Longitudinal and transverse meson correlators in the deconfined phase

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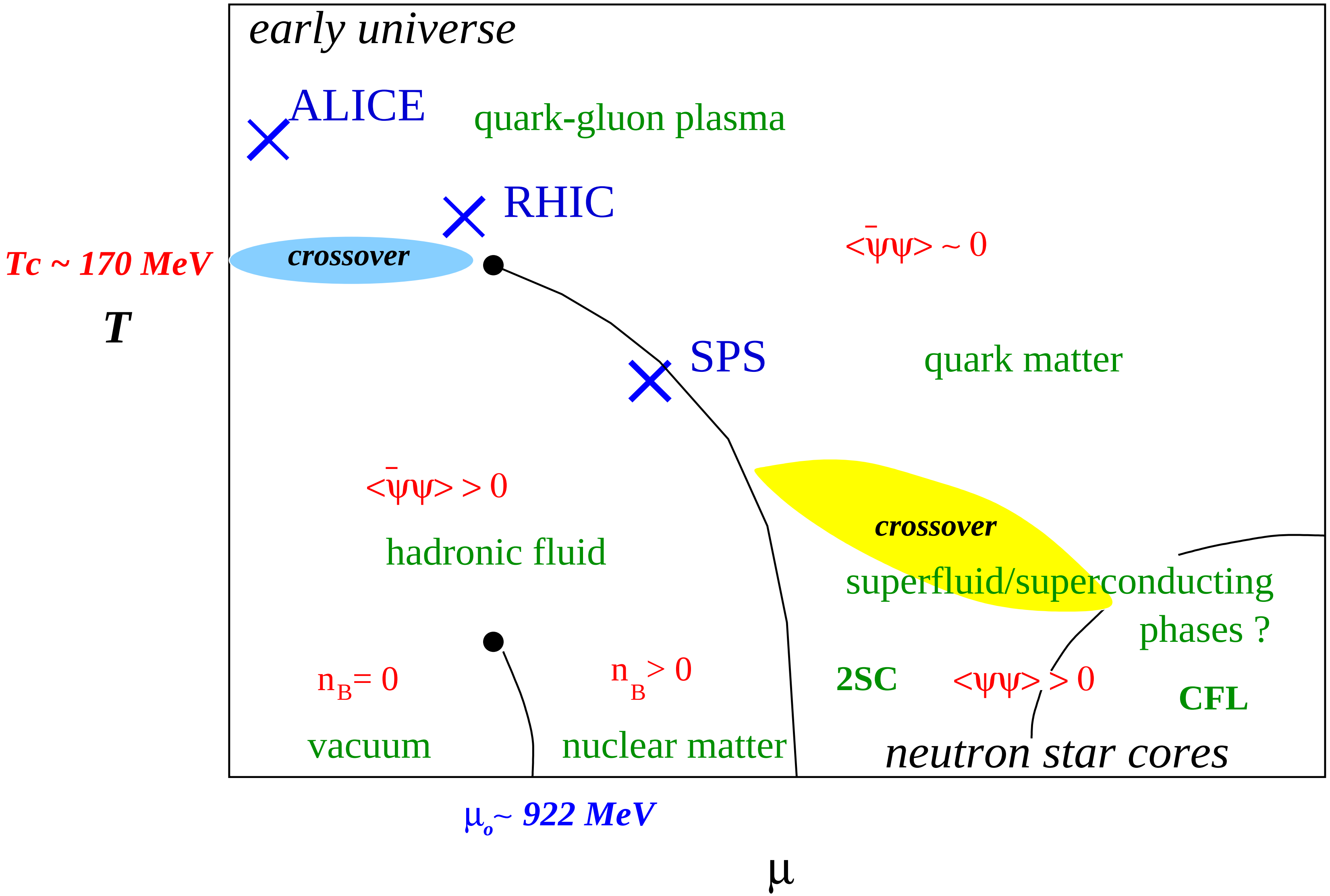
QCD phase diagram



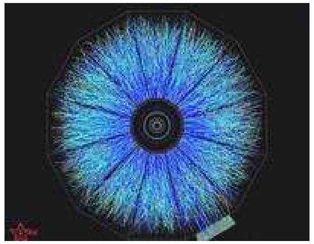
from S. Hands

Orientation...

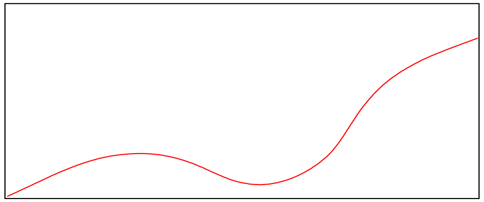
Continuum | **Lattice**

$T \neq 0$

Extreme QCD

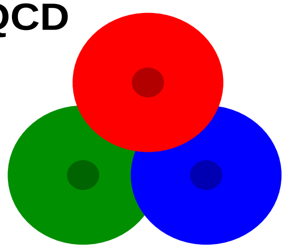


Spectral F'ns

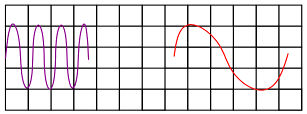


$T = 0$

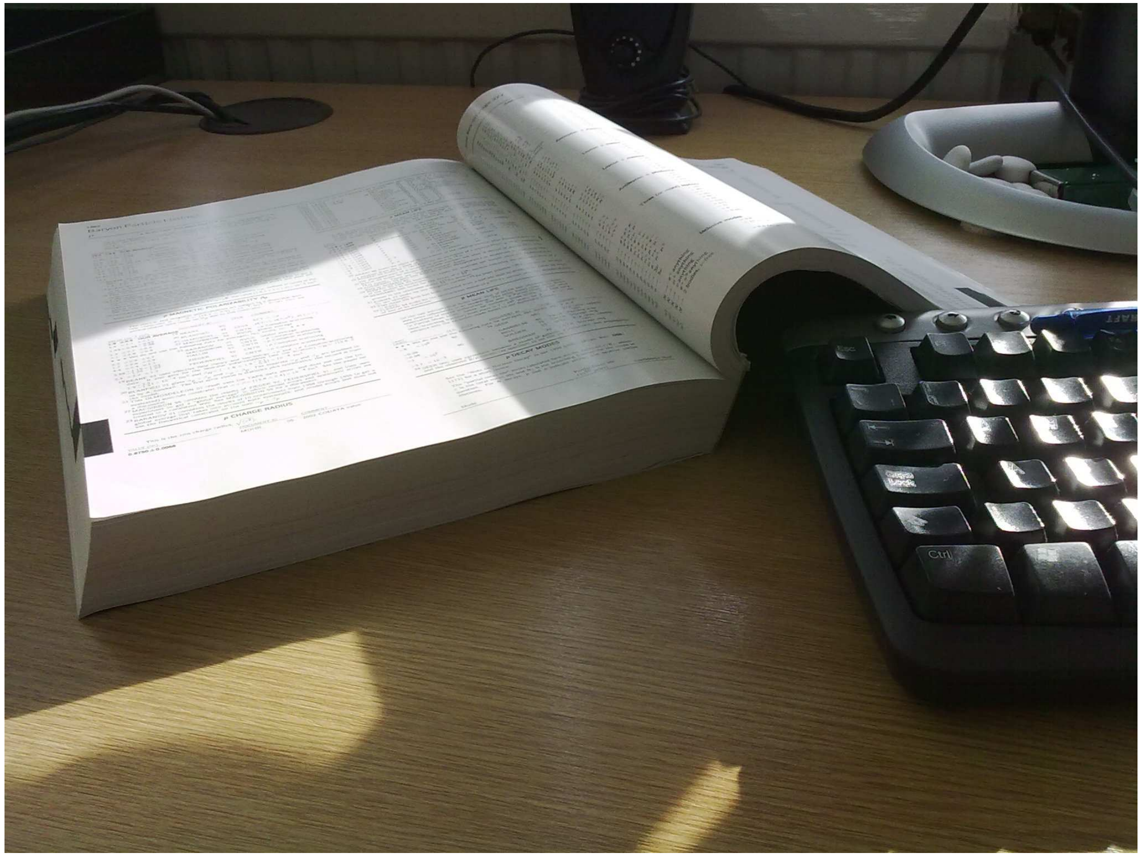
Ordinary QCD



Bound States



Particle Data Book



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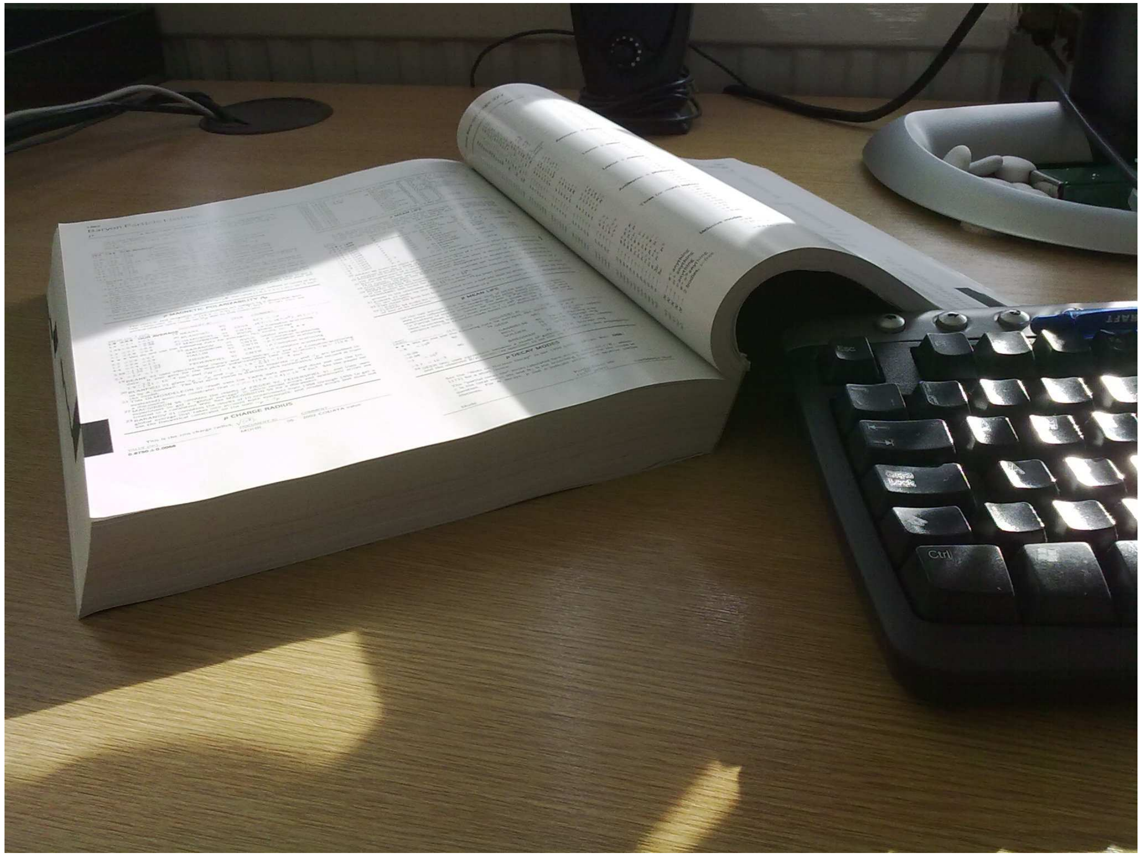

$\sim 1.5 \times 10^3$ pages

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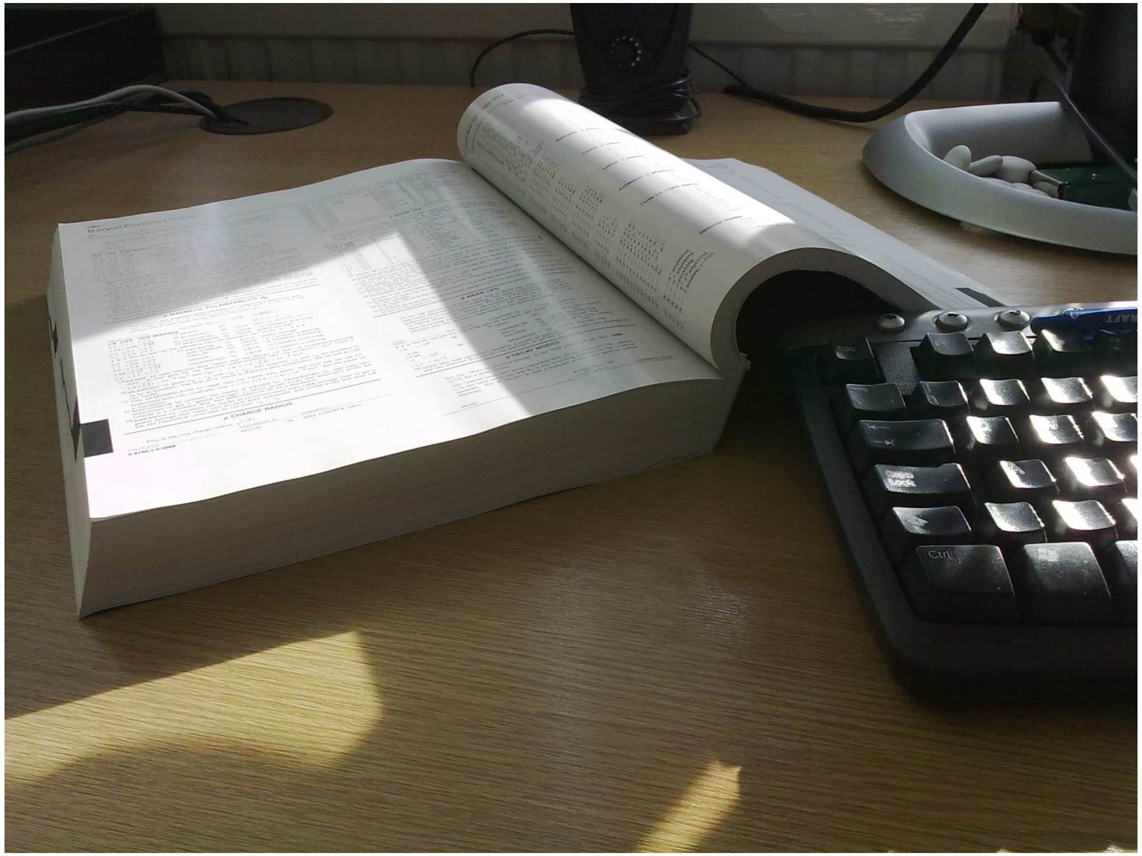


$\sim 1.5 \times 10^3$ pages

zero pages on Quark-Gluon Plasma...

Motivation

Do bound hadronic states persist into the “quark-gluon” plasma phase?

How can we extract transport coefficients?

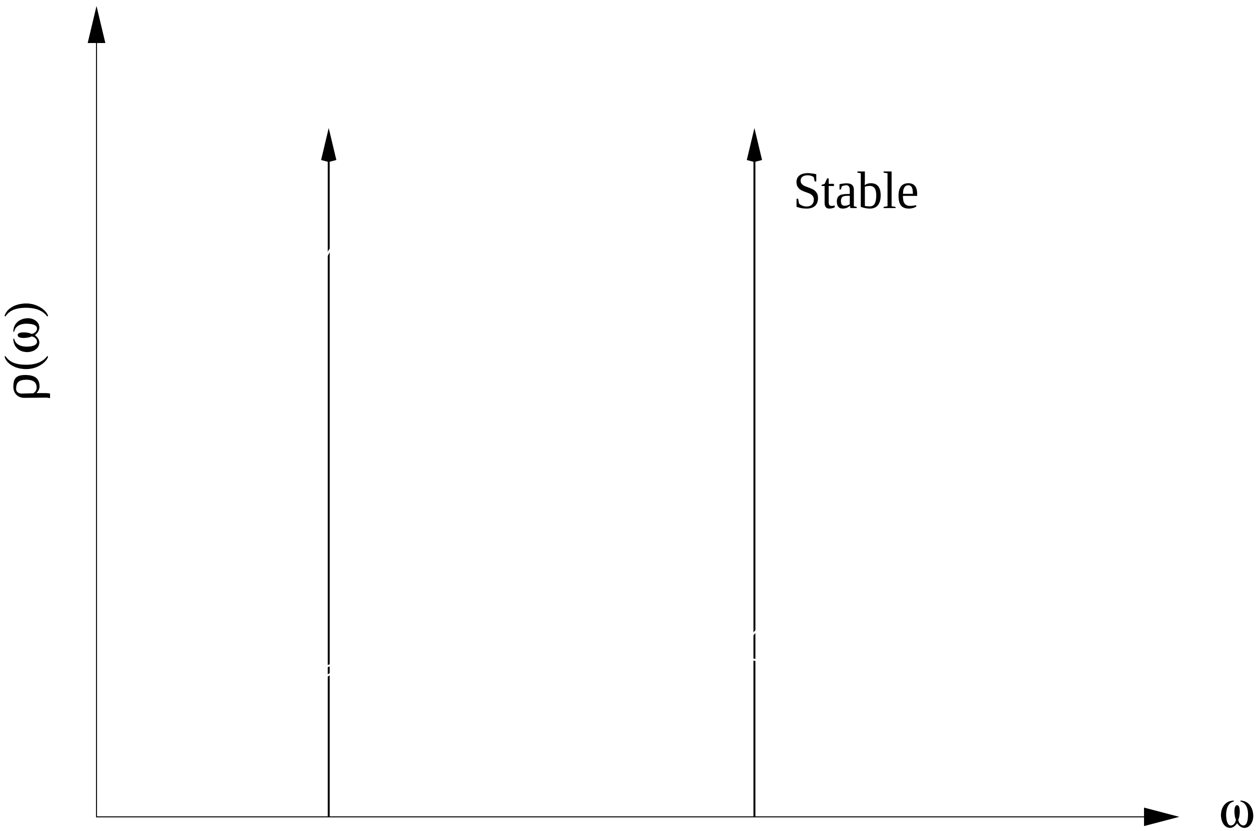
- *Spectral functions* can answer this!

$$G(t,\vec{p}) \;=\; \int \rho(\omega,\vec{p})\, K(t,\omega)\, d\omega$$

$\uparrow$	$\downarrow$	$\nwarrow$
Euclidean Correlator	Spectral Function	(Lattice) Kernel

Example Spectral Functions

$$G(t) \sim \int \rho(\omega)\, e^{-\omega t}\, d\omega$$



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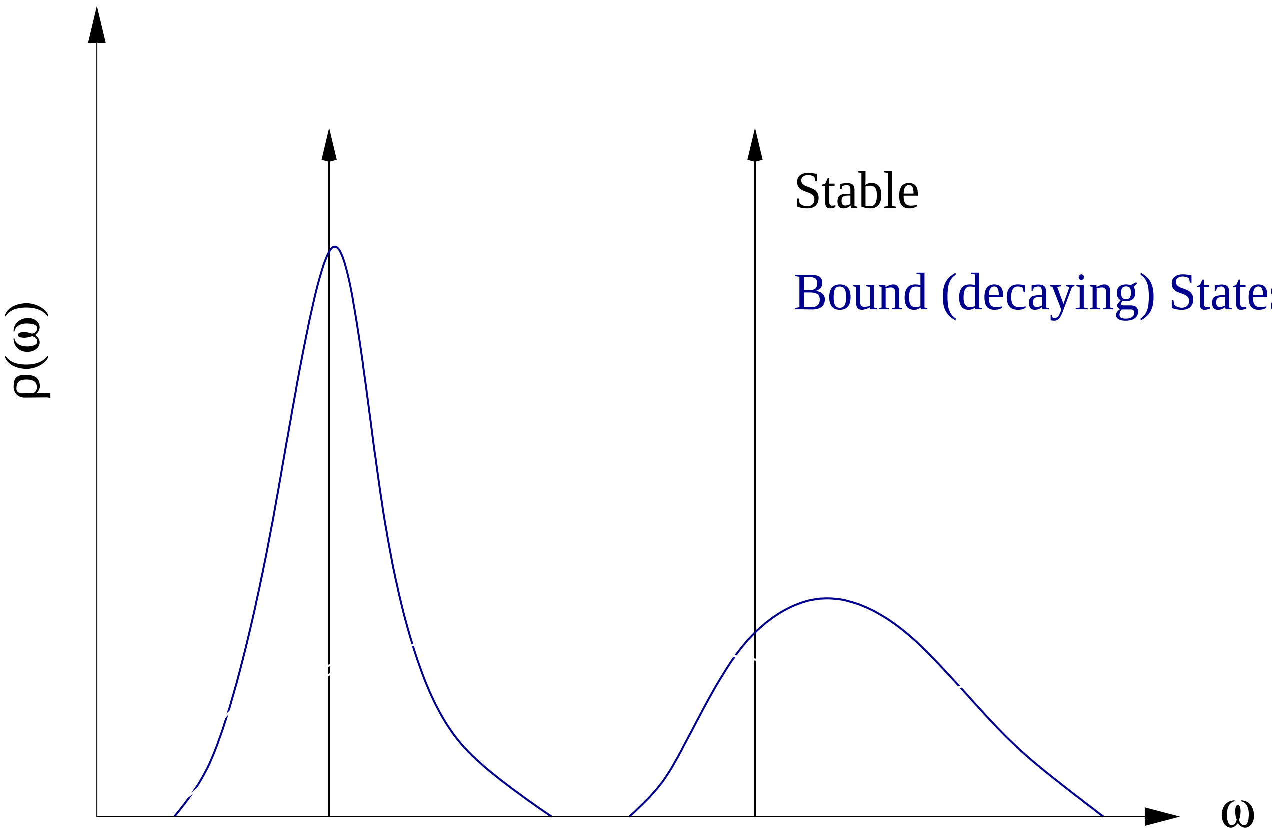

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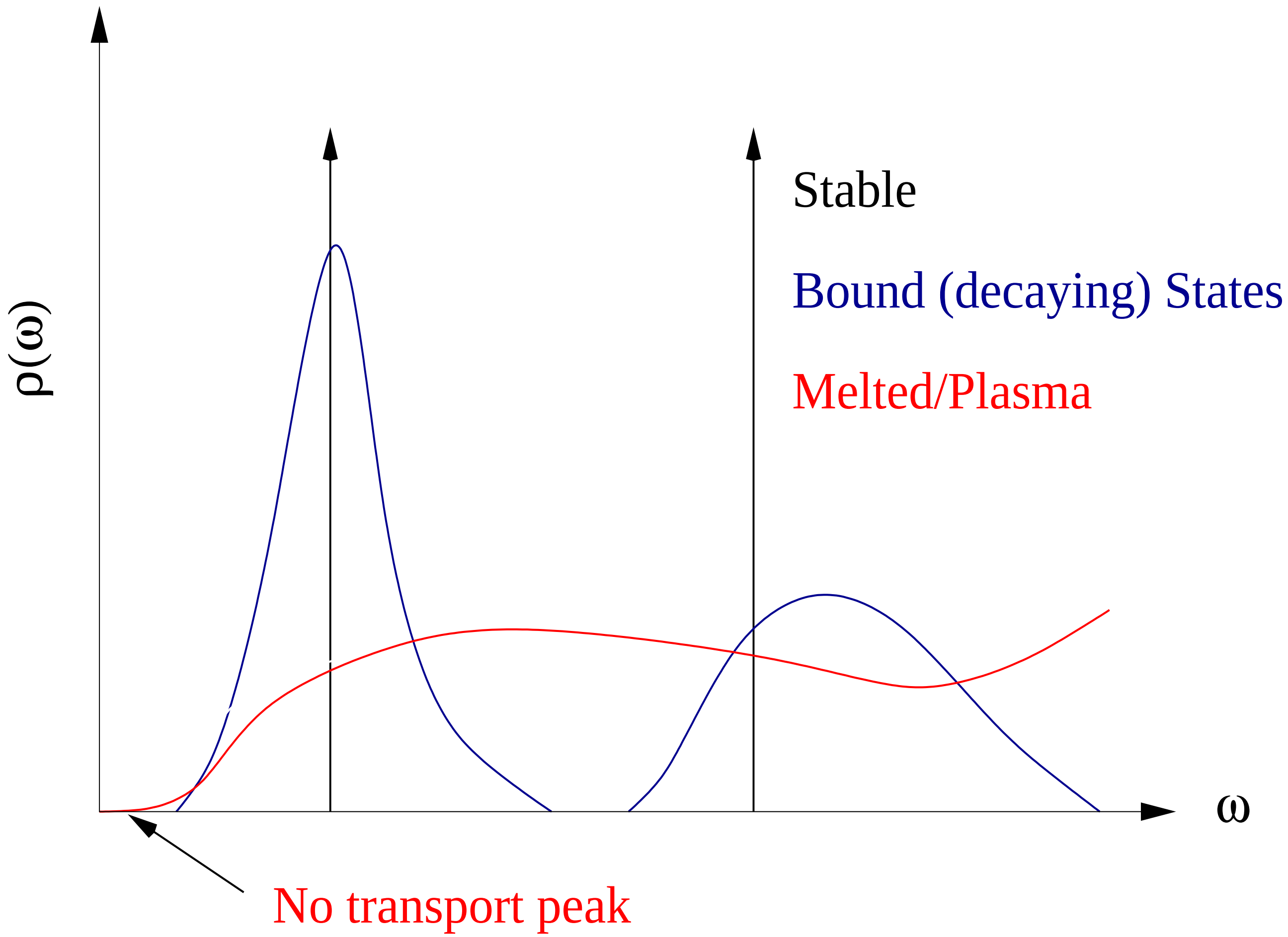

Example Spectral Functions

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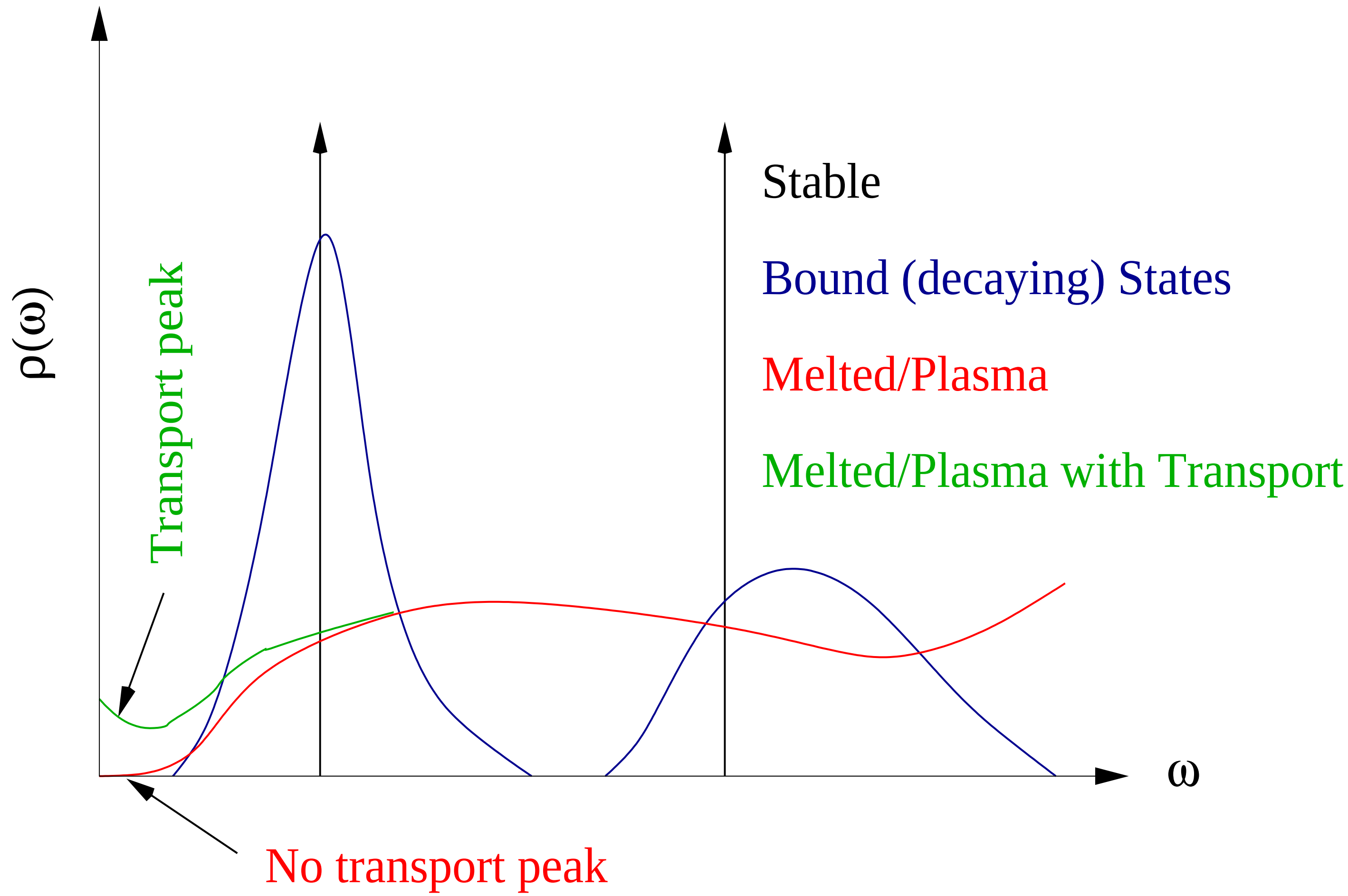


What's special about the Spectral Function?

- $\rho(\omega, \vec{p})$ contains info on
  - (in)stability of hadrons
  - transport coefficients
  - dilepton production . . .

- Extraction of a spectral density from a lattice correlator is an ill-posed problem:
  - *Given* $G(t)$ *derive* $\rho(\omega)$
  - *More* ω *data points then* t *data points!*

- Requires the use of Bayesian analysis -

  Maximum Entropy Method (MEM)

  - Asakawa, Hatsuda et al

MEM Instability

"Strange" behaviour of $\rho(\omega)$ near $\omega \sim 0$ traced to singular behaviour of $K(\omega,t)$ at $\omega = 0$:

$$\lim_{\omega \to 0} K(\omega,t) = \frac{2T}{\omega} + \mathcal{O}(\omega)$$

This is trivially corrected by defining:

$$\overline{K}(\omega,t) = \frac{\omega}{2T} K(\omega,t)$$

$$\overline{\rho}(\omega) = \frac{2T}{\omega} \rho(\omega)$$

and performing MEM on

$$G(t) = \int \overline{\rho}(\omega)\, \overline{K}(\omega,t)\, d\omega$$

Aarts, CRA, Foley, Hands & Kim, [arXiv:0703008]

we used default model $\overline{m}(\omega) = m_0(b + a\omega)$

Lattice Action

- Gluon Action
  - Wilson
- Quenched, Staggered
- Twisted Boundary Conditions
  - large range of momenta available
  - able to study longitudinal and transverse correlators
- Isotropic
- Run on Undergraduate Laboratory PC’s

Lattice Parameters

COLD

Lattice spacings	a^{-1}	~ 4 GeV
Spatial Volume	$N_s^3 \times N_t$	$48^3 \times 24$
T	$1/(aN_t)$	$T \sim 160 MeV \sim 0.62 T_c$
Statistics	N_{cfg}	~ 100

HOT

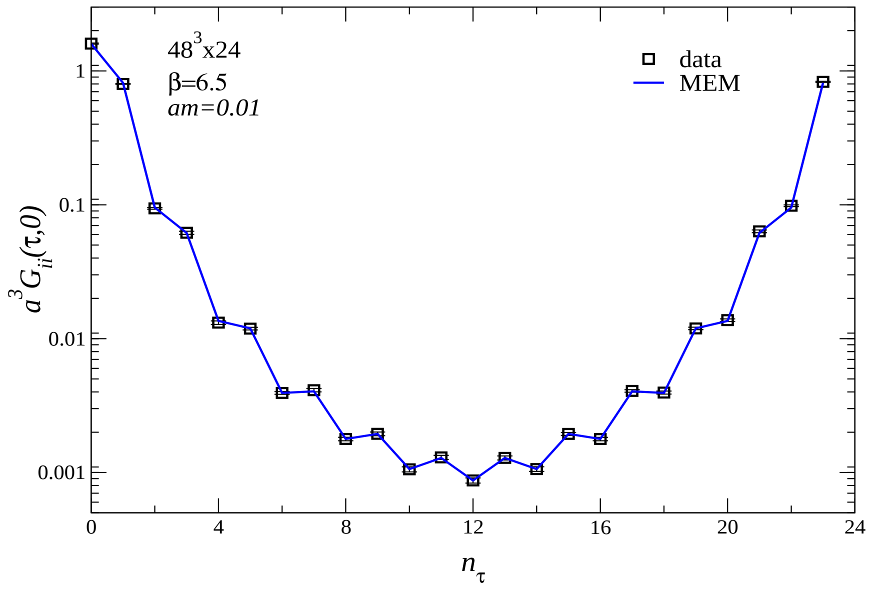
Lattice spacings	a^{-1}	~ 10 GeV
Spatial Volume	$N_s^3 \times N_t$	$64^3 \times 24$
T	$1/(aN_t)$	$T \sim 420 MeV \sim 1.5 T_c$
Statistics	N_{cfg}	~ 100

Momenta

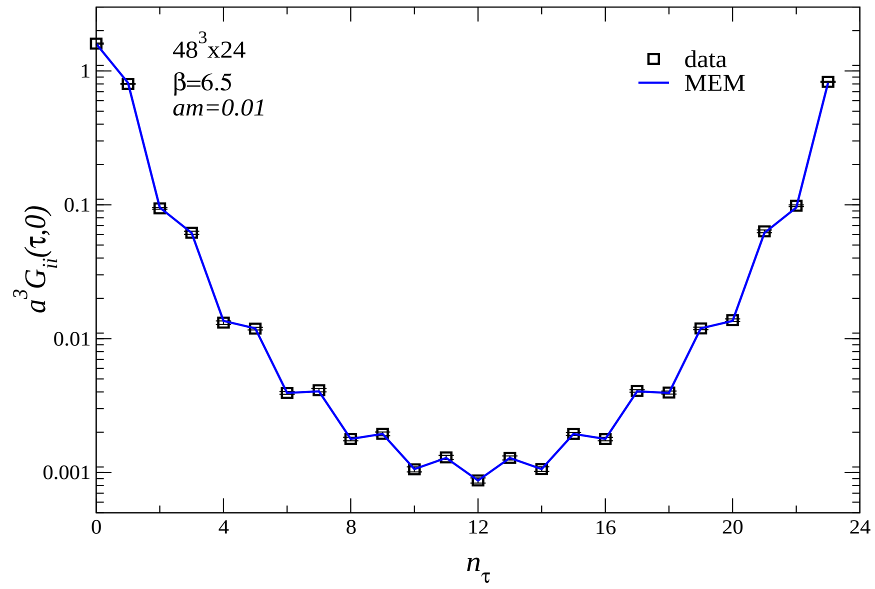
Flynn, Juttner, Sachrajda [hep-lat/0506016]

label	$\mathbf{p}L$	$\lvert p\rvert L$	Longitudinal	Transverse
zaa	$(0,\ 0,\ 0)$	0	any	any
zab	$(2,\ 0,\ 0)$	2	V1	V2 & V3
zac	$(0,\ \pi,\ 0)$	π	V2	V1 & V3
mac	$(0,\ -\pi,\ 0)$	π	V2	V1 & V3
zbc	$(-2,\ \pi,\ 0)$	$\sqrt{4+\pi^2} = 3.72$	-	V3
mbc	$(-2,\ -\pi,\ 0)$	$\sqrt{4+\pi^2} = 3.72$	-	V3
zcd	$(3,\ 3-\pi,\ 3)$	$\sqrt{18+(3-\pi)^2} = 4.25$	-	-
zbd	$(1,\ 3,\ 3)$	$\sqrt{19} = 4.36$	-	-
mbd	$(1,\ 3-2\pi,\ 3)$	$\sqrt{10+(3-2\pi)^2} = 4.56$	-	-
zad	$(3,\ 3,\ 3)$	$3\sqrt{3} = 5.20$	-	-
mad	$(3,\ 3-2\pi,\ 3)$	$\sqrt{18+(3-2\pi)^2} = 5.36$	-	-
paa	$(0,\ 2\pi,\ 0)$	$2\pi = 6.28$	V2	V1 & V3
maa	$(0,\ -2\pi,\ 0)$	$2\pi = 6.28$	V2	V1 & V3
pab	$(2,\ 2\pi,\ 0)$	$2\sqrt{1+\pi^2} = 6.59$	-	V3
mab	$(2,\ -2\pi,\ 0)$	$2\sqrt{1+\pi^2} = 6.59$	-	V3
pcd	$(3,\ 3+\pi,\ 3)$	$\sqrt{18+(3+\pi)^2} = 7.46$	-	-
mcd	$(3,\ 3-3\pi,\ 3)$	$3\sqrt{2+(1-\pi)^2} = 7.70$	-	-
pac	$(0,\ 3\pi,\ 0)$	$3\pi = 9.42$	V2	V1 & V3
pbc	$(-2,\ 3\pi,\ 0)$	$\sqrt{4+9\pi^2} = 9.63$	-	V3
pbd	$(1,\ 3+2\pi,\ 3)$	$\sqrt{10+(3+2\pi)^2} = 9.81$	-	-
pad	$(3,\ 3+2\pi,\ 3)$	$\sqrt{18+(3+2\pi)^2} = 10.21$	-	-

Staggered Correlators



Staggered Correlators



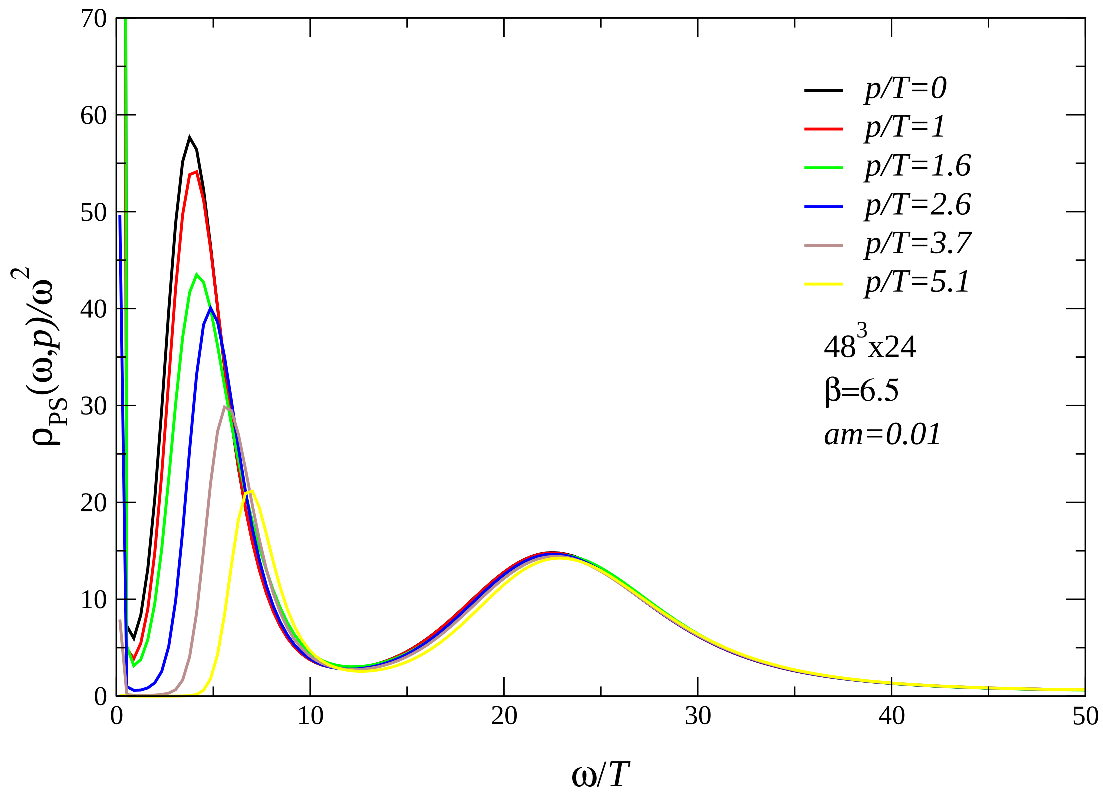
$$G(t) = 2\int \frac{d\omega}{2\pi} K(t,\omega)\left(\rho(\omega) - (-1)^t\,\tilde{\rho}(\omega)\right)$$

$\longrightarrow$ Have to fit to even & odd times separately, then use

$$\rho^{\text{phys}} = \frac{1}{2}\left(\rho^{\text{even}} + \rho^{\text{odd}}\right)$$

MEM results below T_c

Momentum dependence of spectral function (below T_c)



- Can see it moving
- Above was with un-corrected MEM
  - Evidence of MEM singularity at $\omega \sim 0$

Electrical Conductivity (Old Result)

$$\frac{\sigma}{T} = \lim_{\omega \to 0} \frac{\rho(\omega)}{6\omega T}$$

$\sigma =$ Conductivity

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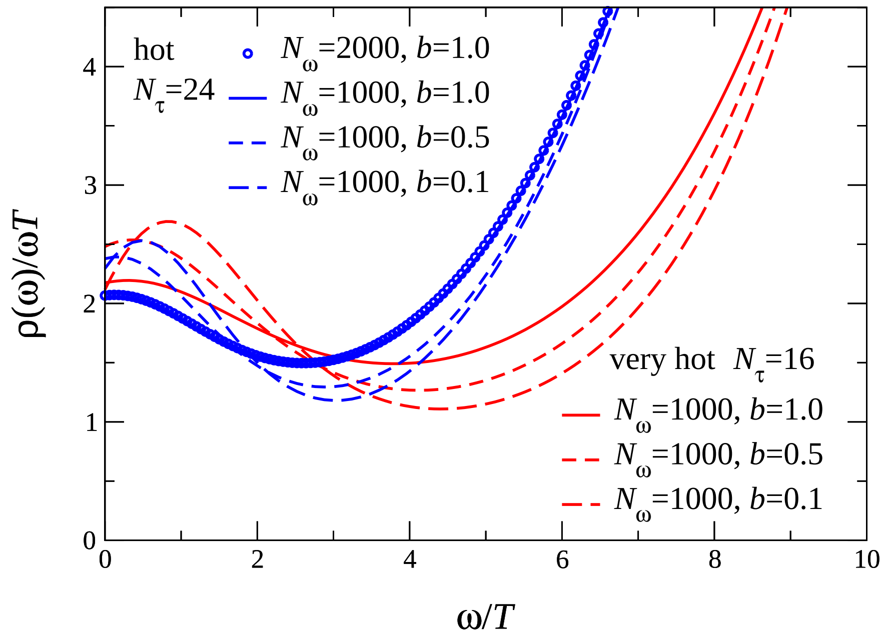
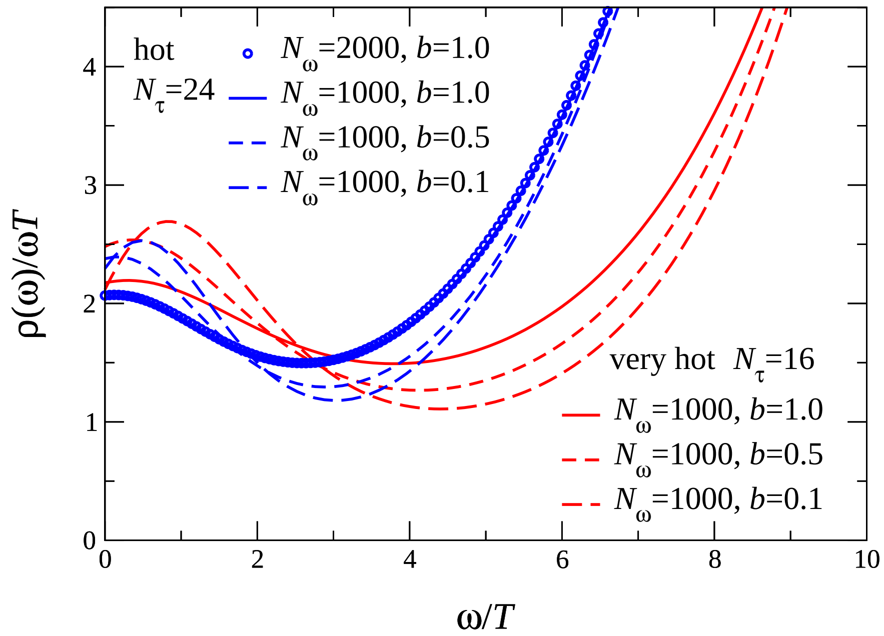


Electrical Conductivity (Old Result)

$$\frac{\sigma}{T} = \lim_{\omega \to 0} \frac{\rho(\omega)}{6\omega T} \qquad \sigma = \text{Conductivity}$$



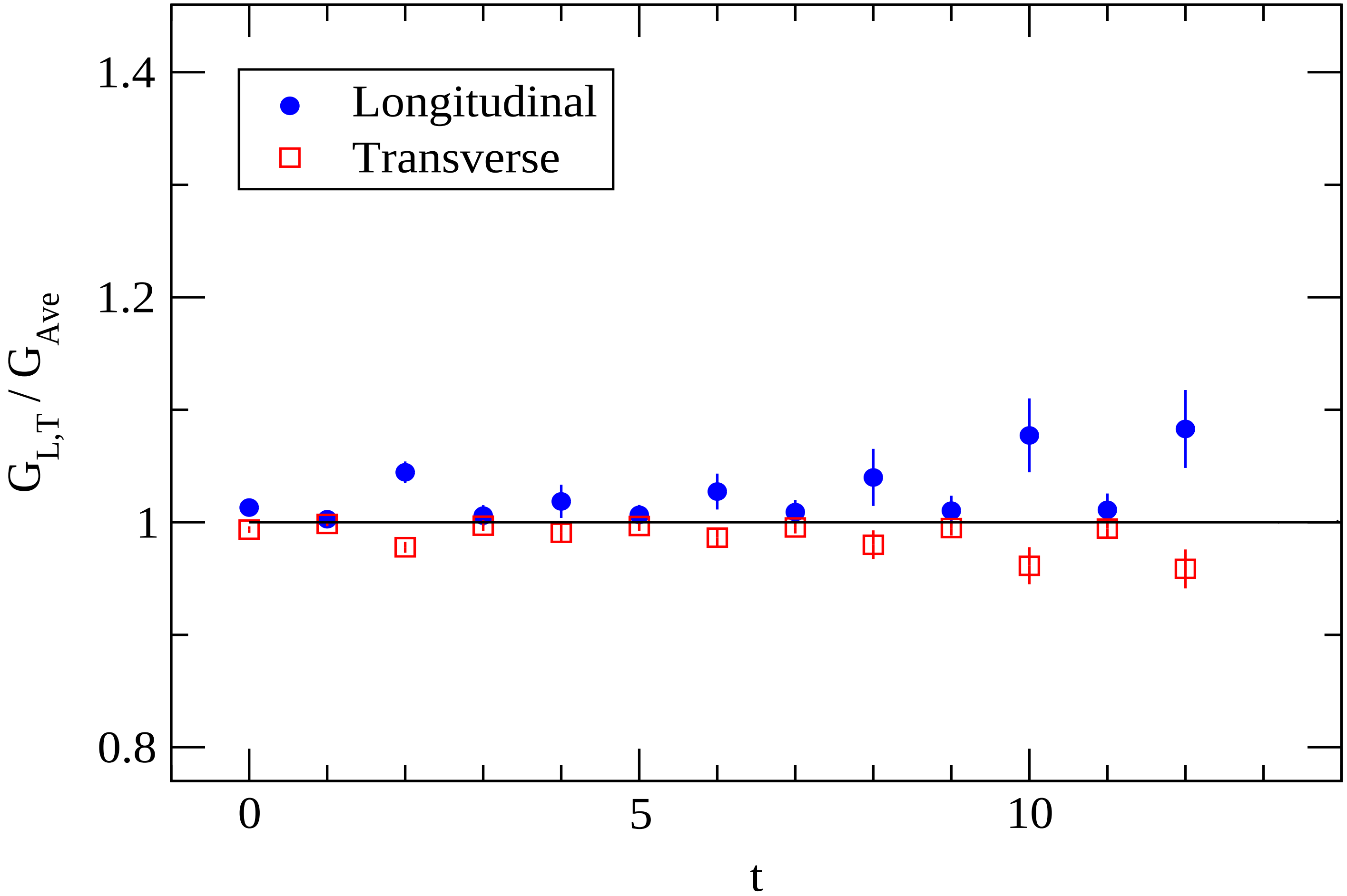
$$\longrightarrow \quad \sigma/T = 0.4 \pm 0.1$$

Aarts, CRA, Foley, Hands & Kim, [arXiv:0703008]

See also Karsch, Tuesday’s talk and
S. Gupta, Phys. Lett. B597, 57(2004)

Longitudinal versus Transverse $[T \sim 1.5T_c]$

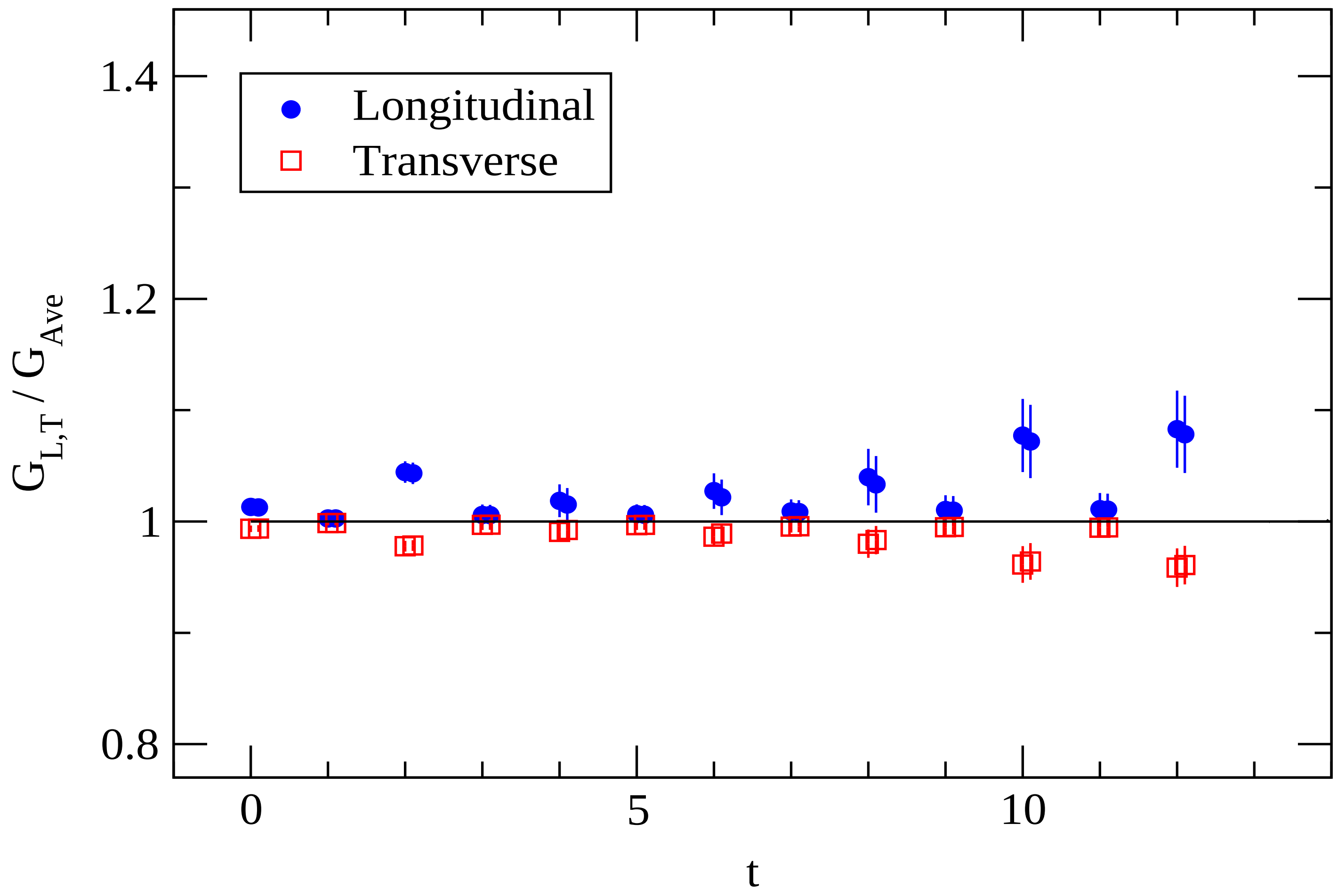
Vector correlator, for $pL = 0$:



$$G_{Ave}(t) = (G_L + 2G_T)/3$$

Longitudinal versus Transverse $[T \sim 1.5T_c]$

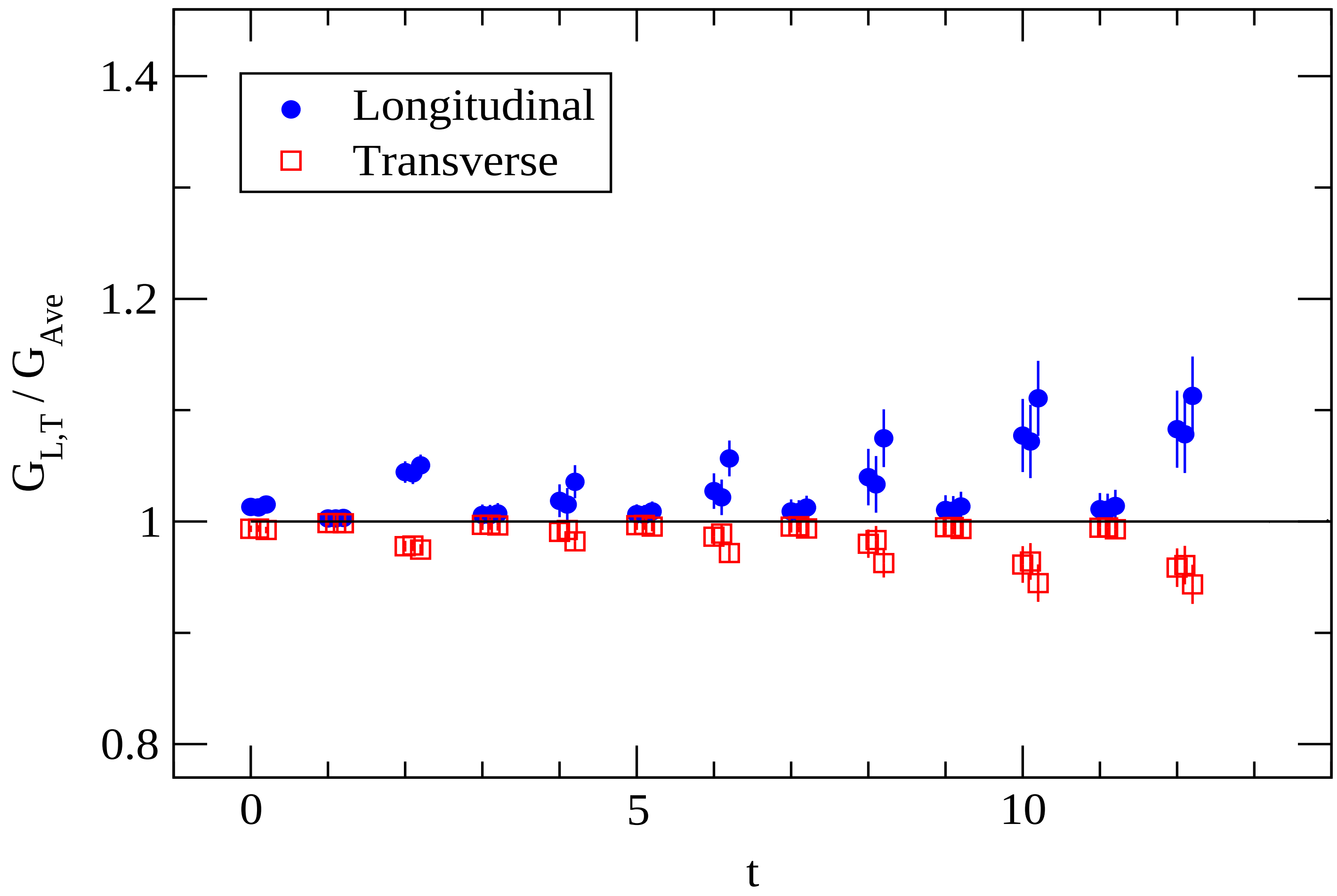
Vector correlator, for $pL = 0, 2$:



$$G_{Ave}(t) = (G_L + 2G_T)/3$$

Longitudinal versus Transverse $[T \sim 1.5T_c]$

Vector correlator, for $pL = 0, 2, \pi$:



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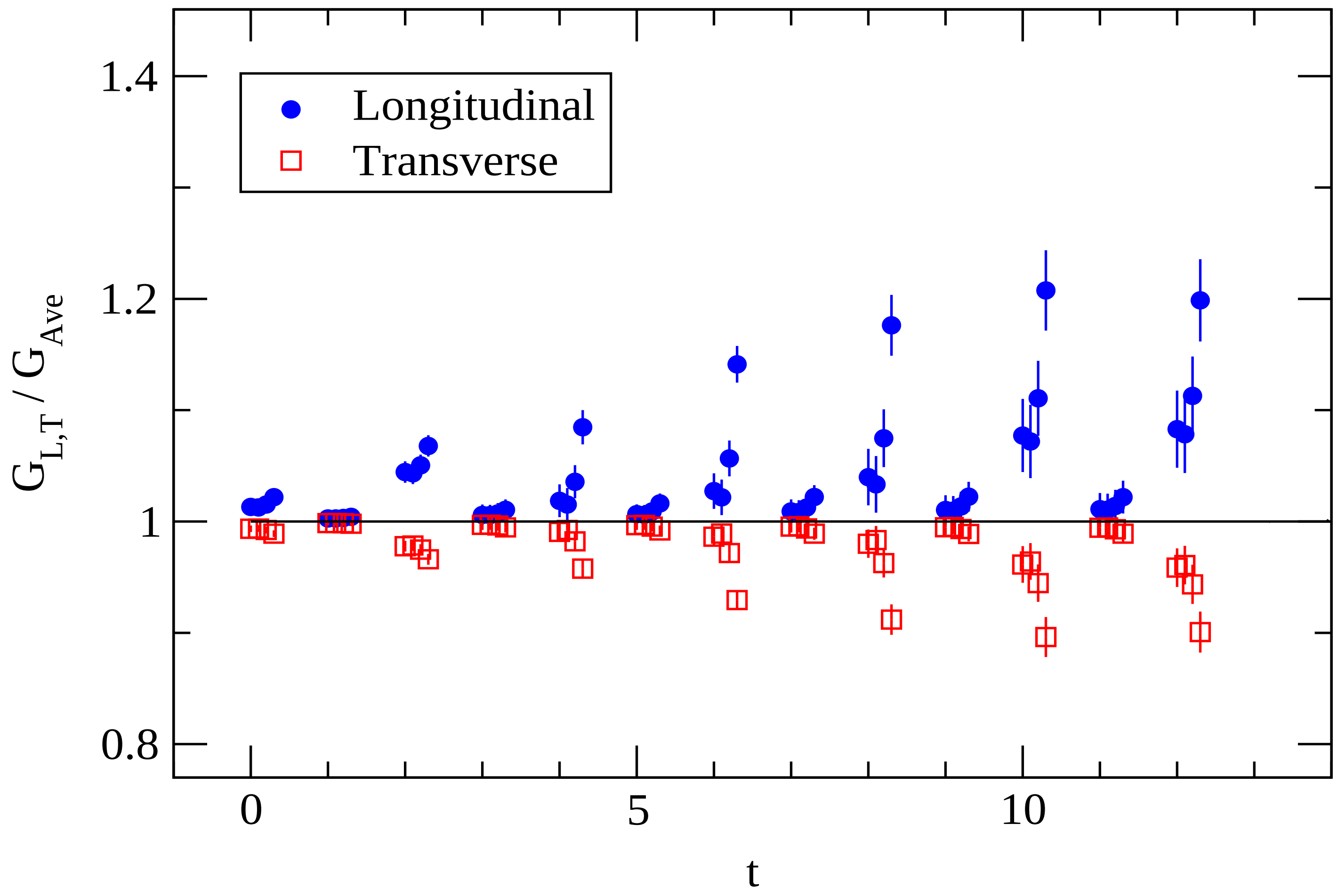

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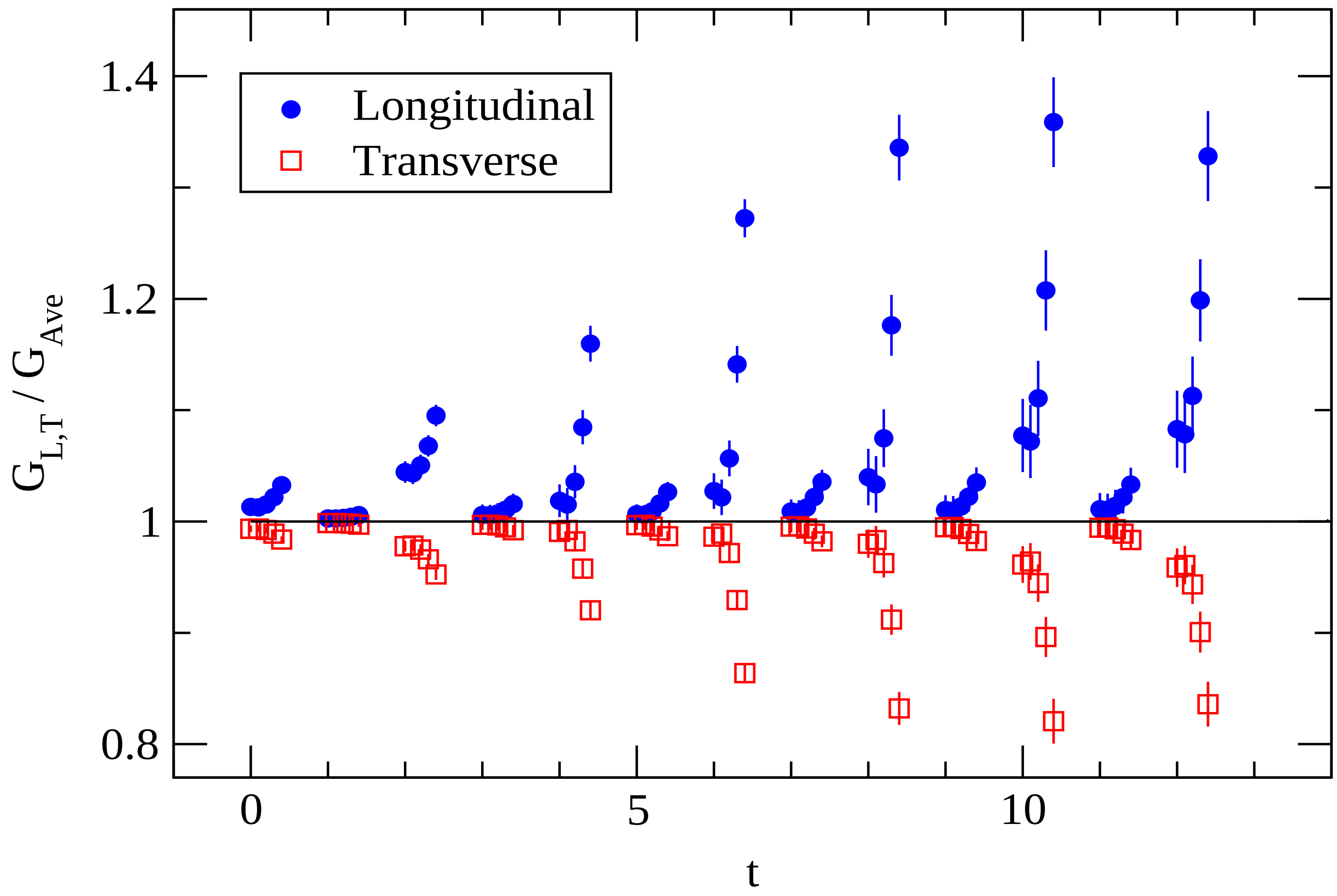
Longitudinal versus Transverse $[T \sim 1.5T_c]$

Vector correlator, for $pL = 0, 2, \pi, 2\pi, 3\pi$:



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Diffusivity

D obtained from the momentum dependency of $\rho^{\text{Tran}}_{\text{Vector}}$ (at small mass).

Note Hong & Teaney arXiv:1003.0699 prediction:

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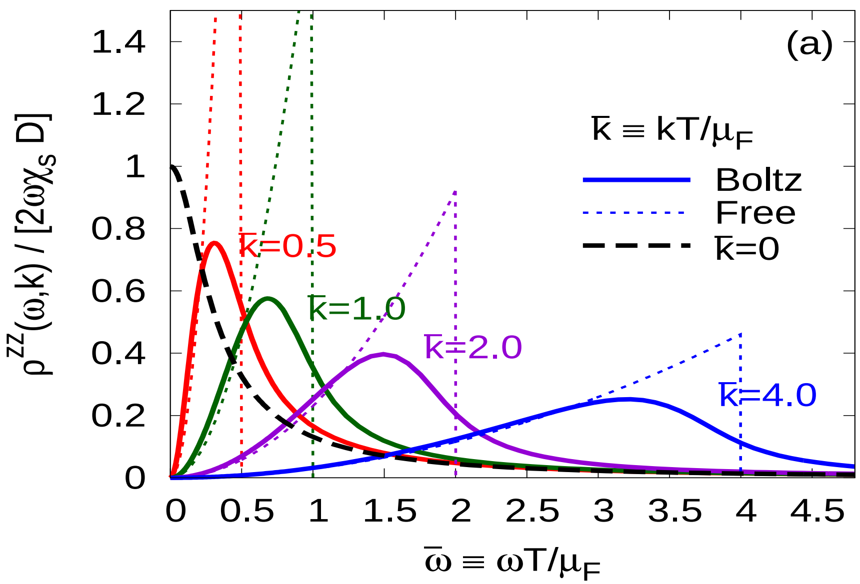
Longitudinal



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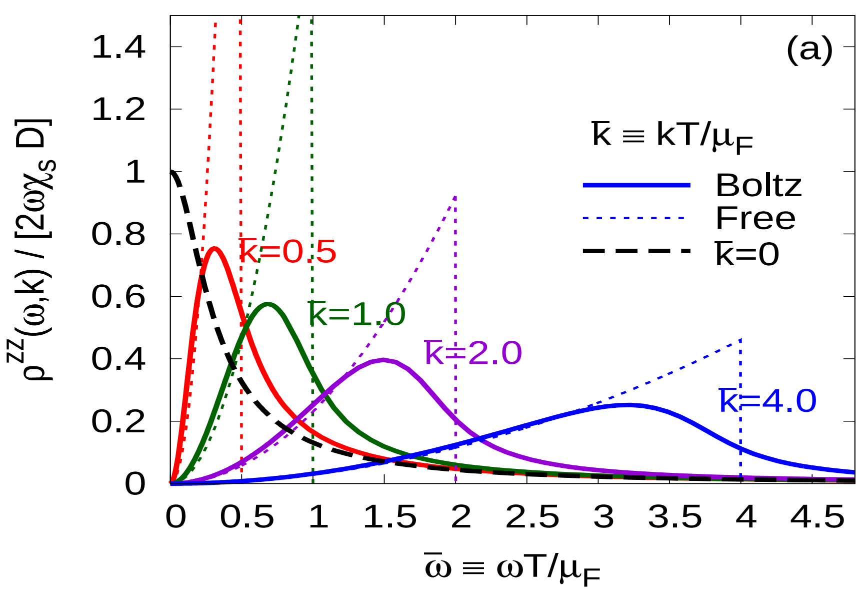
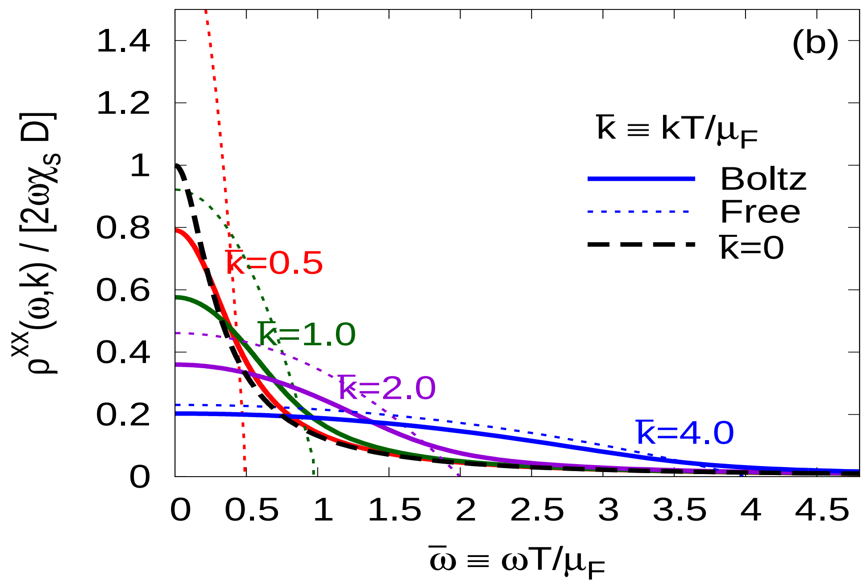


Transverse



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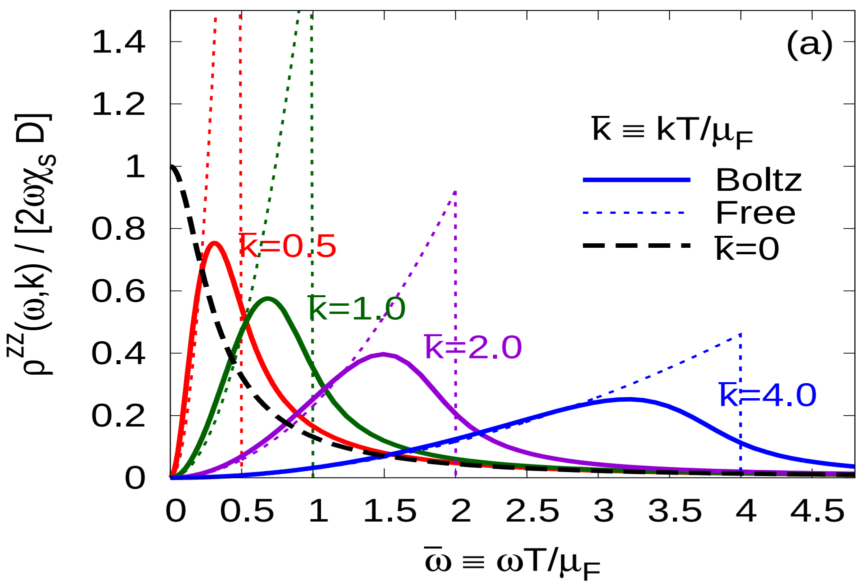

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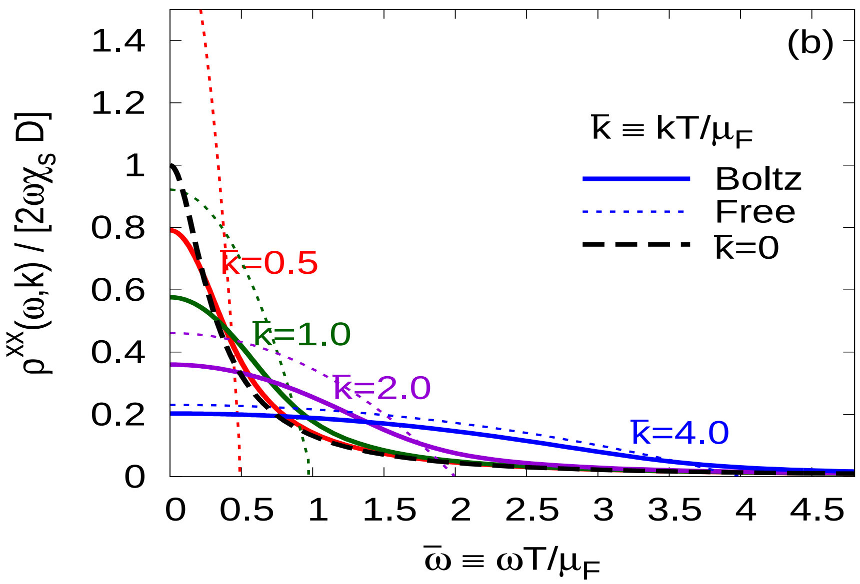
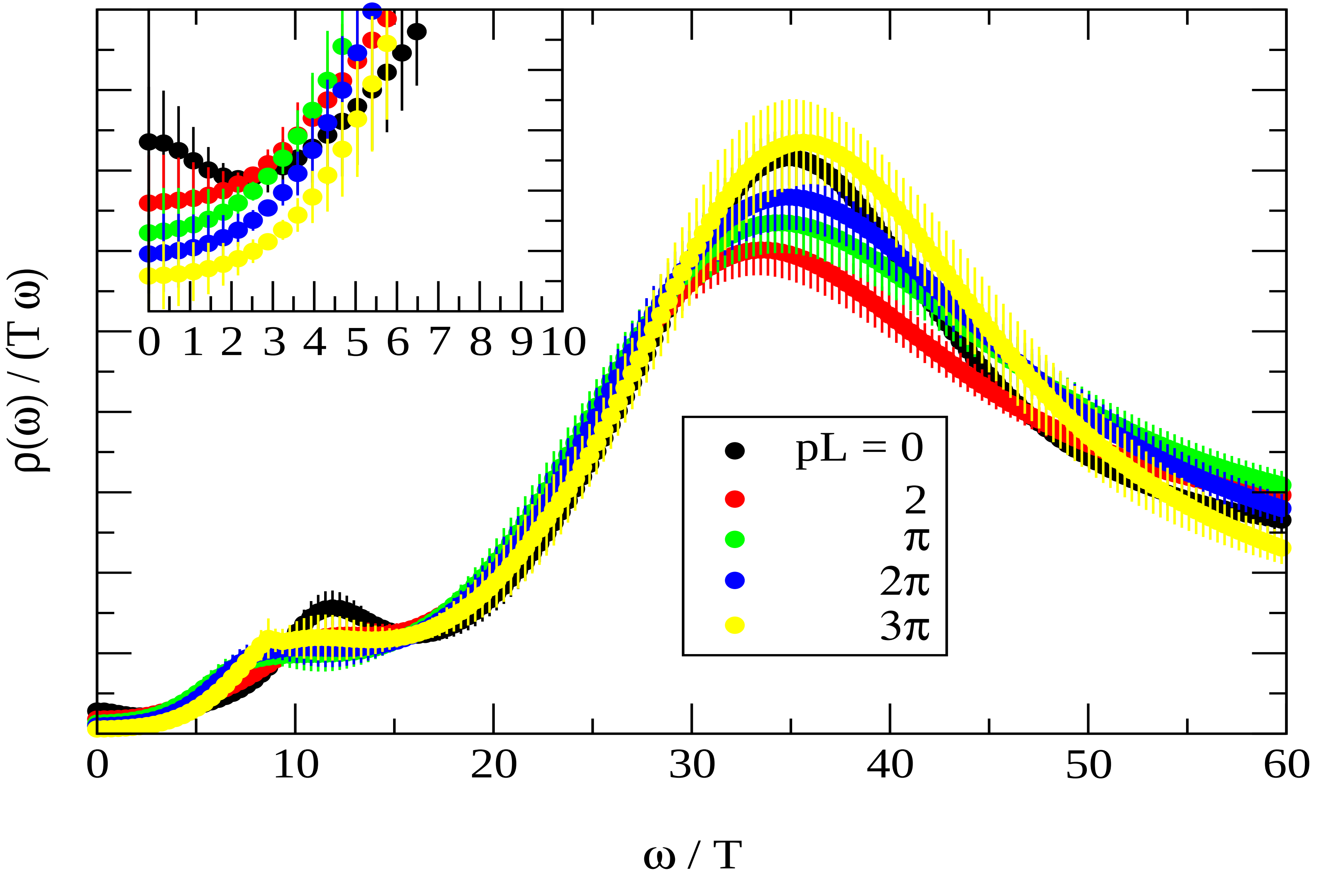


i.e. as $\omega \to 0$ (for $k \neq 0$):

$\rho^{\text{Long}}_{\text{Vector}} \to 0$ but $\rho^{\text{Tran}}_{\text{Vector}} \neq 0$

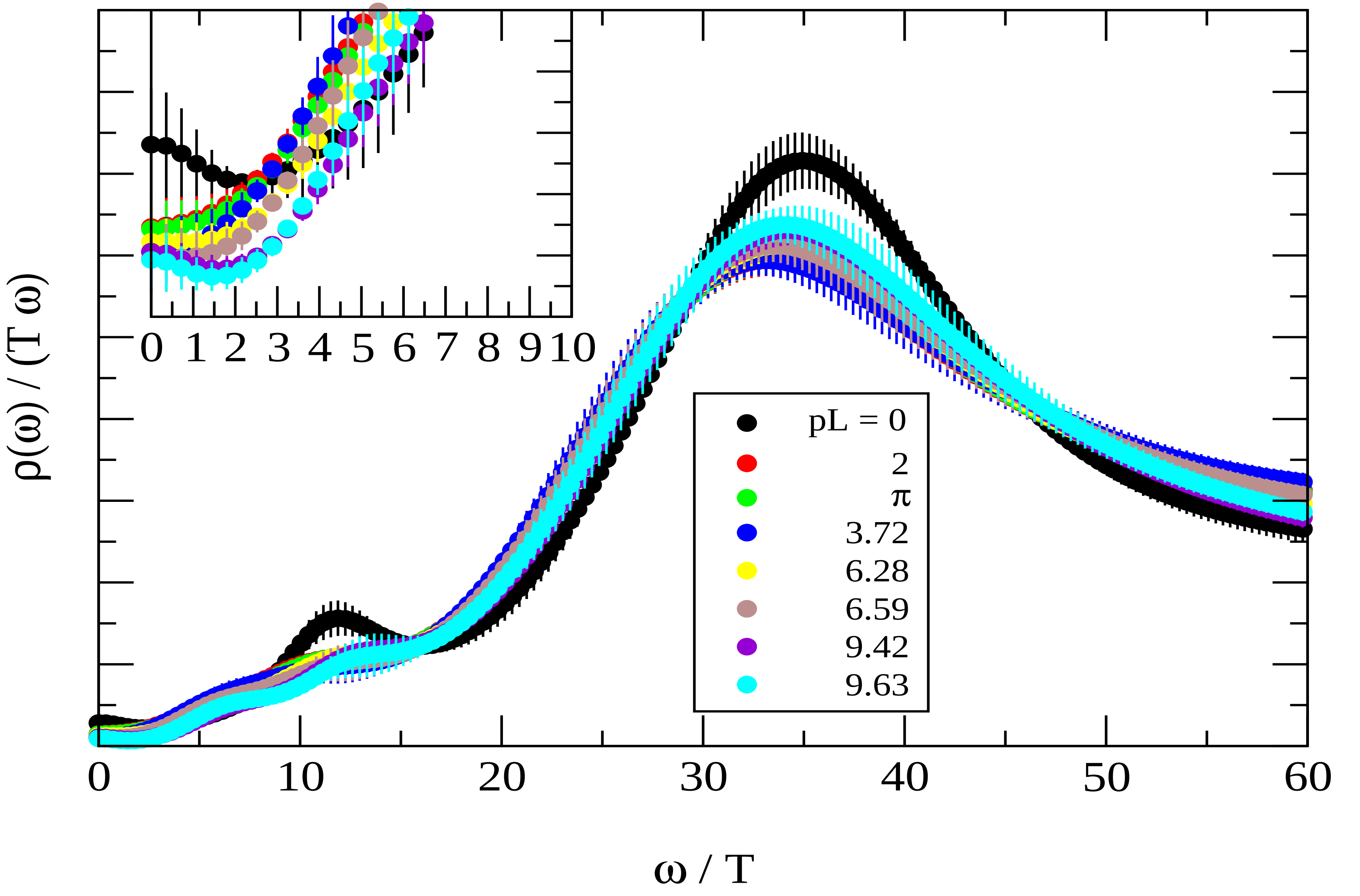
Longitudinal, light, hot

$m/T = 0.24 \quad T \sim 1.5T_c$



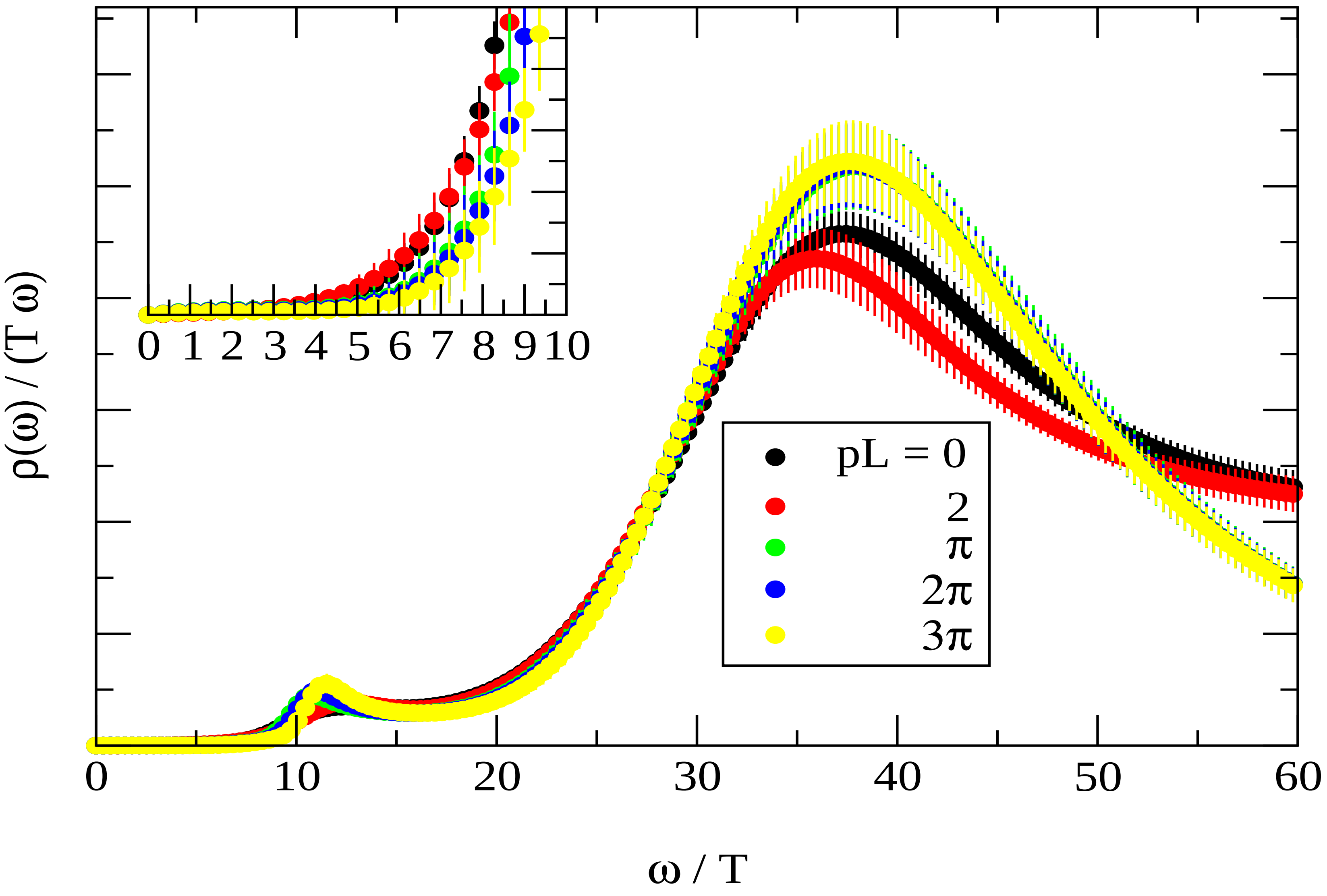
Transverse, light, hot

$m/T = 0.24 \quad T \sim 1.5T_c$



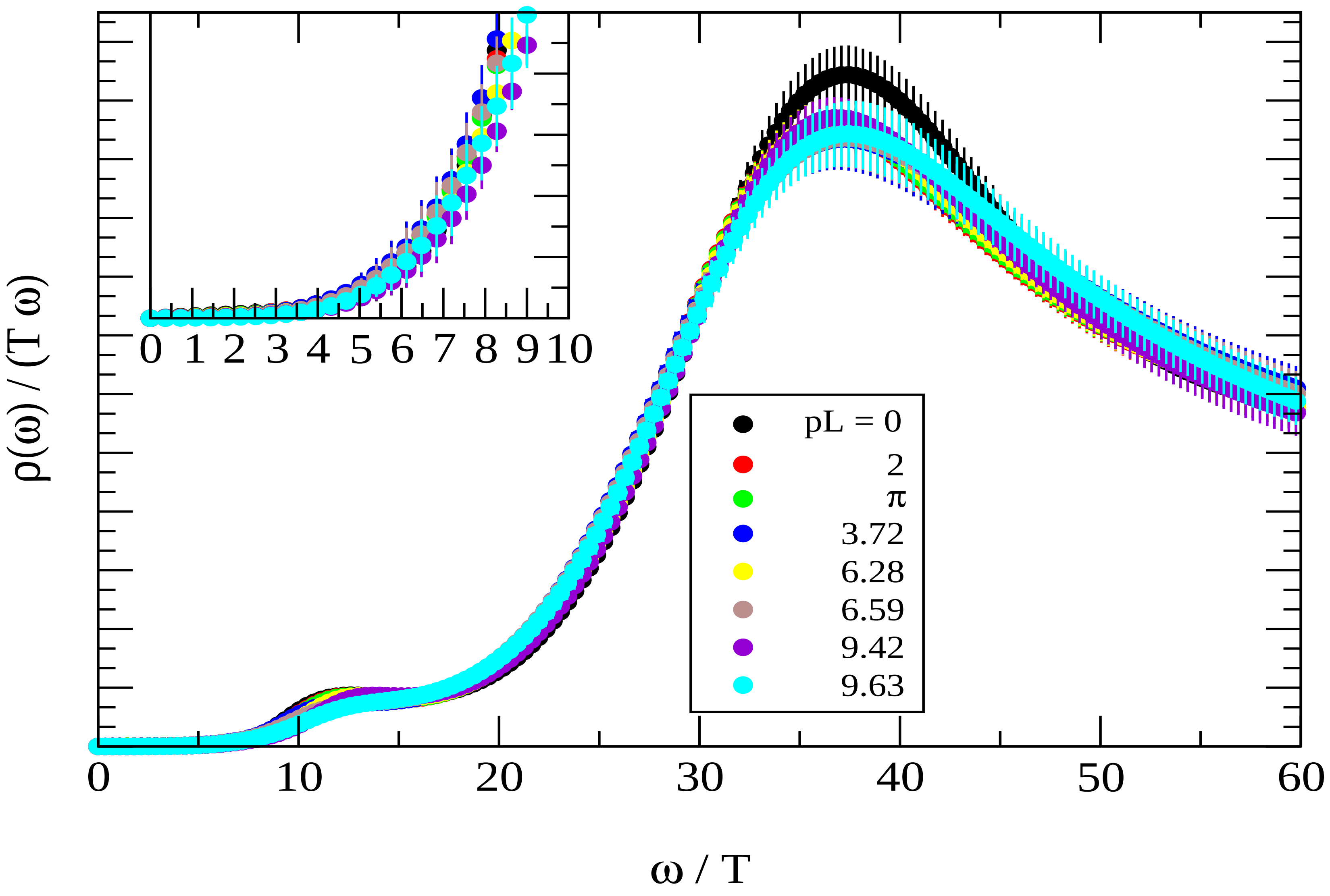
Longitudinal, heavy, hot

$m/T = 1.2 \quad T \sim 1.5T_c$

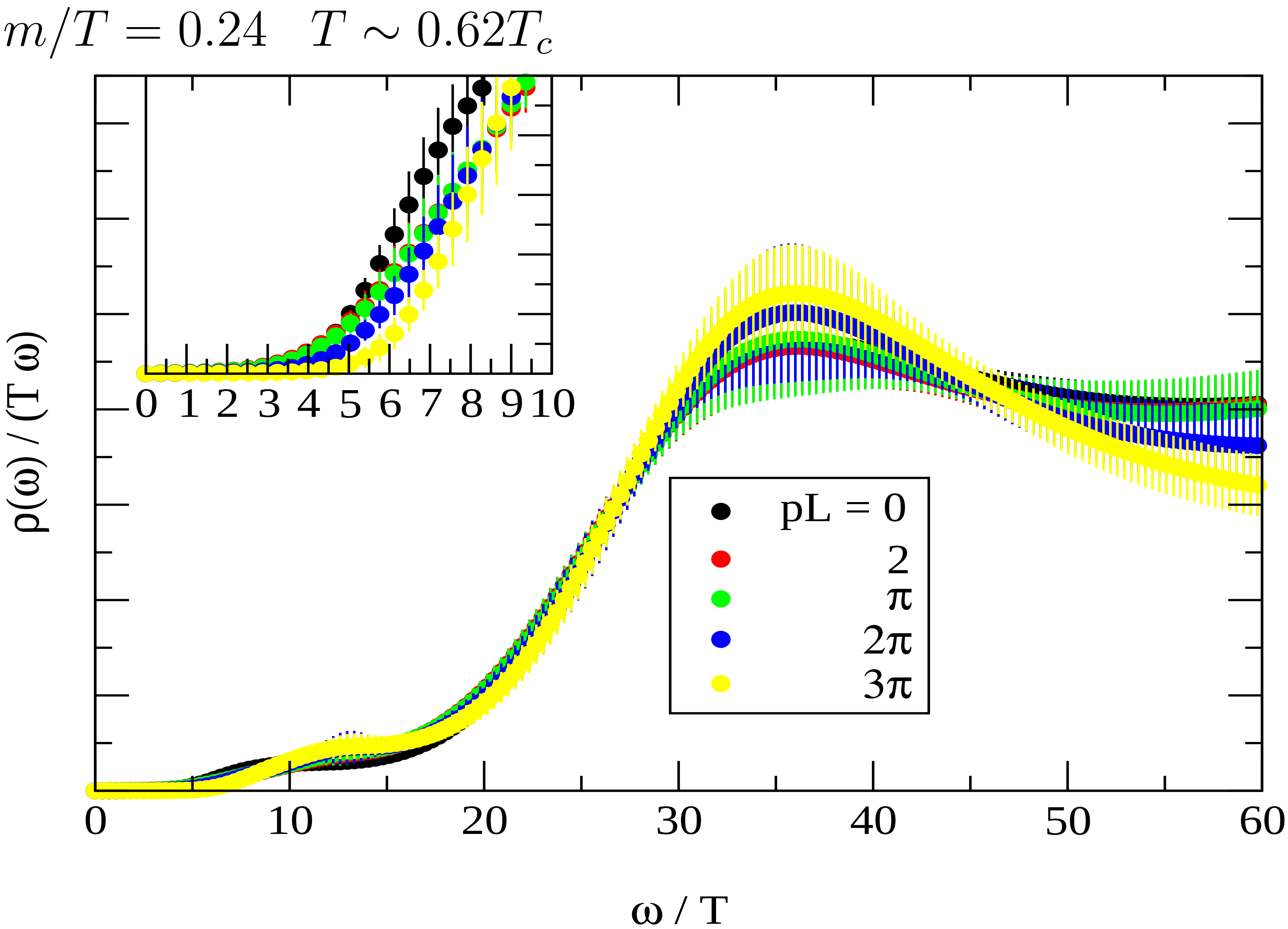


Transverse, heavy, hot

$m/T = 1.2 \quad T \sim 1.5T_c$

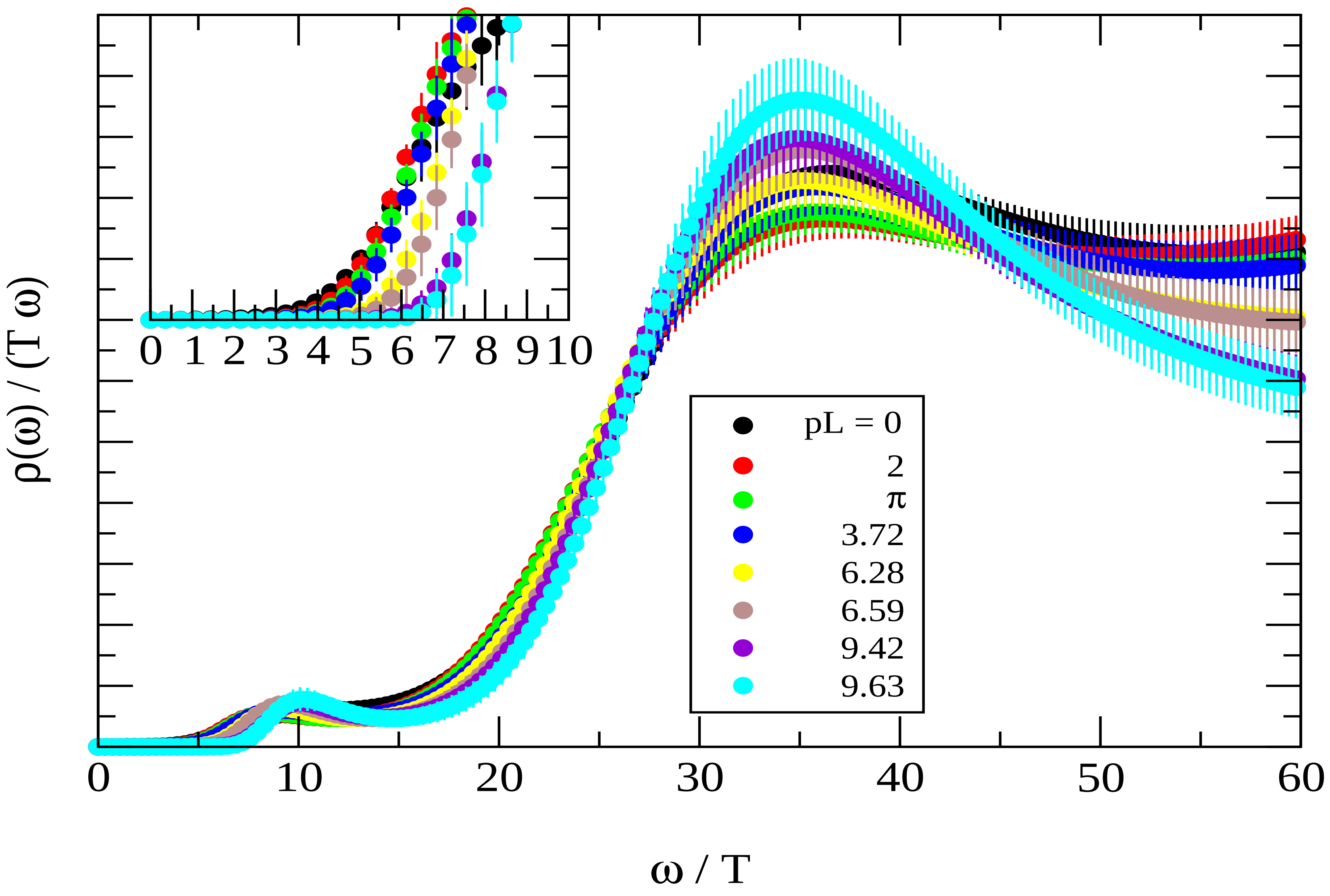


Longitudinal, light, cold



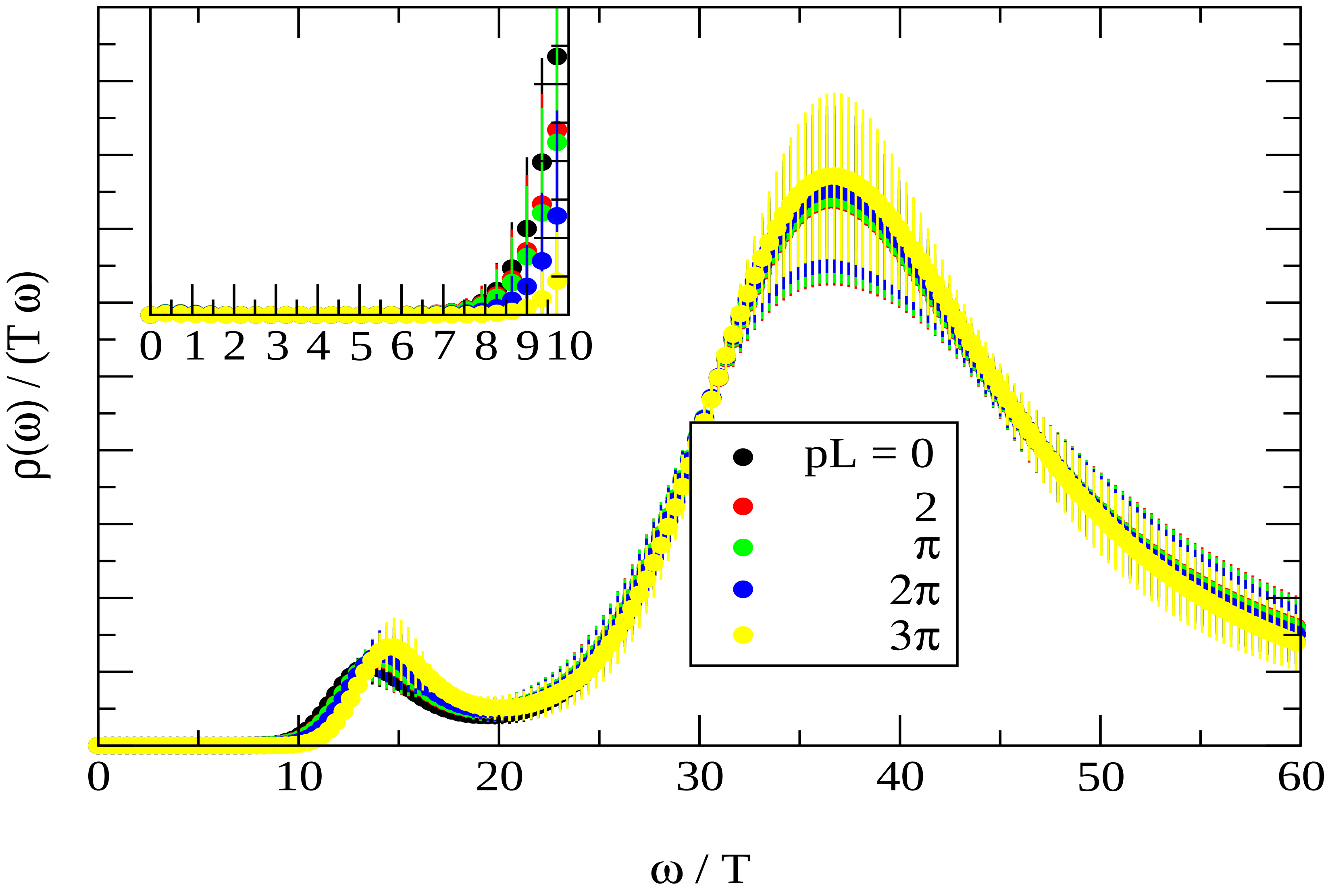
Transverse, light, cold

$m/T = 0.24 \quad T \sim 0.62T_c$



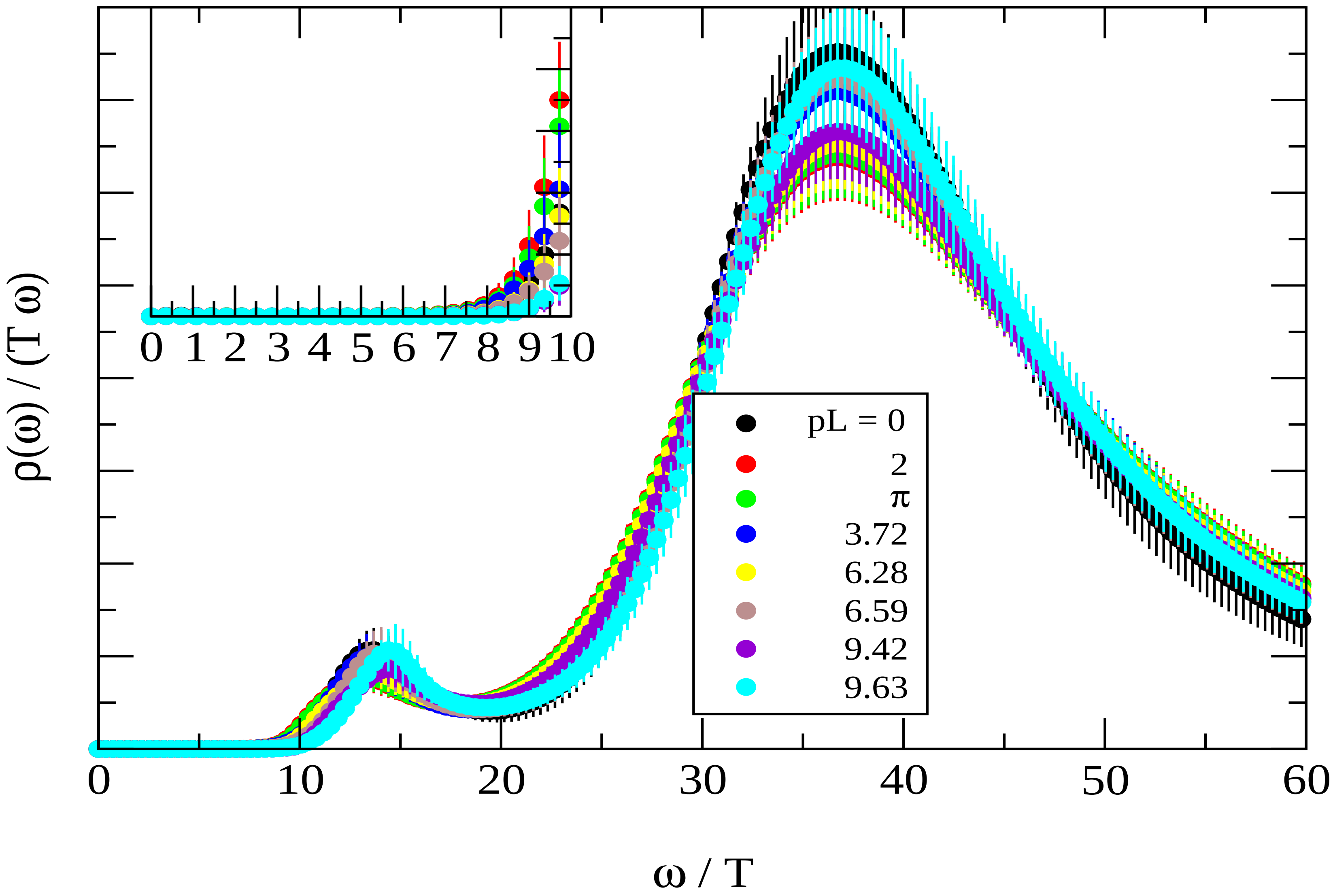
Longitudinal, heavy, cold

$m/T = 1.2 \quad T \sim 0.62T_c$



Transverse, heavy, cold

$m/T = 1.2 \quad T \sim 0.62T_c$

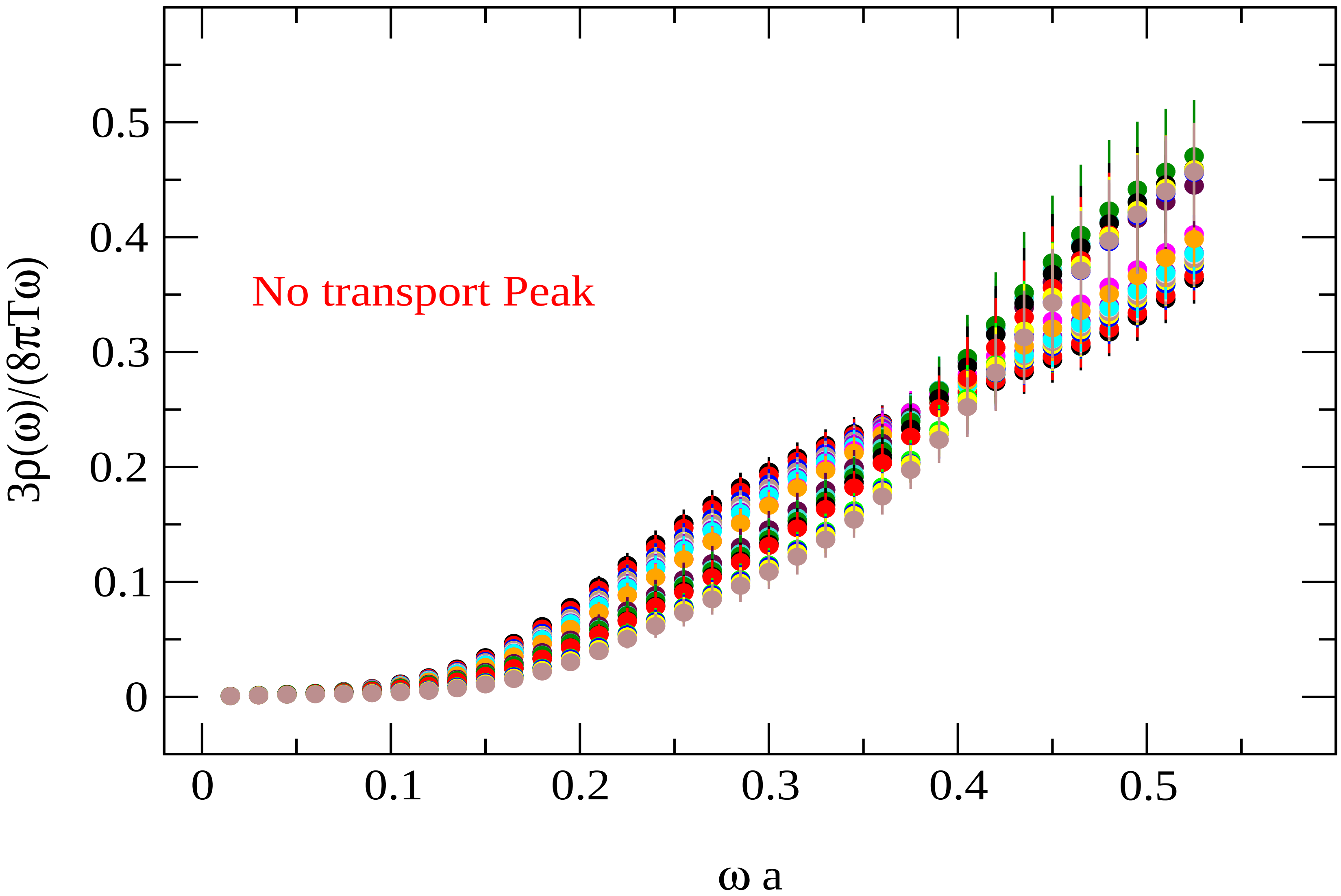


Summary of Long v Tran

T	Polarisation	m_q	$\rho(\omega \to 0)$	
$1.5T_c$	Long.	light	$\neq 0$	Contradicting Hong & Teaney
	Tran.	light	$\neq 0$	Agreeing with Hong & Teaney
	Long.	heavy	$= 0$	
	Tran.	heavy	$= 0$	
$0.62T_c$	Long.	light	$= 0$	
	Tran.	light	$= 0$	
	Long.	heavy	$= 0$	
	Tran.	heavy	$= 0$	

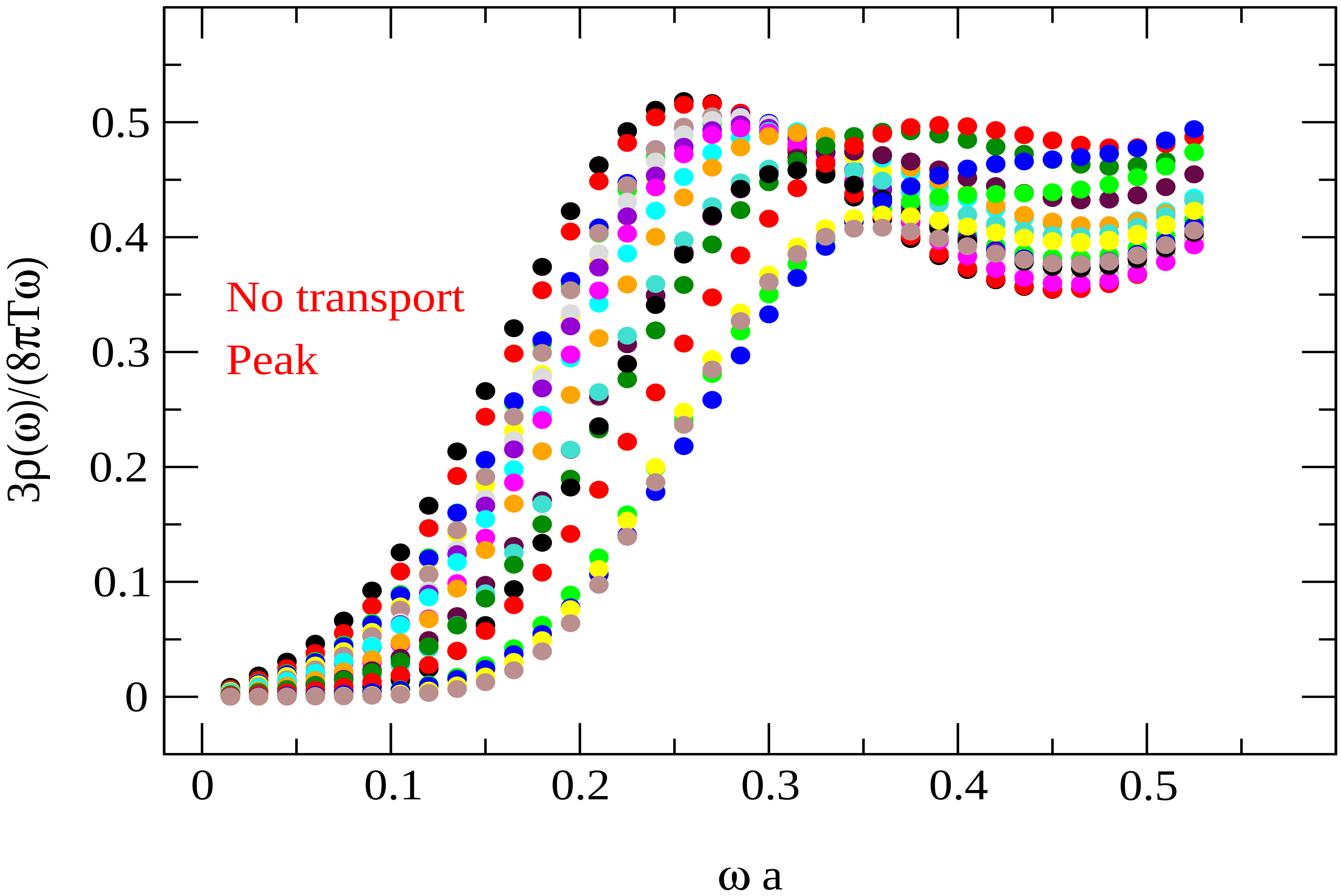
Pseudoscalar, hot

$m/T = 0.24 \quad T \sim 1.5T_c$



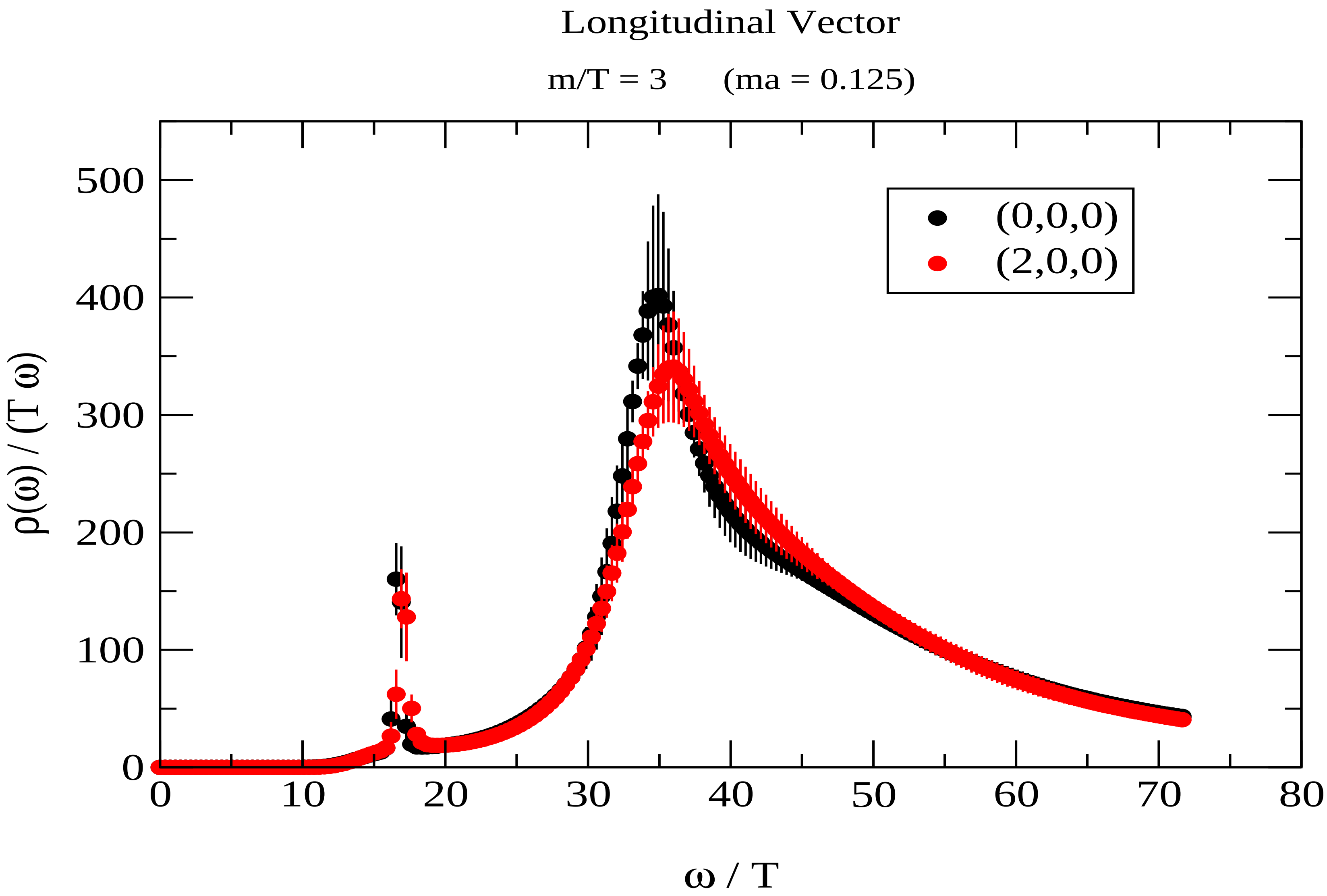
Pseudoscalar, cold

$m/T = 0.24 \quad T \sim 0.62T_c$



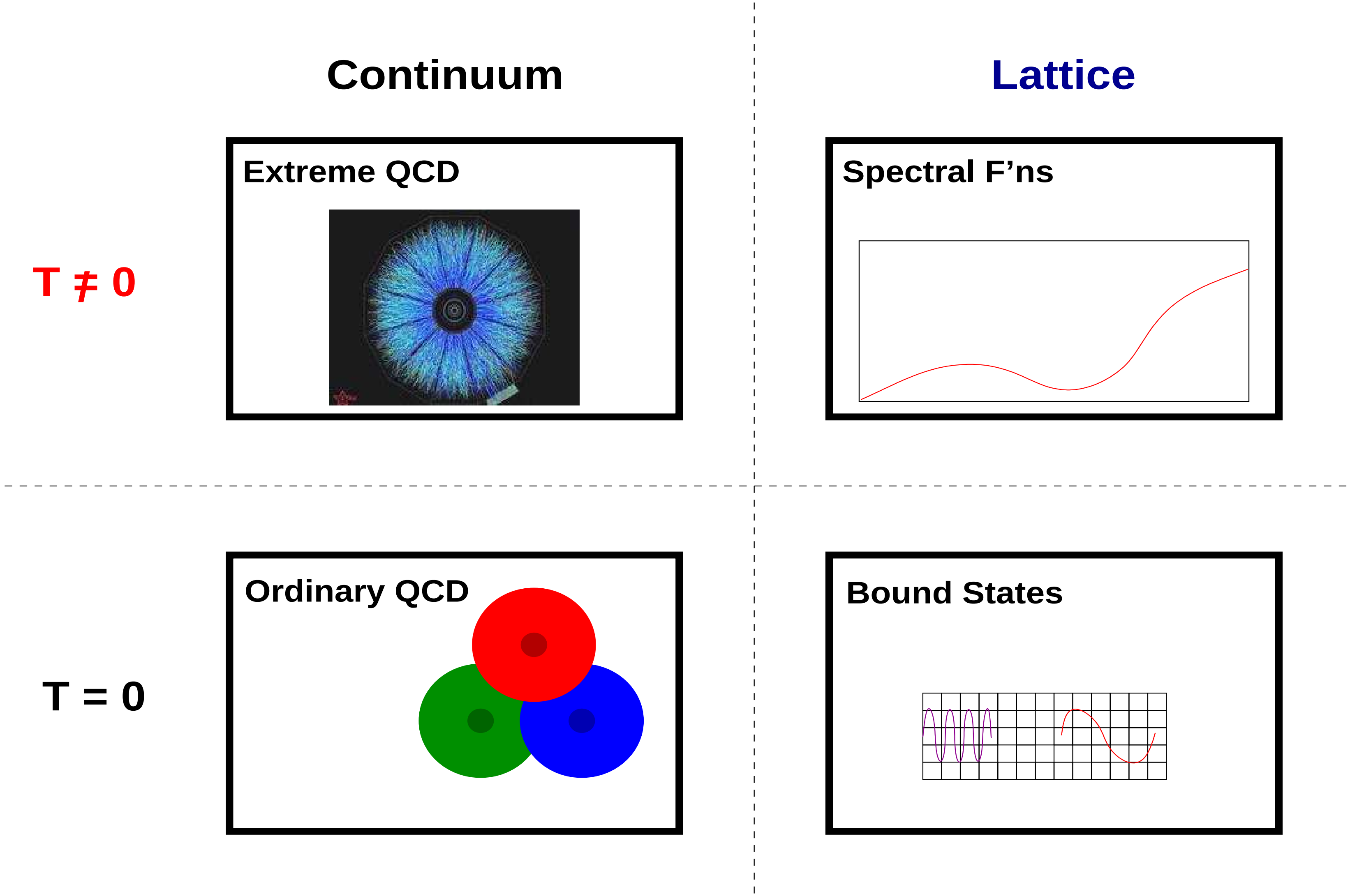
Lack of Melting? [Preliminary]

$m/T = 3$ Hot



Summary

- Have extended previous work on mesonic correlators at $T \neq 0$ to non-zero momentum
- Used twisted B.C. to access finer momentum resolution
- Found distinct difference in $G(t)$ for longitudinal versus transverse in $G(t)$
- This does not correspond to difference in “transport peak”, i.e. $\rho(\omega \to 0)$ behaviour
- This work performed on modest computer power
  - Runs planned on supercomputers
- Concerns about constant term in correlator from quark wrapping around boundary... Umeda, hep-lat/0701005


Continuum
Lattice
T ≠ 0
T = 0
Extreme QCD
Spectral F'ns
Ordinary QCD
Bound States

MEM Limitations

If the correlation function has a large dynamic range then there are numerical instabilities in MEM.

$$G(t) \;=\; \int \rho(\omega)\; K(t,\omega)\; d\omega$$

where

$$K(t,\omega) \;=\; \frac{\cosh[\omega(t - N_t/2)]}{\sinh[\omega/(2T)]}$$
$$\sim\; \exp[-\omega t]$$

i.e. it is *almost* a Laplace transform:

$$G(t) \sim \int \rho(\omega)\; e^{-\omega t}\; d\omega$$

Convolution Rules

Aim is to get $G(t)$ expressed as an exact Laplace transform, then can use convolution rules:

$$G(t) \times e^{+\Omega t} \;=\; \int \rho(\omega + \Omega)\, e^{-\omega t}\, d\omega$$

$\uparrow$

smaller
dynamic
range

Possible Solution

$$\begin{aligned} G(t) &= e^{-Mt} + e^{-M(T-t)} \\ &= e^{-MT/2}(e^{-M(t-T/2)} + e^{M(t-T/2)}) \\ &= e^{-MT/2}(x^i + x^{-i}) \end{aligned}$$

where $x = e^{-M(t-T/2)}$ and $i = t - T/2$.

Define

$$\begin{aligned} \widetilde{G}(t) &= e^{-MT/2}(x + x^{-1})^i \\ &= e^{-MT/2} \sum_{j=0}^{i} {}^iC_j \; x^{2j-i} \\ &= e^{-MT/2} \sum_{j=0}^{[i/2]} {}^iC_j \; (x^{2j-i} + x^{i-2j}) \\ &= \sum_{j=0}^{[i/2]} {}^iC_j \; G(2j-i) \end{aligned}$$

Possible Solution contd.

We can now write $x + x^{-1} \equiv e^y \quad \longrightarrow$

$$\begin{aligned} \widetilde{G}(t) &= e^{-MT/2}(x + x^{-1})^i \\ &= e^{-MT/2} e^{yi} \\ &\equiv \text{linear combination of } G(t) \end{aligned}$$

Fleming et al, Phys.Rev.D80 (2009) 074506 [arXiv:0903.2314]

i.e. by defining a linear combination of $G(t)$ correlators, the lattice kernel is transposed into a pure exponential.

$\longrightarrow$ Laplace Transform Convolution Rule is now viable