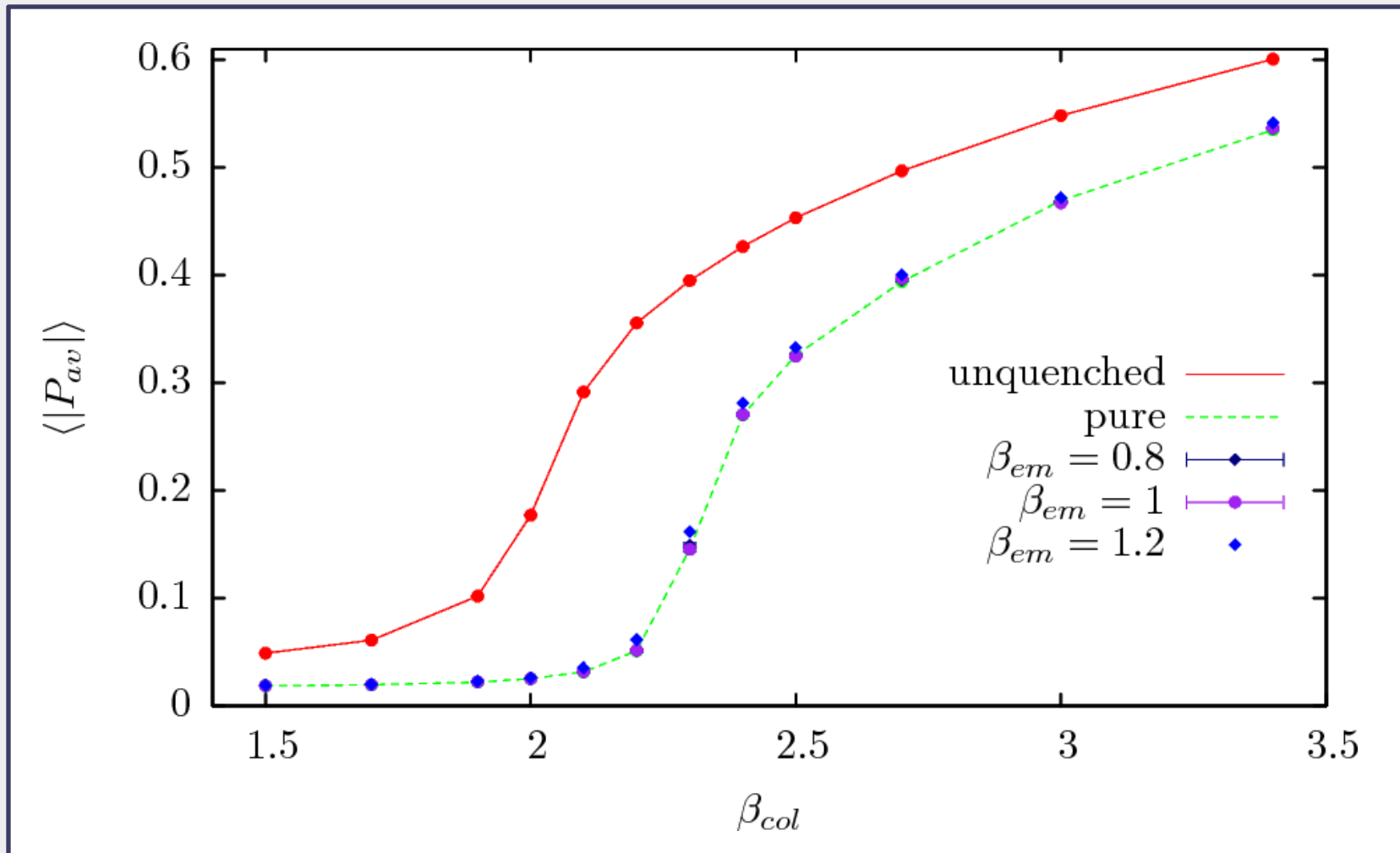
OUTLOOK

- Production runs – respectable masses and volumes
- Related spin models – random field Ising/Potts
- Hopping expansion – baby simulations

- **Twisted boundary conditions!**
 - in presence of dynamical fermions
 - combined **vortices** carrying both color and EM flux

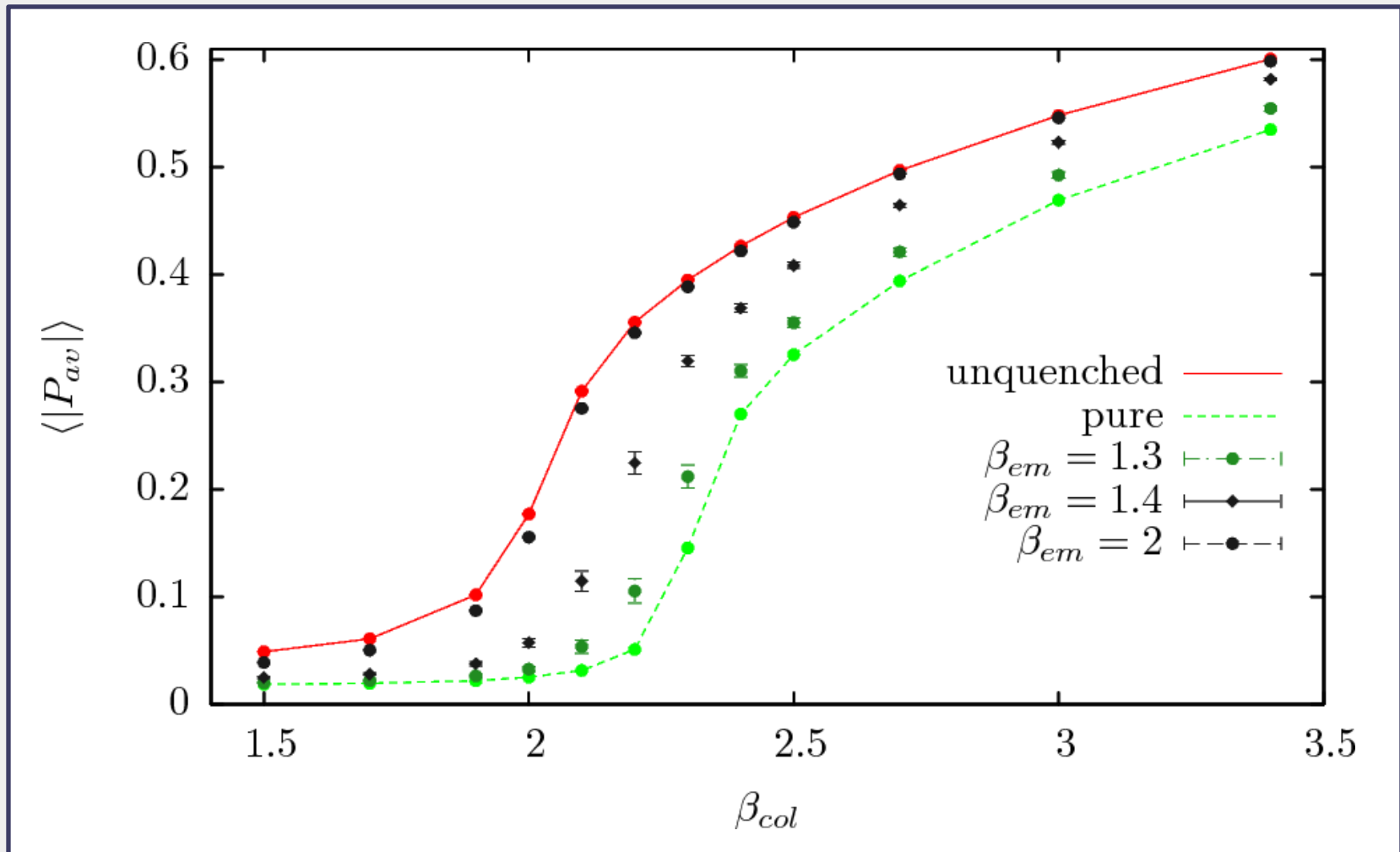


COLD START - $P_{\text{SU}(2)}$



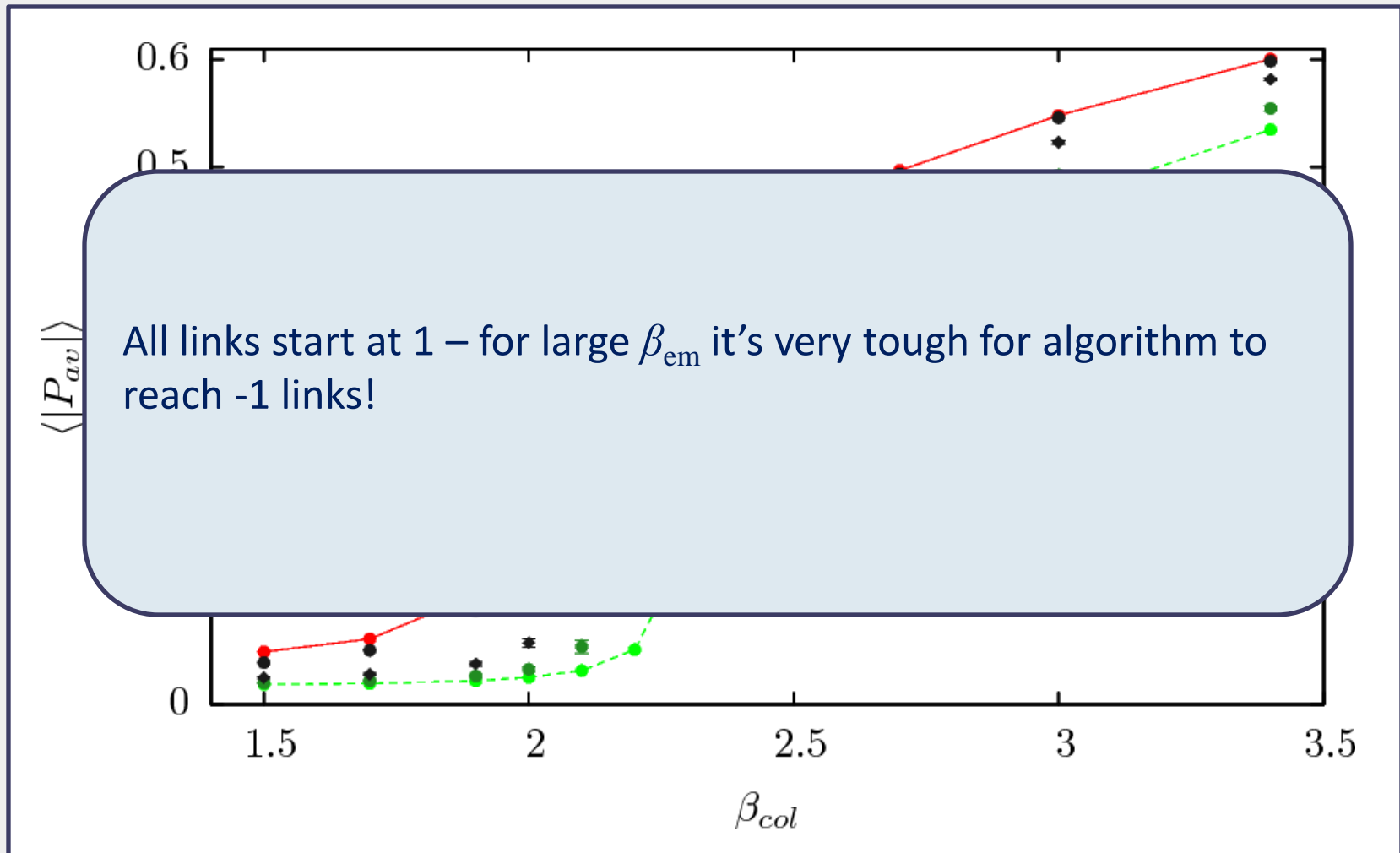
$8^3 \times 4$, $\kappa = 0.15$, ~ 40000 traj.

COLD START - $P_{SU(2)}$



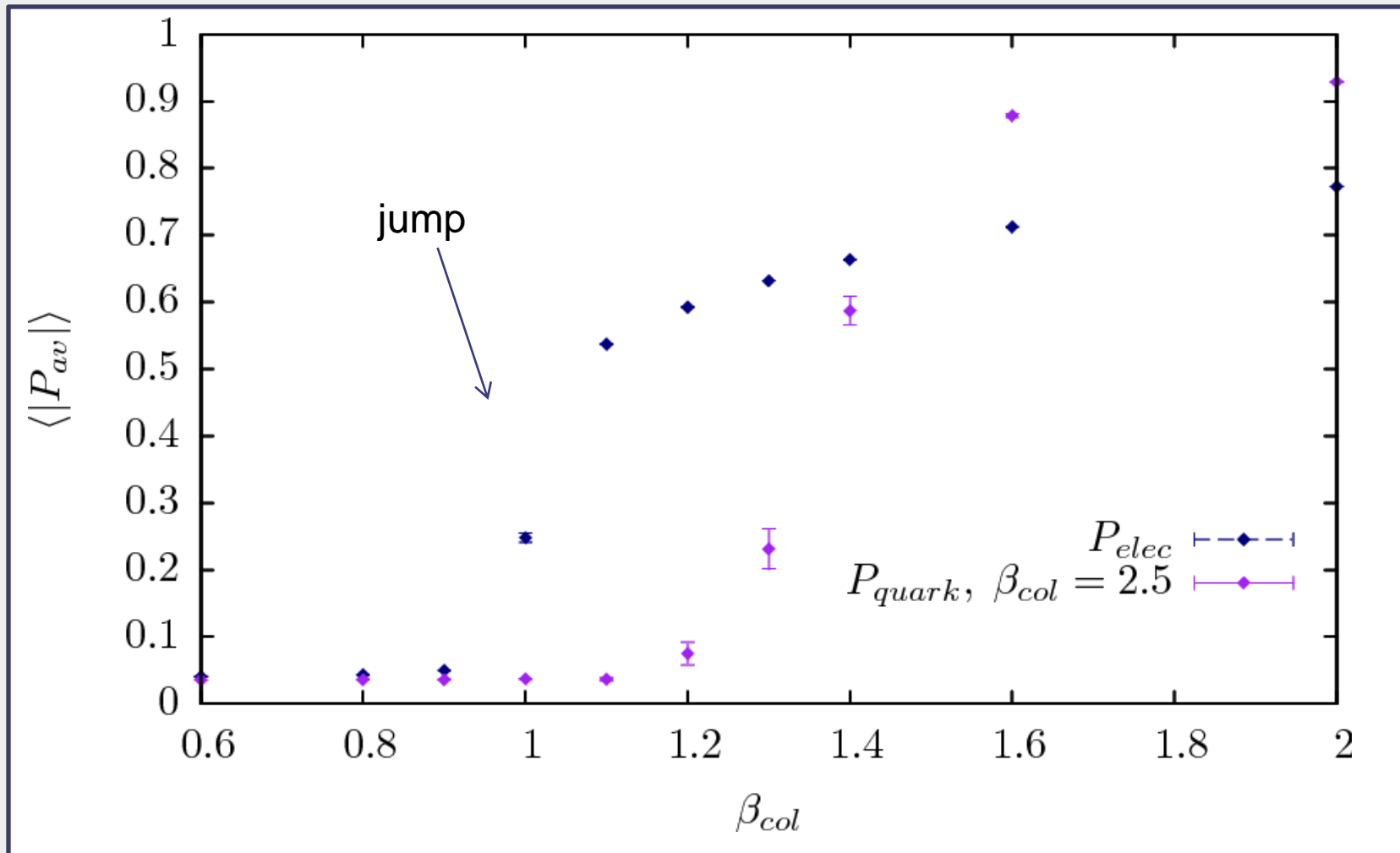
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COLD START - $P_{SU(2)}$



$8^3 \times 4$, $\kappa = 0.15$, ~ 40000 traj.

CHECKING U(1) POLYAKOV LOOPS – COLD START



PLAQ-PLAQ COMPETITION

$$\propto -\kappa^4 \operatorname{Re} \operatorname{Tr} \square_{\text{color}} \operatorname{Re} \square_{\text{EM}} \theta/2$$

