Overlap Valence Quarks on a Twisted Mass Sea

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The XXVIII International Symposium on Lattice Field Theory
Villasimius, Sardinia
14 June 2010

Motivation

Motivation for the mixed action setup of overlap valence and MTM sea quarks:

- profit from good chiral properties of overlap fermions, but
- avoid the high computational cost of generating dynamical overlap configurations.

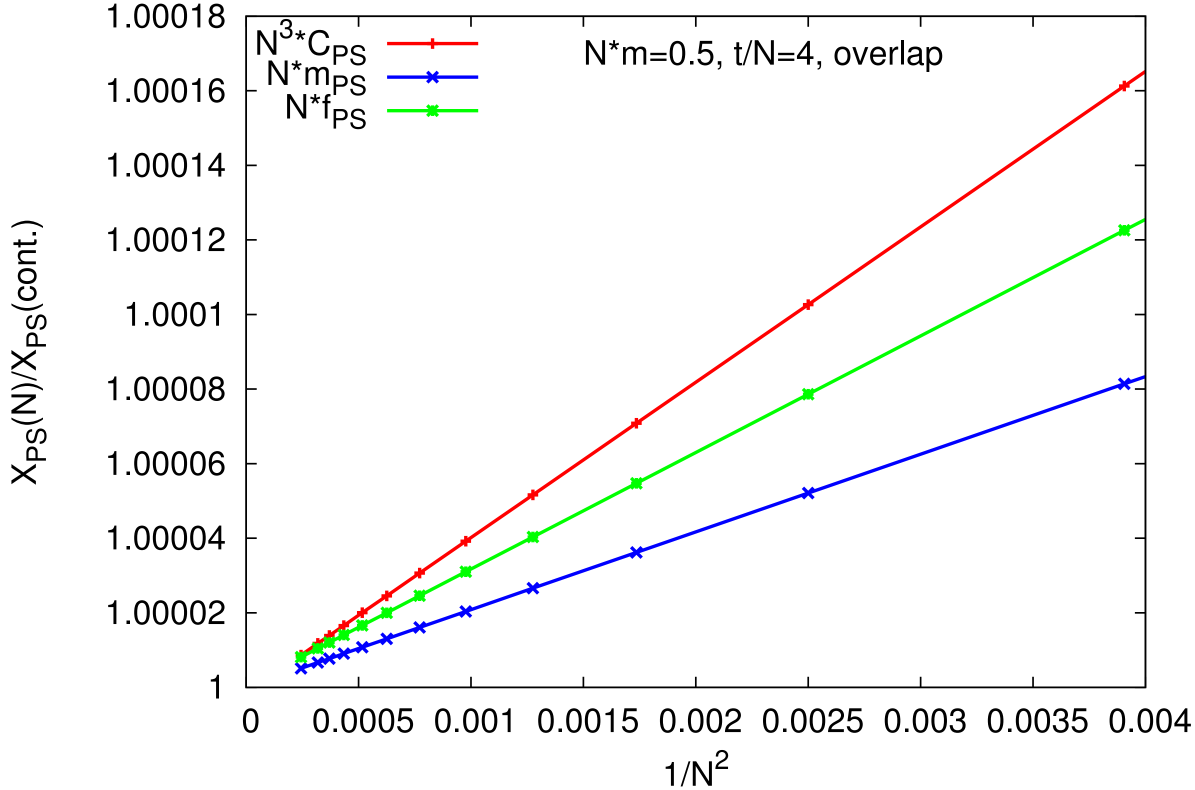
This work:

- Continuum limit scaling test of overlap fermions (free & interacting case)
- The role of chiral zero modes of the overlap operator
  (When) Are we safe against the effects of zero modes?

Tree-level test – $O(a)$-improvement

K.C., J. Gonzalez Lopez, K. Jansen, A. Kujawa, A. Shindler

Nucl.Phys. B 800, 94-108 (2008)



N plays the role of $1/a$ and introduces the scaling – thus $1/N^2 \rightarrow a^2$.

Figure 1: The relative cutoff effects of the pseudoscalar correlator at a fixed physical distance, pseudoscalar mass and decay constant.

Overlap in the interacting case

We want to study the scaling behaviour also in the interacting case.

Main disadvantage of overlap fermions – the cost of computation with respect to e.g. Twisted Mass (TM) fermions:

$$D_{TM} = D_{Wilson}\mathbb{1}_f + i\mu\gamma_5\tau^3,$$

where: μ – twisted mass, $\mathbb{1}_f$ and τ^3 act in flavour space.

The number of applications of the Wilson kernel operator during inversion (in thousands) (Chiarappa et al., 2006):

V, m_π	TM	overlap	relative cost
12^4, 720	0.60	18.1	30
555	0.77	27.9	36
390	0.94	52.9	56
230	1.05	96.7	92
16^4, 720	0.65	22.3	34
555	0.96	34.5	36
390	1.32	66.0	50
230	1.69	198.5	118

Relative cost – 30-120 times more than for TM fermions

Overlap in the interacting case – scaling test

We would like to test the scaling behaviour towards the continuum limit of overlap fermions in fixed volume.

- e.g. for $L = 2.4$ fm:
  $24^3 \times 48$, $a = 0.1$ fm $\longrightarrow 48^3 \times 96$, $a = 0.05$ fm   OUT-OF-REACH

- hence, we have to go to smaller volume – $L \approx 1.3$ fm
  - $16^3 \times 32$, $a \approx 0.079$ fm ($\beta = 3.9$)
  - $20^3 \times 40$, $a \approx 0.063$ fm ($\beta = 4.05$)
  - $24^3 \times 48$, $a \approx 0.051$ fm ($\beta = 4.2$)

- sea fermions – $N_f = 2$ TM fermions at maximal twist – also $O(a)$-improved

- gauge configurations available from ETMC

Locality of the overlap operator

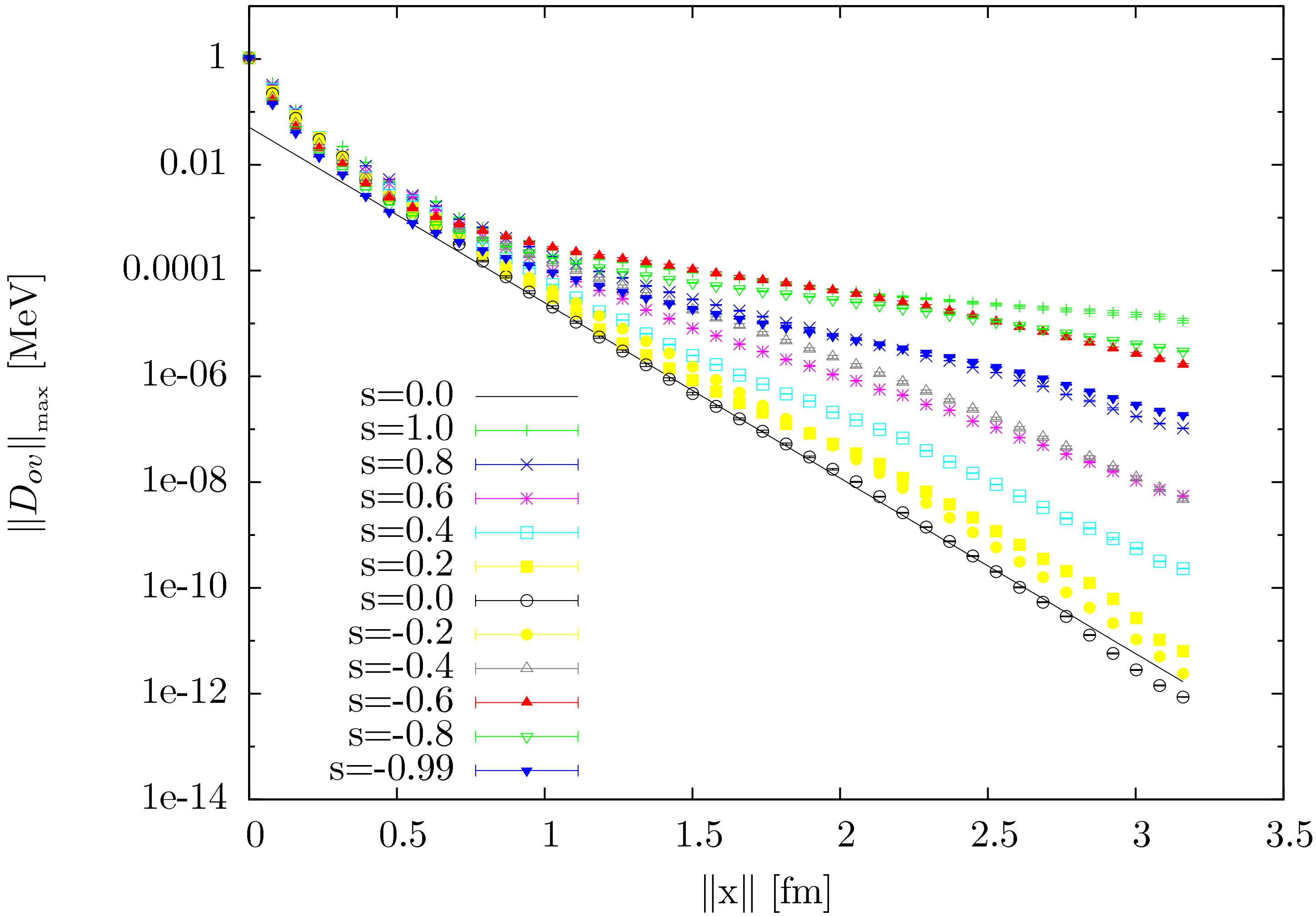


Figure 2: Exponential decay of the overlap Dirac operator for different values of the parameter s.

Locality of the overlap operator

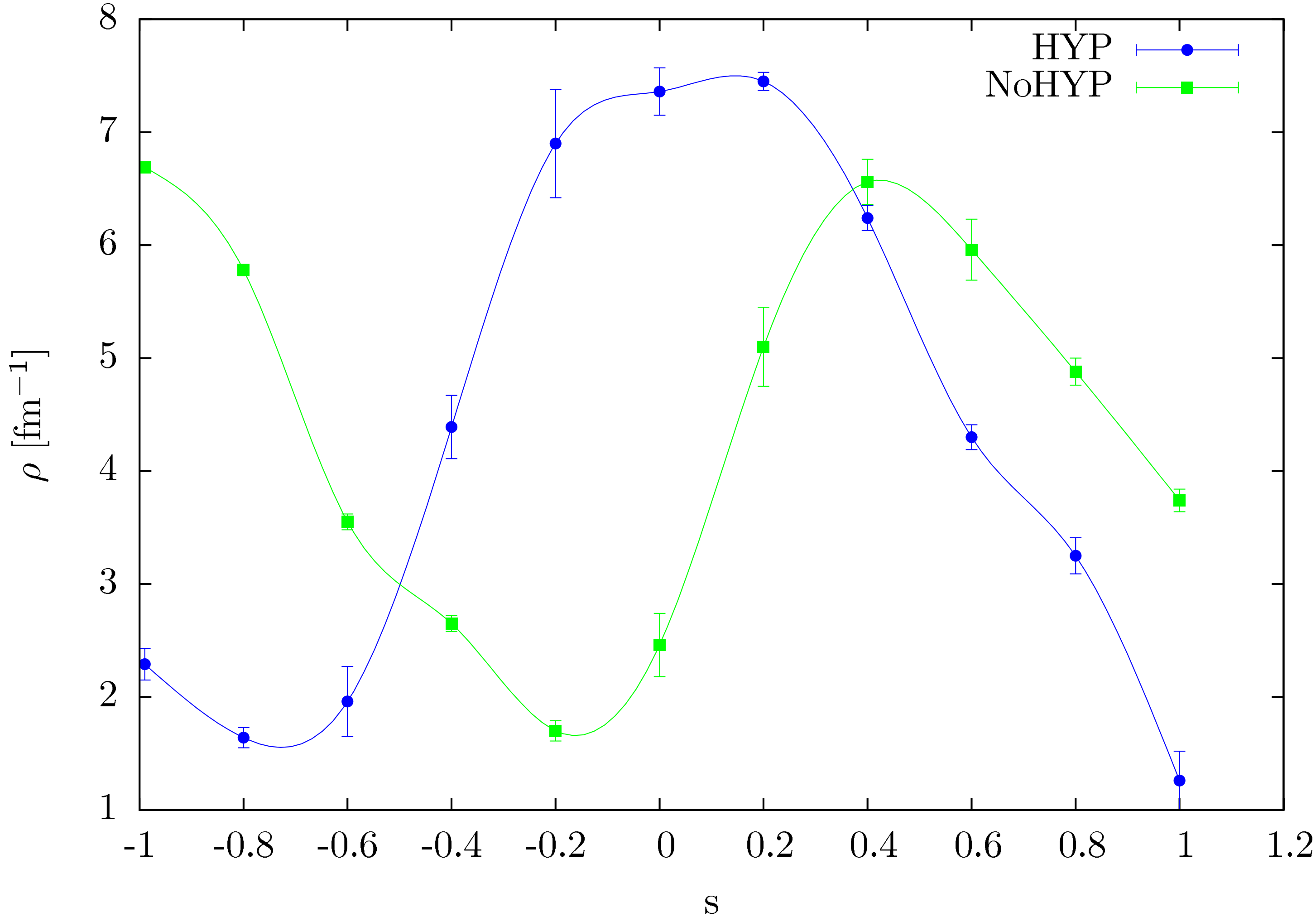


Figure 3: Decay rate of the overlap operator. The values of ρ extracted from the fit to the following formula: $||D_{overlap}||_{max}(d) = Ce^{-\rho d}$.

Locality of the overlap operator

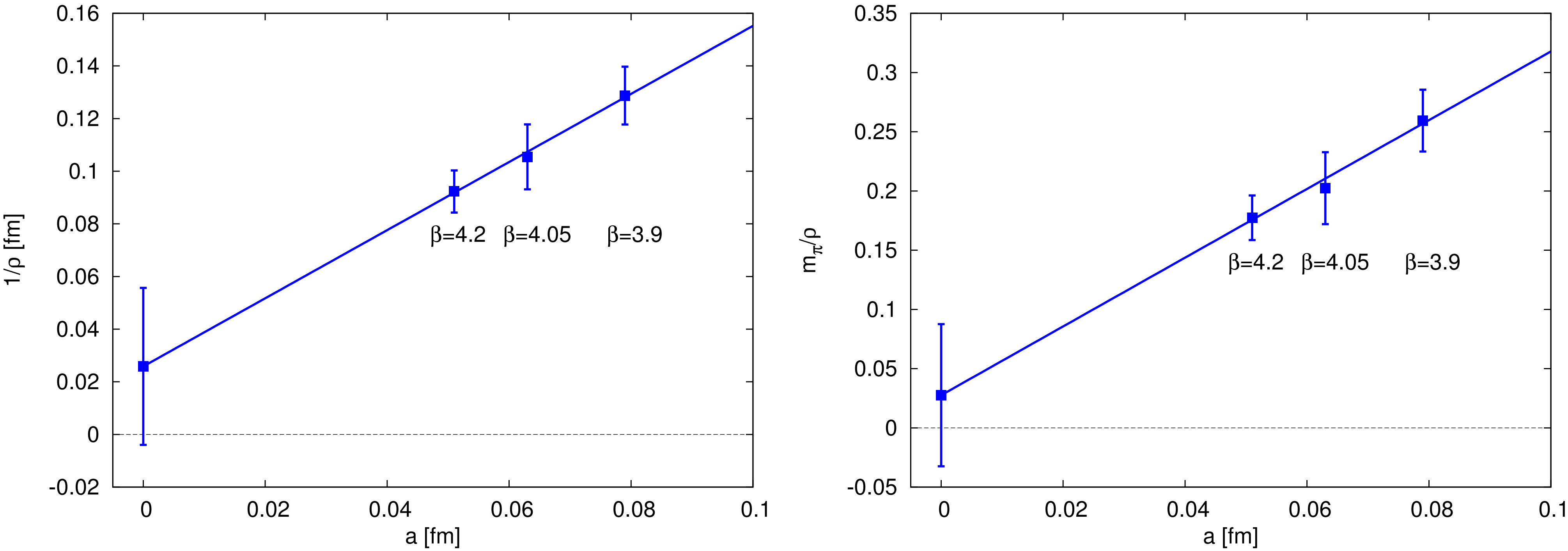


Figure 4: Continuum limit scaling of the inverse decay rate of the overlap operator $1/\rho$ and the ratio m_π/ρ.

Mixed action approach

- The mixed action approach has potential difficulties, originating from the fact that the fermionic determinant comes from an action which is different than the one of the observables and the spectra of D_{TM} and $D_{overlap}$ are different.

- We have many different competing effects: standard FSE, topological FSE, discretization effects (standard ones, isospin violation, zero modes, . . .).

- However, the continuum limit of this approach should be the same as of the unitary approach – continuum limit scaling test should check universality.

- One needs a matching of quark masses – the matching condition can be (for fixed lattice spacing and volume) e.g.:
  - $m_\pi^{VV} = m_\pi^{SS}$ or
  - $m_{q,ren}^{valence} = m_{q,ren}^{sea}$.

  At the matching point, other observables should be matched up to $\mathcal{O}(a^2)$ lattice artefacts.

Matching the pion mass – tree level

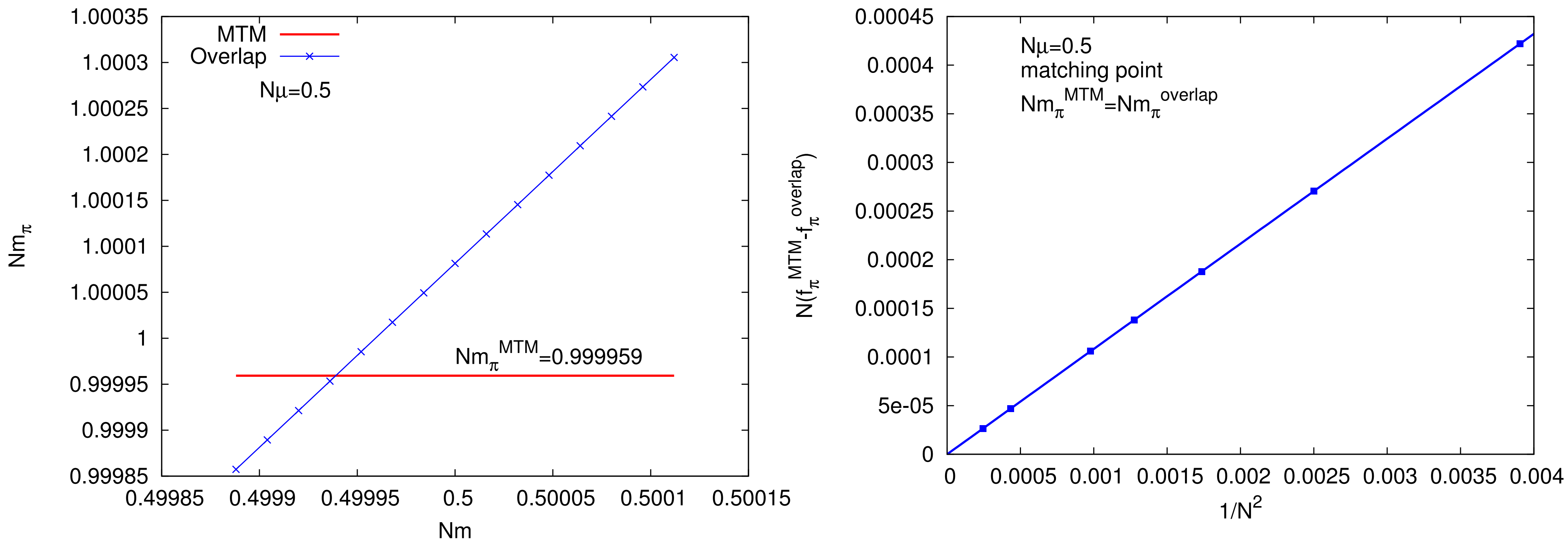


Figure 5: $\mathcal{O}(a^2)$ difference in the pion decay constant at the matching point.

Matching the pion mass – interacting case

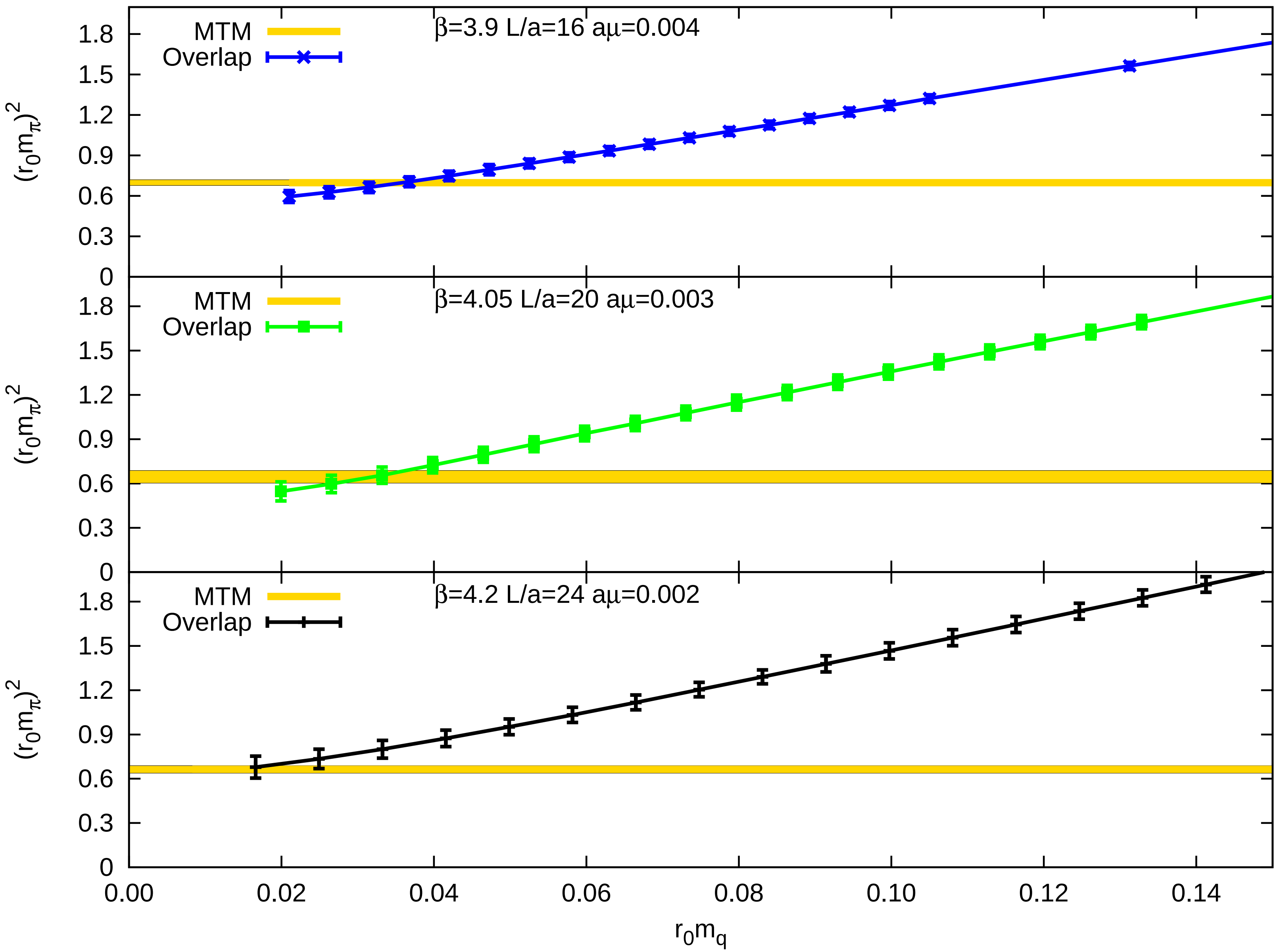


Figure 6: Matching the pion mass – fixed volume $L \approx 1.3$ fm.

Pion decay constant scaling

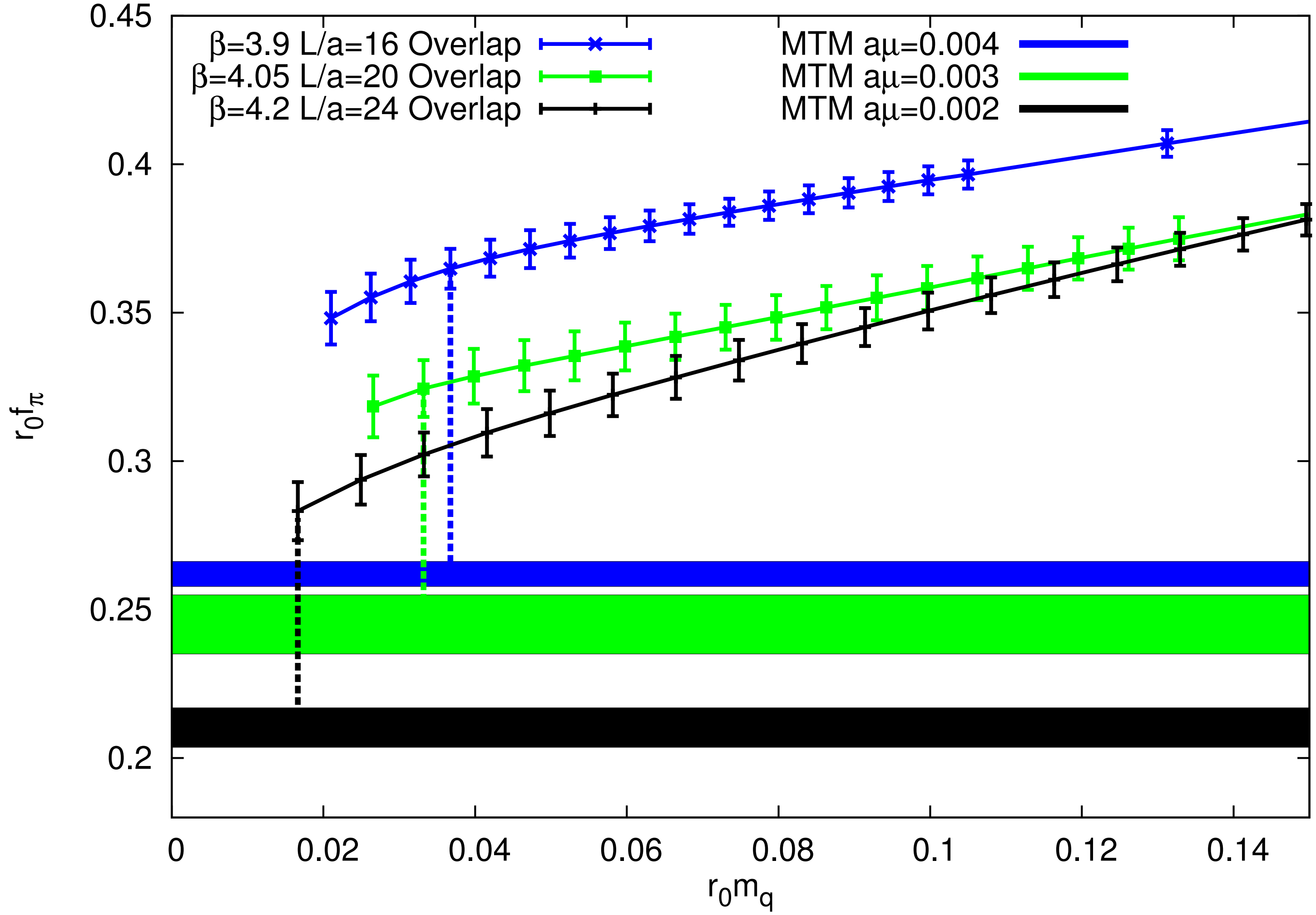


Figure 7: The quark mass dependence of the pion decay constant – fixed volume $L \approx 1.3$ fm.

Pion decay constant scaling

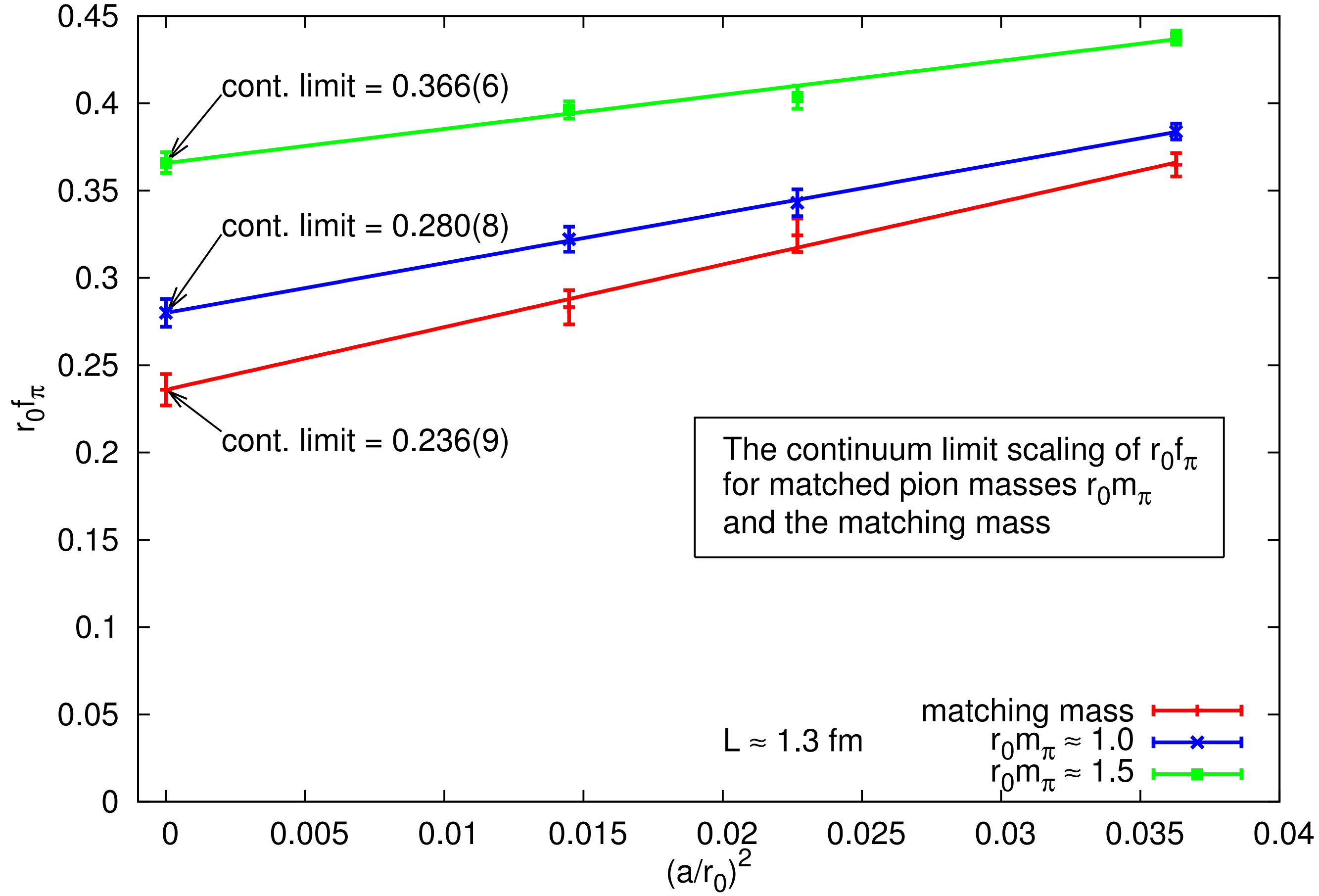


Figure 8: Pion decay constant scaling – fixed volume $L \approx 1.3$ fm.

Pion decay constant scaling

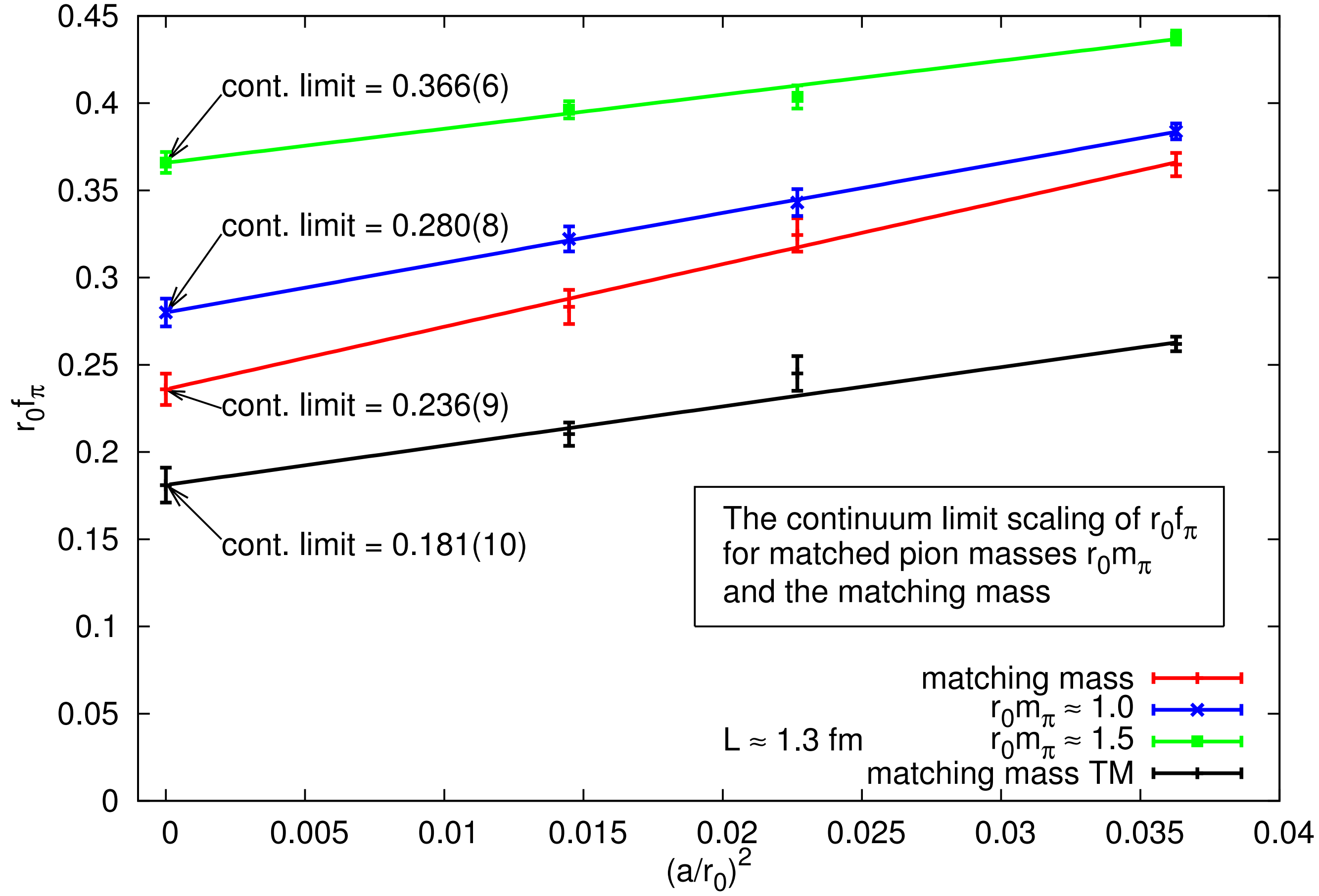


Figure 9: Pion decay constant scaling – fixed volume $L \approx 1.3$ fm.

Pion decay constant mismatch

At the matching point, we should have:

$$f_\pi^{overlap} = f_\pi^{TM} + O(a^2)$$

However, we observe quite large discrepancies between $f_\pi^{overlap}$ and f_π^{TM}.

They seem to go away very slowly when we approach the continuum limit at fixed volume $L \approx 1.3$ fm.

WHY IS IT SO???

Chiral zero modes of the overlap Dirac operator

- The overlap Dirac operator admits chiral zero modes at any value of the lattice spacing.

- The MTM Dirac operator needs sufficiently small lattice spacing to develop chiral zero modes (by far smaller than the values currently reached).

- Hence, in our mixed action setup the contribution of the zero modes of the overlap operator is not suppressed by the fermionic determinant.

- This can give large artefacts in some correlation functions, such as PP and SS.

- The leading contribution is proportional to $1/m^2$ and also it should vanish in the infinite volume limit, but can be very important in small volume.

- The zero modes contribute equally to the PP and SS correlators. Thus, their contribution vanishes in the difference $C_{PP-SS}(t) = C_{PP}(t) - C_{SS}(t)$. This was first proposed in *T. Blum et al. hep-lat/0007038*.

- However, the SS correlator may introduce enhanced unitarity violations.

Subtracting the scalar correlator

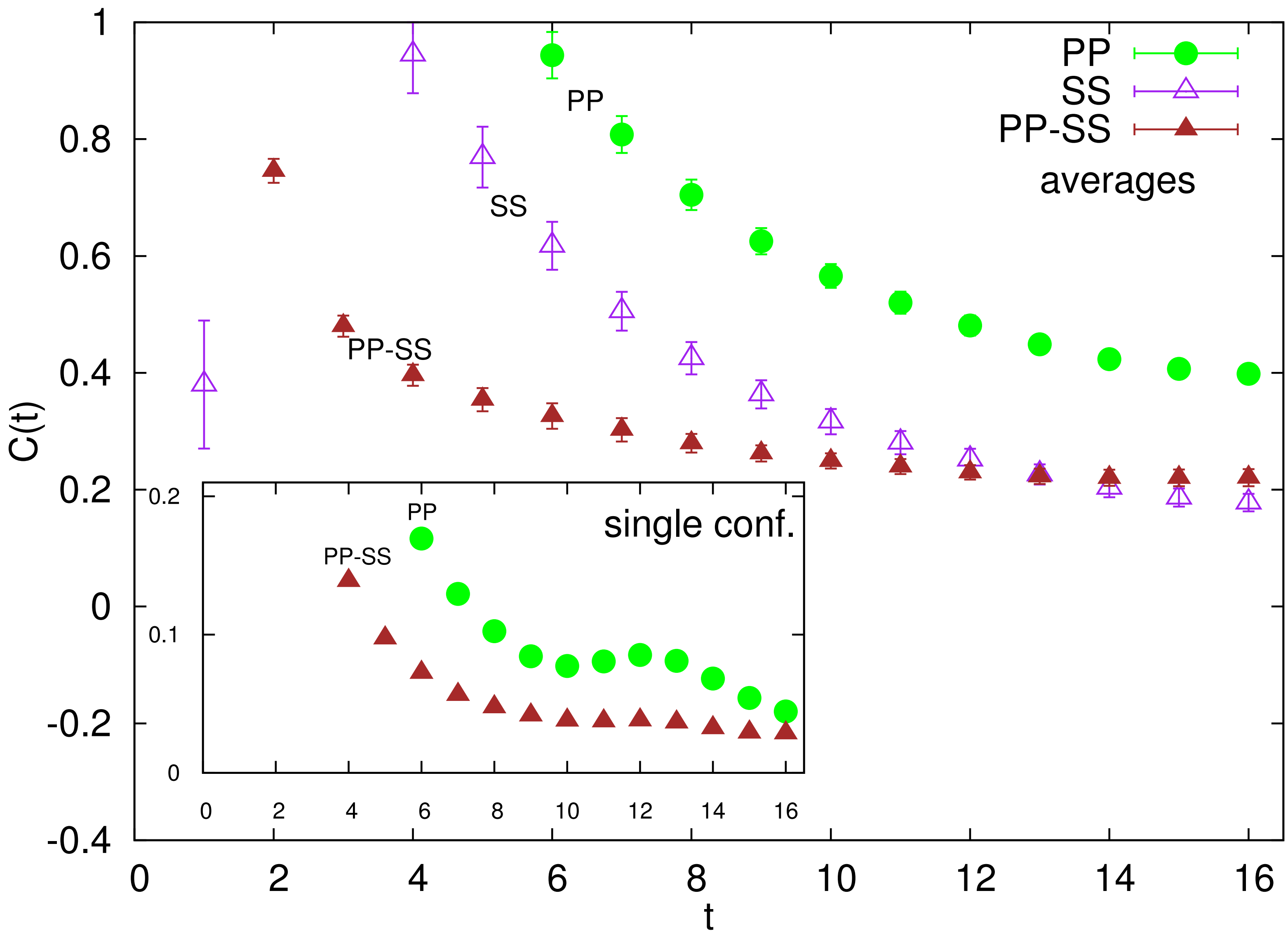


Figure 10: Effect of subtracting the SS correlator at the level of correlation functions.

Subtracting the scalar correlator

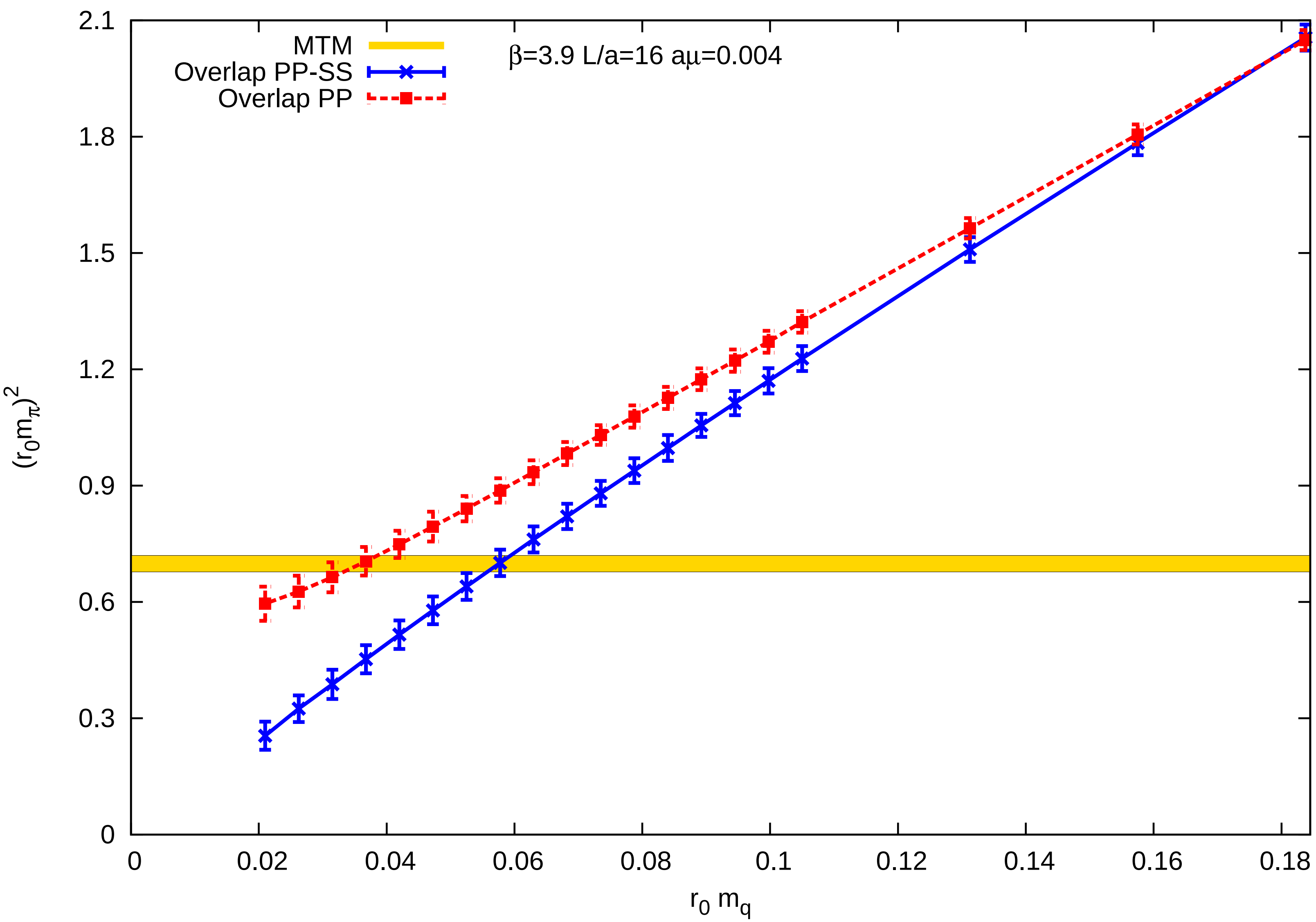


Figure 11: Effect of subtracting the SS correlator at the level of pion masses.

Matching the pion mass – interacting case, PP-SS

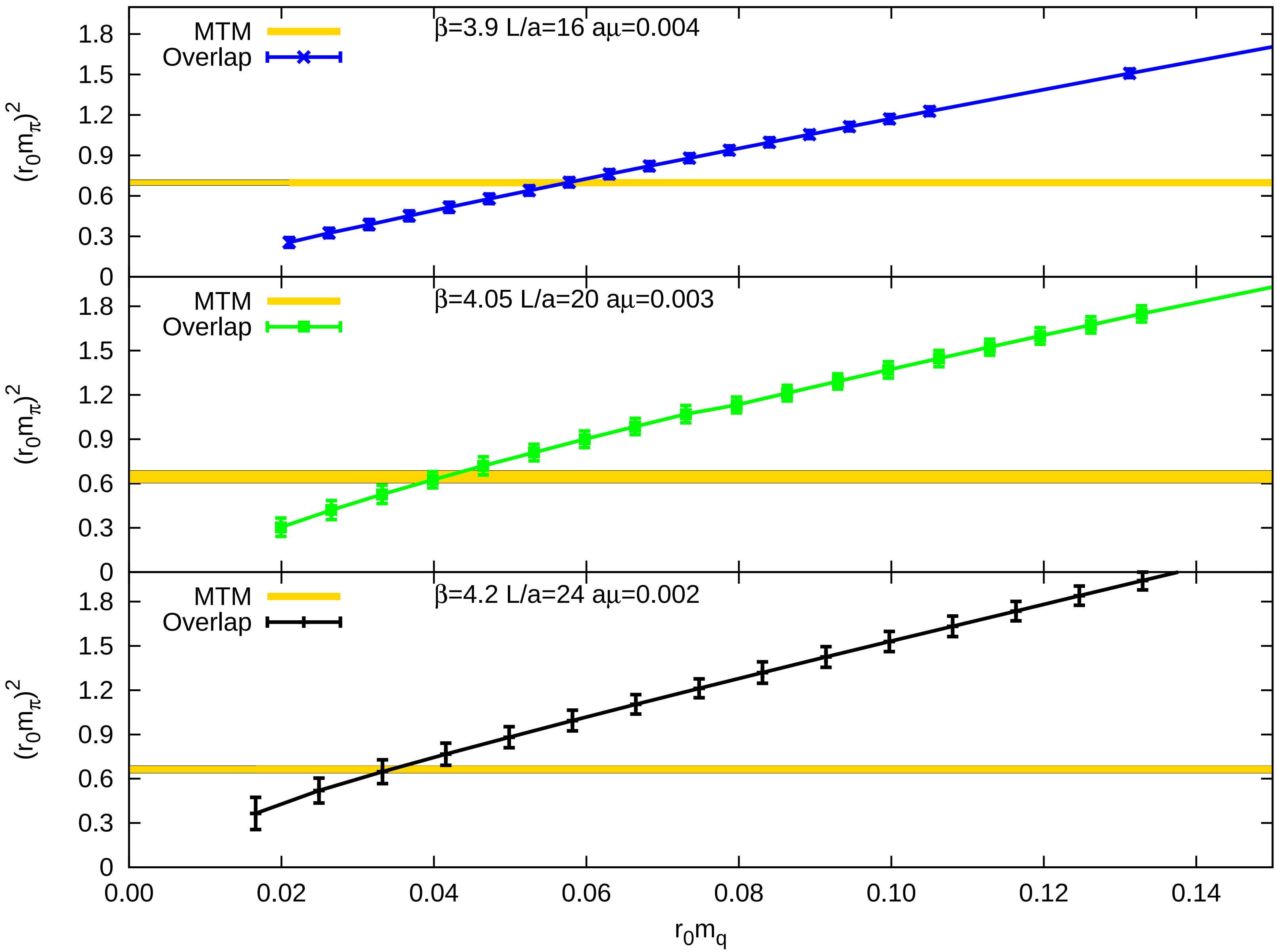


Figure 12: Matching the pion mass from the PP-SS correlator – fixed volume $L \approx 1.3$ fm.

Pion decay constant scaling – $C_{PP-SS}(t)$

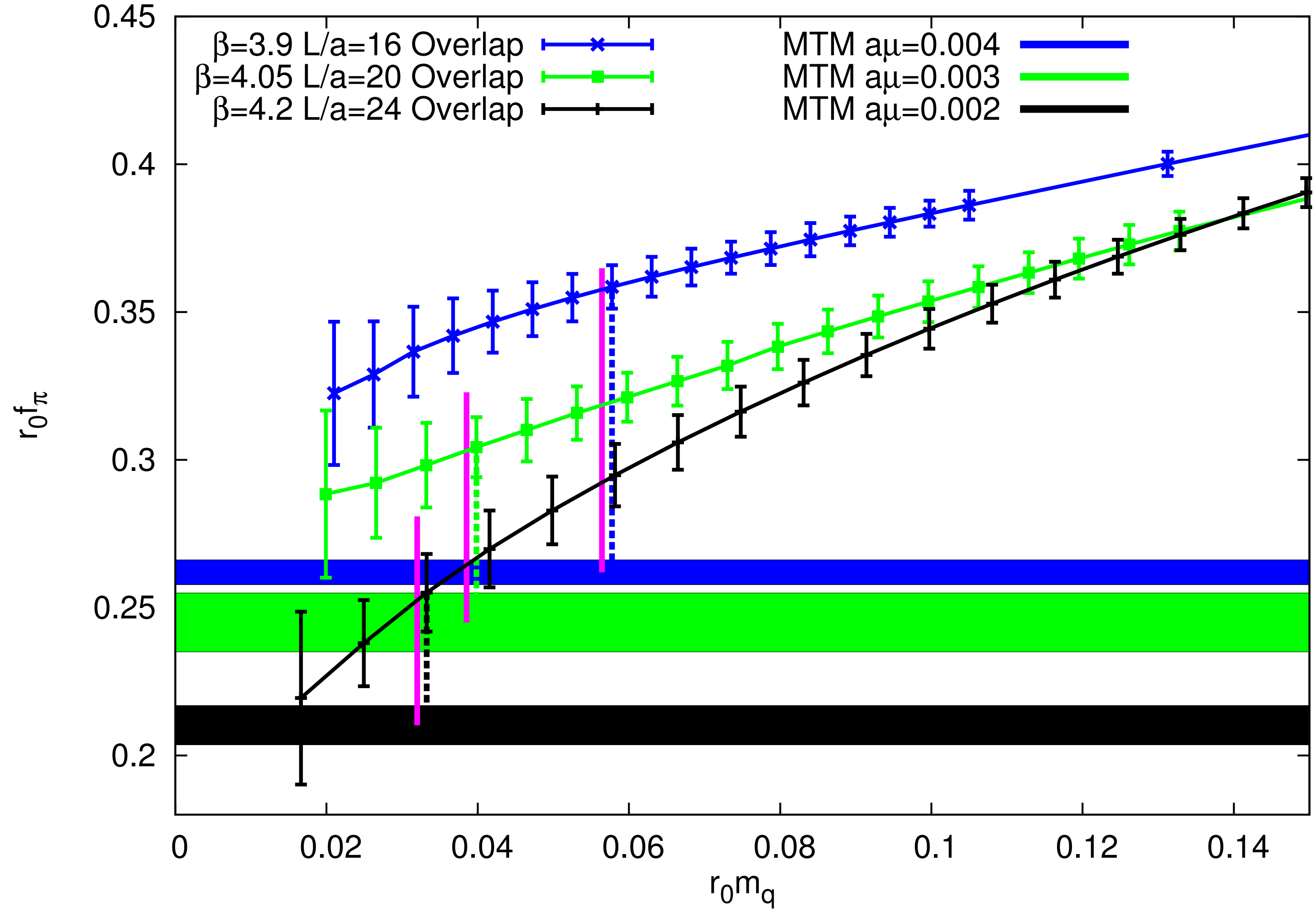


Figure 13: The quark mass dependence of the pion decay constant extracted from the PP-SS correlator – fixed volume $L \approx 1.3$ fm.

Pion decay constant scaling – $C_{PP-SS}(t)$

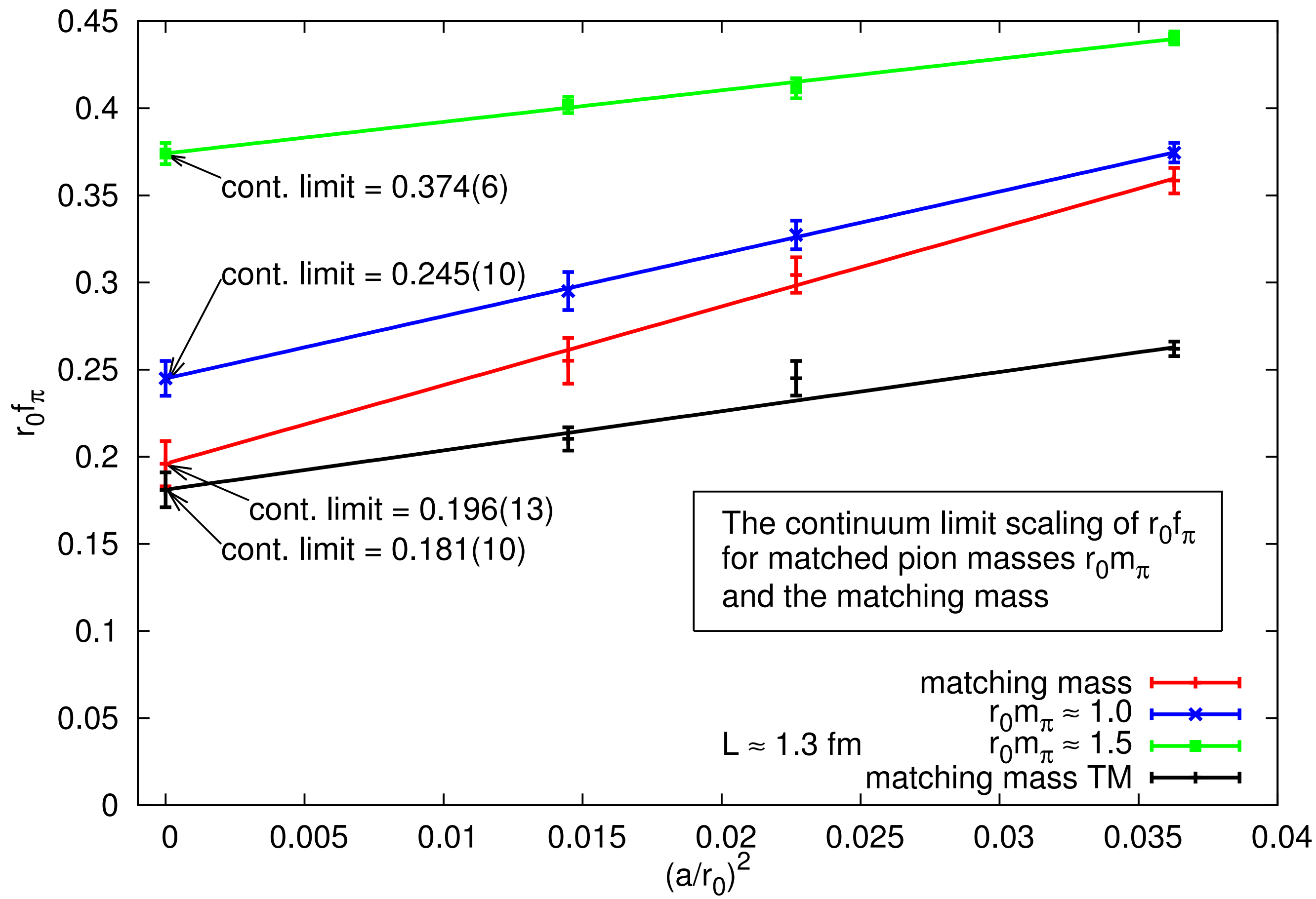


Figure 14: Pion decay constant scaling – fixed volume $L \approx 1.3$ fm.

Finite Volume Effects

We also want to study finite volume effects for a given lattice spacing – we choose $a \approx 0.079$ fm and the following lattice sizes: $16^3 \times 32$ ($L \approx 1.3$ fm), $20^3 \times 40$ ($L \approx 1.7$ fm), $24^3 \times 48$ ($L \approx 2.0$ fm).

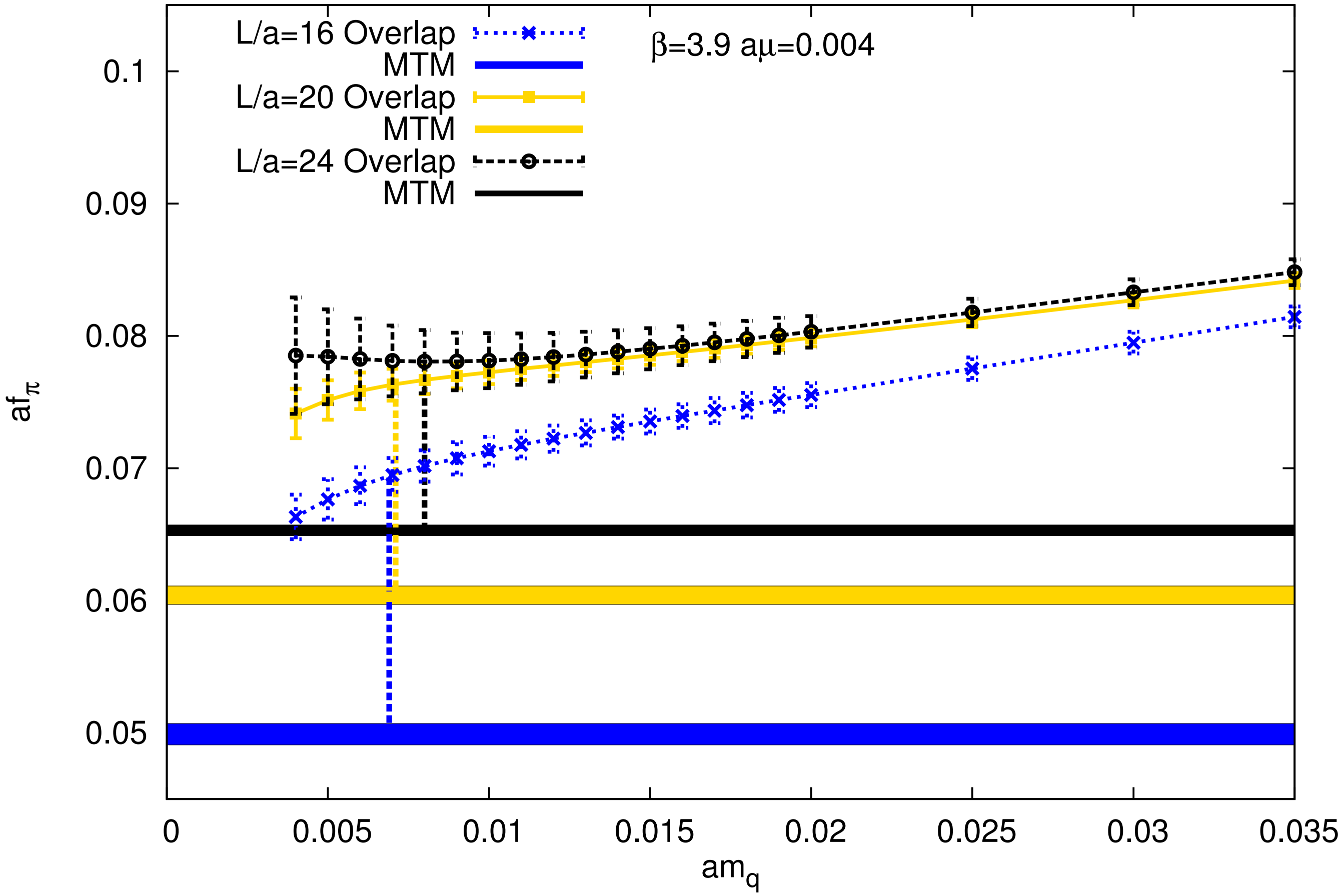


Figure 15: FVE in f_π – fixed lattice spacing $a \approx 0.079$ fm.

When are we safe against the zero modes?

- The role of the zero modes is decreasing as we increase the volume.

- Hence, we can estimate the volume for which their contribution is negligible.

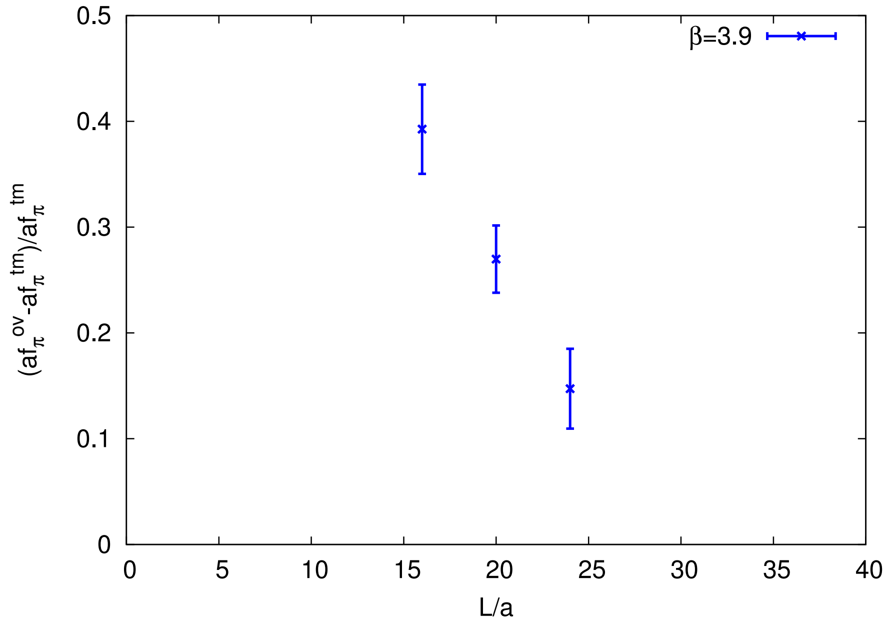


Figure 16: Mismatch between af_π^{TM} and af_π^{ov} at the matching point.

- However, the role of the zero modes also decreases as we increase the sea quark mass.

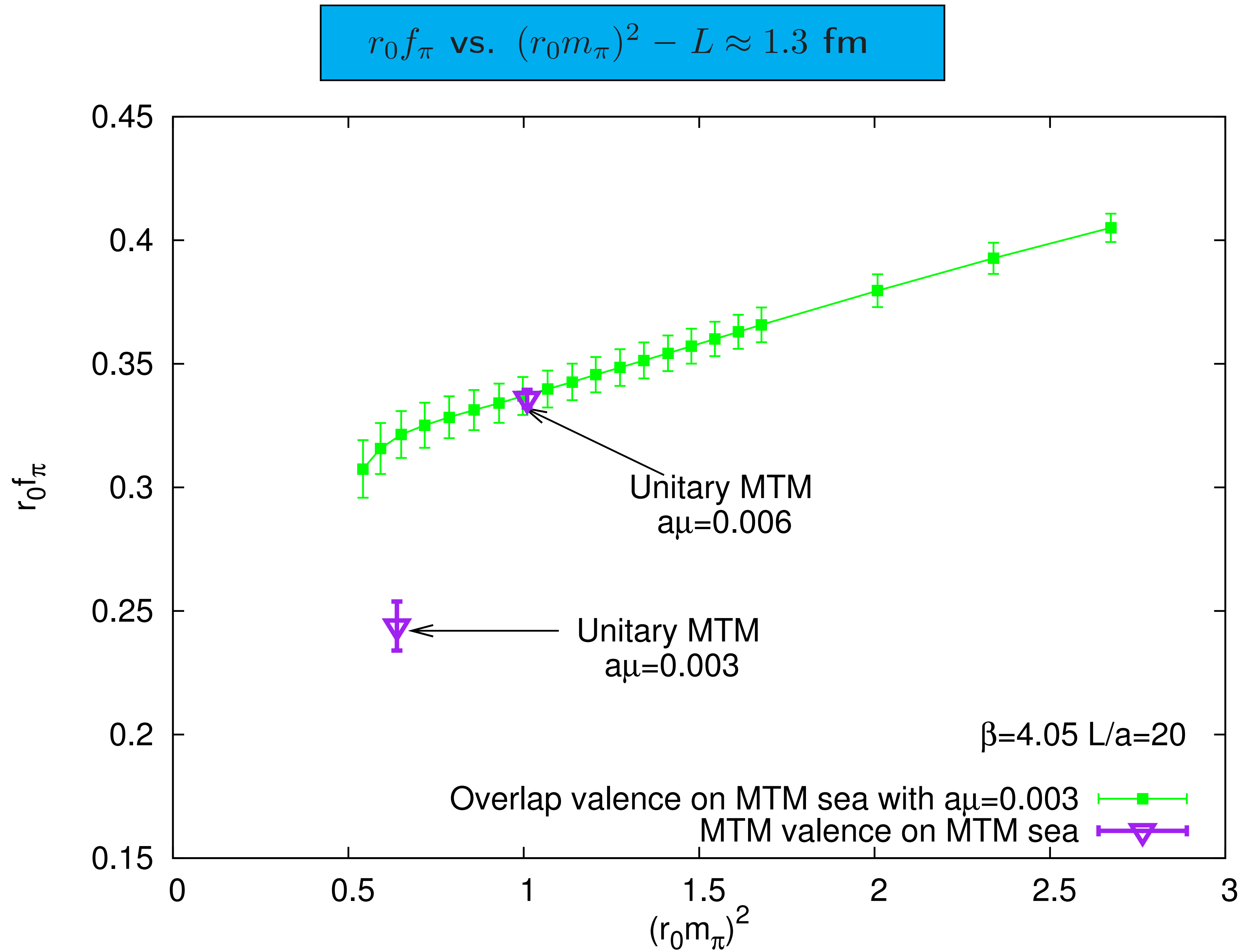


Figure 17: Overlap and unitary TM data.

Heavier quark mass

Let us ask the following question:

What happens at a higher sea quark mass?

We consider the following ensembles:

- $16^3 \times 32$, $a \approx 0.079$ fm ($\beta = 3.9$, $a\mu = 0.0074$)
- $20^3 \times 40$, $a \approx 0.063$ fm ($\beta = 4.05$, $a\mu = 0.006$)
- $24^3 \times 48$, $a \approx 0.051$ fm ($\beta = 4.2$, $a\mu = 0.005$)

which correspond to pion mass ca. 450 MeV.

Pion decay constant scaling

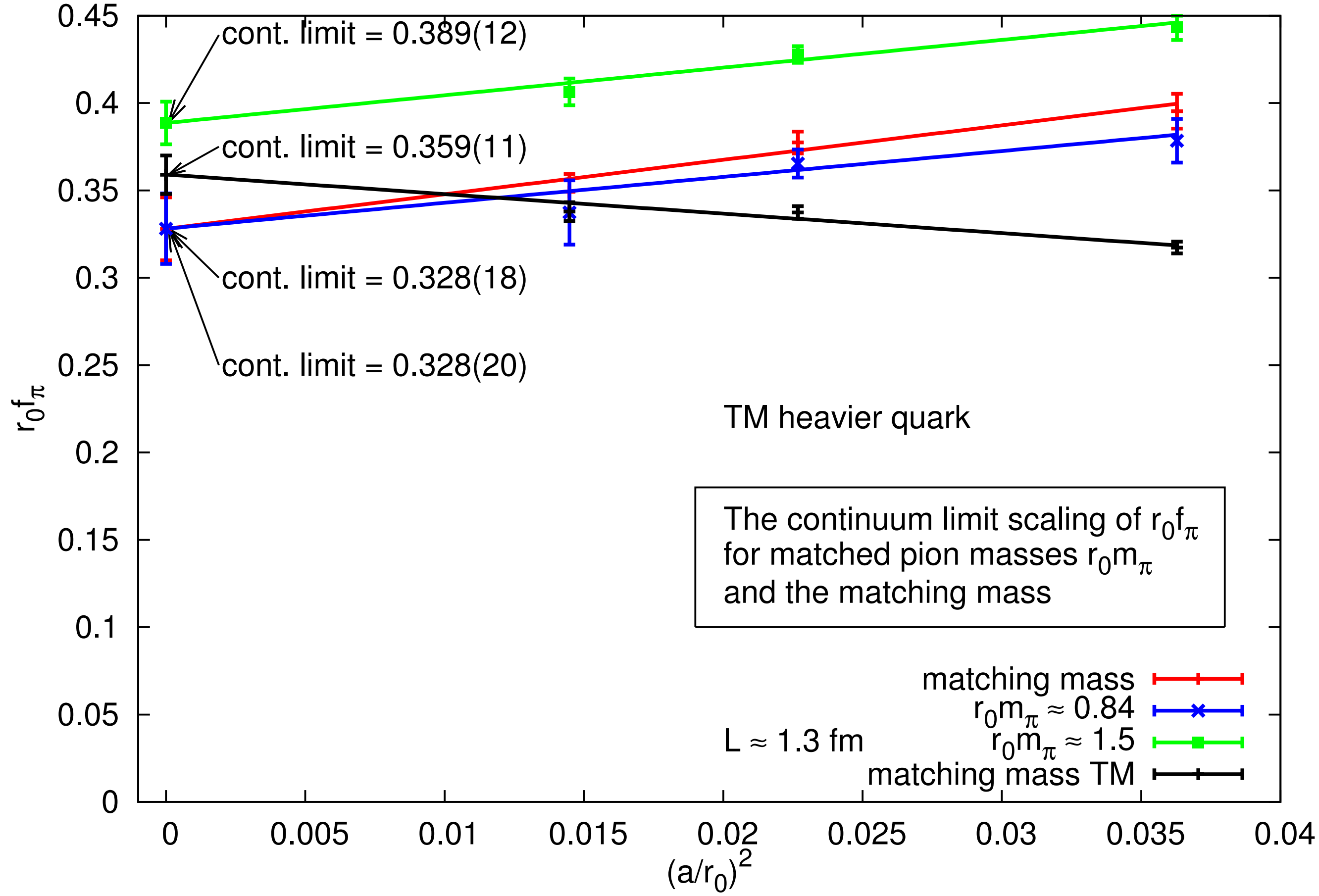


Figure 18: Pion decay constant scaling – fixed volume $L \approx 1.3$ fm, heavier sea quark mass μ.

Pion decay constant scaling

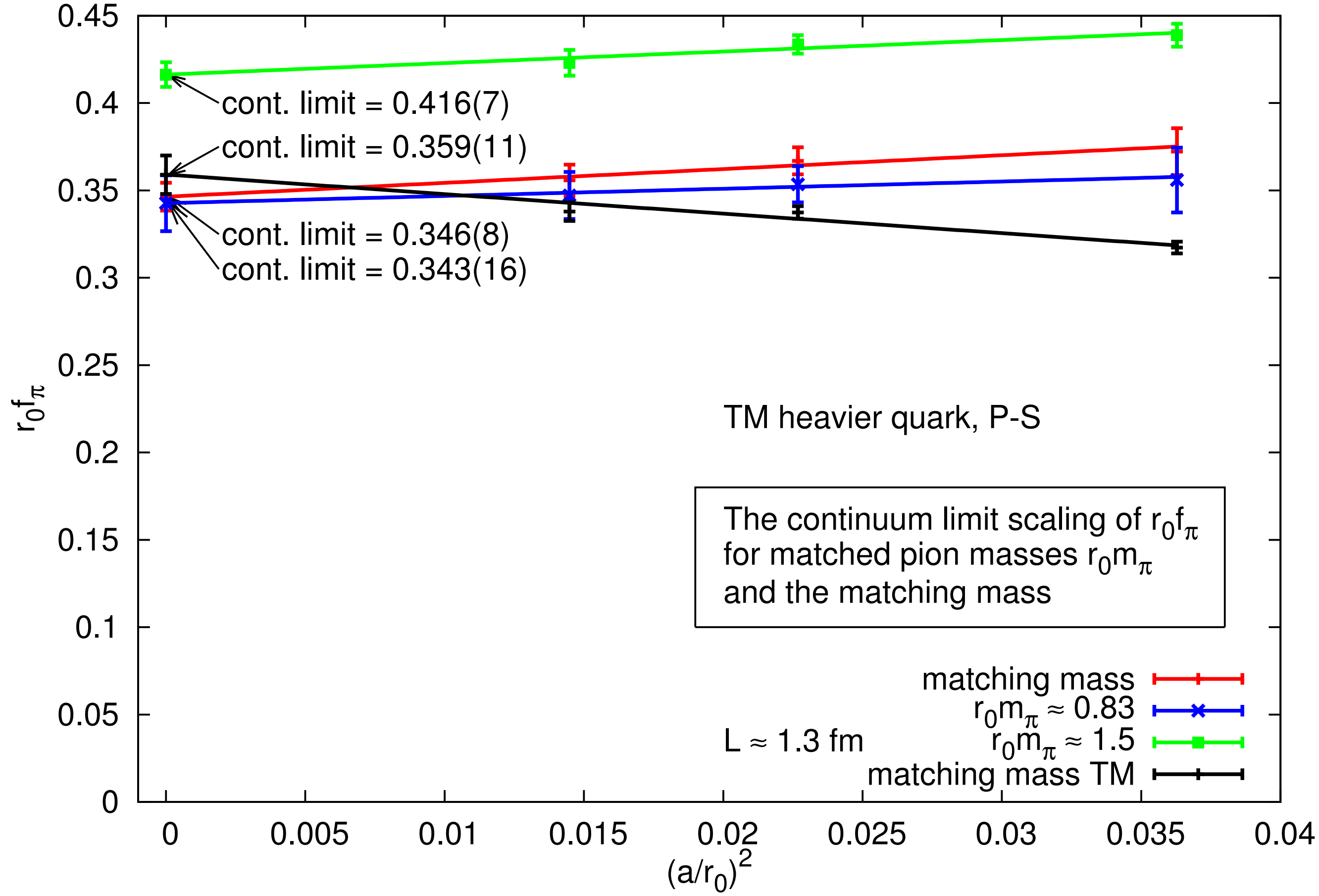


Figure 19: Pion decay constant scaling – fixed volume $L \approx 1.3$ fm, heavier sea quark mass μ.

When are we safe against the zero modes? – part 2

Hence, we seem to be safe against the zero modes when:

- The volume is large enough – at $\beta = 3.9$ (and $m_\pi \approx 300$ MeV) we need something of the order of 2.6 fm, i.e. $32^3 \times 64$!

- The sea quark mass is large enough – at $m_\pi \approx 450$ MeV we need something of the order of 1.8 fm, i.e. $24^3 \times 48$.

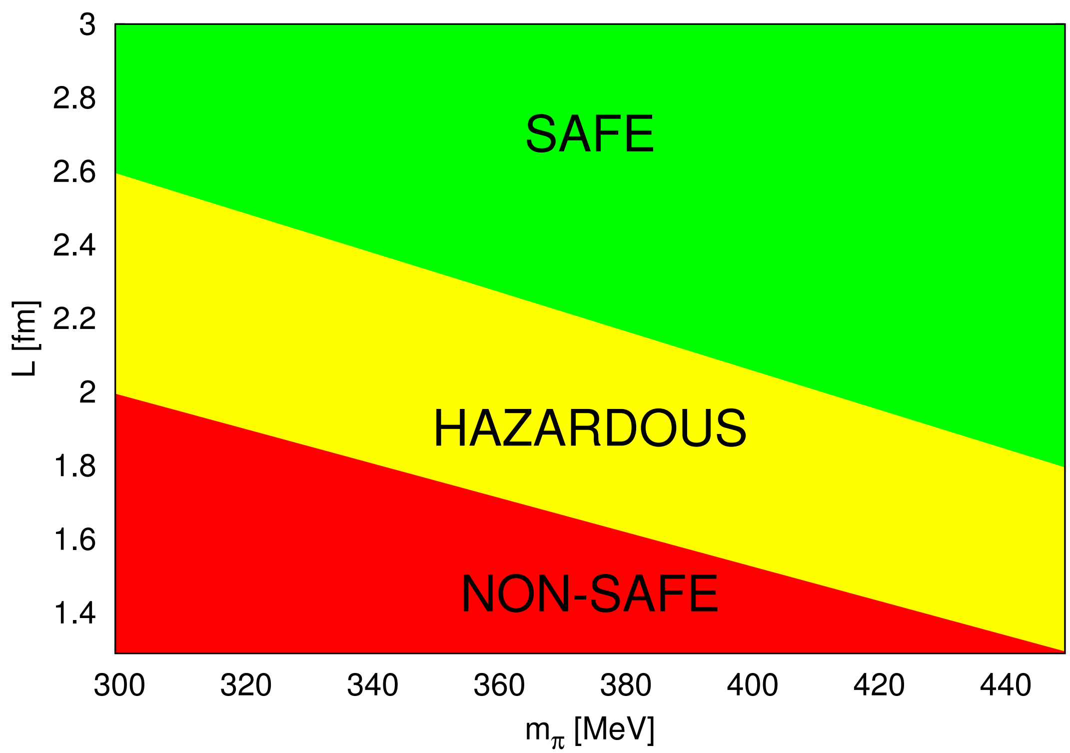


Figure 20: SAFE $\iff$ $m_\pi L > 4$, HAZARDOUS $\iff$ $m_\pi L \in [3, 4]$

Light baryon masses

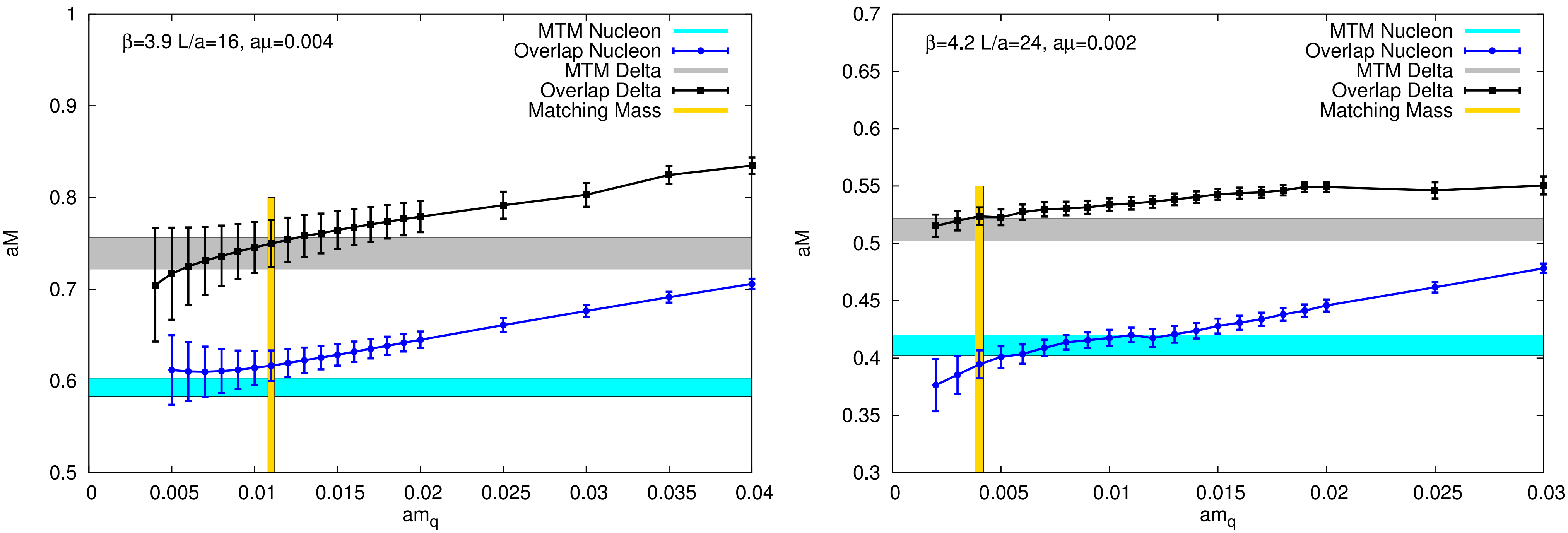


Figure 21: The quark mass dependence of the nucleon and delta mass.

Light baryon masses

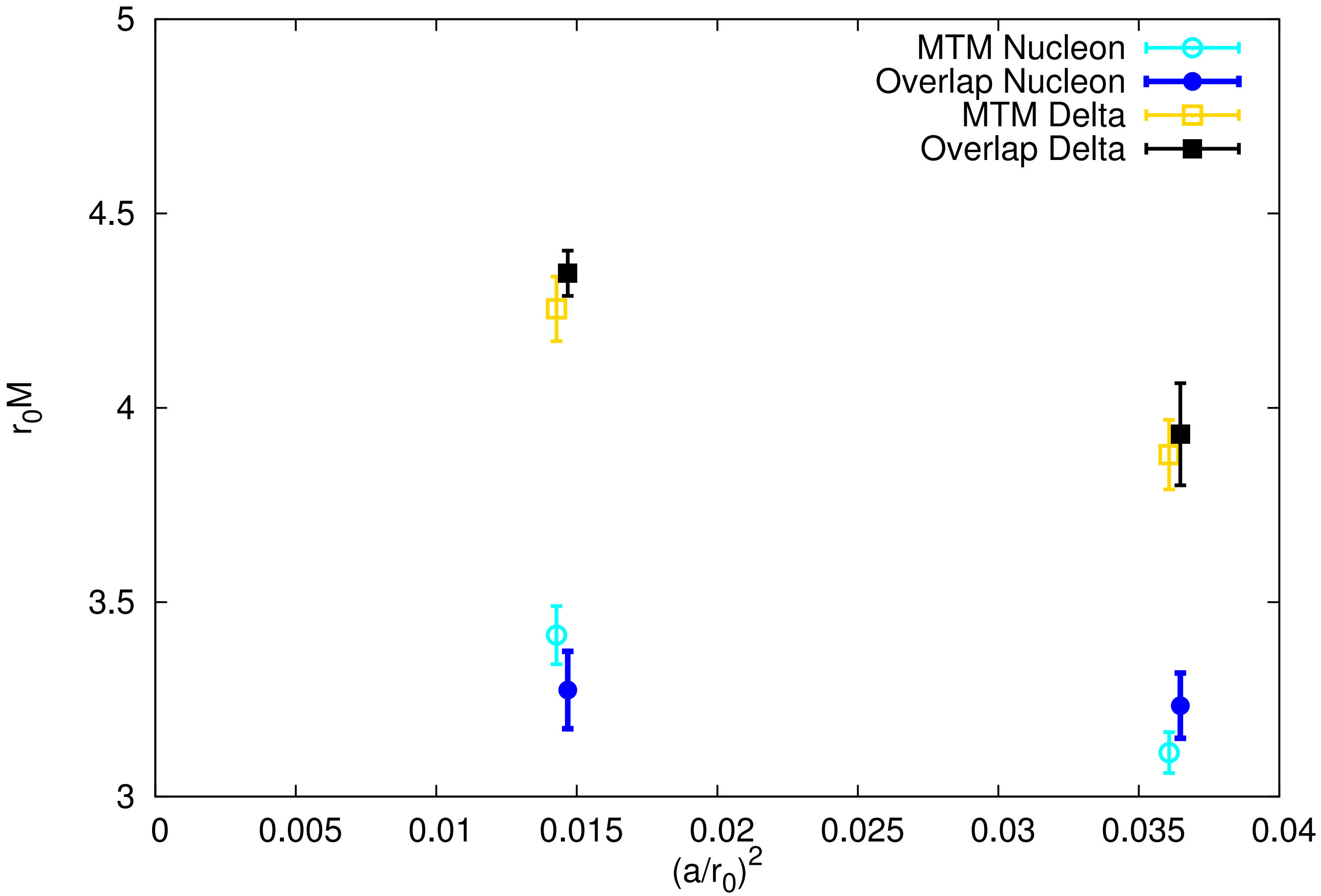


Figure 22: Continuum limit scaling of the nucleon and delta mass – $L \approx 1.3$ fm.

Unitarity violations

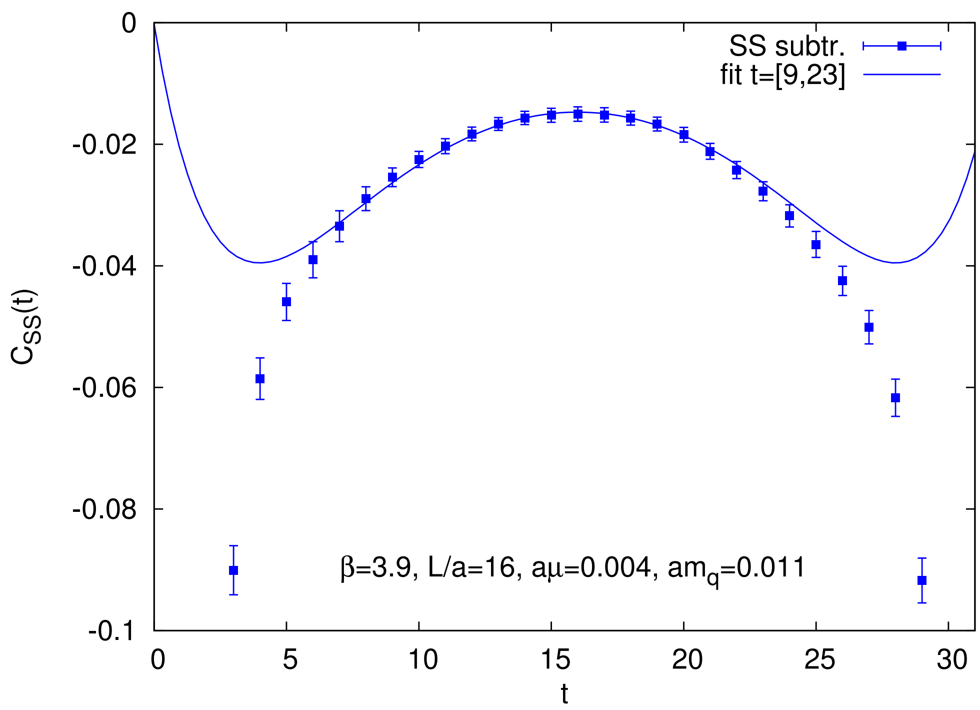
The scalar correlator is especially vulnerable to a double pole in the meson propagator, which can lead to an unphysical negative contribution to this correlator, even at the matching point (*Golterman, Izubuchi, Shamir hep-lat/0504013*):

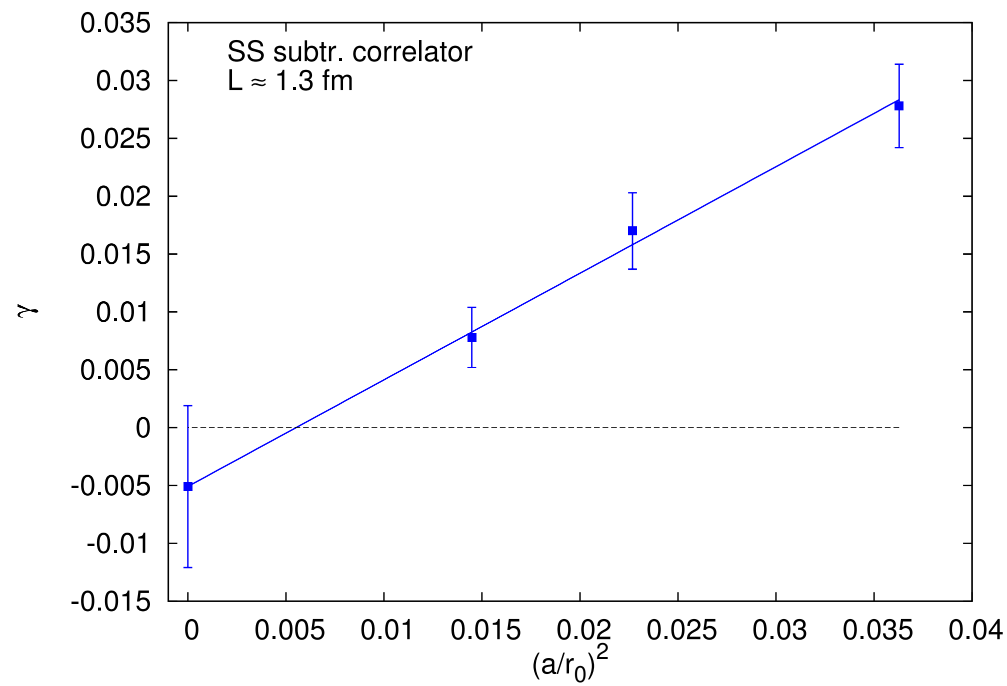
$$C_{SS}(t) \stackrel{t\to\infty}{=} -\frac{B_0^2}{2L^3}\frac{e^{-2M_{VV}t}}{M_{VV}^3}\gamma_{SS}a^2 t.$$

Define: $\gamma \equiv \frac{B_0^2\gamma_{SS}}{2(M_{VV}L)^3}a^2$. Thus: $C_{SS}(t) \stackrel{t\to\infty}{=} -\gamma\left(t\, e^{-2M_{VV}t} + (T-t)\, e^{-2M_{VV}(T-t)}\right)$.

Use the SS correlator with explicitly subtracted zero modes to isolate the effect.

We want to extract γ for each light-quark, small-volume ensemble.





Conclusions

- We observe good scaling behaviour of overlap fermions.
- However, the pion decay constant computed with overlap fermions is at the matching mass significantly larger than its TM value.
- The main reason for this can be the chiral zero modes of the overlap operator.
- The effects of the zero modes are observable-dependent and operator-dependent.
- We have estimated the parameters for which one seems to be safe against the effects of the zero modes:
  - $L \approx 2.6$ fm at $m_\pi \approx 300$ MeV,
  - $L \approx 1.8$ fm at $m_\pi \approx 450$ MeV.
- With the knowledge of these parameters it will be possible to address various physical questions.

Prospects

- compute observables for which good chiral properties of valence fermions are essential – e.g. the kaon bag parameter B_K, or the decay $K \to \pi\pi$;
- investigate questions that are related to topology, i.e. the computation of topological susceptibility and the determination of the singlet meson mass η';
- analyze in the mixed action setup unitarity violations in the scalar correlator and in mixed correlation functions (with one valence and one sea quark);
- confront the simulation results with (Mixed Action) Partially Quenched Chiral Perturbation Theory formulas to extract the corresponding LECs;
- perform a continuum limit scaling test of the pion decay constant (and other observables) at larger volume.

Prospects

Moreover, it would also be interesting to further investigate the role of the zero modes, by:

- testing alternative matching conditions (e.g. the matching condition of equal renormalized quark masses);
- investigating the role of the zero modes in baryonic observables;
- performing an analysis of topological aspects by explicitly computing the zero modes.

Thank you for your attention!