Sextet QCD: slow running and the mass anomalous dimension

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SU(3) gauge theory with $N_f = 2$ fermions in the 6 rep — Wilson–clover fermions, nHYP fat links

1. In the conformal window?

2. Phase diagram on a finite lattice ($m, T \neq 0$)

3. The running coupling at $m = 0$: Schrödinger Functional
   $\Rightarrow$ runs slowly;
   $\Rightarrow$ IRFP? Inconclusive

4. Mass anomalous dimension $\gamma(g^2)$: Too small for walking technicolor!

(arXiv:1006.0707)
Our work: $N = 3$, $REP = SYM = 6$, $N_f = 2$

Is there an IRFP?  Ladder approx says NO: $g_*^2 \simeq 10$. 
PHASE DIAGRAM:

Bulk / finite temperature transition

**FIRST ORDER**

cf. SU(3) with large $N_f$ fund rep

(Damgaard, Heller, Krasnitz, Olesen 1997)

(Iwasaki *et al.* 2002)
PHASE DIAGRAM:

No critical point

confining

non-conf.

m_q < 0

m_q = 0

1st

2nd

Cf. QCD

m_q = 0

m_{\pi} = 0

1st xover

N_t
AS $N_t$ INCREASES:

Cf. QCD
AS $N_t$ INCREASES:

Intersection point moves slowly (or not at all)

$\rightarrow$ slow running (or conformal phase on $\kappa_c$ line!)

- $m_q < 0$
- $m_q = 0$
- $1$st move?

1st

confining

non-conf.

$\kappa$

$\kappa_c$

$\beta$

$N_t$
The DISCRETE BETA FUNCTION (2008)

$u = 1/g^2 (4^4)$

Thin links $4^4 \rightarrow 8^4$

$B(u, 2)$ crosses zero at $g^2 \simeq 2.0$ — a weak coupling.

$\Rightarrow$ IRFP
The DISCRETE BETA FUNCTION (2009–10)

1. Rules out old IRFP
2. Go to stronger coupling? Stopped by 1st order phase transition!
3. No clear IRFP
4. SLOW running

FAT links $6^4 \rightarrow 12^4$
The DISCRETE BETA FUNCTION (2009–10)

\[ u = \frac{1}{g^2} (4^4, 6^4, \text{or} 8^4) \]

FAT links $8^4 \rightarrow 16^4$

Not much help …
How SLOW is SLOW?

\[
\frac{a}{L} \quad 0.05 \quad 0.1 \quad 0.15 \quad 0.2
\]

\[
g^{-2}(L) \quad 0.2 \quad 0.4 \quad 0.6 \quad 0.8 \quad 1.0
\]

\[
\beta = 8.0 \quad \beta = 5.8 \quad \beta = 5.4 \quad \beta = 5.0 \quad \beta = 4.8 \quad \beta = 4.6 \quad \beta = 4.4 \quad \beta = 4.3
\]

< 15% variation as \( L = 6 \rightarrow 16 \)

Remember this . . .
MASS ANOMALOUS DIMENSION

A technicolor theory needs to enhance the techniquark condensate from $\Lambda_{TC} \rightarrow \Lambda_{ETC}$ according to

$$\langle \bar{\Psi}\Psi \rangle_{ETC} = \langle \bar{\Psi}\Psi \rangle_{TC} \times \exp \left[ \int_{\Lambda_{TC}}^{\Lambda_{ETC}} \frac{d\mu}{\mu} \gamma(g^2(\mu)) \right]$$

[enhance $m_q$ while suppressing FCNC]

Walking technicolor: $g^2 \simeq g_*^2$ nearly constant during running, so

$$\langle \bar{\Psi}\Psi \rangle_{ETC} \simeq \langle \bar{\Psi}\Psi \rangle_{TC} \times \left( \frac{\Lambda_{ETC}}{\Lambda_{TC}} \right) \gamma(g_*^2)$$

WANTED: $\gamma(g_*^2) = 1$ (or very close to 1)  

(Chivukula & Simmons, arXiv:1005.5727)

What does the lattice say about this?
MASS ANOMALOUS DIMENSION — the calculation

(Bursa et al. arXiv:0910.4535)

Correlation functions on lattice:

$$\langle P^{b}(t) O^{b}(t' = 0) \rangle_{t=L/2} = Z_{P} Z_{O} e^{-m_{\pi}L/2}$$

$$\langle O^{b}(t = L) O^{b}(t' = 0) \rangle = Z_{O}^{2} e^{-m_{\pi}L}$$

Take ratio, extract $Z_{P}(L)$, whence

$$\frac{Z_{P}(L)}{Z_{P}(L_{0})} = \left( \frac{L}{L_{0}} \right)^{-\gamma}$$

assuming $\gamma \simeq \text{const}$ as $L_{0} \to L$, since the coupling (almost) doesn’t run:
MASS ANOMALOUS DIMENSION — result

Mass renormalization

slope $= -\gamma_m(g^2)$

Cf. one loop: $\gamma = \frac{6C_2(R)}{16\pi^2} g^2$

WHAT HAVE WE LEARNED?

Two possibilities:

1. We have NO IR fixed point:
   • 1st order transition will slide towards $\beta = \infty$ as $N_t \to \infty$
   • Theory has confinement $\chi_{SB}$
   • $\beta$ function doesn’t approach zero; it’s just smaller than two loops, and much smaller than in QCD.
     The theory doesn’t walk (but it runs slowly).
   • $\gamma_m \lesssim 0.6$: no good for extended technicolor

2. We have an IR fixed point:
   • 1st order transition ($m_q = 0$) is stuck: can’t penetrate conformal phase
   • The zero of the $\beta$ function is around the corner (change lattice action?)
   • $\gamma_m \lesssim 0.6$ at IRFP: The theory is well inside the conformal window.*

* since $\gamma_m$ should = 1 at the edge of the window
(Cohen & Georgi 1989; Kaplan, Lee, Son, Stephanov arXiv:0905.4752)