D to K semi-leptonic decay form factors from HISQ light and charm quarks

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In collaboration with:

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HPQCD collaboration

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Outline

- Introduction: result
  - Scalar current: no need for operator matching
- Improvement strategy
  - Random-wall source
  - Simultaneous multi-T fit
- Tuning quark masses (m_c and m_s)
  - Tuning and discretization error
- Decay constants (f_D, $f_{K,}$ f_{Ds}, f_π, and $f_{\eta s}$)
- ChiPT and chiral/continuum extrapolations
  - Bayesian fit and pseudo-prior
- Results, errors, and discussions
  - $f_+(0)$
  - $|V_{cs}|$ and unitarity check
  - f_0/f_{Ds}
- Summary and future plan

• Introduction: D to K l ν semi-leptonic decay

- Using HISQ action for both charm and light quarks
- Form factors and PCVC

$$< V^{\mu} > = [(p_D^{\mu} + p_{\pi}^{\mu}) - \frac{m_D^2 - m_{\pi}^2}{q^2} q^{\mu}] f_+(q^2) + \frac{m_D^2 - m_{\pi}^2}{q^2} q^{\mu} f_0(q^2)$$

$$q^{\mu} < V_{\mu}^{conti} > = (m_c - m_q) < S^{conti} >$$

$$\text{where} < S > = < K | \bar{s} c | D >$$

$f_0(q^2)$ and $f_+(0)$ can be determined just from <S> with no need for Z factors

$$< S > = \frac{m_D^2 - m_{\pi}^2}{m_c - m_q} f_0(q^2)$$

$$f_0(q^2) = \frac{m_c - m_q}{m_D^2 - m_{\pi}^2} < S >$$

$$f_+(0) = f_0(0) = \frac{m_c - m_q}{m_D^2 - m_{\pi}^2} < S > \Big|_{q^2=0}$$

- Simulations

  - 600 MILC 2+1 asqtad lattices

    Three coarse: 24^3x64, $am_l u_0$=0.005, $am_s u_0$=0.05

    20^3x64, $am_l u_0$=0.01, $am_s u_0$=0.05

    20^3x64, $am_l u_0$=0.02, $am_s u_0$=0.05

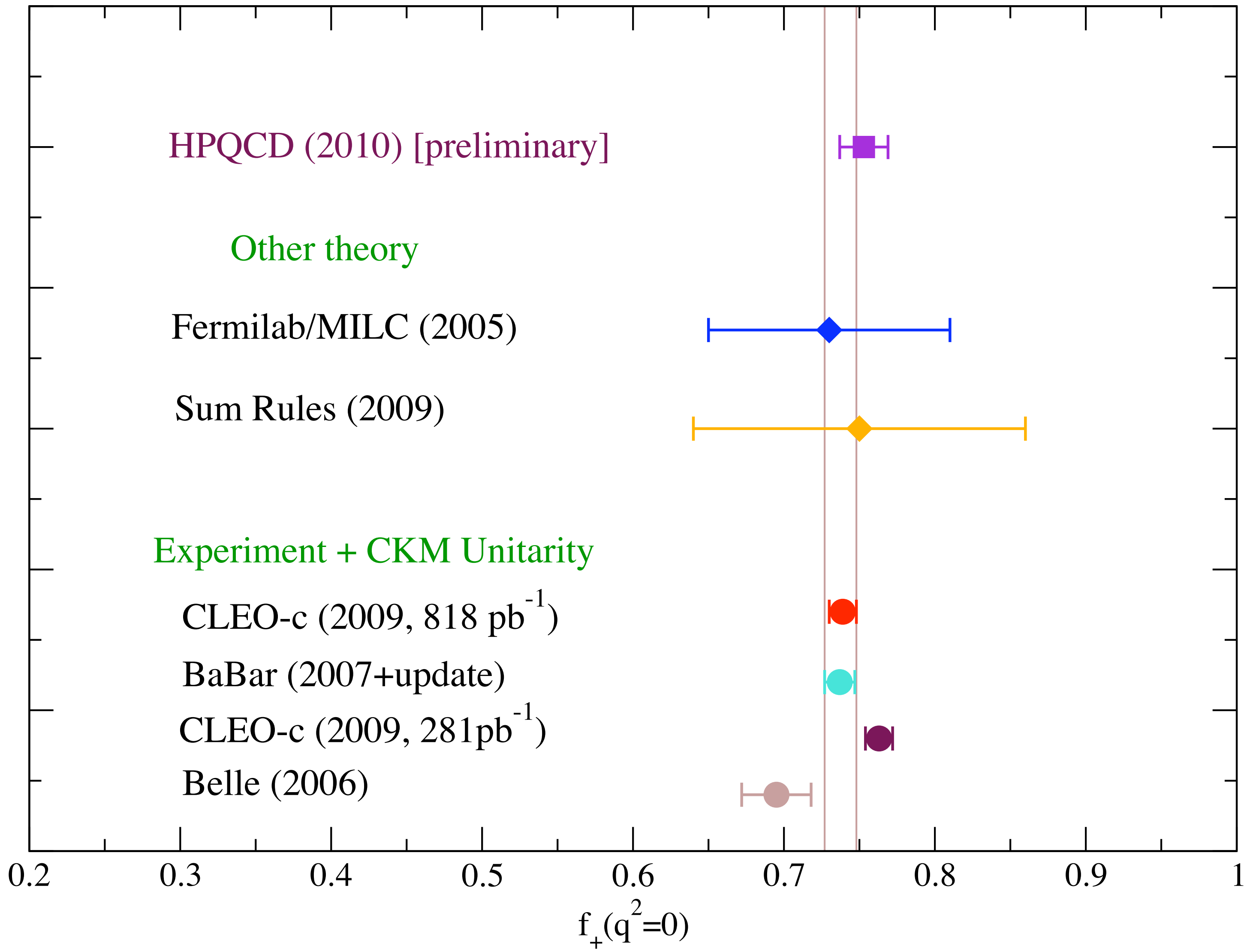
    Two fine: 28^3x96, $am_l u_0$=0.0062, $am_s u_0$=0.031

    28^3x96, $am_l u_0$=0.0124, $am_s u_0$=0.031

  - two (coarse) and four (fine) time sources
  - Random-wall source, periodic boundary condition
  - p=(0,0,0), (1,0,0), (1,1,0), (1,1,1)

    [p=(1,1,1) is already equivalent to negative q^2]

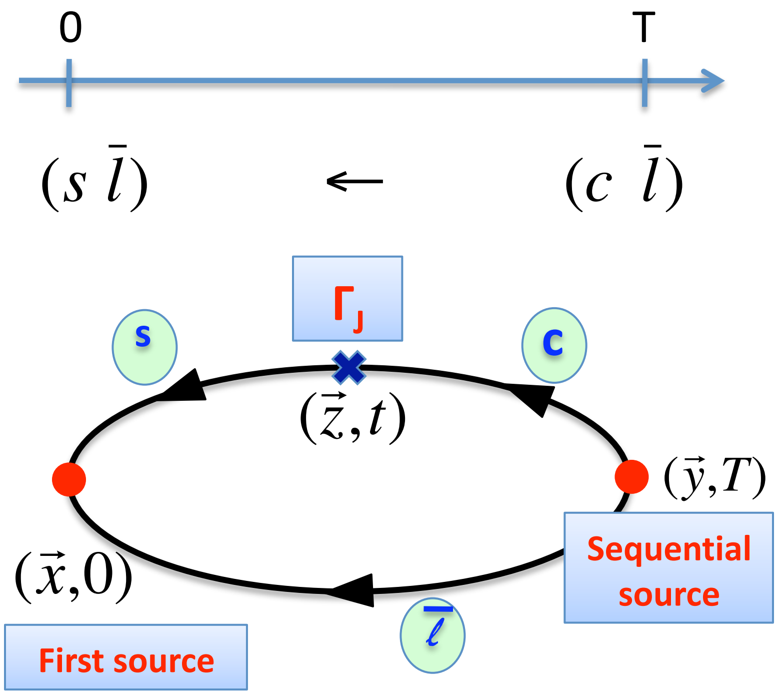
- Result: $f_+(q^2=0)$



We obtain $f_+(q^2=0)$ with ~ 2% total errors.

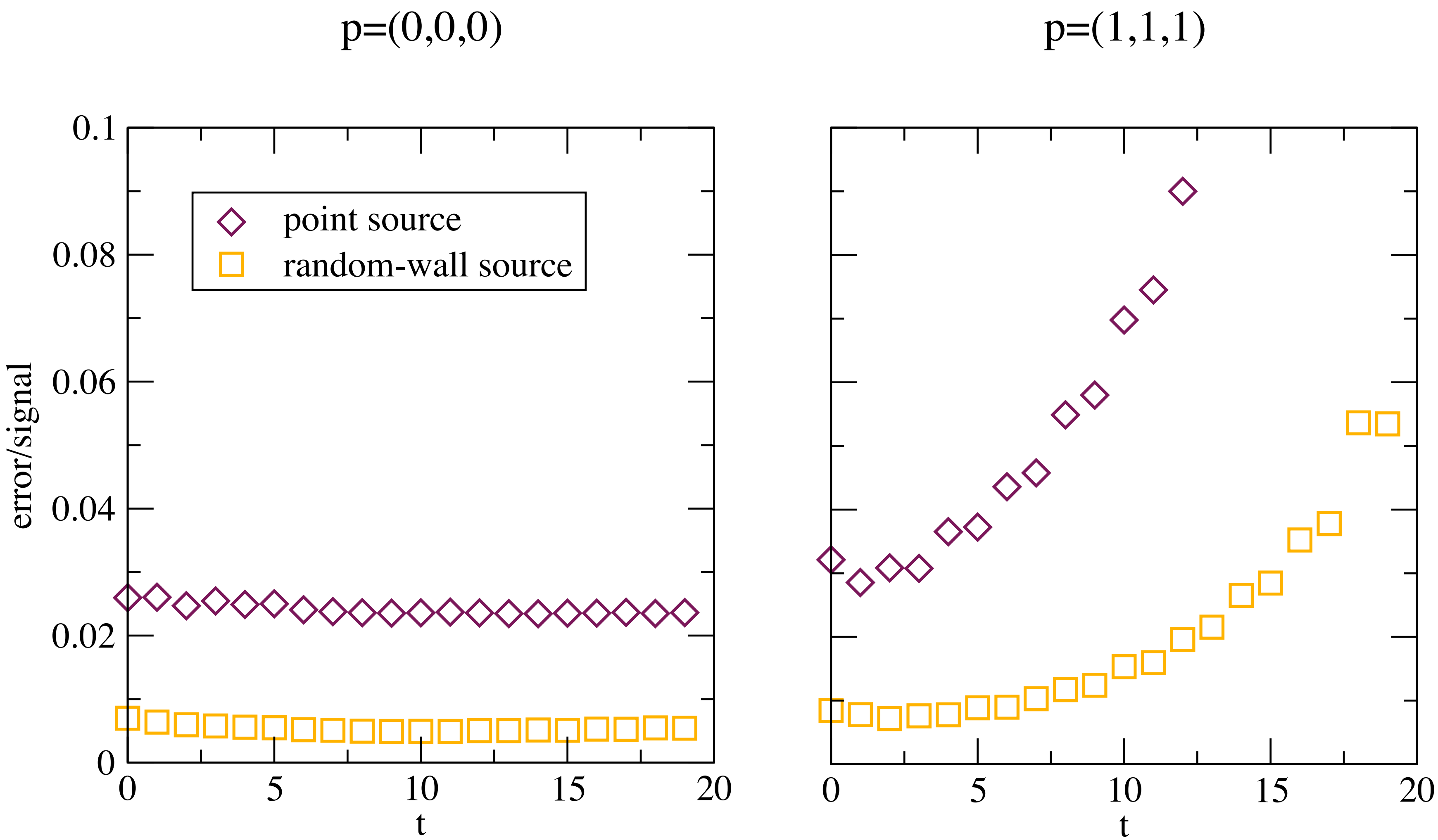
• Improvement strategy I: Random-wall source

- Random-wall source



- Using U(1) noise
- For p = (0,0,0) to (1,1,1)
- Need to calculate strange quark propagators for every equivalent momenta.
- T is the sequential source point.

- Random-wall source: error/signal ratio for the three-point function



• Improvement strategy II: Simultaneous multi-T fit

- Simultaneous fit with:
  - D and K meson two-point functions
  - <S> three-point functions with multiple T (sequential source)

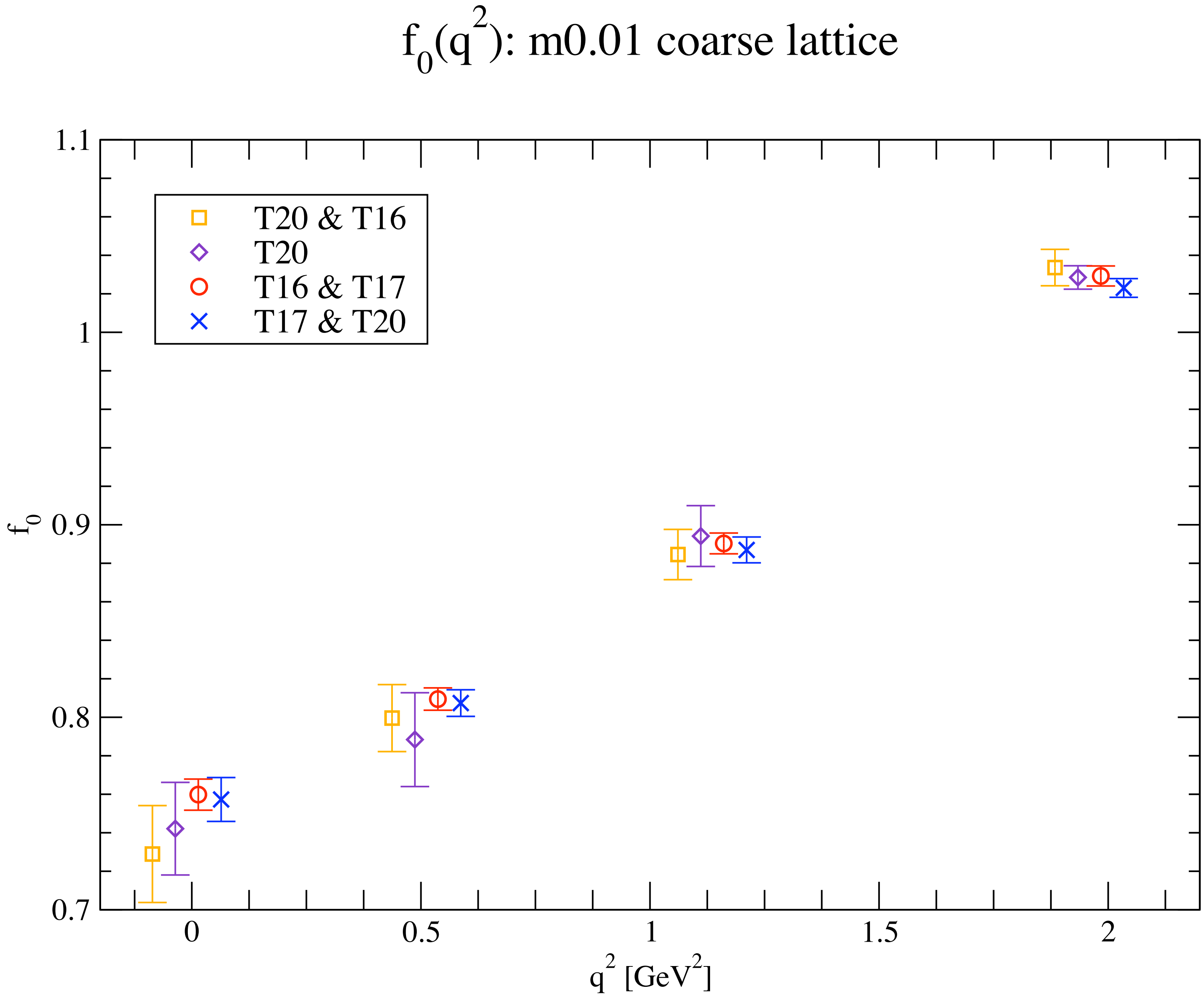
$$C_K(t) = \sum_n^{\text{nexp}} \mathrm{a}_\mathrm{K}^2 (e^{-E_K t} + e^{-E_K (T-t)}) + \sum_n^{\text{nexpo}} \mathrm{s}(-1)^\mathrm{t} \mathrm{a}_\mathrm{Ko}^2 (e^{-E_{Ko} t} + e^{-E_{Ko}(T-t)})$$

$$C_D(t) = \sum_n^{\text{nexp}} \mathrm{a}_\mathrm{D}^2 (e^{-E_D t} + e^{-E_D (T-t)}) + \sum_n^{\text{nexpo}} \mathrm{s}(-1)^\mathrm{t} \mathrm{a}_\mathrm{Do}^2 (e^{-E_{Do} t} + e^{-E_{Do}(T-t)})$$

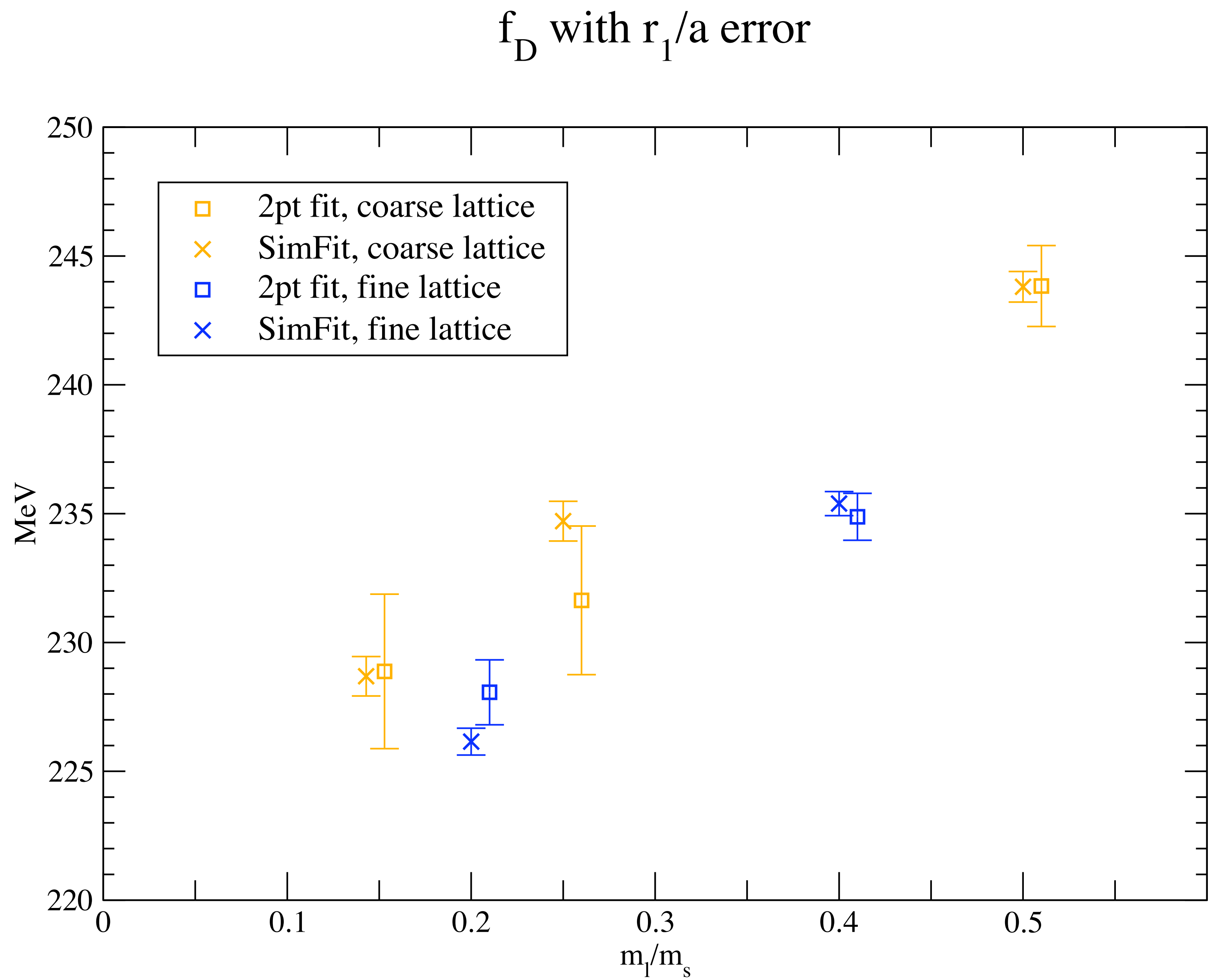
$$\begin{aligned} C_3(t) = {} & A_{nn} e^{-E_K t} e^{-E_D (T-t)} + s(-1)^{T-t} A_{no} e^{-E_K t} e^{-E_{Do}(T-t)} \\ & + s(-1)^t A_{on} e^{-E_{Ko} t} e^{-E_D (T-t)} + (-1)^t (-1)^{T-t} A_{oo} e^{-E_{Ko} t} e^{-E_{Do}(T-t)} \\ & + \text{backward in time terms} \end{aligned}$$

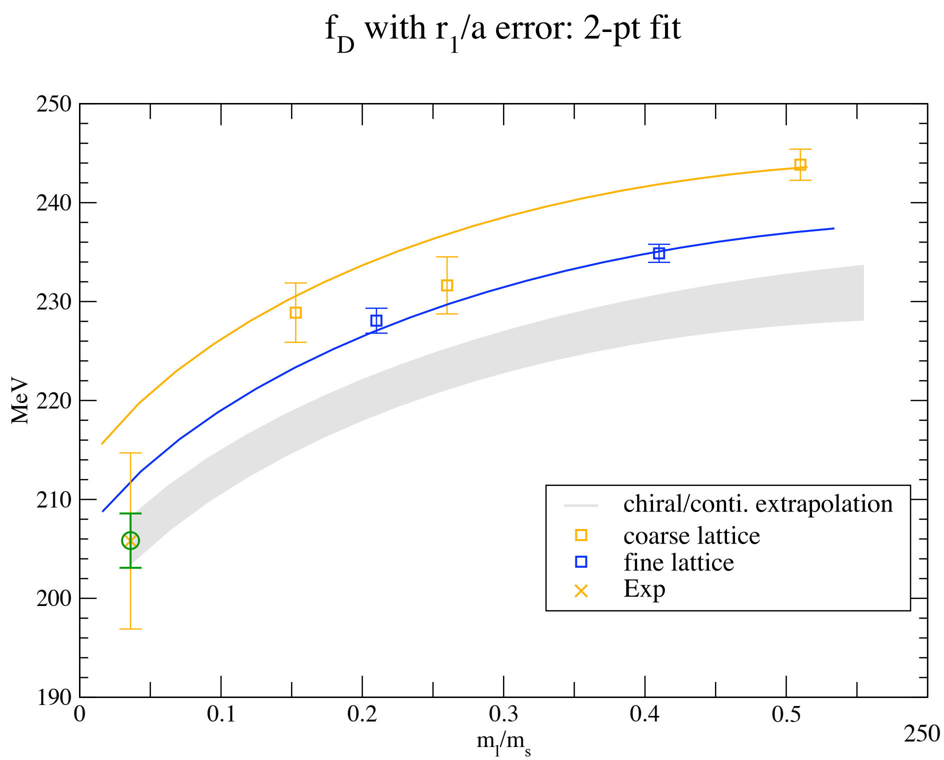
- Advantages:
  - Effect of putting more information
  - Increase the number of data points without increase the number of parameters
  - Even-Odd combination of T
  - Cheap price

• Simultaneous multi-T fit: fit results (f_0)



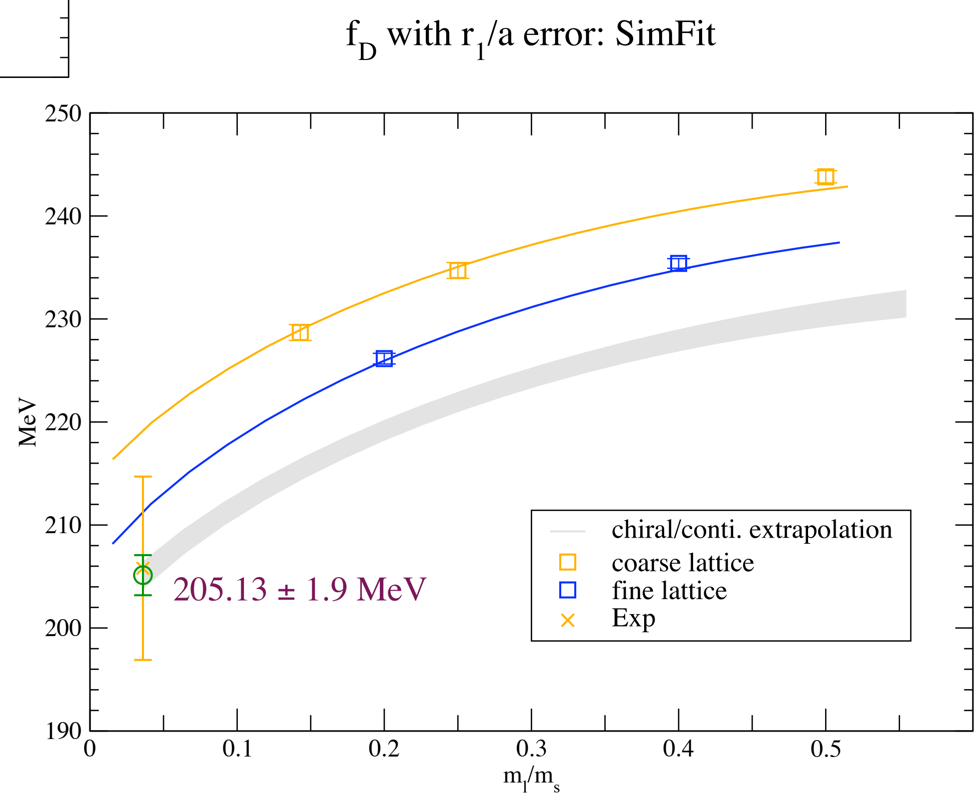
• Simultaneous multi-T fit: fit results (f_D)





Two-point fit

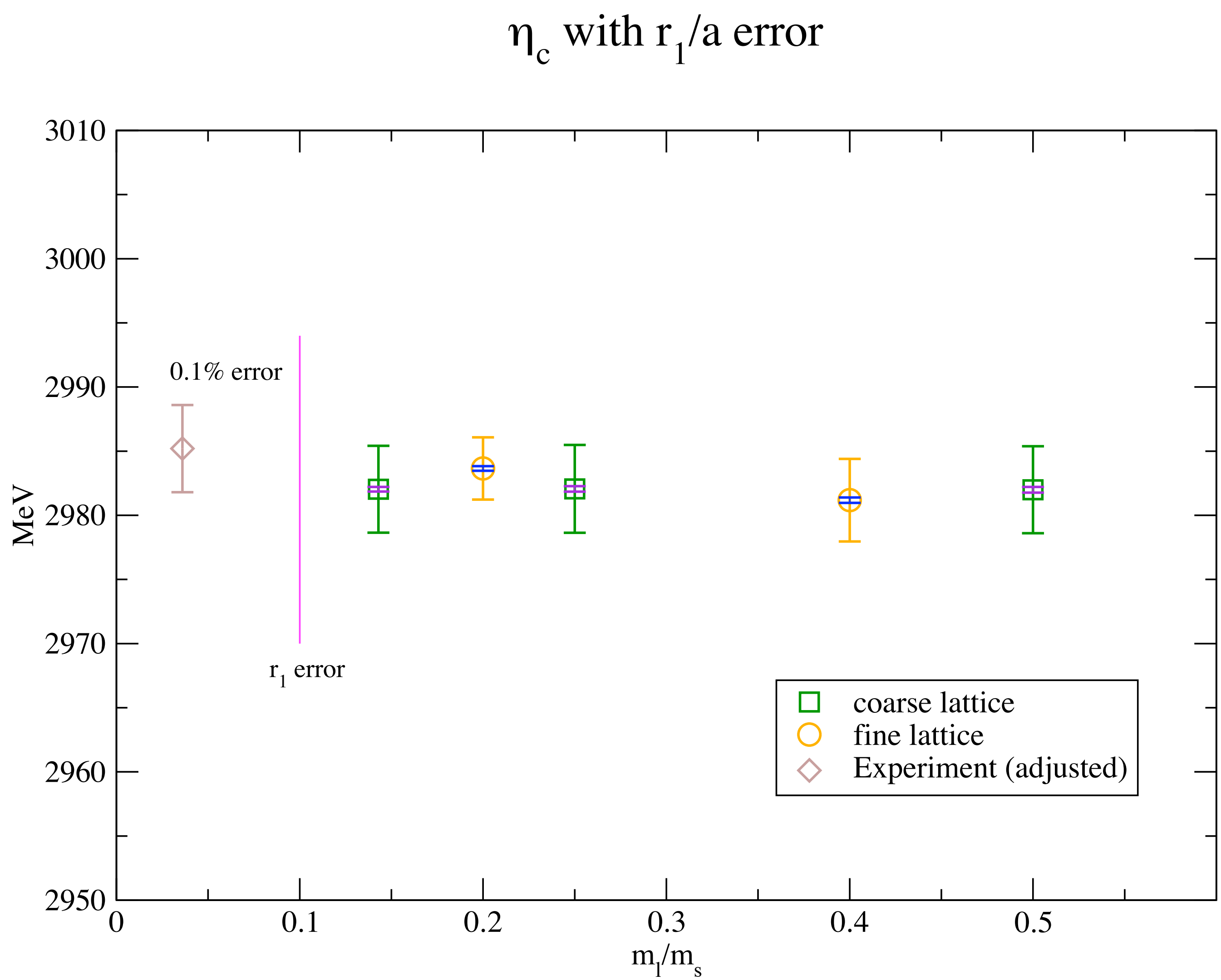
$\chi^2/dof = 0.99$



SimFit

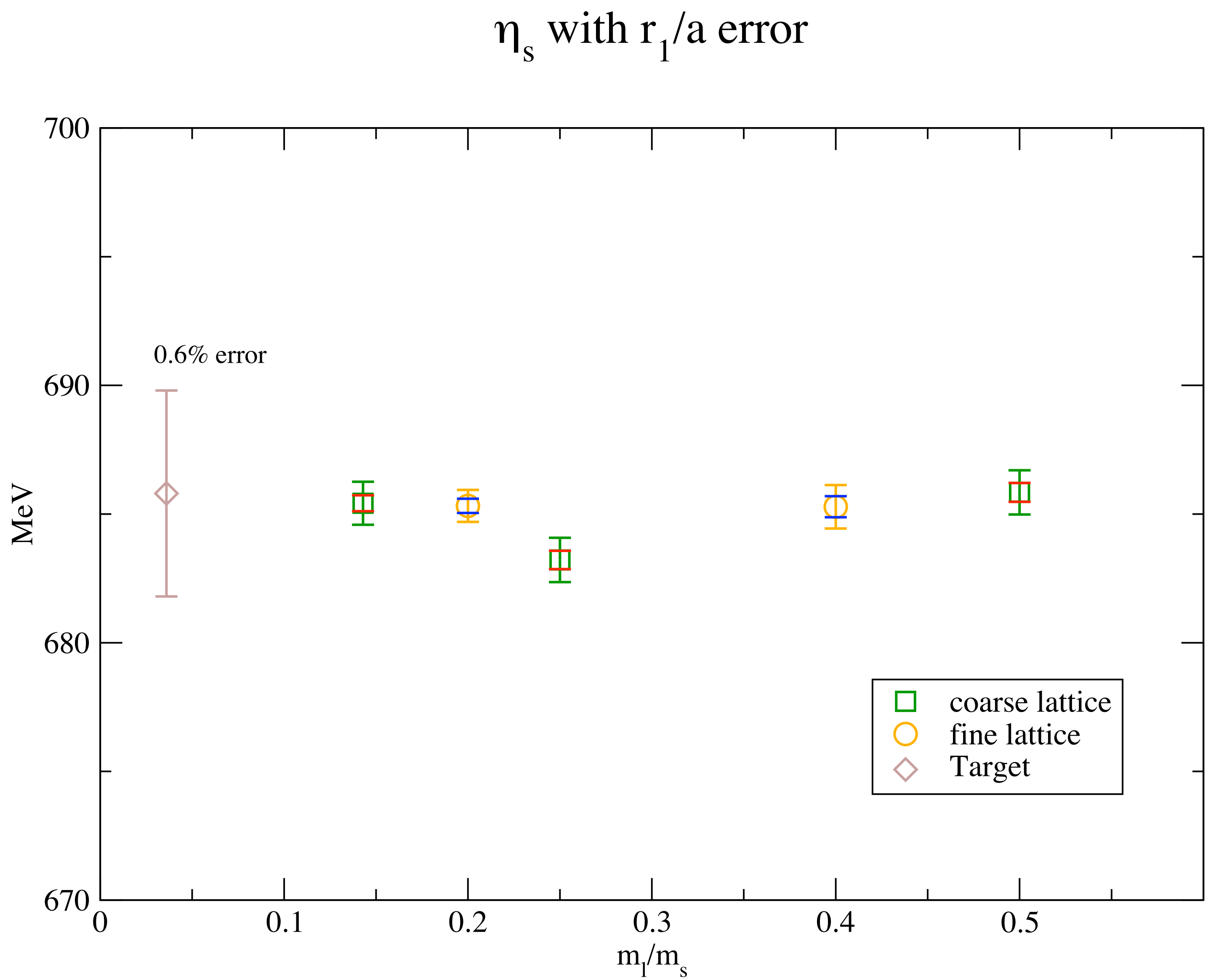
$\chi^2/dof = 0.88$

• Tuning quark mass: m_c and m_s



r_1/a error ~ 0.1% r_1 error ~ 0.8%

- For charm, tree level ε is used. Using η_c for m_c, and η_s for m_s



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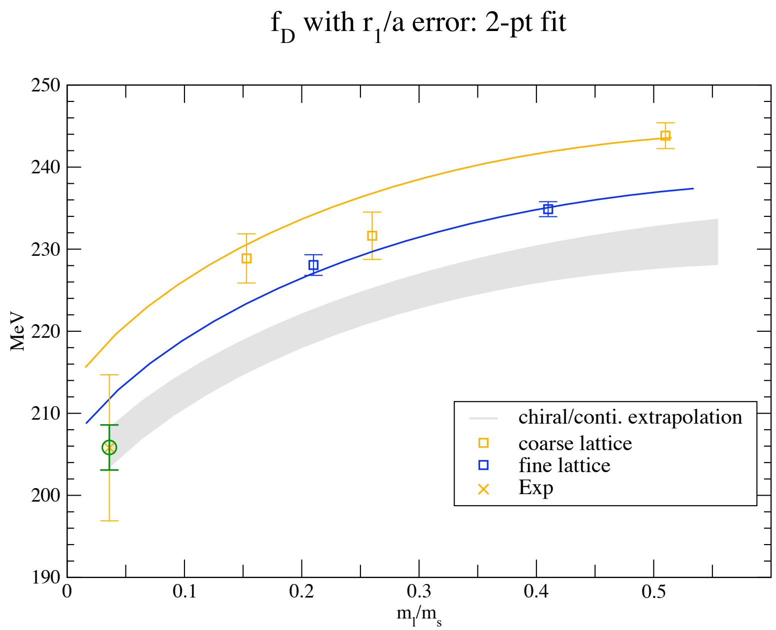
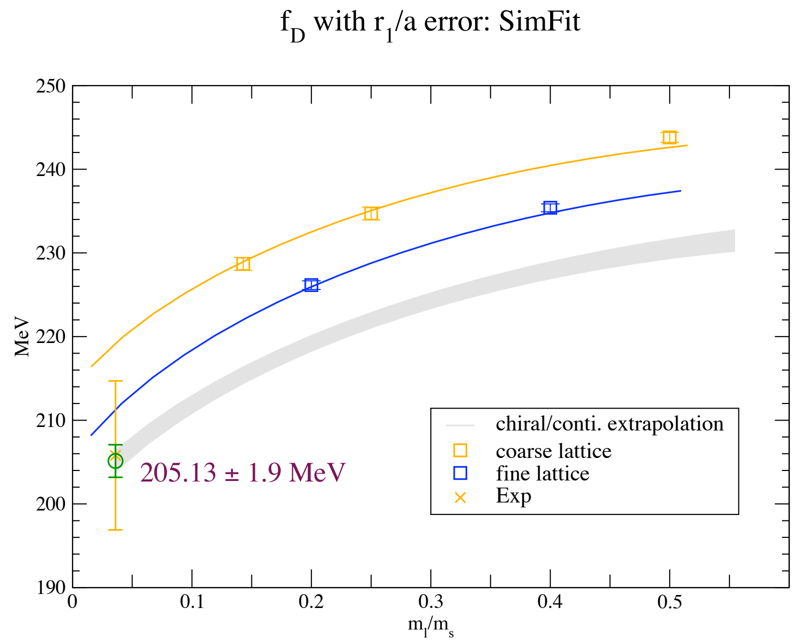
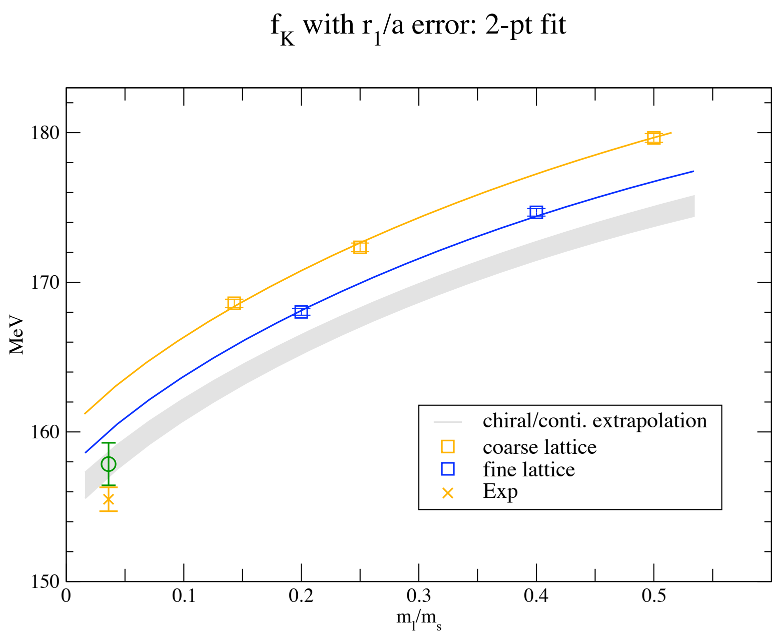
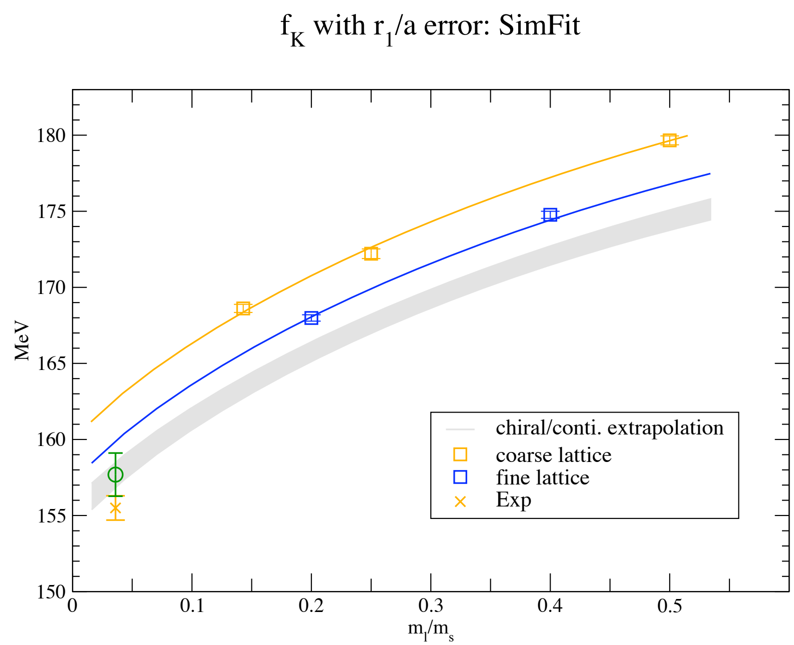
• Decay constants - f_D and f_K

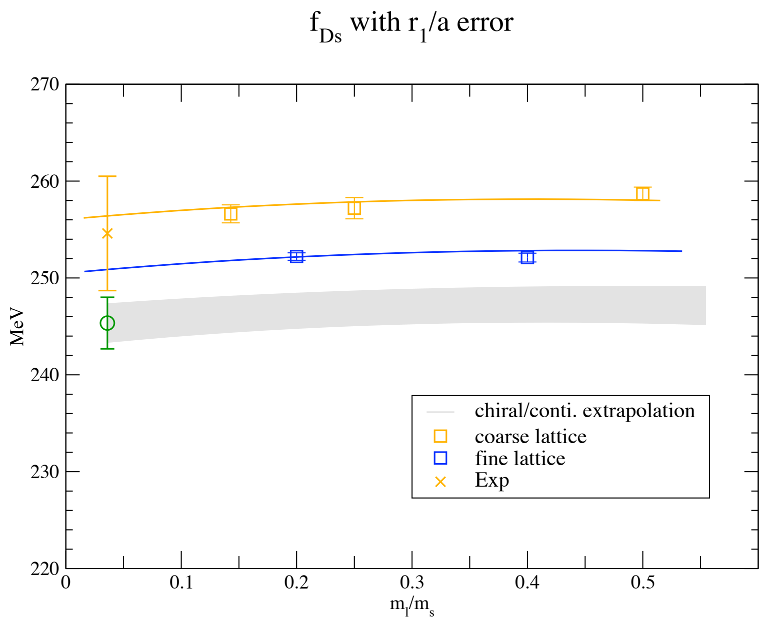


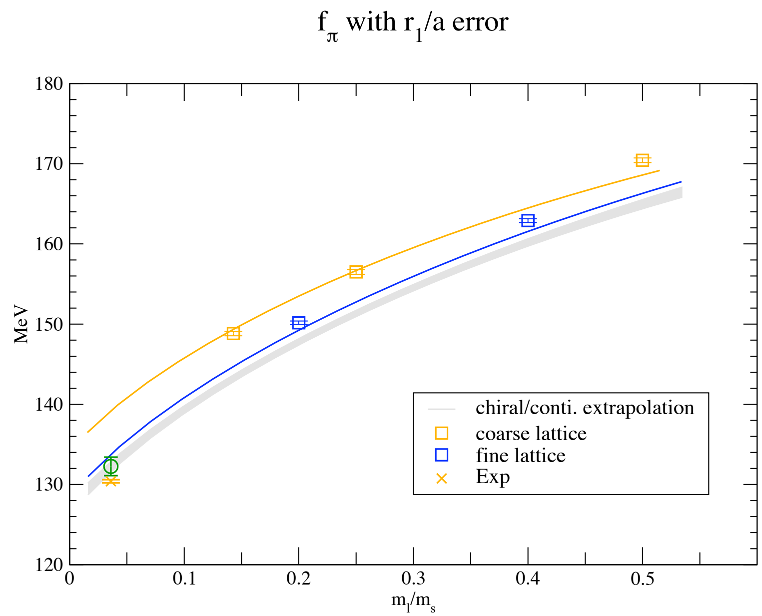


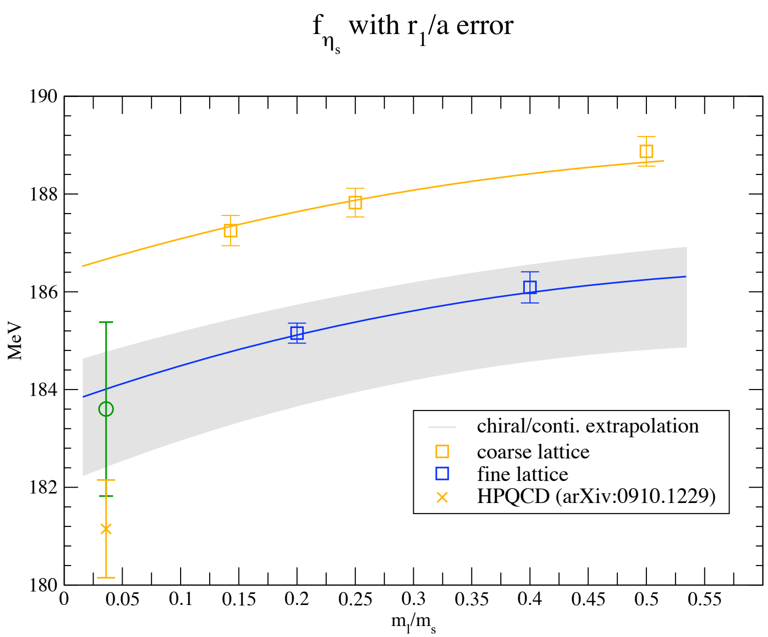




• Decay constants - f_{Ds}, f_{π}, $f_{\eta s}$







- All parameters and tuning are reasonable!

• Chiral perturbation theory

- We calculate f_0 directly, but we only have ChiPT with $f_{||}$ and $f_{\perp}$.

$$f_0(q^2) = \frac{\sqrt{2m_D}}{m_D^2 - m_\pi^2}[(m_D - m_\pi)f_{||}(E_\pi) + (E_\pi^2 - m_\pi^2)f_\perp(E_\pi)]$$

$$f_{||} = \frac{\kappa}{f}\left[1 + \delta f_{||} + c_l^{||}m_l + c_{s'}^{||}m_{s'} + c_{sea}^{||}(2m_u + m_s) + c_1^{||}E_K + c_2^{||}E_K^2\right](1 + c_0 a^2)$$

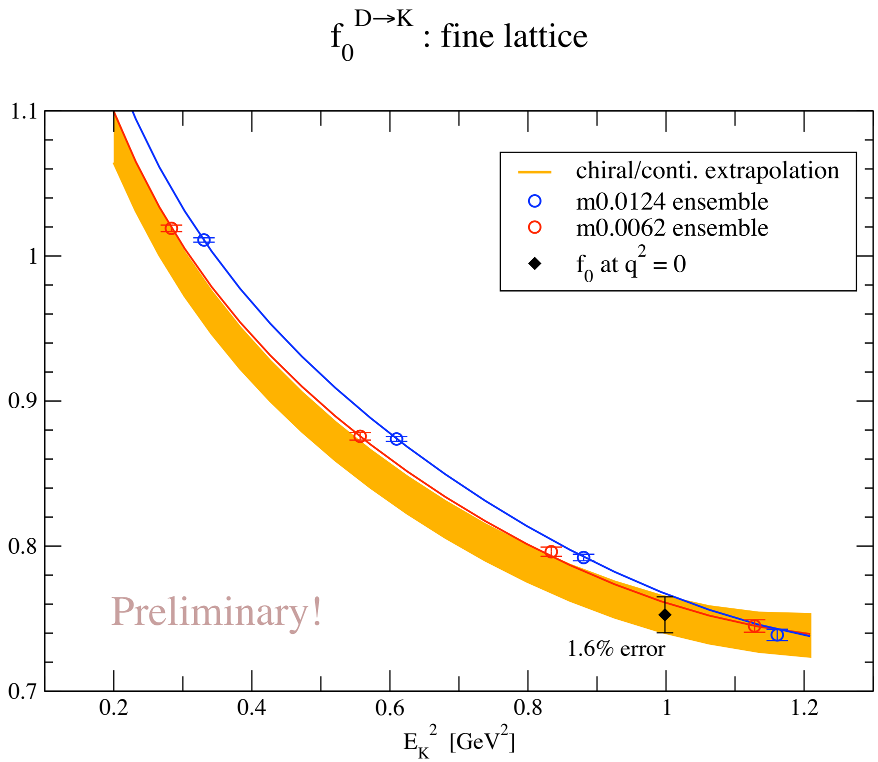
$$f_\perp = \frac{\kappa}{f}\frac{g_\pi}{E_K + \Delta^* + D}\left[1 + \delta f_\perp + c_l^\perp m_l + (c_l^{||} + c_l^\perp - c_{s'}^{||})m_{s'} + c_{sea}^\perp(2m_u + m_s) + c_1^\perp E_K + c_2^\perp E_K^2\right](1 + c_0' a^2)$$

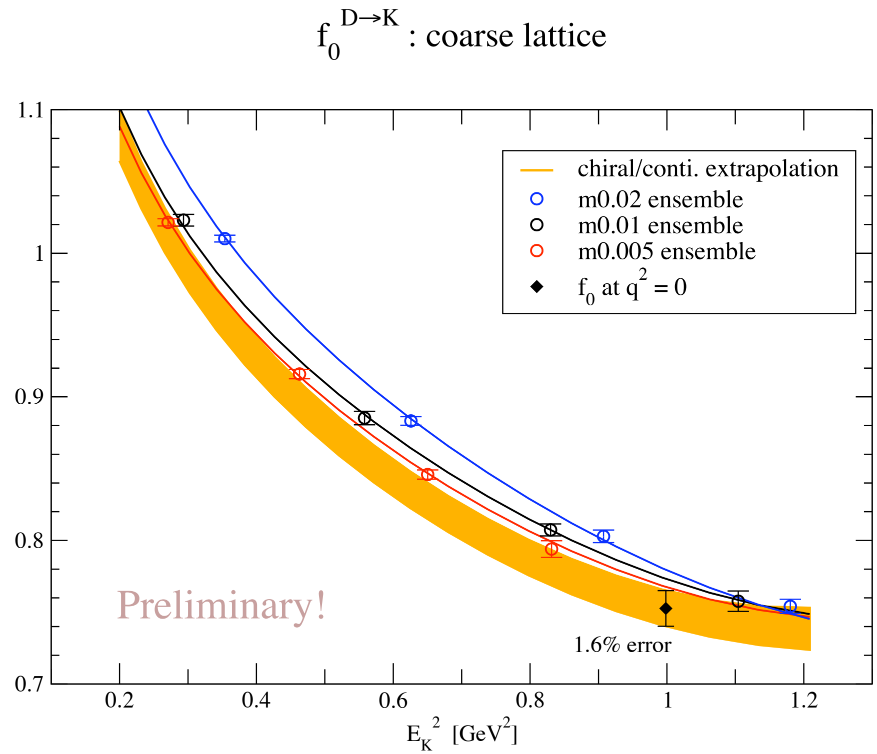
- Aubin and Bernard, PRD 76 (2007) 014002

- Using meson masses instead of the quark masses

$$(2m_u + m_s) \propto \frac{1}{2}m_\pi^2 + m_K^2 \quad \text{or,} \quad m_\pi^2 + \frac{1}{2}m_{\eta_s}^2$$

• Chiral / continuum extrapolation: results





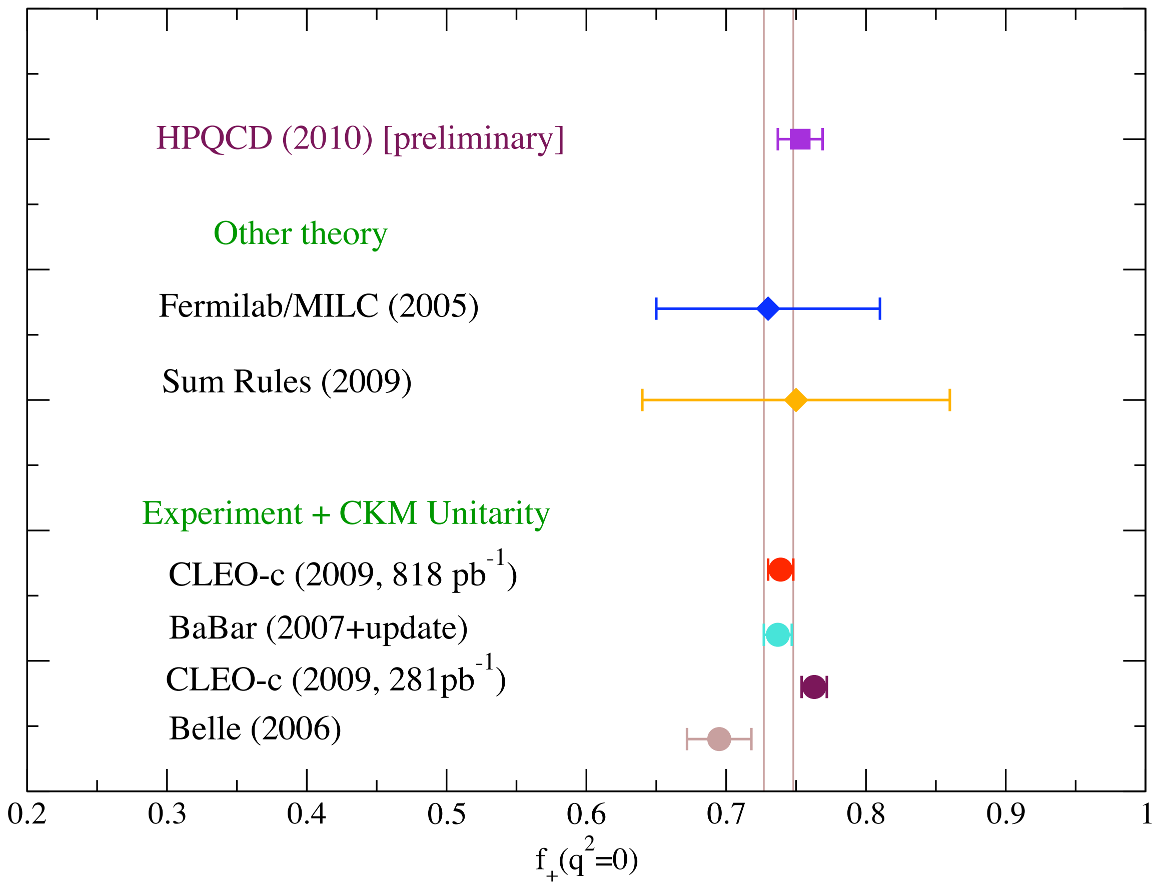
- χ^2/dof = 0.62

- Included the error involved with the determination of $q^2 = 0$

• Results, errors, and discussions

• $f_+(0) = 0.753\ (12)(10)$ total 2.1% error

stat. syst.

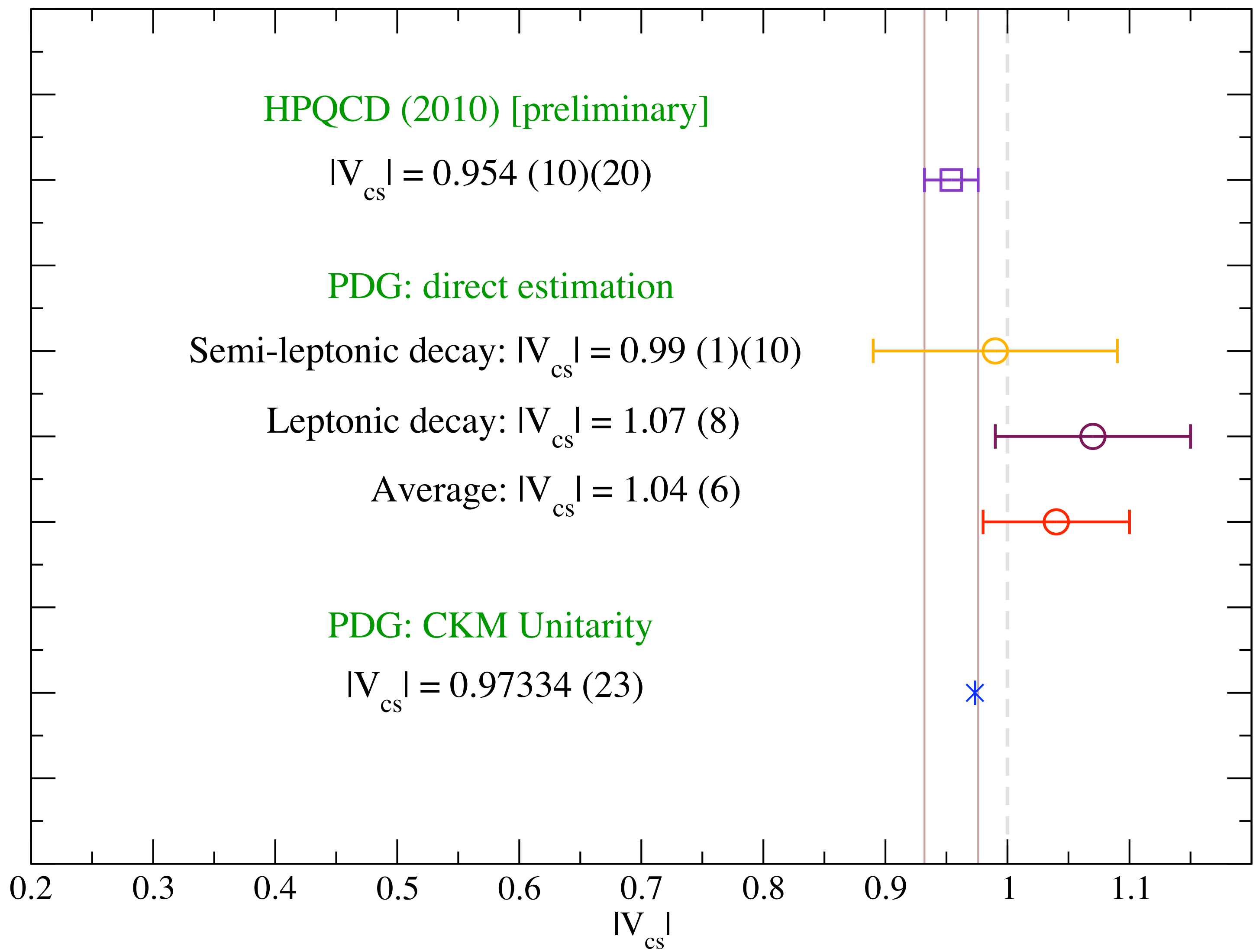


Preliminary error budget

Statistics+fit (others)	1.6%
Chiral extrapolation	1.2%
Finite V	0.5%
Charm sea quark	0.2%
Quark mass tuning	0.2%

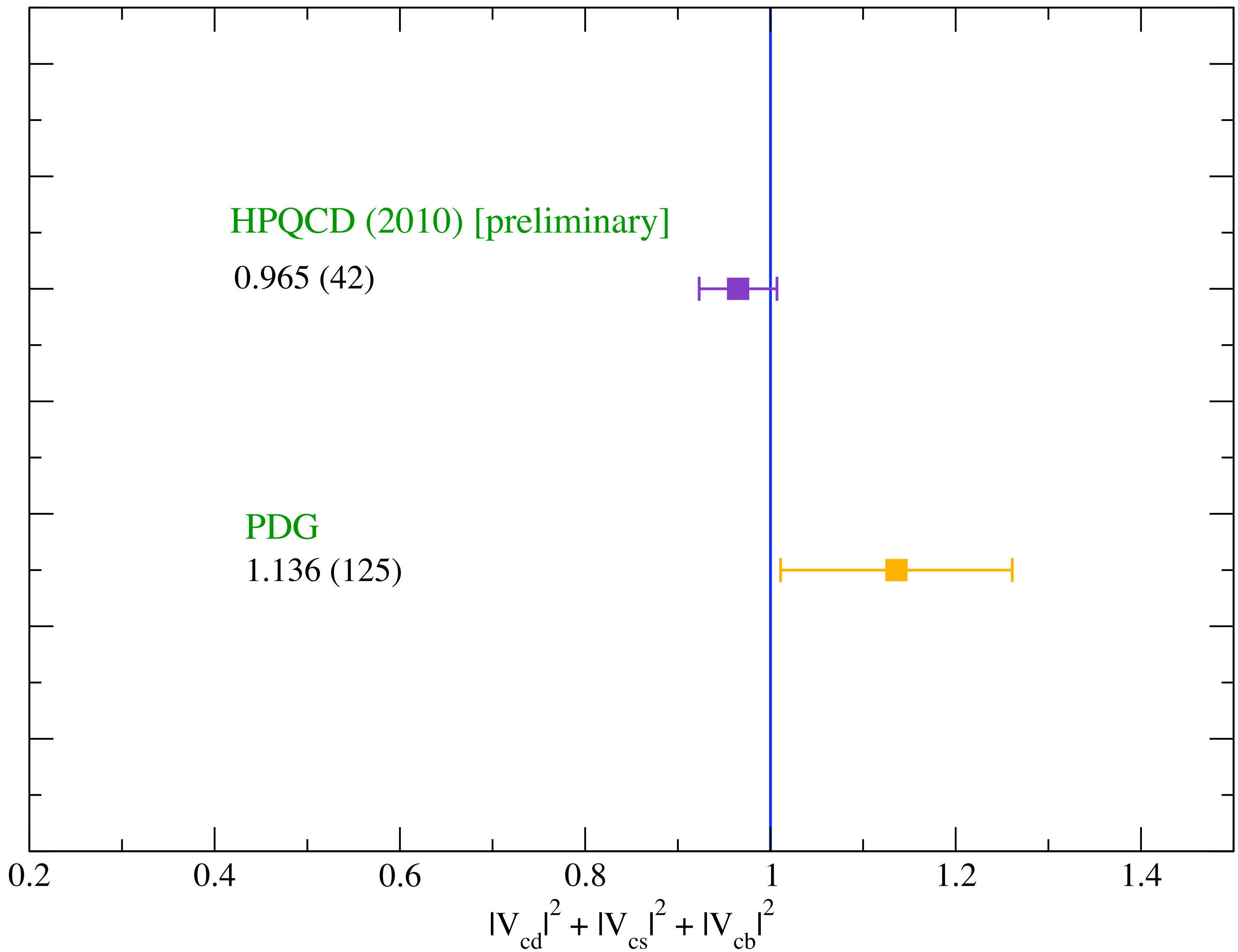
• Results, errors, and discussions

• $|V_{cs}|$ = 0.954 (10)(20) [(exp)(lattice) errors]

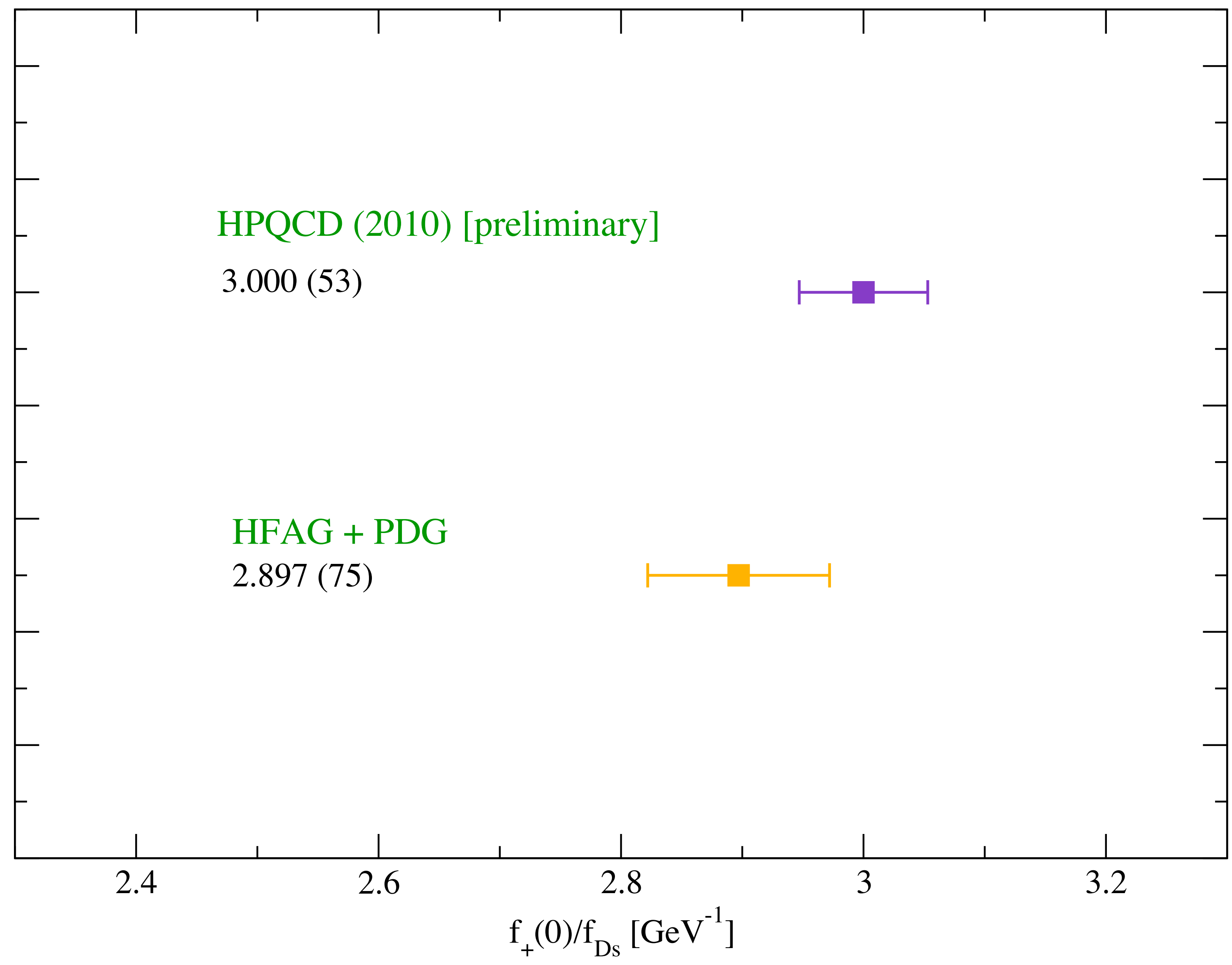


- Unitarity check

$|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 = 0.965\ (42)$



- $f_+(0)/f_{Ds}$: independent of $|V_{cs}|$



• Summary and future plan

- Using HISQ action for the light and charm quark is successful to simulate D to K semi-leptonic form factors.
- We obtain a preliminary result, $f_+(0) = 0.753\ (12)(10)$, which is ~ 2% total error, and a factor of five times smaller error than the previous calculations.
- We also estimate $|V_{cs}| = 0.954\ (10)(20)$.

- Future plan
  - Understand systematic errors more
  - D to π semi-leptonic decay using the same method
  - D semi-leptonic decay using the fully non-perturbative matching method with the vector current.
  - B semi-leptonic decay

Backup slides

• Chiral / continuum extrapolation: Bayesian fit

- A method to include systematic errors from known (input) parameters, such as
  - ChiPT params: g_π, B_0, f, Δ^*
  - Meson masses: $m_D, E_K, m_{\eta_s}, m_\pi$
  - Quark masses: $m_u, m_s, m_l, m_{s'}$
  - Lattice scaling: r_1
- by using "pseudo-priors"

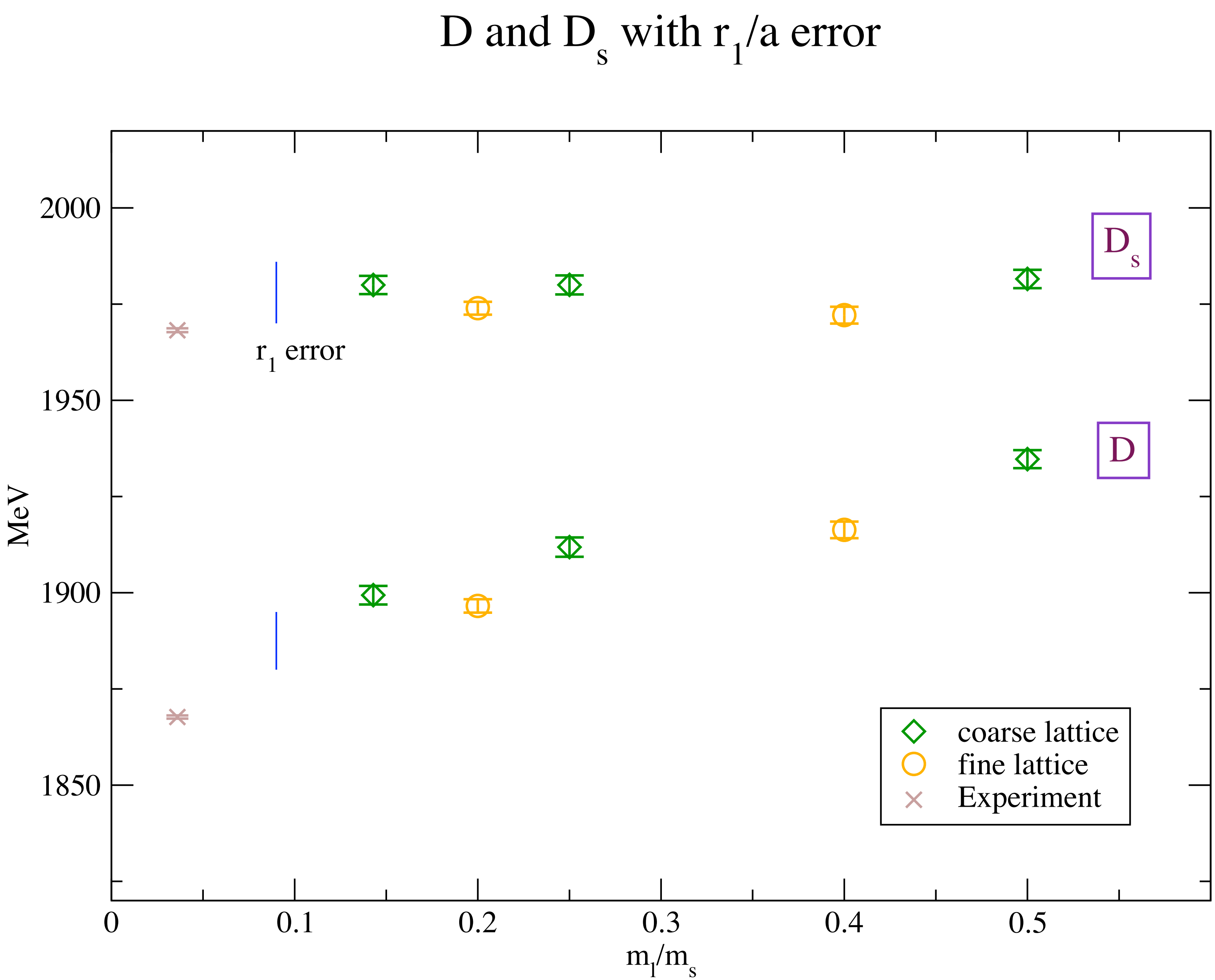
C. Davies et al, arXiv:0807.1687

$$\chi^2 \equiv \sum_{i=1}^{20} \frac{(f_0^i - f_0(c_n, g_\pi, B_0, \cdots))^2}{\sigma_{f_o}^{i\ 2}} + \sum_{n=1}^{11} \frac{(c_n - \bar{c}_n)^2}{\sigma_n^2} + \sum_{j=1}^{74} \frac{(p_j - \bar{p}_j)^2}{\sigma_j^2}$$

Classical χ^2 | Priors | Pseudo-priors

$$\sigma_{tot}^2 = \sum c_{f_0} \sigma_{f_0}^2 + \sum c_{c_n} \sigma_n^2 + \sum c_{p_1} \sigma_{p_1}^2 + \cdots + \sum c_{p_4} \sigma_{p_4}^2$$

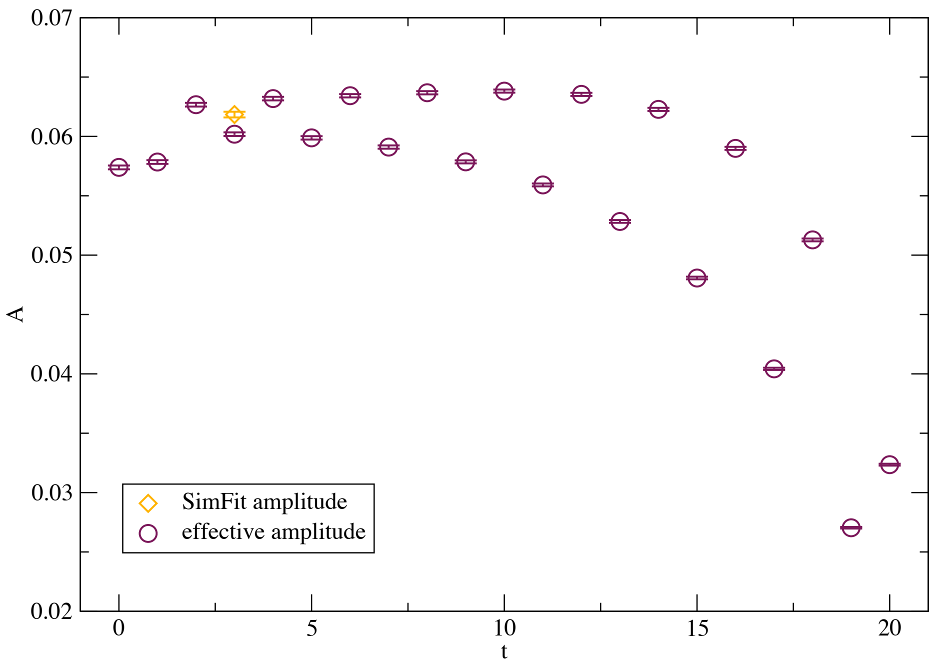
- Tuning and discretization error



• Simultaneous multi-T fit: effective amplitude

$$A = C_3(t)\, e^{E_K t} e^{E_D (T-t)}$$

Effective amplitude for p = (0,0,0) and T = 20



Effective amplitude for p = (1,1,1) and T=20