Wilson Fermions with Brillouin Improvement

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Outline

- Motivation
- Stencils and Improvement
  - Laplacian operator
  - Dirac operator
- Eigenvalue spectra
- Tests in quenched QCD
  - Computational cost
  - Physical observables
- Summary / Outlook

Motivation

$$D(x,y) = \sum_\mu \gamma_\mu \nabla_\mu(x,y) - \frac{1}{2}\Delta(x,y) + m_0$$

Can one do better in discretizing the first-derivative and Laplacian terms?

- Perfect Actions (Hasenfratz, Niedermayer, etc.)
  - Very expensive computationally
- Truncation (Gattringer, etc.)
  - Tunable parameters for each β, κ etc.
- Our approach
  - Consider effect on rotational symmetry
  - Consider EV spectrum

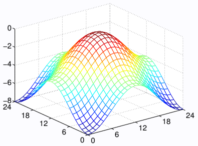
2D Case - Laplacian

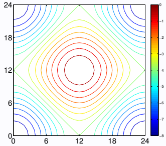
"Standard" stencil

0	1	0
1	-4	1
0	1	0

/1

$$\Delta_s = 2\cos(k_1) + 2\cos(k_2) - 4$$



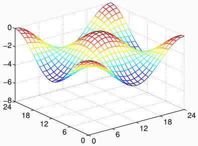


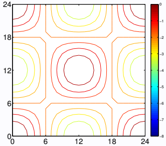
"Tilted" stencil

1	0	1
0	-4	0
1	0	1

/2

$$\Delta_t = 2\cos(k_1)\cos(k_2) - 2$$





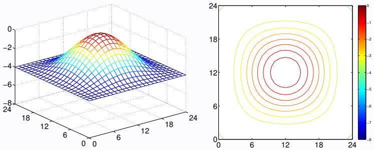
Combine in the form: $\alpha\Delta_s + (1-\alpha)\Delta_t$

2D Case - Laplacian

Improved Laplacian: $\alpha\Delta_s + (1-\alpha)\Delta_t$

$\alpha = 1/2$ "Brillouin" stencil

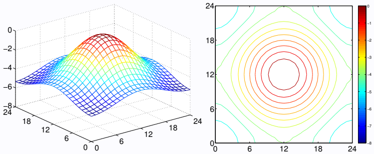
1	2	1	
2	-12	2	/4
1	2	1	



- Same "lift" for all doublers

$\alpha = 2/3$ "Isotropic" stencil

1	4	1	
4	-20	4	/6
1	4	1	



- Isotropic around origin

2D Case - First derivative

Standard derivative in x:

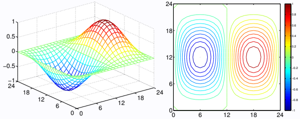
0	0	0
-1	0	1
0	0	0

/2

"Brillouin" in x

-1	0	1
-2	0	2
-1	0	1

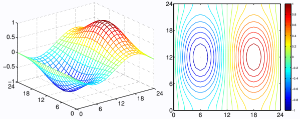
/8



"Isotropic" in x

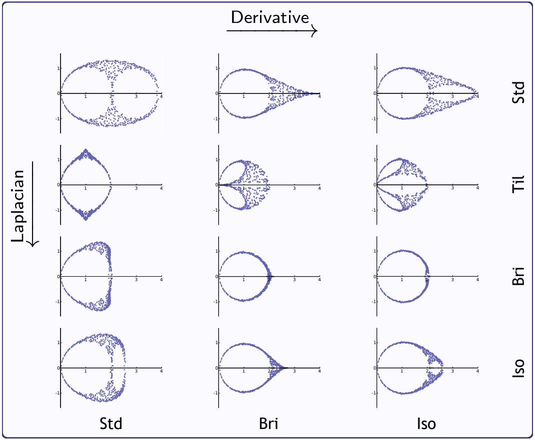
-1	0	1
-4	0	4
-1	0	1

/12



2D Case - Fermion Operator Eigenvalue Spectrum

Operator combinations [U(1), 16×16, $\beta = 4.4$, $c_{sw} = 1$]

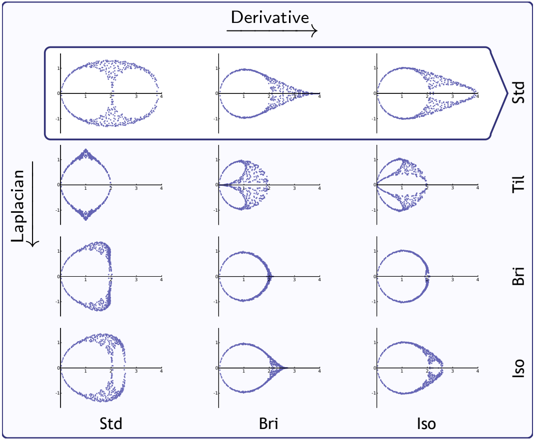


12 Combinations
3 for derivative
$\times$
4 for Laplacian

Variation of Laplacian most significant

2D Case - Fermion Operator Eigenvalue Spectrum

Operator combinations [U(1), 16×16, $\beta = 4.4$, $c_{\mathrm{sw}} = 1$]

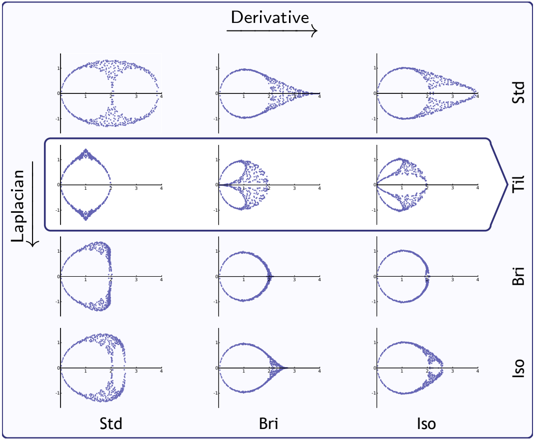


Standard Laplacian

Generally larger C.N.

2D Case - Fermion Operator Eigenvalue Spectrum

Operator combinations [U(1), 16×16, $\beta = 4.4$, $c_{\text{sw}} = 1$]



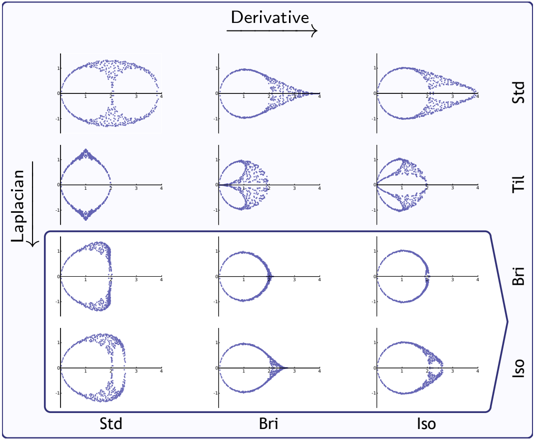
Tilted Laplacian

←

Remaining doublers

2D Case - Fermion Operator Eigenvalue Spectrum

Operator combinations [U(1), 16×16, $\beta = 4.4$, $c_{sw} = 1$]



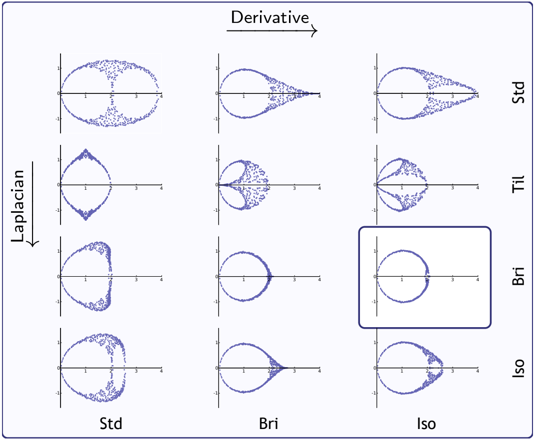
Bril. & Isotr. Laplacians

Expect better properties

(C.N., Shifted op, rotational symmetry)

2D Case - Fermion Operator Eigenvalue Spectrum

Operator combinations [U(1), 16×16, $\beta = 4.4$, $c_{sw} = 1$]

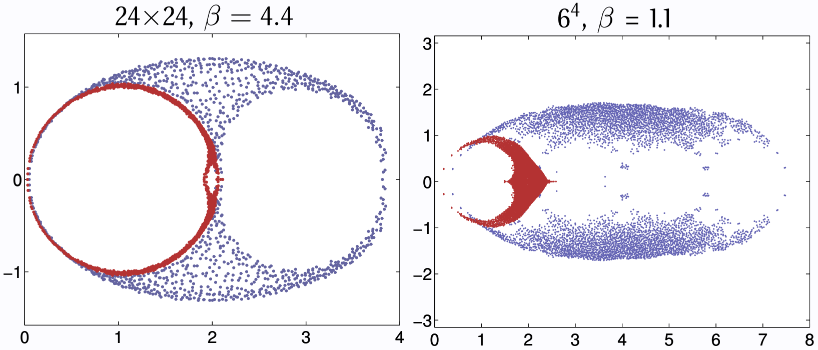


Bril. Laplacian & Isotr. derivative

Our choice

The “Beast” operator

Brillouin Laplacian, Isotropic derivative



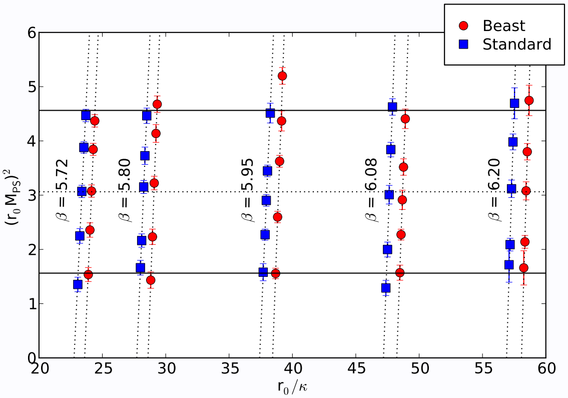
- Smaller condition number (especially as Overlap kernel)
- Smaller additive quark mass renormalization ⇒ better chirality
- Expect less rotational symmetry violation
- Good candidate for Overlap kernel

Practical tests in quenched QCD - Setup

size	β	a (fm)	a^{-1} (GeV)
$10^3 \times 20$	5.72	0.160	1.236
$12^3 \times 24$	5.80	0.133	1.479
$16^3 \times 32$	5.95	0.100	1.978
$20^3 \times 40$	6.08	0.080	2.463
$24^3 \times 48$	6.20	0.067	2.964

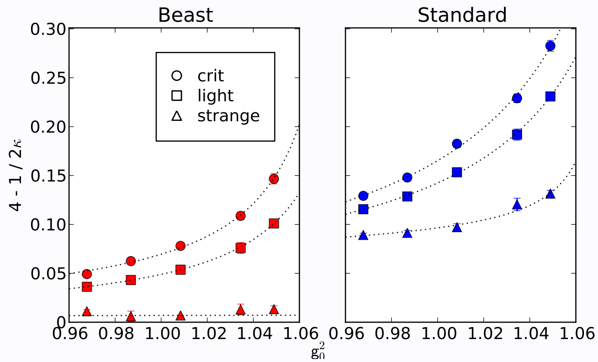
- Constant box size $L \simeq 1.6$ fm
- One iteration of APE smearing $\alpha = 0.72$
- $c_{SW} = 1$ throughout
- Targets: $r_0 M_{ud}^{PS} = 1.25$, $r_0 M_{s\bar{s}}^{PS} = 2.136$ and $r_0 M_{c\bar{c}}^{PS} = 6.812$ ($M_\pi = 500$ MeV & $M_K = 700$ MeV)

Tuning for κ_{crit}



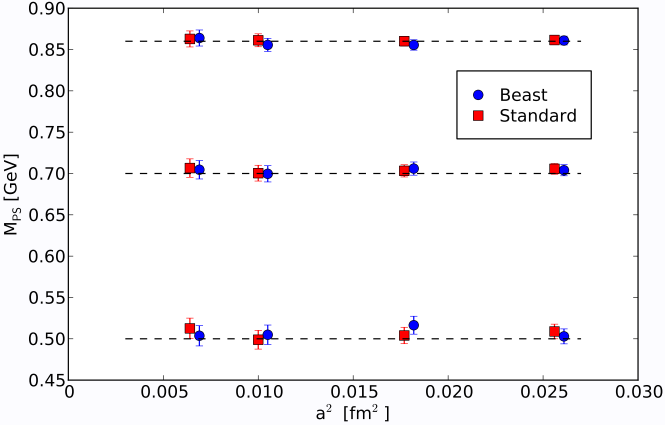
- Linear extrapolation in the range $(r_0 M_{\mathrm{PS}})^2 \in [1.25^2,\ 2.136^2]$
- Also done for charm quark

Determining target κ



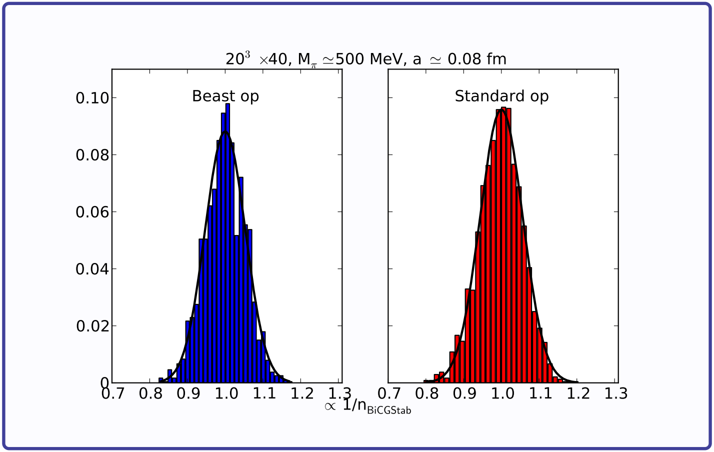
- Rational Ansatz: $-am_{\mathrm{crit}} = \frac{c_1 g_0^2 + c_2 g_0^4}{1 + c_3 g_0^2}$
- $c_1 = S/(12\pi^2)$, S available for standard Wilson case
- Interpolate for κ_{light} and $\kappa_{\mathrm{strange}}$

Pseudoscalar masses



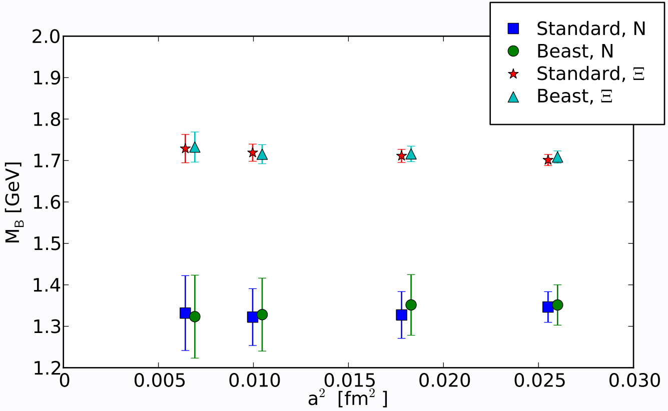
- Consistent masses for all spacings

Inversion Convergence



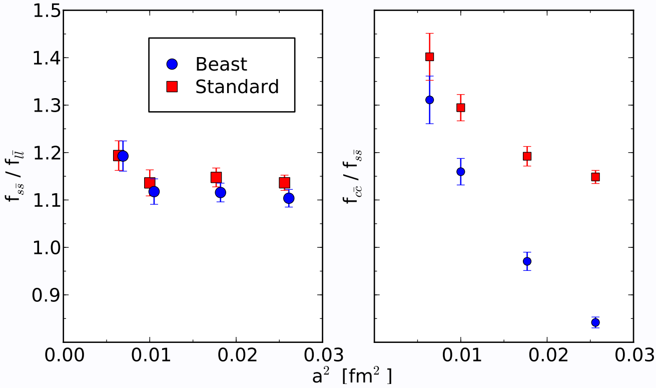
- $\sim$ 1.7 less iterations for improved op
- Approx. same width
- Improved operator inversion $\sim$ 10 times more expensive (unoptimized code)

Nucleon and Ξ



- No visible difference for these observables

Decay constant ratios



- Ratios of decay constants to eliminate Z_A
- Hint of larger scaling region for heavy quarks

Summary / Outlook

Summary / Conclusions

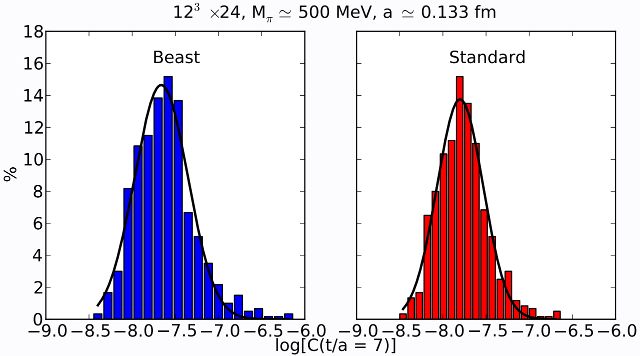
- Extended Laplacian and first-derivative operators with no tunable parameters
- Observed reduction of condition number for plain operator
- Carefully tuned κ_{crit} for various lattice spacings
- Extended scaling region for some observables? – Further analysis required

Further investigation

- Suitability for Charm physics
- Properties as Overlap kernel

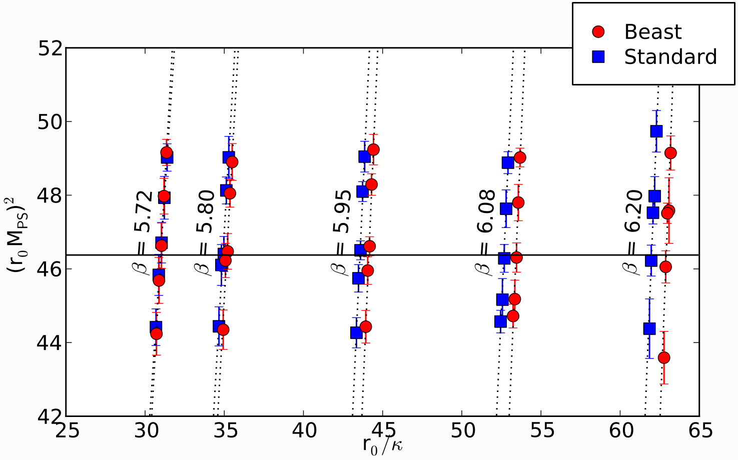
Correlator distribution

Pseudoscalar correlator at given time - slice



- No significant difference between distributions

Tuning κ_{charm}



Interpolating for $f_{c\bar{c}}$