Near-Integrability of Yang-Mills Theories

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. . . Lattice 2010

1. **What is near-integrability?**

In d=2+1, make β of plaquettes $\mu = 0$, $\nu = 1$ HUGE. Result is d=1+1 integrable QFT's.

2. **Why?**

Solvable model with confinement. It is the high-energy (large s, small t) limit.

3. **What are the problems?**

The scaling limit is not the standard one (CROSSOVER).

Longitudinal rescaling:

Verlinde+Verlinde('93), McLerran+Venugopalan('94)

$$x^L = (x^0, x^3)\ ,\quad x^\perp = (x^1, x^2)$$
$$x^L \to \lambda x^L,\quad x^\perp \to x^\perp$$

C.M. energy squared: $s \to \lambda^{-2} s$. **For high energies, take** $\lambda \ll 1$.

Rescaled action:

$$S = \frac{1}{2g_0^2}\int d^4x\, \mathrm{Tr}\left(\sum_{j=1}^{2} F_{0j}^2 - \sum_{j=1}^{2} F_{j3}^2 + \lambda^{-2} F_{03}^2 - \lambda^2 F_{12}^2\right),$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - \mathrm{i}[A_\mu, A_\nu].$$

Hamiltonian:

$$H = \int d^3x \left[\frac{g_0^2}{2}\mathcal{E}_\perp^2 + \frac{1}{2g_0^2}\mathcal{B}_\perp^2 + \lambda^2 \left(\frac{g_0^2}{2}\mathcal{E}_3^2 + \frac{1}{2g_0^2}\mathcal{B}_3^2 \right) \right]$$

$$= H_0 + \lambda^2 H_1 .$$

$$(\partial_\perp \cdot \mathcal{E}_\perp + \partial_3 \mathcal{E}_3 - \rho)\, \Psi_{\rm Physical} = 0 \ ,$$

If d= 2+1, the last term ($\mathcal{B}_3^2$) is gone!

Latticize!

H_0 **is many 1+1-dimensional SU(N) principal-chiral sigma models,**

$$\mathcal{L} = \frac{1}{2g_0^2}\int d^2x \, \mathrm{Tr}\, \partial^\mu U^\dagger \partial_\mu U, \quad U \in \mathbf{SU(N)}, \quad \mu = 0, 3.$$

So $\lambda \to 0$ **limit of the YM theory is integrable.**

H_1 **is "small", provided**

$$\lambda^2 \ll g_0^{-3} \exp -4\pi/(g_0^2 N) \,.$$

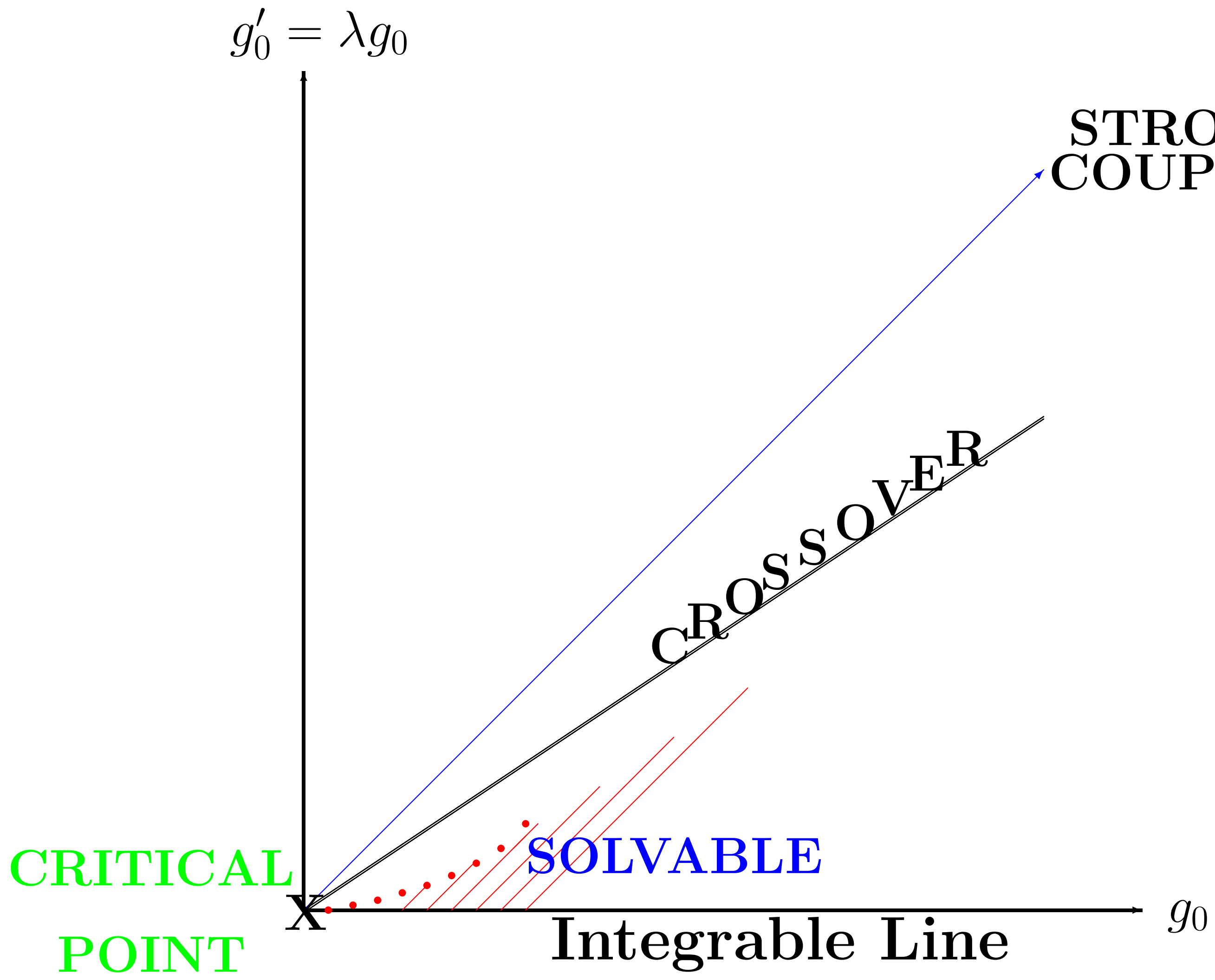
Gauss Law + Mass Gap $\Longrightarrow$ Confinement

For d=3+1, one coupling $g_0'' = g_0/\lambda$, is strong (small β).

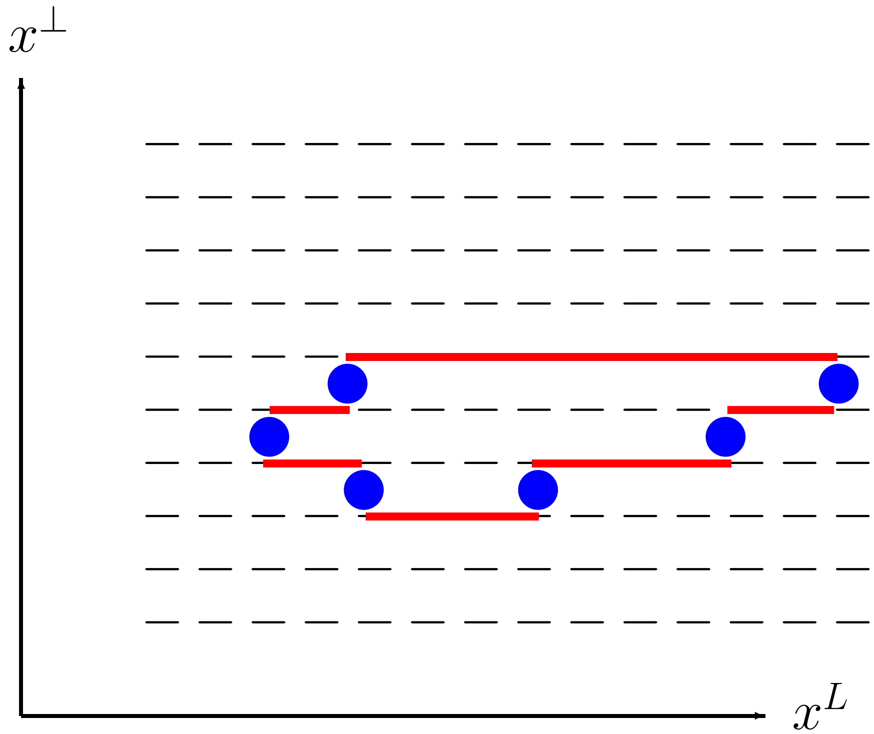
For $d = 2 + 1$ dimensions, all couplings are weak (all β's are large).

If $\lambda \ll 1$, scattering is diffractive.

d=2+1 PHASE DIAGRAM



Glueball for SU(2)



Bullets are massive "solitons" of transverse electric flux. Red lines are longitudinal electric flux. Flux can terminate on quarks.

Details of PC Sigma Model I

(95% of you should sleep now...)

Spectrum: $m_r = m_1 \frac{\sin(\pi r/N)}{\sin(\pi/N)},\ r = 1, \ldots, N-1$.

Species $r = 1$ are color dipoles ($q\bar{q}$), and $r > 1$ are bound states. Elementary antiparticle has $r = N - 1$.

(r=1) by (r=1) S-matrix:

$$\mathfrak{S}_{11}(\theta) = \frac{\sin(\theta/2 - \pi \mathrm{i}/N)}{\sin(\theta/2 + \pi \mathrm{i}/N)}\ S_{\mathrm{CGN}}(\theta) \otimes S_{\mathrm{CGN}}(\theta),$$

$$S_{\mathrm{CGN}}(\theta) = \frac{\Gamma(\mathrm{i}\theta/2\pi + 1)\Gamma(-\mathrm{i}\theta/2\pi - 1/N)}{\Gamma(\mathrm{i}\theta/2\pi + 1 - 1/N)\Gamma(-\mathrm{i}\theta/2\pi)}\left(\mathbb{1} - \frac{2\pi \mathrm{i}}{N\theta}P\right).$$

All S-matrix elements found by fusion and crossing ($\theta \to \pi\mathrm{i} - \theta$).

Details of PC Sigma Model II

Form Factor, Karowski and Weisz (1977):

$$\langle 0|j_0^{L,R}(x)_b|\theta_2, j_2, \theta_1, j_1\rangle = \mathrm{i}\sqrt{2}G\left(\delta_{j_1 4}\delta_{j_2 b} - \delta_{j_2 4}\delta_{j_1 b} \pm \epsilon_{b j_1 j_2}\right)$$
$$\times m(\mathrm{ch}\theta_1 - \mathrm{ch}\theta_2)\exp\{-\mathrm{i}m[x^0(\mathrm{ch}\theta_1 + \mathrm{ch}\theta_2) - x^1(\mathrm{sh}\theta_1 + \mathrm{sh}\theta_2)]\}$$
$$\times\ F(\theta_2 - \theta_1)\ ,$$

$$F(\theta) = \exp 2\int_0^\infty \frac{d\xi}{\xi}\,\frac{e^{-\xi}-1}{e^{\xi}+1}\,\frac{\sin^2\frac{\xi(\pi\mathrm{i}-\theta)}{2\pi}}{\mathrm{sh}\xi}\ .$$

(Please wake up!)

RESULTS: Mostly d=2+1 and SU(2)

Form factor/S-matrix $\Longrightarrow$ subleading.

1. Longitudinal string tension (2006):

$$\sigma_{\rm L} = \frac{3(g_0')^2}{2a^2}\left\{ 1 \; + \; \frac{4}{3m^2\pi^2}\frac{(g_0')^2}{g_0^4}\, 0.726 \right\}^{-1} .$$

$\uparrow$ **leading** $\qquad$ $\uparrow$ **subleading**

RESULTS: (CONTINUED)

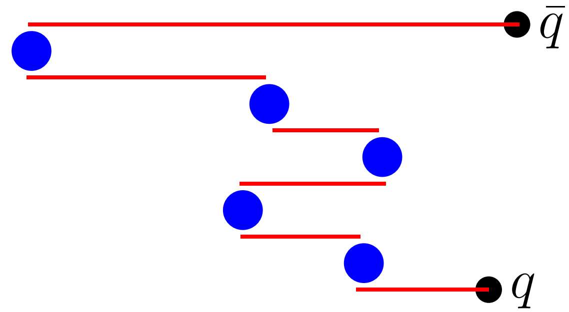
2. Transverse string tension (2008):

$$\sigma_\perp = \frac{m}{a} - \frac{2\sqrt{3}}{\pi}\frac{g_0'}{g_0^2 a^2}.$$

$\uparrow$ $\uparrow$

leading **subleading**

$\updownarrow$



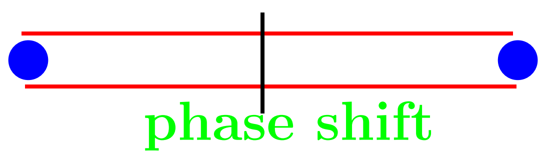
RESULTS: (CONTINUED)

3. Low-lying mass spectrum (2007):

$$M_n = 2m_1 + \left[\epsilon_n^{1/3} - \frac{3(3-2\ln 2)\sigma_\perp}{4\pi m}\epsilon_n^{-1/3}\right]^2, \text{ where}$$

$\uparrow$ **leading** $\uparrow$ **subleading**

$$\epsilon_n = \frac{3\pi\sigma_\perp(n+\frac{1}{2})}{4m^{1/2}} + \left\{\left[\frac{3\pi\sigma_\perp}{4m^{1/2}(n+\frac{1}{2})}\right]^2 + \frac{1}{8}\left[\frac{3(3-2\ln 2)\sigma_\perp}{2\pi m}\right]^3\right\}^{1/2}.$$



Inspired by McCoy and Wu (1978).

RESULTS: (CONTINUED)

4. k**-string tensions (for** $N > 2$**) (2006):**

Casimir law for longitudinal string tensions.

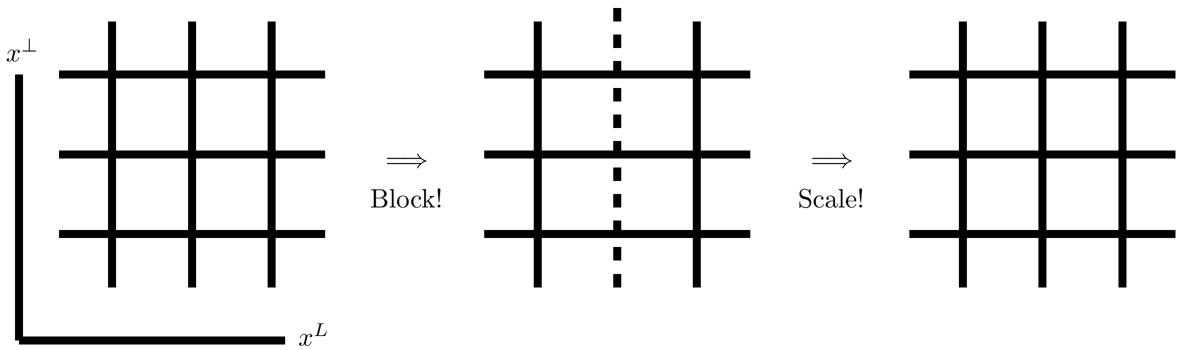
Sine law for transverse string tensions.

RESULTS: (CONTINUED)

5. **Question:** **The rescaling $x^L \longrightarrow \lambda x^L$, is classical. How can we do a quantum rescaling?**

Answer: **Anisotropic renormalization (with J. Xiao) (2009)**.

RESULTS: (CONTINUED)



Lattice rescaling, $\lambda = 1/2$.

First apply a Kadanoff transformation.

Then restore lattice spacing by longitudinal rescaling.

RESULTS: (CONTINUED)

$$\mathcal{L}_{\rm eff} = \frac{1}{4g_{\rm eff}^2}\,{\rm Tr}\big(F_{01}^2 + F_{02}^2 - F_{13}^2 - F_{23}^2 + \lambda^{-2+\frac{17C_N}{48\pi^2}\tilde{g}_0^2}F_{03}^2 - \lambda^{2+\frac{7C_N}{48\pi^2}\tilde{g}_0^2}F_{12}^2\big) + \cdots \ .$$

$$\frac{1}{g_{\rm eff}^2} = \frac{1}{g_0^2} - \frac{11C_N}{48\pi^2}\ln\frac{\Lambda}{\tilde{\Lambda}} + \frac{C_N\ln\lambda}{128\pi^2} = \frac{1}{\tilde{g}_0^2}\,\lambda^{\frac{C_N}{128\pi^2}\tilde{g}_0^2} + \cdots \ .$$

UNDER INVESTIGATION

1. BEAT THE CROSSOVER!!!

Konik and Adamov, Phys. Rev. Lett. (2009) did this (!) via DMRG, for $\mathbb{Z}_2$ (3DIM). Critical exponent: $\nu = 0.622 \pm 0.019$.

For (d=2+1) SU(2), do the longitudinal renormalization, $\lambda \to 0$, but keep finite part of action.

UNDER INVESTIGATION (CONTINUED)

2. For forward elastic scattering amplitude, find soliton distributions in transverse plane and in rapidity space.

3. Exact SU(N) sigma-model form factors, to extend to N=3 or N=∞.

THANK YOU!

Papers...

1., 2. **discuss (3+1)-dimensional collisions. The others concern confinement in 2+1 dimensions.**

1. **Longitudinal Rescaling and High-Energy Effective Actions**, with Jing Xiao,
Phys. Rev. **D80** (2009) 016005, arXiv:0901.2955

2. **Near-Integrability and Confinement for High-Energy Hadron-Hadron Collisions**,
Phys. Rev. **D77** (2008) 056004, arXiv:0801.0389

3. **Composite Strings in (2+1)-Dimensional Anisotropic Weakly-Coupled Yang-Mills Theory**,
Phys. Rev. **D77** (2008) 025035, arXiv:0710.3733

Papers...

4. Glueball Masses in (2+1)-Dimensional Anisotropic Weakly-Coupled Yang-Mills Theory,
Phys. Rev. **D75** (2007) 101702, arXiv:0704.0940

5. String Tensions and Representations in Anisotropic 2+1-Dimensional Weakly-Coupled Yang-Mills Theory,
Phys. Rev. **D75** (2007) 025001, hep-th/0608067

6. Integrable Models and Confinement in (2+1)-Dimensional Weakly-Coupled Yang-Mills Theory,
Phys.Rev.**D74** (2006) 085001, hep-th/0607013

7. Lattice QCD_{2+1},
Phys. Rev. **D71** (2005) 054503, hep-lat/0501026