Hadron electric polarizability
- finite volume corrections -

Andrei Alexandru, Frank Lee

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THE GEORGE WASHINGTON UNIVERSITY
WASHINGTON DC

Outline

- Motivation
- Background field method
- Fitting strategy
- Finite volume effects
- Numerical results
- Conclusions

Motivation

- To lowest order the hadron mass changes when placed in a electromagnetic field

$$\Delta E = -\vec{p} \cdot \vec{E} - \vec{\mu} \cdot \vec{B} - \frac{1}{2}(\alpha E^2 + \beta B^2) + ...$$

- p & μ - the electric & magnetic dipole
- α & β - the electric & magnetic polarizability
- The polarizability measures the dipole moment induced by the field
- The polarizabilities are measured in Compton scattering experiments

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Electric polarizability

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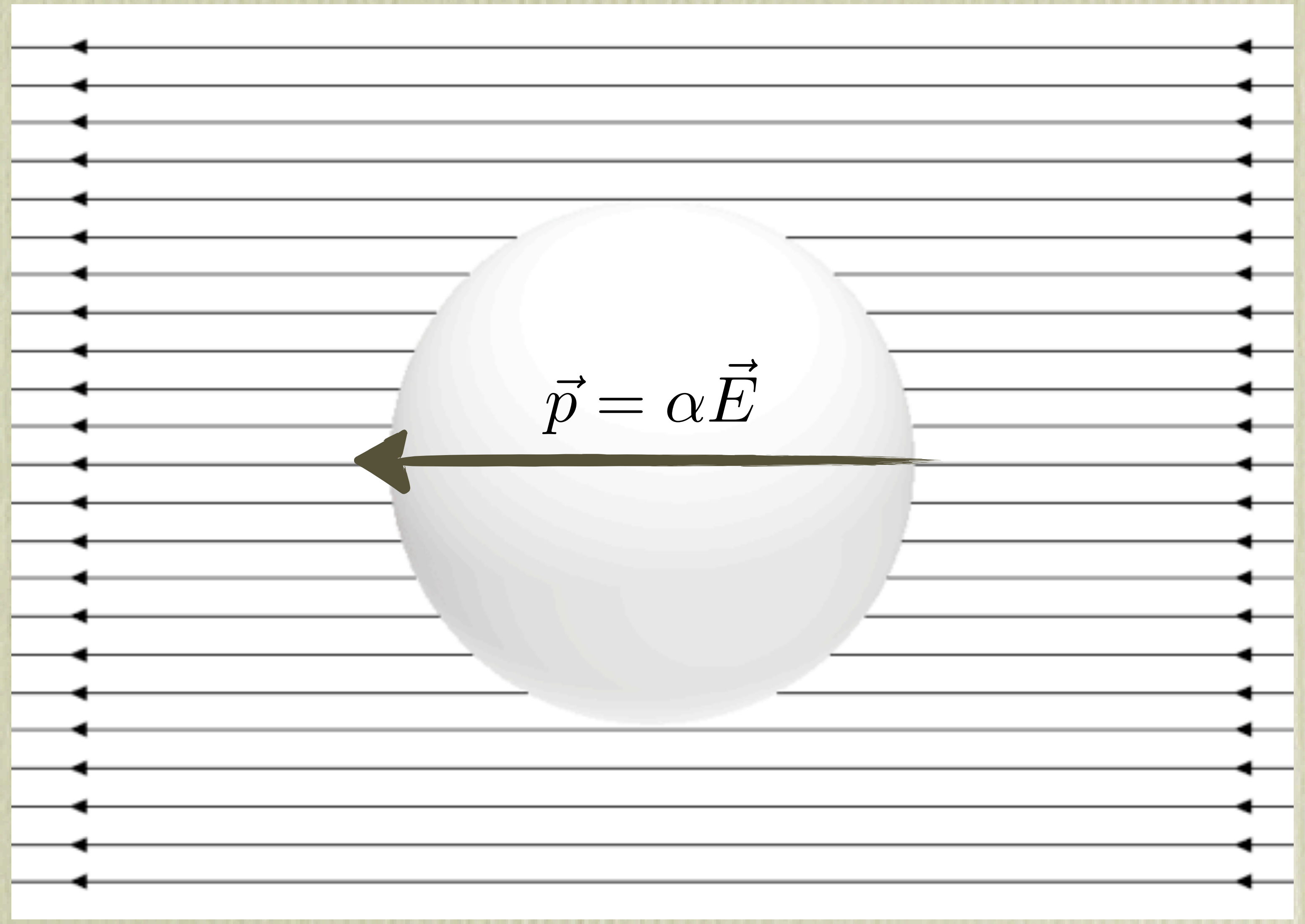
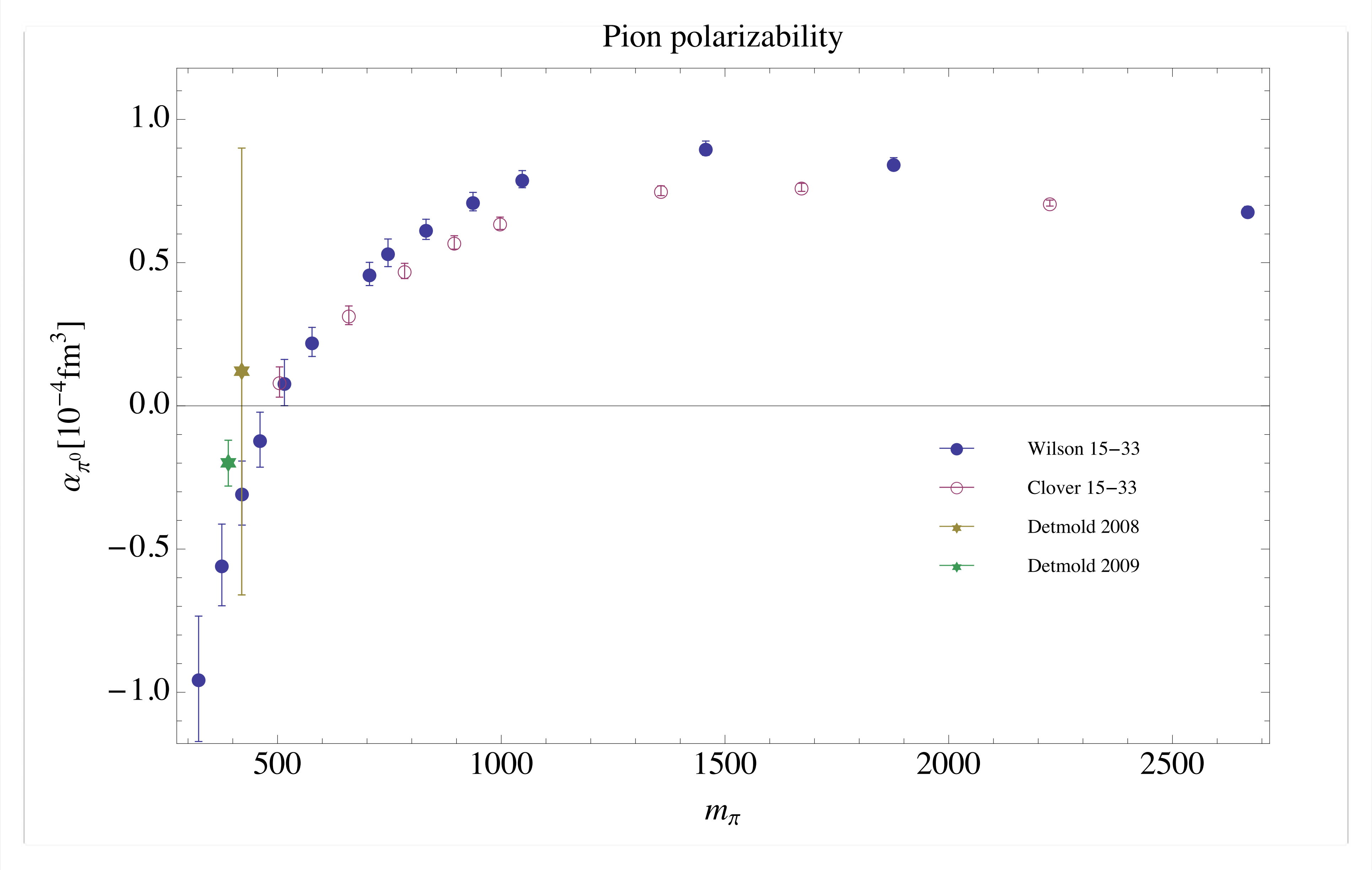


$$\Delta E = -\frac{1}{2}\alpha E^2$$

Motivation
"neutral pion" polarizability



“Neutral Pion”

- The physical π° correlator has disconnected contributions in the presence of a background field
- The disconnected diagrams cancel only in the isospin limit -- the electric field breaks isospin symmetry
- Our calculation doesn’t include disconnected contributions
- The particle we study is more like $\bar{d}u$ when u and d have the same charge
- In this version of QCD the pions are all uncharged and χPT predicts a flat behavior (to leading order)

Background field method

- The electro-magnetic field is introduced as a background field via minimal coupling

$$D_\mu = \partial_\mu - igG_\mu - iqA_\mu$$

- The U(1) field A_μ is static; on the lattice this amounts to changing the links

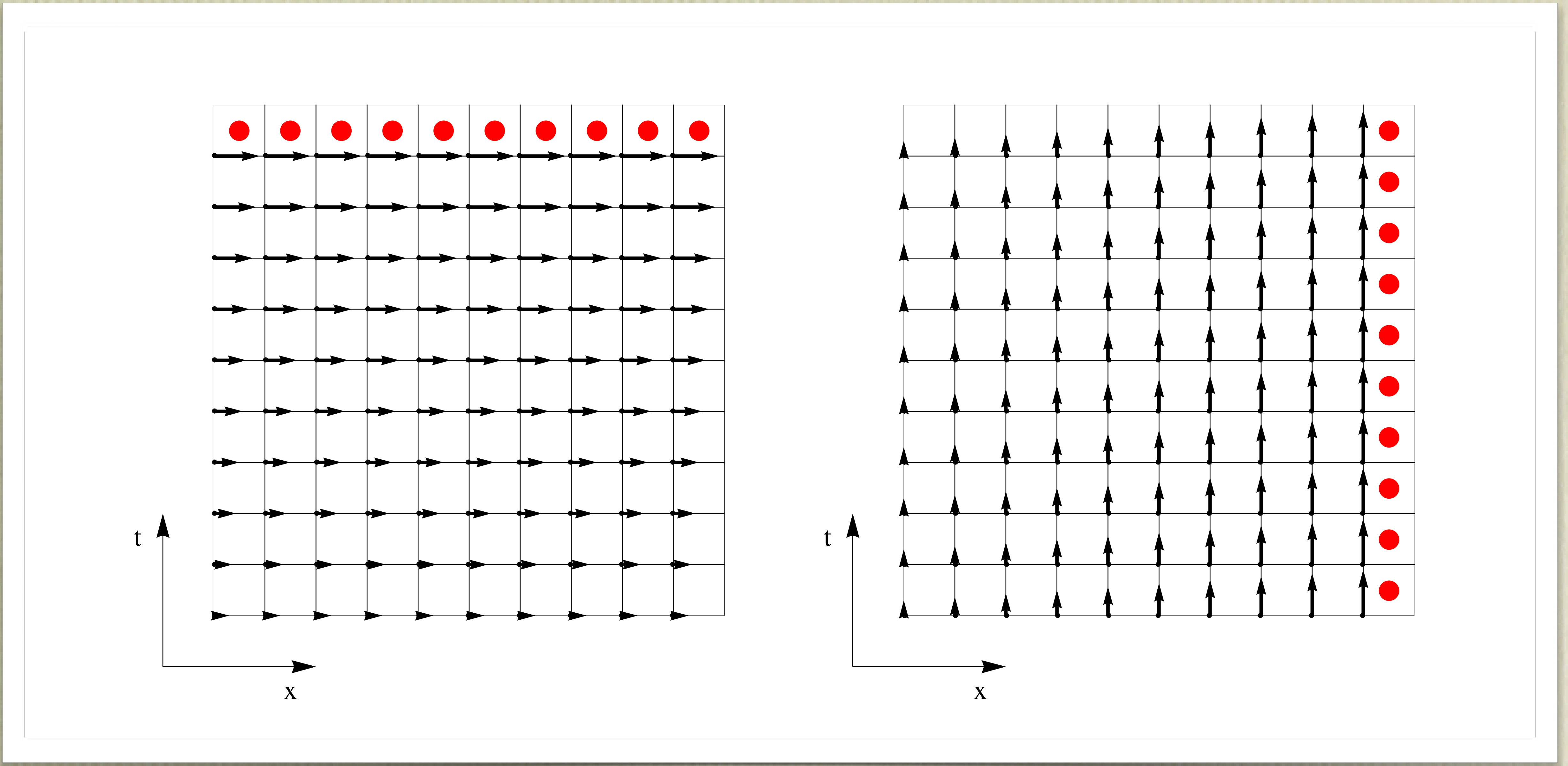
$$U_\mu \to e^{-iqaA_\mu} U_\mu$$

- Measure the dipole moments and polarizabilities form the hadron mass shift

- Normally, a positive electric polarizability corresponds to a negative mass shift but the electric field introduced this way is imaginary

$$U_1 \to U_1 e^{-iaqEt} \Rightarrow \Delta m = +\frac{1}{2}\alpha E^2$$

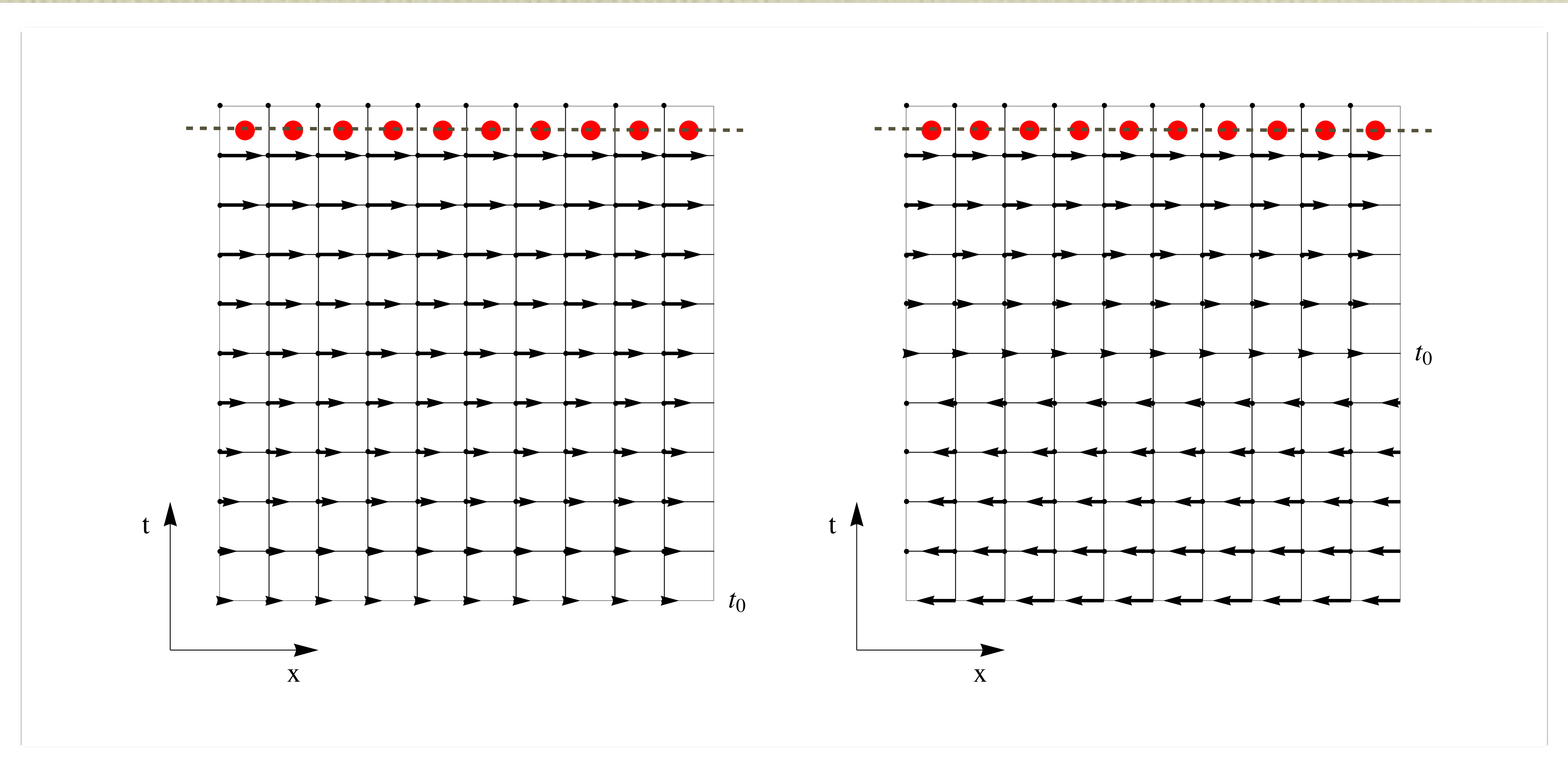
Boundary conditions



$A_x = Et$

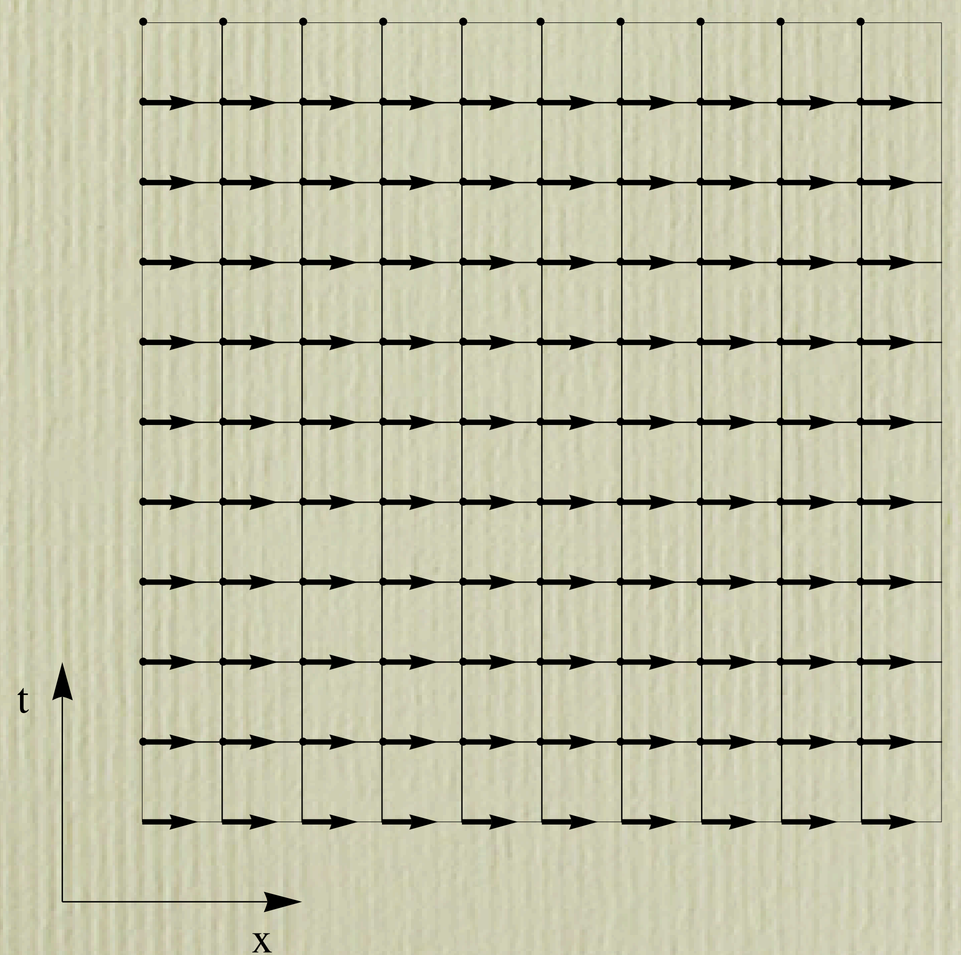
$A_t = -Ex$

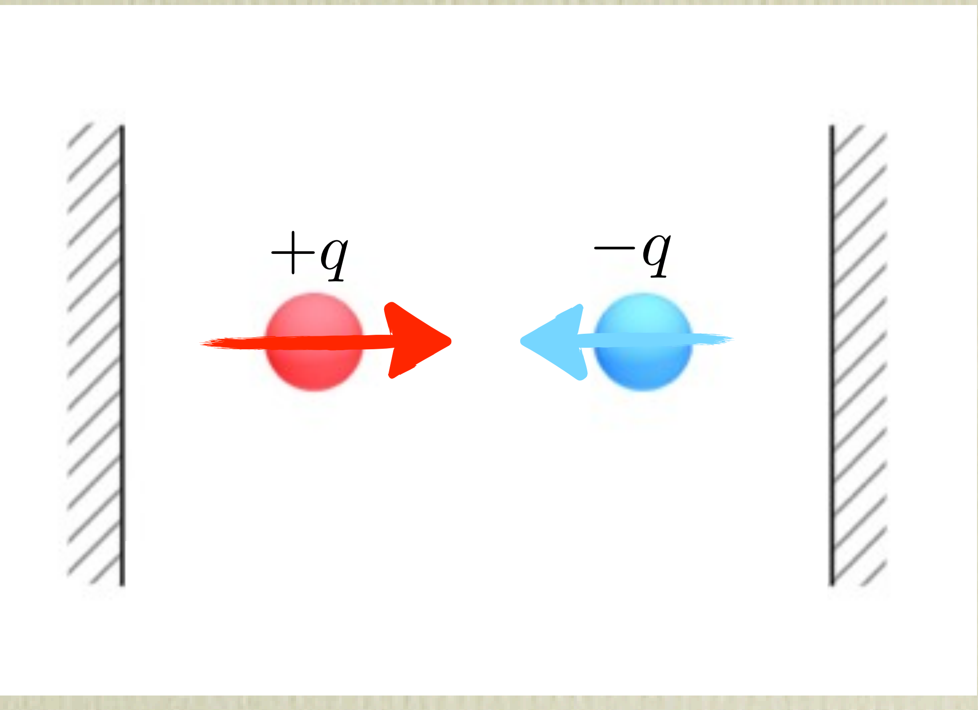
Periodic boundary conditions



$A_x = Et$ $\qquad$ $A_x = E(t - t_0)$

Periodic boundary conditions

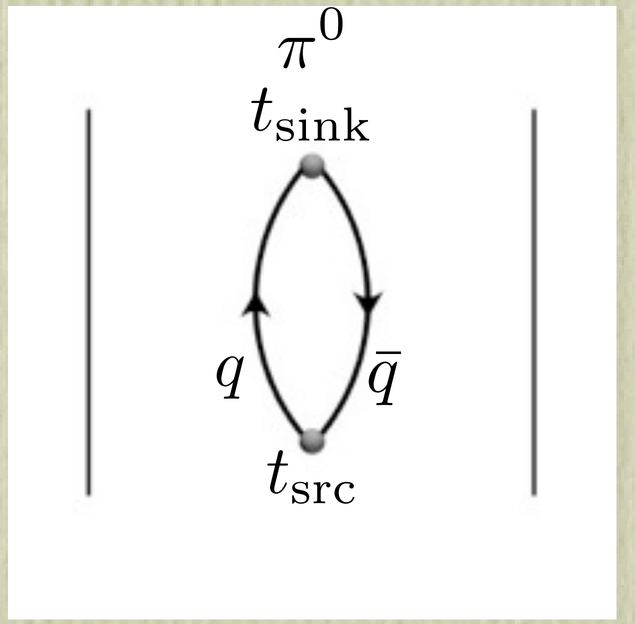


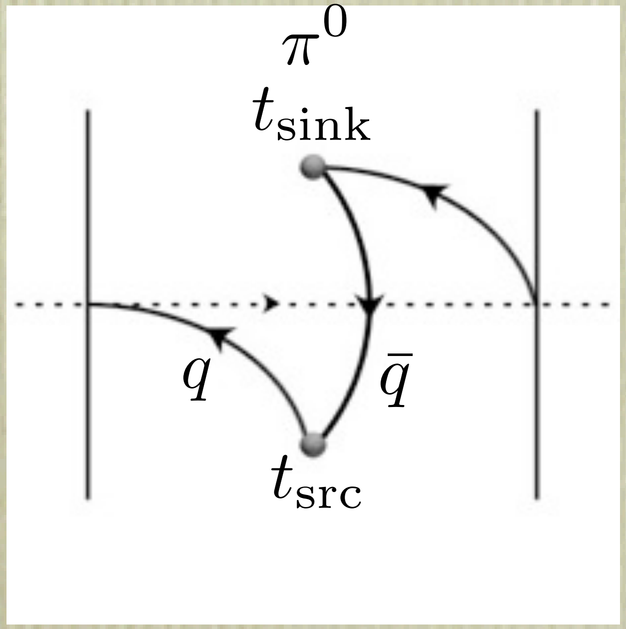


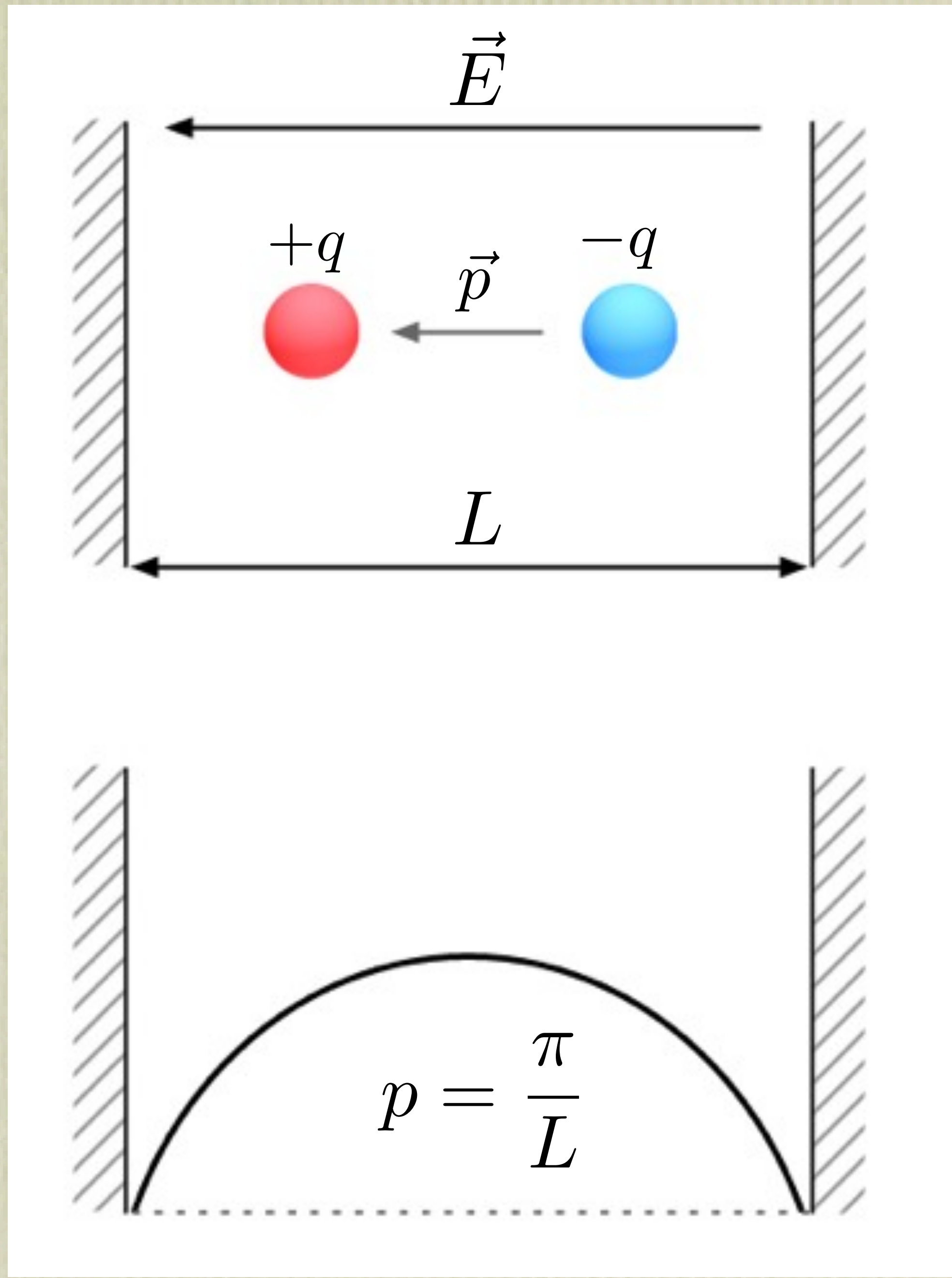
$U_x \to e^{iaqV_0} U_x$

$p = qV_0 N_x$

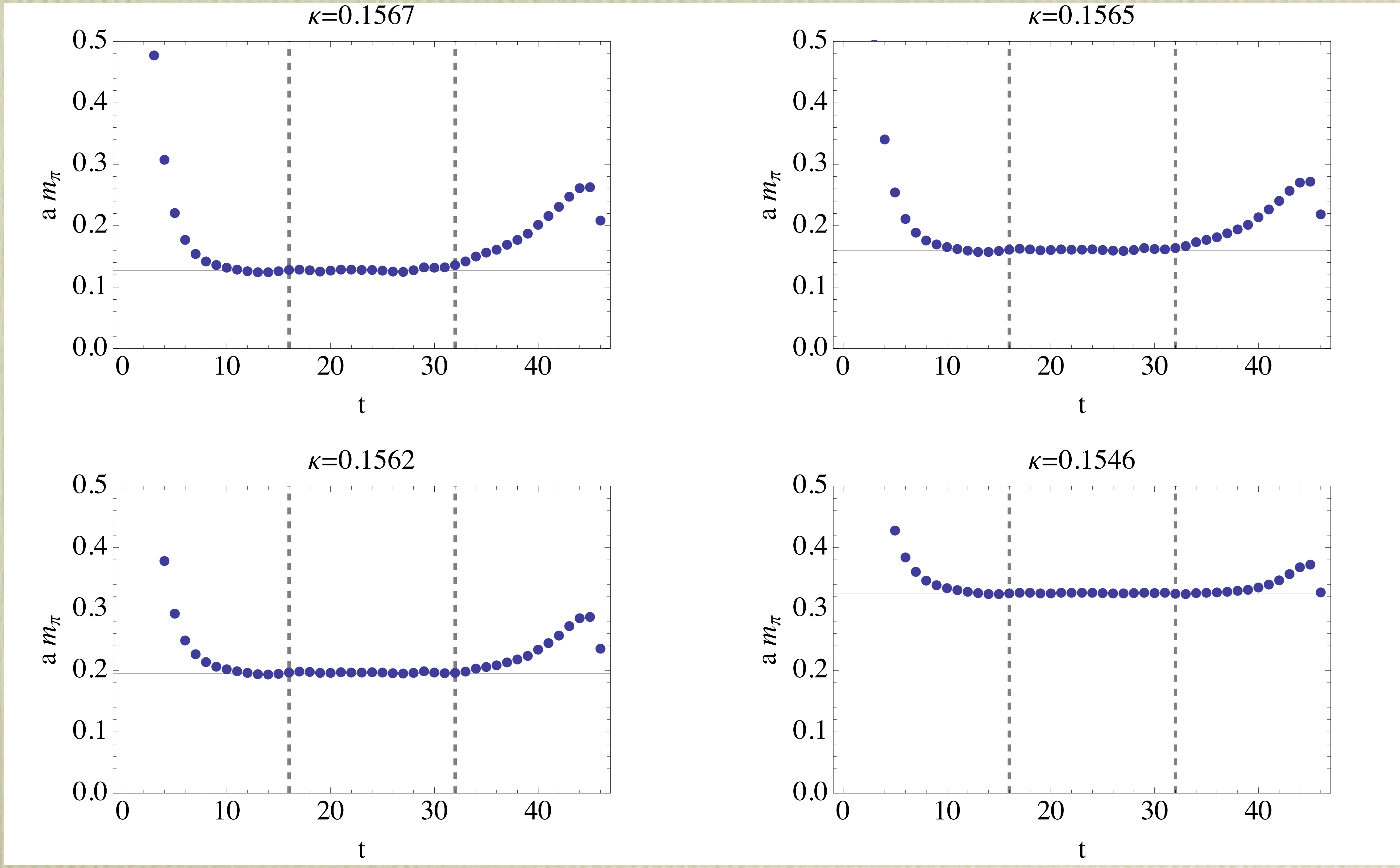
Dirichlet boundary conditions







Fitting strategy



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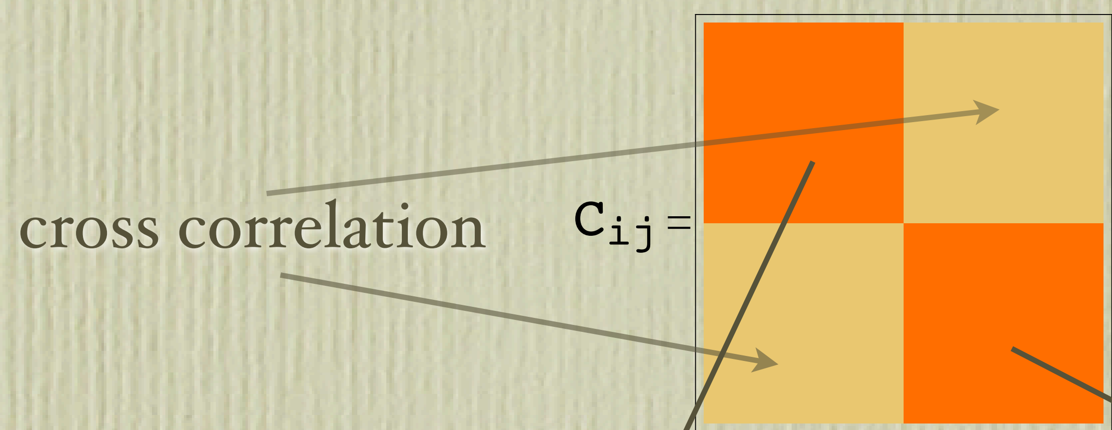
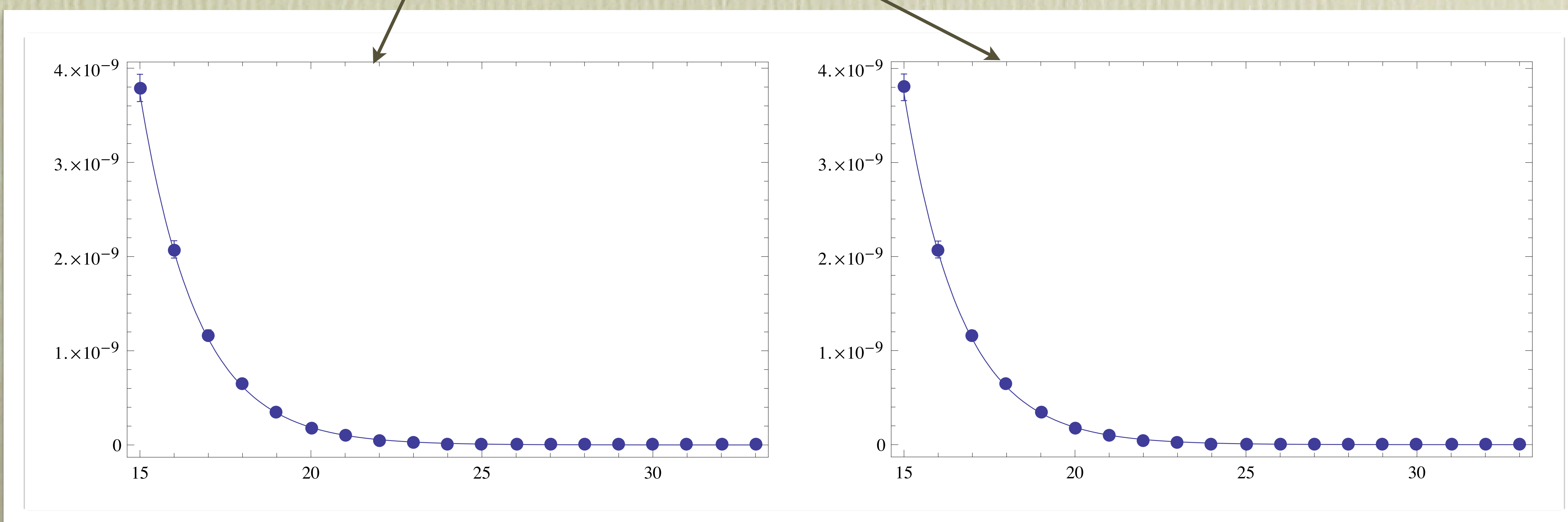


$$\chi^2 = \sum_{i,j} (f(t_i) - y_i) C_{ij}^{-1} (f(t_j) - y_j)$$

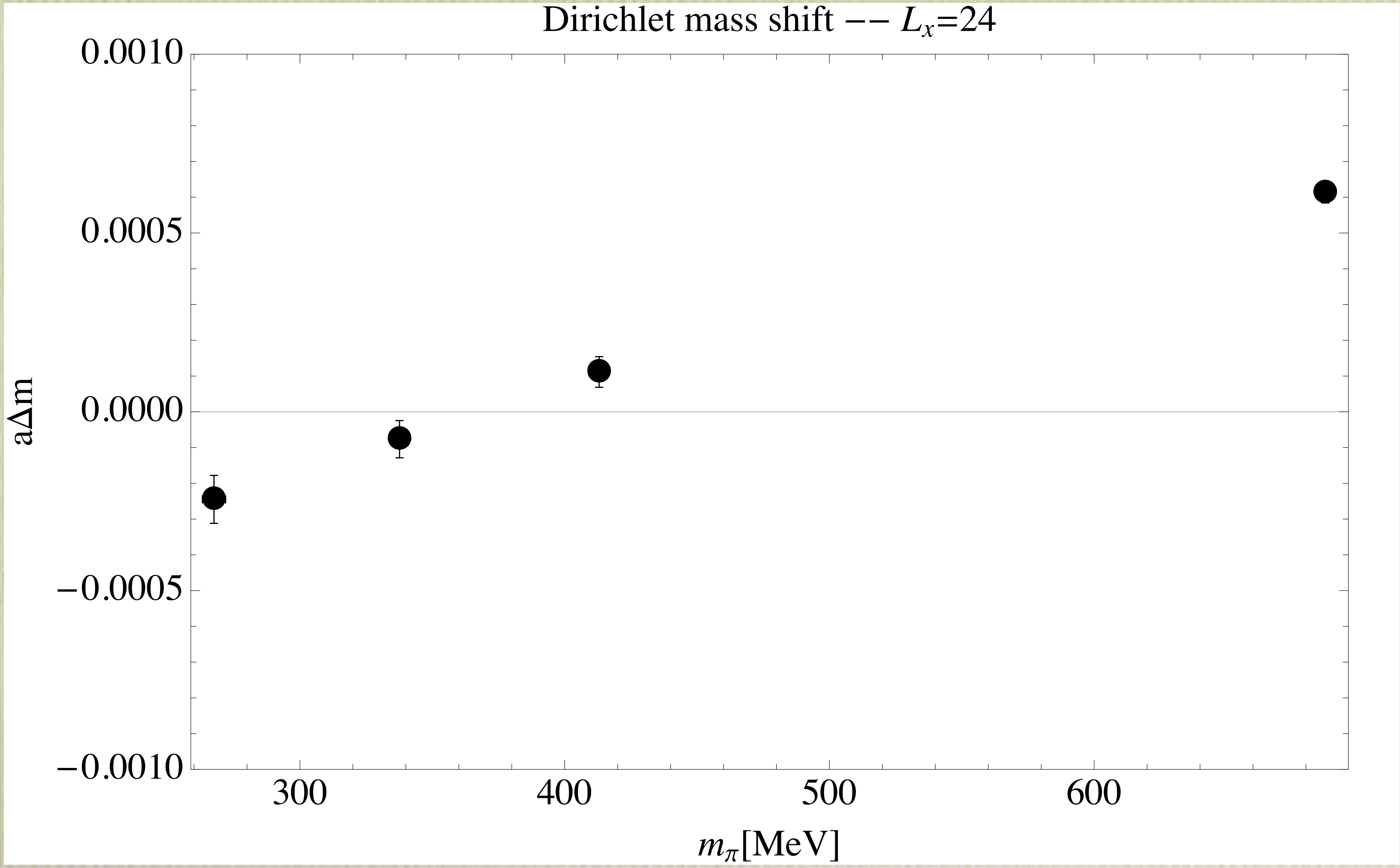
$$f(t) = Ae^{-mt} + (A + \delta A)e^{-(m+\delta m)t}$$



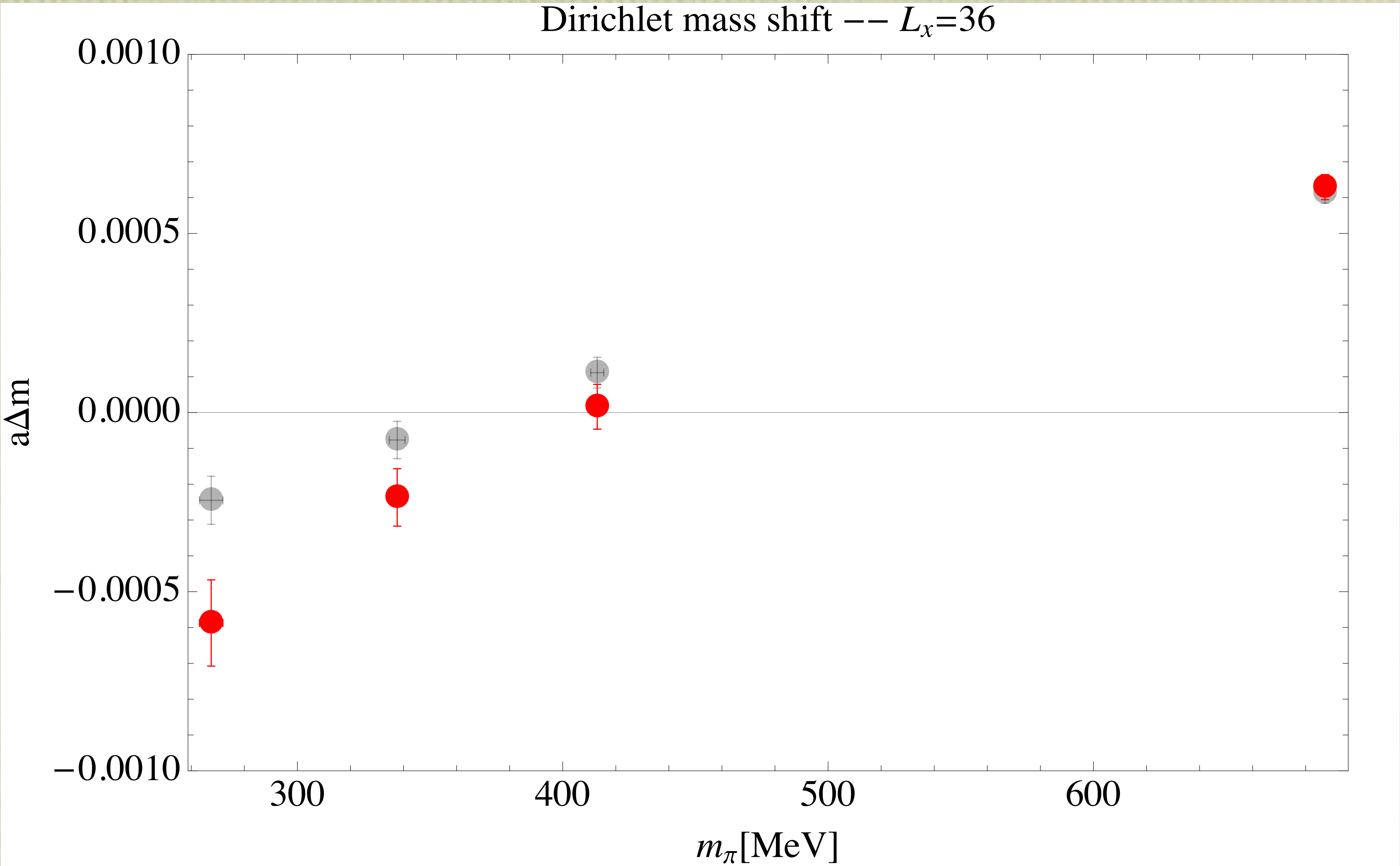
Lattice parameters - Dirichlet bc

- Quenched ensemble - Wilson action β=6.0, a=0.093 fm
- Wilson fermions: 270 - 700 MeV ($m_\pi L$: 3.2-8)
- Electric field - $\eta = a^2 qE = 0.00576$
- Three different lattice sizes: 24, 36, 48 x 24^2 x 48 -- 1000 configurations

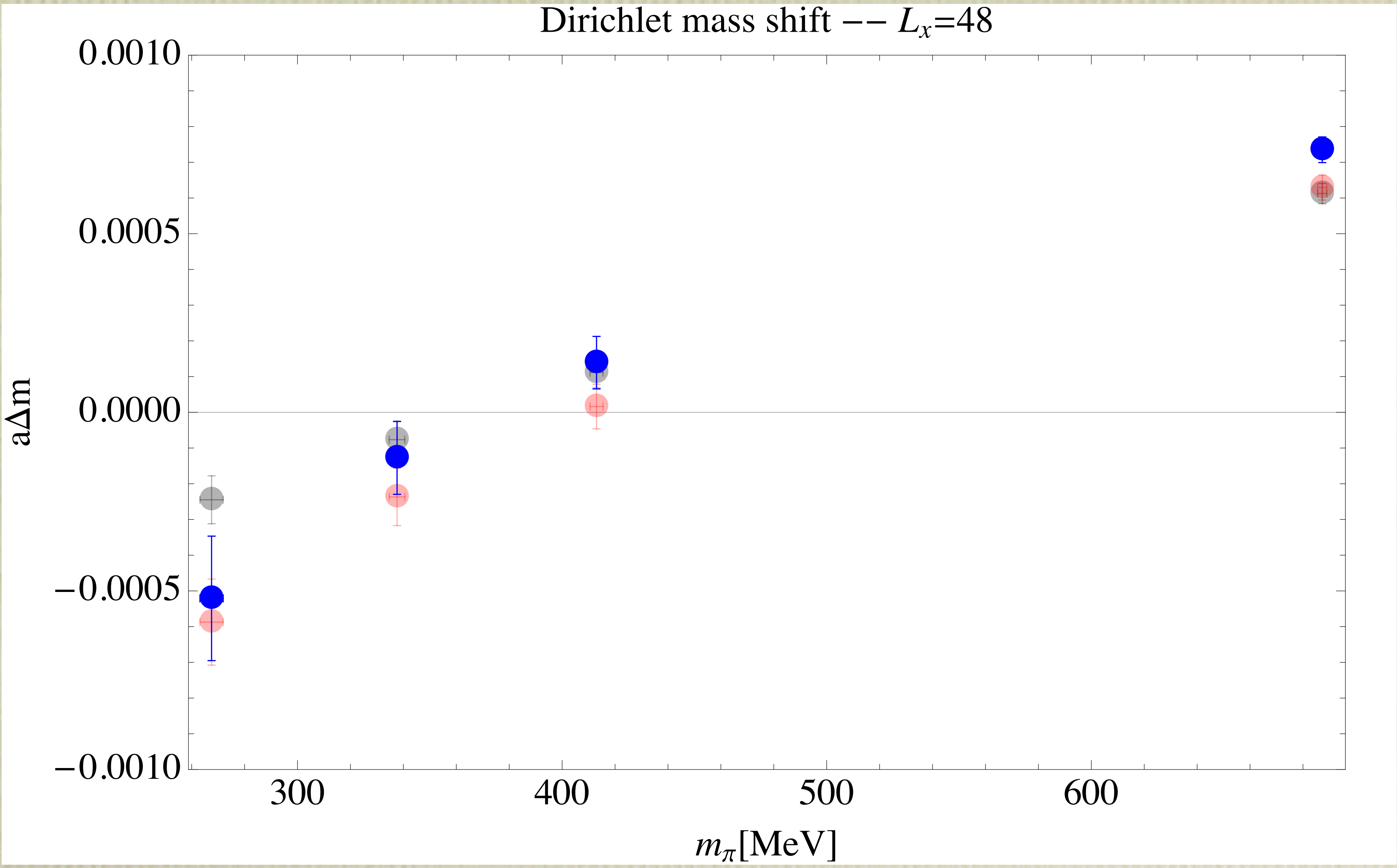
Dirichlet bc -- mass shift



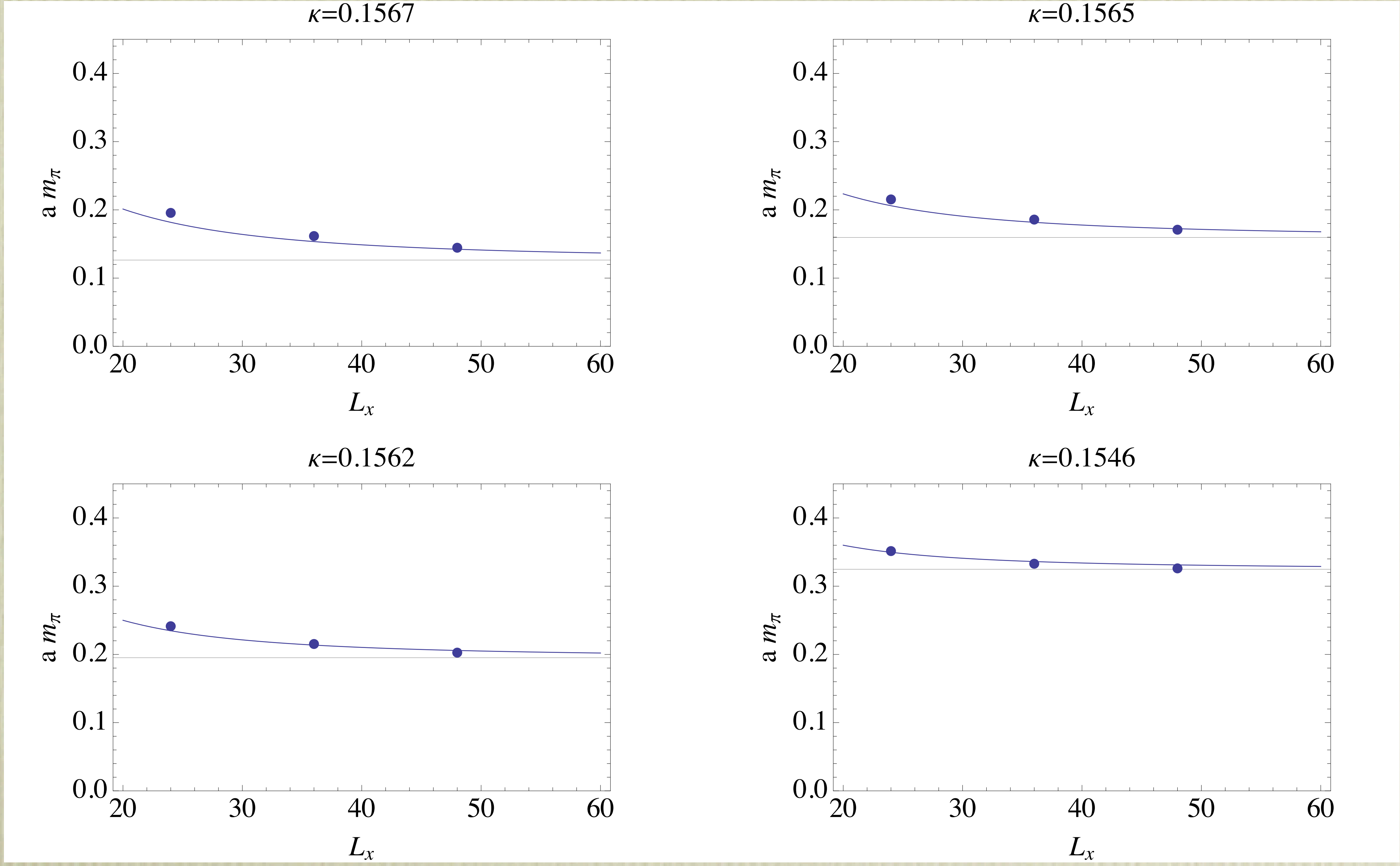
Dirichlet bc -- mass shift



Dirichlet bc -- mass shift



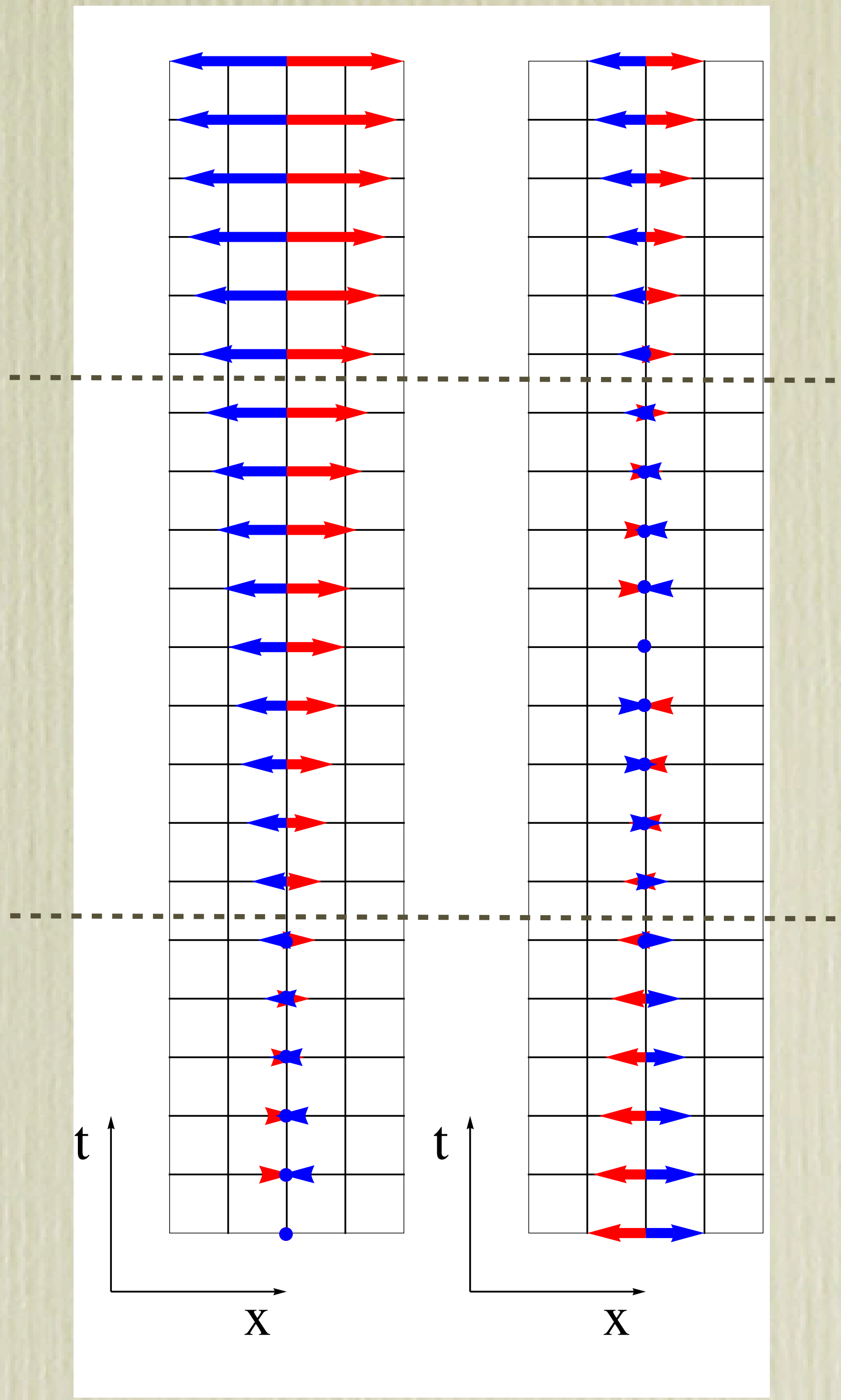
Pion mass



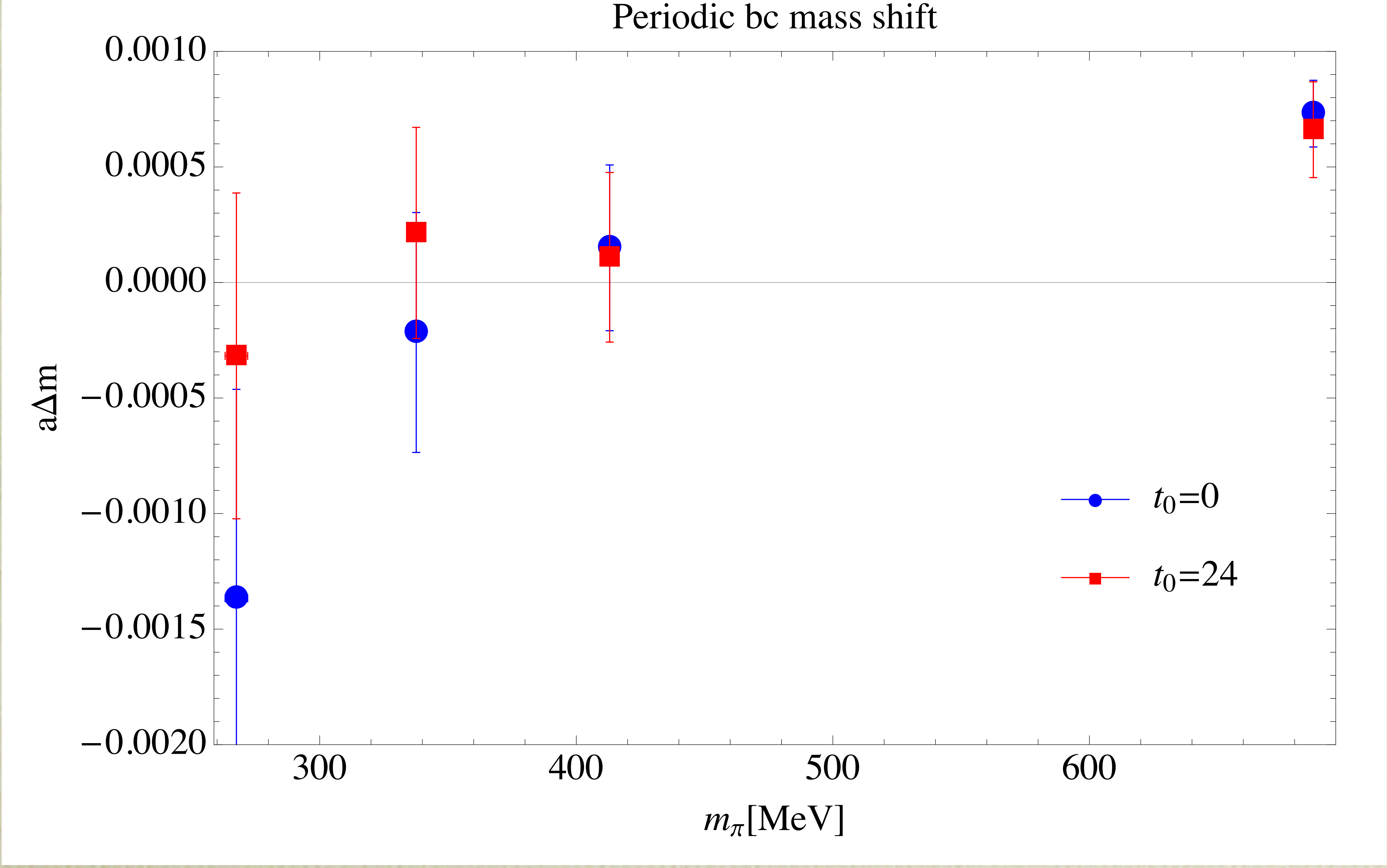
$$m_\pi(L) = \sqrt{m_\pi^2 + (\pi/L_x)^2}$$

Lattice parameters - periodic bc

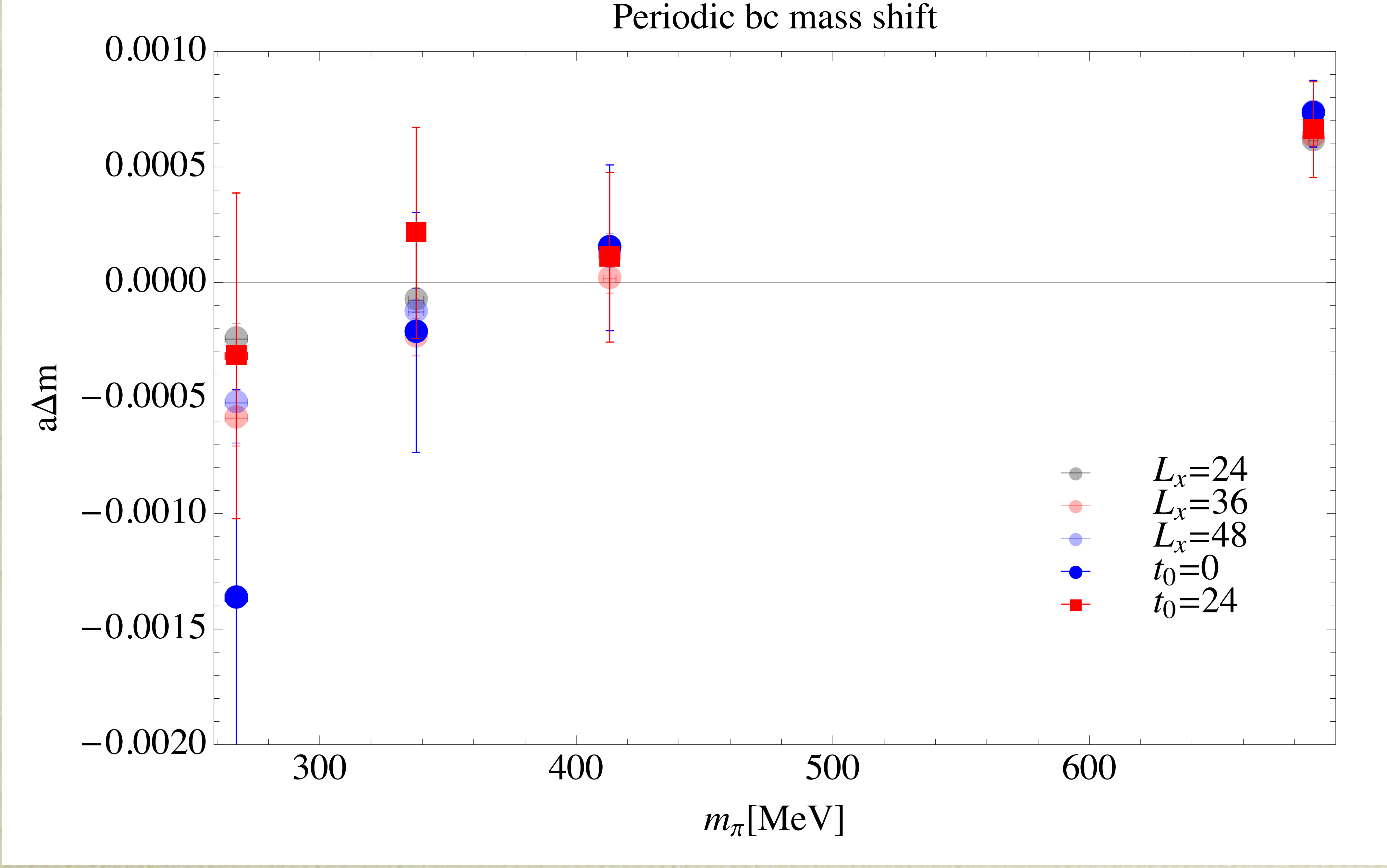
- Quenched ensemble - Wilson action β=6.0, a=0.093 fm
- Wilson fermions: 270 - 700 MeV ($m_\pi L$: 3.2-8)
- Electric field - $\eta = a^2 qE$ = 0.00576
- Lattice size: 24^3 x 48 -- 600 configurations
- t_0=0, 24



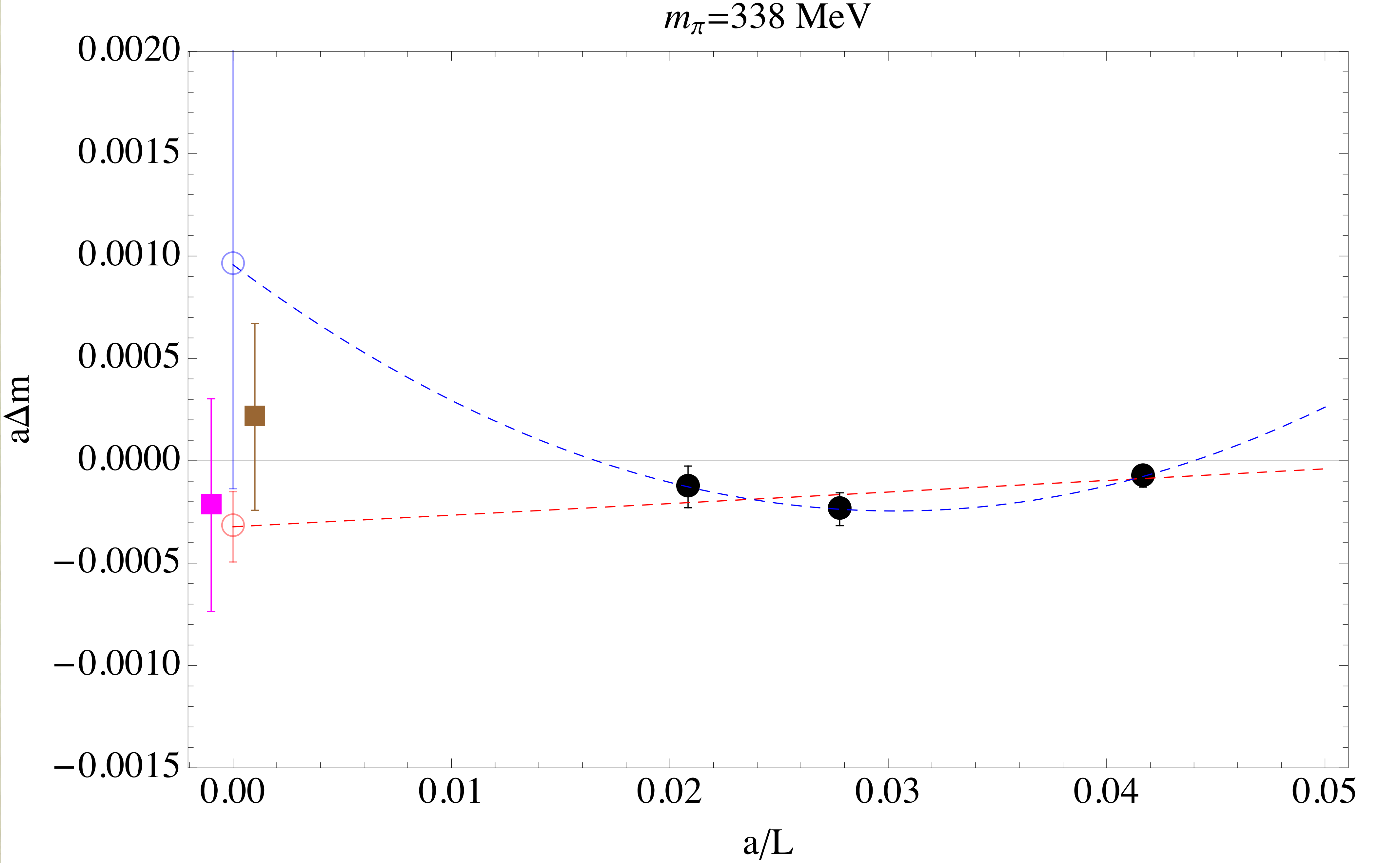
Periodic bc -- mass shift



Periodic bc -- mass shift



Infinite volume extrapolation



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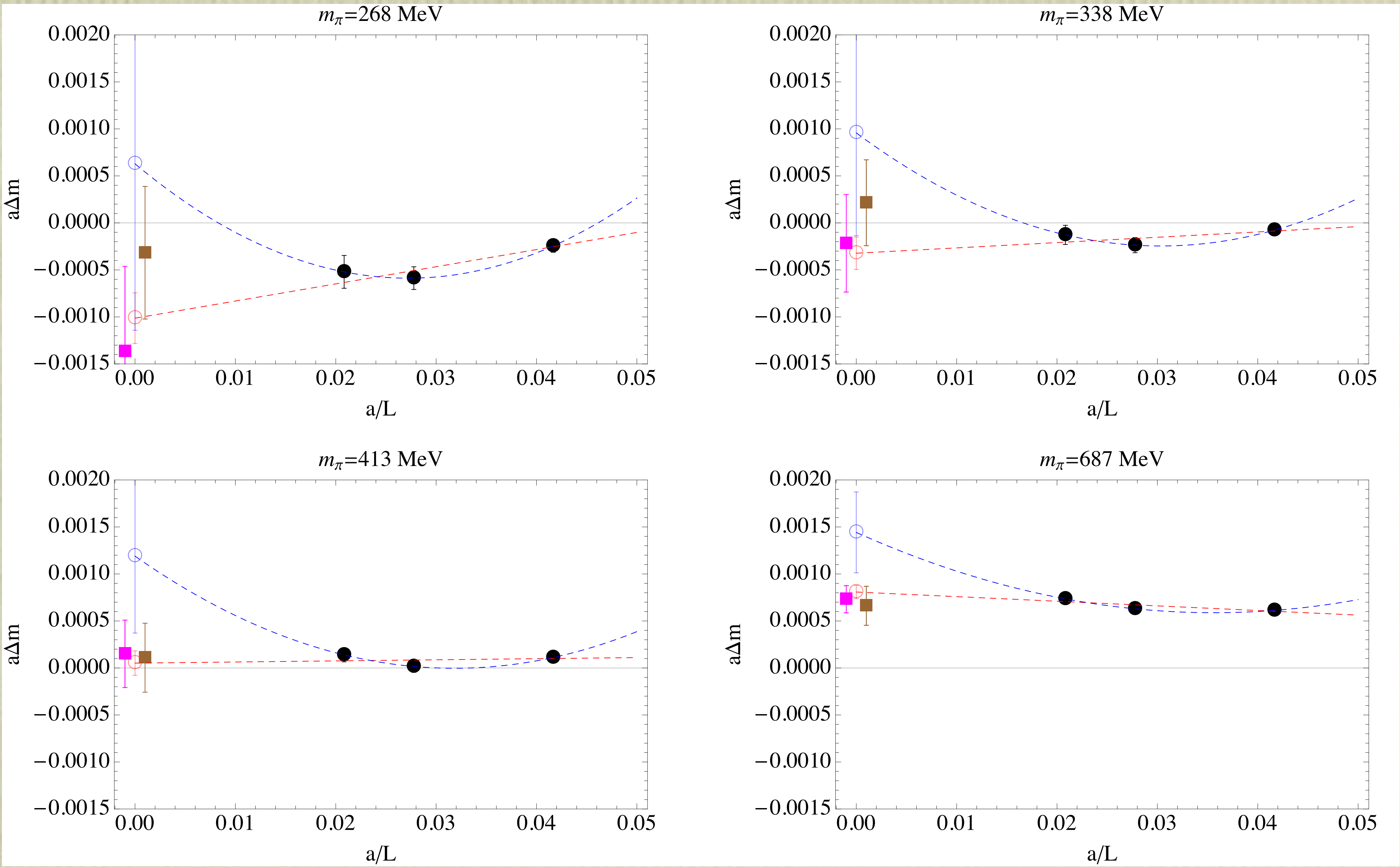

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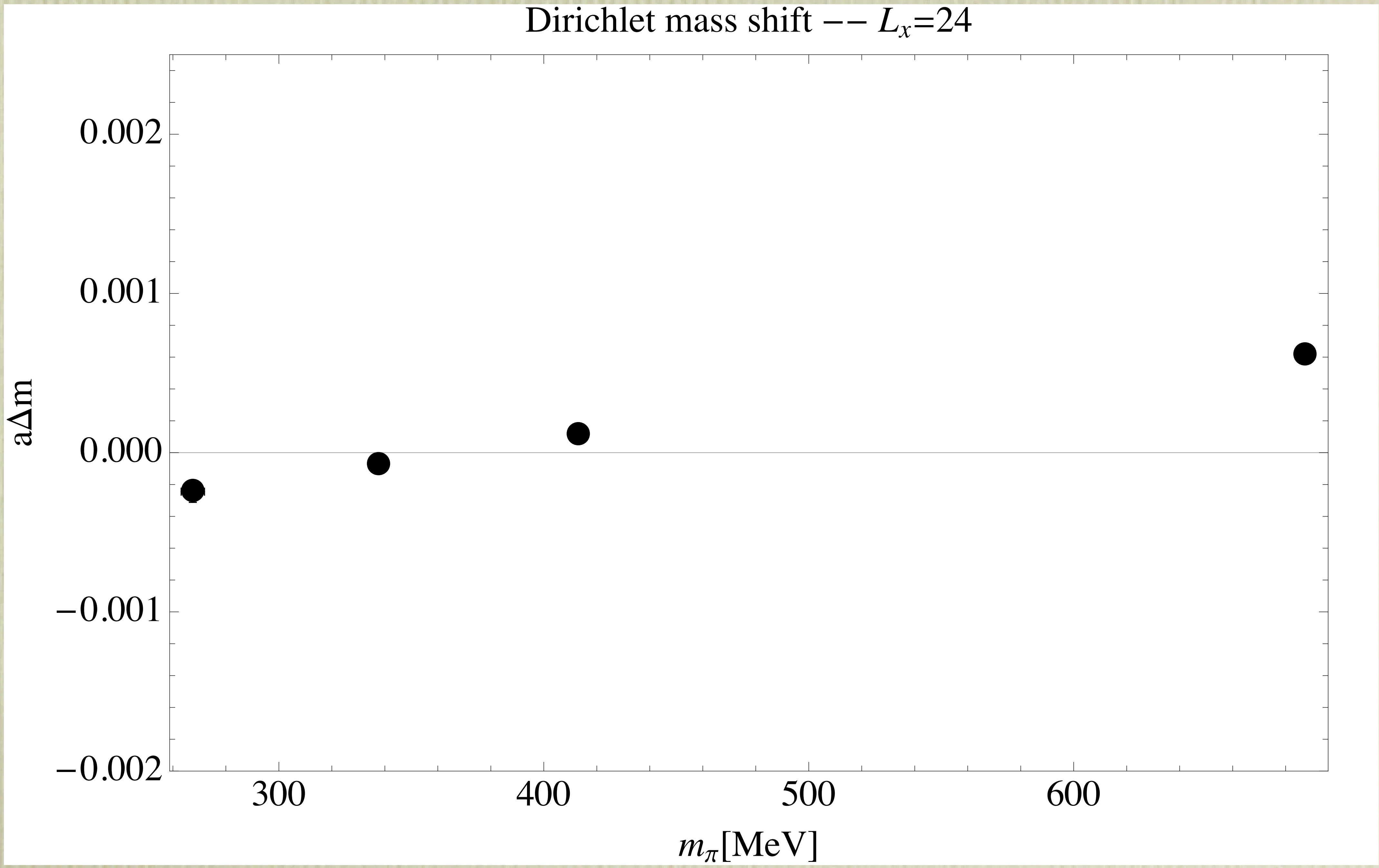

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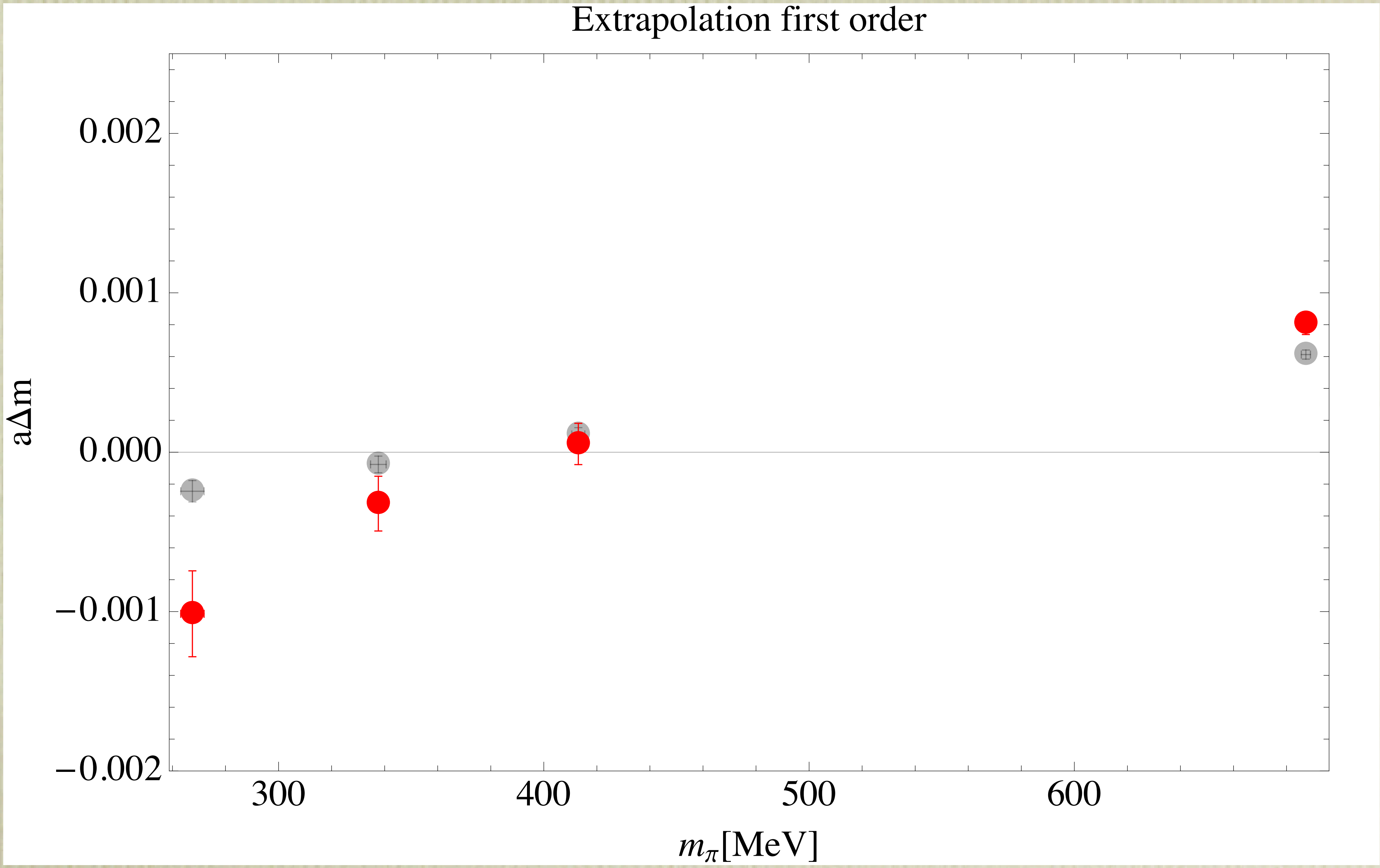

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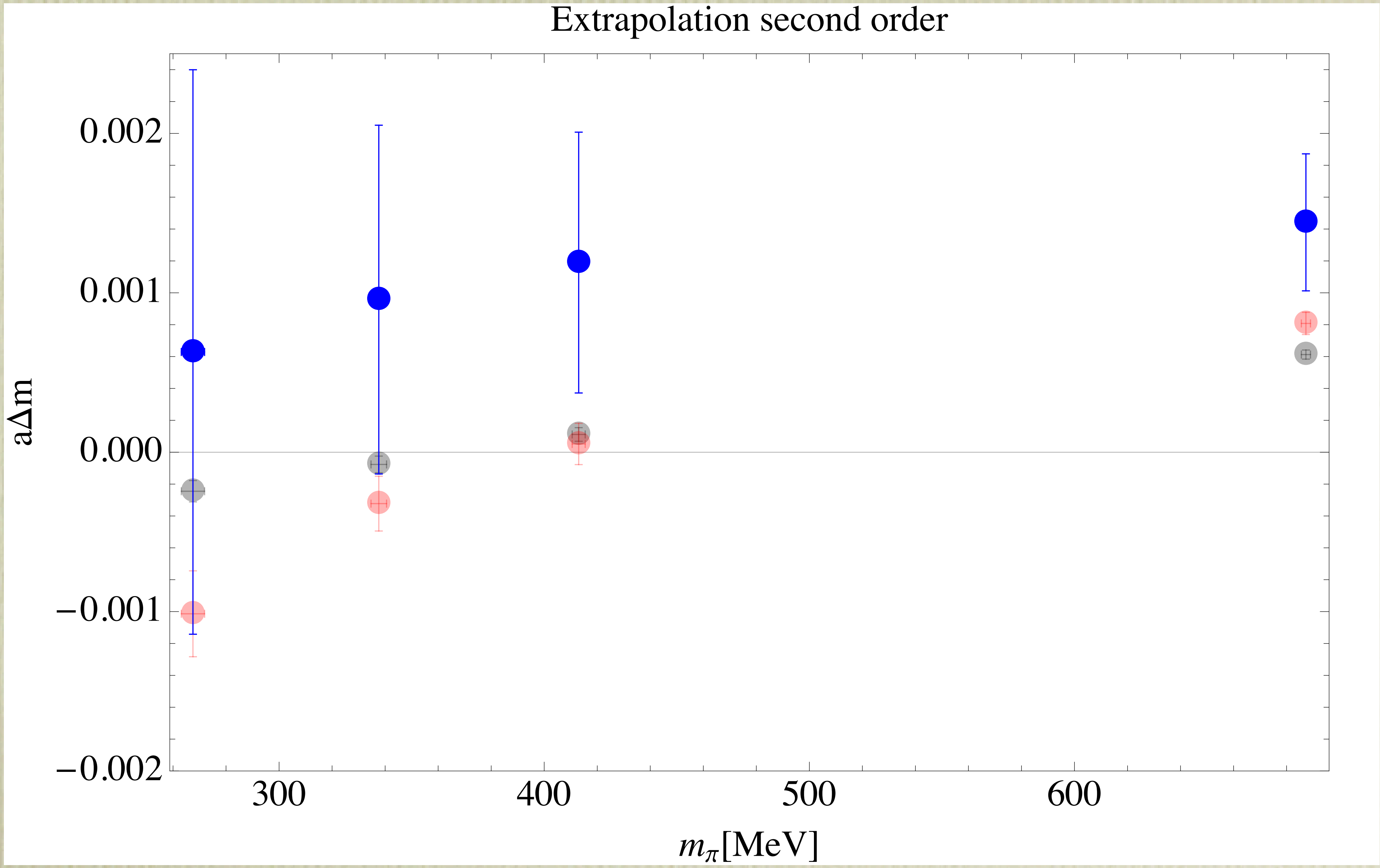

Conclusions

- We computed "pion" polarizability using Dirichlet and periodic boundary conditions
- The mass shifts computed with both methods are consistent -- we need to reduce the error bar in the periodic bc case to make the statement sharper
- For the periodic case the change in value when we shift the time origin is consistent with zero
- In the Dirichlet case the mass shift changes with volume -- it is possible that the negative value of the polarization is a finite volume effect
- The extrapolation is very sensitive to the order of the extrapolation -- we need to understand the functional dependence to extrapolate reliably