Towards a Direct Lattice Determination of $m_d - m_u$

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College of William and Mary

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Work in collaboration with

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- Will Detmold (William and Mary)
- Kostas Orginos (William and Mary)
- Brian Tiburzi (UMD)

Preview

Motivation

- parameter of the standard model
- possible solution to the strong *CP* problem ($m_u = 0$?)
- neutron lifetime and BBN
- ...
- parameter of the standard model

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Neutron Lifetime:

$$n \longrightarrow p + e + \bar{\nu}_e$$

$$\frac{1}{\tau_n} = \frac{1}{4\pi^3}\left(\frac{g_W}{2M_W}\right)^4 m_e^5 \left[\frac{1}{15}(2a^4 - 9a^2 - 8)\sqrt{a^2-1} + a\ln(a + \sqrt{a^2-1})\right]$$

$$a \equiv \frac{m_n - m_p}{m_e}$$

Griffiths

10% change in $m_n - m_p \longrightarrow$ 100% change in τ_n!

Idea:

- Use partially quenched lattice qcd to introduce an isospin breaking mass parameter to the valence quarks
- compute the hadron spectrum for various values of $2\delta = m_d - m_u$
- use the isospin mass splittings in the physical spectrum (corrected for electromagnetism) to determine the physical value of δ
- specifically

$$m_{u,val} = \hat{m}_{sea} - \delta \qquad m_{d,val} = \hat{m}_{sea} + \delta$$

Significantly minimizes effects of partial quenching
A.W-L arXiv:0904.2404

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Determining electromagnetic self-energy corrections to hadrons is the most challenging part: two options

- compute with lattice QCD + QED: two challenges
  - have to deal with Coulomb potential
  - large (power law) volume corrections
- improve estimates of QED self-energy corrections from modern knowledge of structure functions

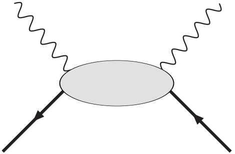
Tuesday

- Baum, I. 9:50
- Freeland, E. 11:50

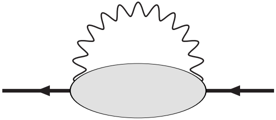
Thursday

- Torok, A. 14:30
- Taku, I. 14:50
- Portelli, A. 15:10

Correcting for electromagnetic self-energy: Cottingham 1963



$\sim \frac{l^{\mu\nu}W_{\mu\nu}}{Q^4}$



$$\Delta M_\gamma^B = \frac{4\pi\alpha_{f.s.}}{2}\int \frac{d^4q}{(2\pi)^4}\frac{g^{\mu\nu}W_{\mu\nu}}{q^2+i\epsilon}$$

$$W_{\mu\nu} = -\,W_1\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) + W_2\left(\frac{p_\mu}{M} - \frac{\nu q_\mu}{q^2}\right)\left(\frac{p_\nu}{M} - \frac{\nu q_\nu}{q^2}\right)$$

$$\nu = \frac{p\cdot q}{M}$$

$$\Delta M_\gamma^B = -\frac{\alpha_{f.s.}}{4\pi^2}\int_0^\infty dQ^2\int_{-Q}^{Q} d\nu\,\frac{\sqrt{Q^2-\nu^2}}{Q^2}\left[3W_1(Q^2,i\nu) - \left(1-\frac{\nu^2}{Q^2}\right)W_2(Q^2,i\nu)\right]$$

Correcting for electromagnetic self-energy: Gasser and Leutwyler

$$\Delta M_\gamma^B = -\frac{\alpha_{f.s.}}{4\pi^2}\int_0^\infty dQ^2 \int_{-Q}^{Q} d\nu \frac{\sqrt{Q^2-\nu^2}}{Q^2}\left[3W_1(Q^2,i\nu) - \left(1-\frac{\nu^2}{Q^2}\right)W_2(Q^2,i\nu)\right]$$

- W_1 requires once-subtracted dispersion relation Harari PRL 17 (1966)
- Gasser-Leutwyler *Quark Masses*, Phys. Rept. 87 (1982)
  - assume no subtraction needed
  - compute only elastic contributions
  - estimate inelastic contributions for uncertainties

$$\Delta M_\gamma^B = -\frac{\alpha_{f.s.}}{\pi}\int_0^\infty dQQ^2 \Bigg\{3\left[\sqrt{1+\frac{Q^2}{4M^2}} - \frac{Q}{2M}\right]\frac{G_M^2(-Q^2)-G_E^2(-Q^2)}{Q^2+4M^2} - \left[\sqrt{1+\frac{Q^2}{4M^2}}\left(1-2\frac{Q^2}{4M^2}\right)+2\frac{Q^3}{8M^3}\right]\frac{4M^2G_E^2(-Q^2)+Q^2G_M^2(-Q^2)}{Q^2(Q^2+4M^2)}\Bigg\}$$

$B-B'$	Experiment	ΔM_γ	QCD
$\Xi^- - \Xi^0$	6.85 ± 0.21	0.86 ± 0.30	5.99 ± 0.37
$n-p$	1.29	-0.76 ± 0.30	2.05 ± 0.30

Baryons: $SU(2)$ Lagrangian

$$\mathcal{L} = \overline{N} iv \cdot DN - \frac{\alpha_N}{(4\pi f)} \overline{N}\mathcal{M}_+ N - \frac{\sigma_N}{(4\pi f)} \overline{N}N \, \mathrm{tr}(\mathcal{M}_+)$$

$$- (\overline{T}^\mu iv \cdot D\, T_\mu) + \Delta\, (\overline{T}^\mu T_\mu) + \frac{\gamma_N}{(4\pi f)} (\overline{T}^\mu \mathcal{M}_+ T_\mu) + \frac{\overline{\sigma}_N}{(4\pi f)} (\overline{T}^\mu T_\mu)\, \mathrm{tr}(\mathcal{M}_+)$$

$$+ 2\, g_A\, \overline{N} S \cdot A\, N + 2\, g_{\Delta\Delta}\, \overline{T}^\mu S \cdot A\, T_\mu + g_{\Delta N} \left[\overline{T}^{kji}_\mu A^{\mu,i'}_i \epsilon_{ji'} N_k + h.c. \right]$$

$$N = \begin{pmatrix} p \\ n \end{pmatrix} \qquad T^\mu_{ijk} = \text{ symmetrix tensor, } T_{111} = \Delta^{++}$$

$$\mathcal{M}_+ = \frac{1}{4} \left(\xi^\dagger (2Bm_q) \xi^\dagger + \xi (2Bm_q)^\dagger \xi \right) \qquad A_\mu = \frac{i}{2} \left(\xi \partial_\mu \xi^\dagger - \xi^\dagger \partial_\mu \xi \right)$$

Baryons: $SU(4|2)$ Lagrangian

$$\mathcal{L}^{(PQ)} = \left(\overline{\mathcal{B}} iv\cdot D\mathcal{B}\right) - \frac{\alpha_N^{(PQ)}}{(4\pi f)}\left(\overline{\mathcal{B}}\mathcal{B}\mathcal{M}_+\right) - \frac{\beta_N^{(PQ)}}{(4\pi f)}\left(\overline{\mathcal{B}}\mathcal{M}_+\mathcal{B}\right) - \frac{\sigma_N^{(PQ)}}{(4\pi f)}\left(\overline{\mathcal{B}}\mathcal{B}\right)\mathrm{tr}(\mathcal{M}_+)$$
$$- (\overline{\mathcal{T}}^\mu iv\cdot D\,\mathcal{T}_\mu) + \Delta\,(\overline{\mathcal{T}}^\mu\mathcal{T}_\mu) + \frac{\gamma_N^{(PQ)}}{(4\pi f)}(\overline{\mathcal{T}}^\mu\mathcal{M}_+\mathcal{T}_\mu) + \frac{\overline{\sigma}_M^{(PQ)}}{(4\pi f)}(\overline{\mathcal{T}}^\mu\mathcal{T}_\mu)\,\mathrm{tr}(\mathcal{M}_+)$$
$$+ 2\alpha^{(PQ)}\left(\overline{\mathcal{B}}S^\mu\mathcal{B}A_\mu\right) + 2\beta^{(PQ)}\left(\overline{\mathcal{B}}S^\mu A_\mu\mathcal{B}\right) + 2\mathcal{H}^{(PQ)}\left(\overline{\mathcal{T}}^\nu S^\mu A_\mu\mathcal{T}_\nu\right)$$
$$+ \sqrt{\frac{3}{2}}\mathcal{C}\left[\left(\overline{\mathcal{T}}^\nu A_\nu\mathcal{B}\right) + \left(\overline{\mathcal{B}}A_\nu\mathcal{T}^\nu\right)\right]$$

$$\mathcal{B}_{ijk} = \frac{1}{\sqrt{6}}\left(\varepsilon_{ij}N_k + \varepsilon_{ik}N_j\right)$$

project to valence sector

$$\alpha_N = \frac{2}{3}\alpha_N^{(PQ)} - \frac{1}{3}\beta_N^{(PQ)}, \qquad \sigma_N = \sigma_N^{(PQ)} + \frac{1}{6}\alpha_N^{(PQ)} + \frac{2}{3}\beta_N^{(PQ)},$$
$$\gamma_N = \gamma_N^{(PQ)}\,,\ \bar{\sigma}_N = \bar{\sigma}_N^{(PQ)},$$
$$g_A = \frac{2}{3}\alpha^{(PQ)} - \frac{1}{3}\beta^{(PQ)}\,, \qquad g_1 = \frac{1}{3}\alpha^{(PQ)} + \frac{4}{3}\beta^{PQ)},$$
$$g_{\Delta\Delta} = \mathcal{H}\,,\ g_{\Delta N} = -\mathcal{C},$$

PQχPT

$$m_p = M_0 - \frac{\hat{\delta}}{(4\pi f_\pi)} \frac{\alpha_N}{2} + \frac{m_\pi^2}{(4\pi f_\pi)} \left(\frac{\alpha_N}{2} + \sigma_N^r(\mu) \right) - \frac{3\pi g_A^2}{(4\pi f_\pi)^2} m_\pi^3 - \frac{8 g_{\Delta N}^2}{3(4\pi f_\pi)^2} \mathcal{F}(m_\pi, \Delta, \mu) + \frac{3\pi \Delta_{PQ}^4 (g_A + g_1)^2}{8 m_\pi (4\pi f_\pi)^2}$$

$$m_n = M_0 + \frac{\hat{\delta}}{(4\pi f_\pi)} \frac{\alpha_N}{2} + \frac{m_\pi^2}{(4\pi f_\pi)} \left(\frac{\alpha_N}{2} + \sigma_N^r(\mu) \right) - \frac{3\pi g_A^2}{(4\pi f_\pi)^2} m_\pi^3 - \frac{8 g_{\Delta N}^2}{3(4\pi f_\pi)^2} \mathcal{F}(m_\pi, \Delta, \mu) + \frac{3\pi \Delta_{PQ}^4 (g_A + g_1)^2}{8 m_\pi (4\pi f_\pi)^2}$$

with

$$\mathcal{F}(m, \Delta, \mu) = (\Delta^2 - m^2 + i\epsilon)^{3/2} \ln \left(\frac{\Delta + \sqrt{\Delta^2 - m^2 + i\epsilon}}{\Delta - \sqrt{\Delta^2 - m^2 + i\epsilon}} \right) - \frac{3}{2} \Delta m^2 \ln \left(\frac{m^2}{\mu^2} \right) - \Delta^3 \ln \left(\frac{4\Delta^2}{m^2} \right) .$$

$$m_n - m_p = \frac{\hat{\delta}}{(4\pi f_\pi)} \alpha_N + \mathcal{O}(\hat{\delta}^2, \hat{\delta} m_\pi^2)$$

The expansion is as good as for mesons

NNLO:

$$\mathcal{L}_M = \frac{1}{(4\pi f)^3}\Big\{ b_1^M \bar{N}\,\mathcal{M}_+^2 N + b_5^M \bar{N}N\,\text{tr}(\mathcal{M}_+^2) + b_6^M \bar{N}\,\mathcal{M}_+ N\,\text{tr}(\mathcal{M}_+) + b_8^M \bar{N}N\,[\text{tr}(\mathcal{M}_+)]^2$$
$$+ t_1^M\,\bar{T}_\mu^{kji}(\mathcal{M}_+\mathcal{M}_+)_i{}^{i'}\,T_{\mu,i'jk} + t_2^M\,\bar{T}_\mu^{kji}(\mathcal{M}_+)_i{}^{i'}(\mathcal{M}_+)_j{}^{j'}\,T_{\mu,i'j'k} + t_3^M\,\bar{T}_\mu T_\mu \text{tr}(\mathcal{M}_+^2)$$
$$+ t_4^M\,(\bar{T}_\mu \mathcal{M}_+ T_\mu)\,\text{tr}(\mathcal{M}_+) + t_5^M\,\bar{T}_\mu T_\mu [\text{tr}(\mathcal{M}_+)]^2 \Big\}$$

here we see the first *error* from having $\delta_{sea} = 0$.

In $SU(2)$, $\quad \delta m_N = \frac{b_5^M(m_\pi^4 + \hat{\delta}^2)}{2(4\pi f_\pi)^3}$, $\quad$ while in $SU(4|2)$ $\quad \delta m_N \to \frac{b_5^M(m_\pi^4)}{2(4\pi f_\pi)^3}$,

$\delta m_\Delta = \frac{t_3^M(m_\pi^4 + \hat{\delta}^2)}{2(4\pi f_\pi)^3}$, $\quad \delta m_\Delta \to \frac{t_3^M(m_\pi^4)}{2(4\pi f_\pi)^3}$.

but in the mass splittings, these *errors* exactly cancel

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$$\mathcal{L}_M = \frac{1}{(4\pi f)^3}\Big\{ b_1^M \bar{N}\, \mathcal{M}_+^2 N + b_5^M \bar{N}N\,\mathrm{tr}(\mathcal{M}_+^2) + b_6^M \bar{N}\,\mathcal{M}_+ N\,\mathrm{tr}(\mathcal{M}_+) + b_8^M \bar{N}N\,[\mathrm{tr}(\mathcal{M}_+)]^2$$

$$+ t_1^M\, \bar{T}_\mu^{kji} (\mathcal{M}_+\mathcal{M}_+)_i{}^{i'}\, T_{\mu,i'jk} + t_2^M\, \bar{T}_\mu^{kji} (\mathcal{M}_+)_i{}^{i'} (\mathcal{M}_+)_j{}^{j'}\, T_{\mu,i'j'k} + t_3^M\, \bar{T}_\mu T_\mu \mathrm{tr}(\mathcal{M}_+^2)$$

$$+ t_4^M\, (\bar{T}_\mu \mathcal{M}_+ T_\mu)\,\mathrm{tr}(\mathcal{M}_+) + t_5^M\, \bar{T}_\mu T_\mu [\mathrm{tr}(\mathcal{M}_+)]^2 \Big\}$$

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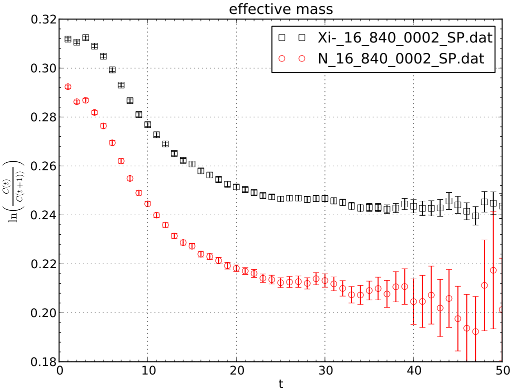
NNLO:

$$
\begin{aligned}
m_n - m_p = \frac{\hat{\delta}}{(4\pi f_\pi)} \Big\{ & \alpha_N \\
& + \frac{m_\pi^2}{(4\pi f_\pi)^2}(b_1^M + b_6^M) + \frac{\mathcal{J}(m_\pi, \Delta, \Lambda)}{(4\pi f_\pi)^2} 4 g_{\Delta N}^2 \left(\frac{5}{9}\gamma_N - \alpha_N \right) \\
& + \frac{m_\pi^2}{(4\pi f_\pi)^2} \left[\frac{20}{9}\gamma_N g_{\Delta N}^2 - 4\alpha_N (g_A^2 + g_{\Delta N}^2) - \alpha_N (6 g_A^2 + 1) \ln\left(\frac{m_\pi^2}{\Lambda^2} \right) \right] \\
& + \frac{\alpha_N \Delta_{PQ}^4}{m_\pi^2 (4\pi f_\pi)^2} \left(2 - \frac{3}{2}(g_A + g_1)^2 \right) \Big\}
\end{aligned}
$$

Baryon Spectrum



$\longrightarrow m_{\Xi^-} - m_{\Xi^0}$ to determine δ^{phys}, predict $m_n - m_p$

choice of action and parameters

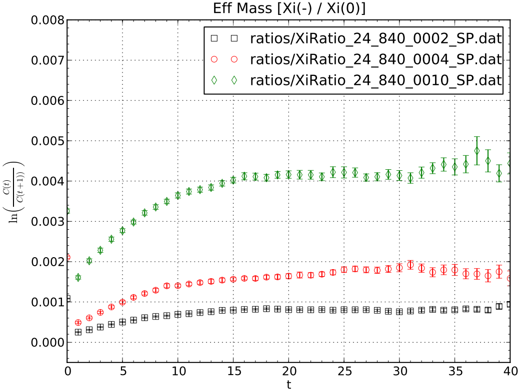
$$a_s \simeq 0.113 \text{ fm}, \qquad \frac{a_s}{a_t} = 3.5 \qquad m_\Omega \text{ used for scale setting}$$

ensemble: $a_t m_s = -0.0743$				m_π	m_K	$a_t\delta$ $[N_{cfg} \times N_{src}]$			
L	T	$a_t m_l$	$a_t m_l^{val}$	[MeV]	[MeV]	0.0002	0.0004	0.0010	0.0020
16	128	-0.0830	-0.0830	490	630	167 × 25	–	167 × 25	–
16	128	-0.0840	-0.0840	420	592	166 × 25	166 × 25	166 × 25	166 × 50
20	128	-0.0840	-0.0840	420	592	120 × 25	–	–	–
24	128	-0.0840	-0.0840	420	592	97 × 25	100 × 10	193 × 25	–
24	128	-0.0840	-0.0849	326		200 × 10	–	–	–
24	128	-0.0860	-0.0854	285	519	206 × 14	–	–	–
24	128	-0.0860	-0.0860	244	506	108 × 25	–	–	–
32	256	-0.0860	-0.0860	244	506	104 × 7	–	–	–

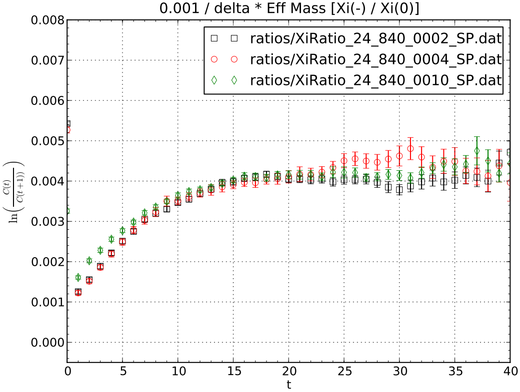
$$1 \lesssim \delta \lesssim 11 \text{ MeV} \qquad 2\delta = m_d - m_u$$

work mostly computed on `Sporades` - local machine at William and Mary

Numerical Results

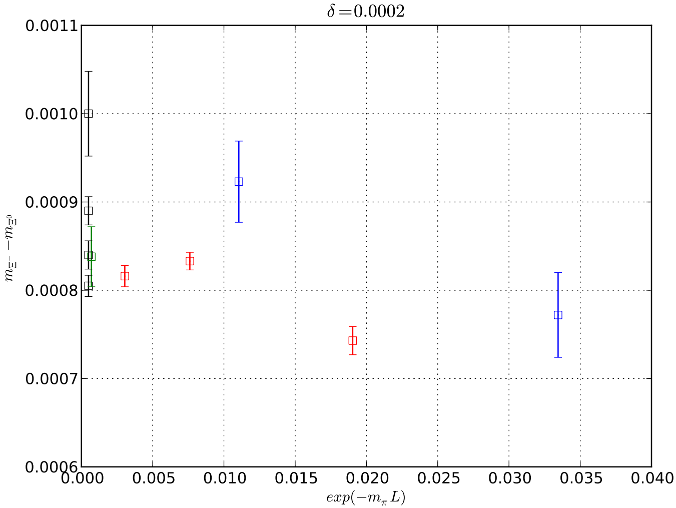


Numerical Results

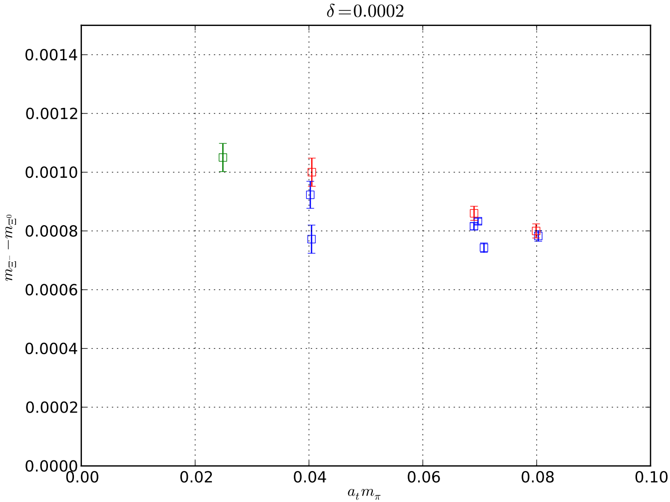


Numerical Results

ensemble: $a_t m_s = -0.0743$				m_π	m_K	$m_{\Xi^-} - m_{\Xi^0}$			
L	T	$a_t m_l$	$a_t m_l^{val}$	[MeV]	[MeV]	0.0002	0.0004	0.0010	0.0020
16	128	-0.0830	-0.0830	490	630	4.8(1)	–	23.2(3)	–
16	128	-0.0840	-0.0840	420	592	4.5(1)	9.0(2)	22.7(5)	46.8(1.7)
20	128	-0.0840	-0.0840	420	592	5.1(1)	–	–	–
24	128	-0.0840	-0.0840	420	592	4.9(1)	10.1(2)	25.0(3)	–
24	128	-0.0840	-0.0849	326		$\otimes$	–	–	–
24	128	-0.0860	-0.0854	285	519	$\otimes$	–	–	–
24	128	-0.0860	-0.0860	244	506	4.7(3)	–	–	–
32	256	-0.0860	-0.0860	244	506	5.6(3)	–	–	–


$\delta = 0.0002$
0.0011
0.0010
0.0009
0.0008
0.0007
0.0006
$m_{\Xi^-} - m_{\Xi^0}$
0.000
0.005
0.010
0.015
0.020
0.025
0.030
0.035
0.040
$exp(-m_\pi L)$

Numerical Results



$$m_{\Xi^-} - m_{\Xi^0} = \frac{\hat{\delta}}{4\pi f}\alpha_{\Xi} + \text{NNLO}$$

$$\downarrow$$

$$\mathcal{Z}_{\delta}^{latt}\,\delta_{phys} = 1.32 \pm 0.03 \pm \text{sys.} \pm 0.10(EM) \quad \text{[MeV]} \quad \text{PRELIMINARY}$$

$$\mathcal{Z}_{\delta}^{latt}\,\delta_{phys} = 0.000216(15)(\text{sys.})(13) \quad \text{[l.u.]} \quad \text{PRELIMINARY}$$

Numerical Results

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$$\longrightarrow m_n - m_p = 3.4 \pm 0.20 \pm \text{ sys.} \pm 0.74 \quad [\texttt{MeV}]$$

NPLQCD : $m_n - m_p = 2.26 \pm 0.72$ [MeV] NPB 768 (2007)

Blum et al. : $m_n - m_p = 2.24 \pm 0.12 \pm$ sys. [MeV] arXiv:1006.1311

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$$\text{Blum et al.}: \quad m_n - m_p = 2.24 \pm 0.12 \pm \text{ sys.} \quad \texttt{[MeV]} \qquad \text{arXiv:1006.1311}$$

$$\frac{\delta}{m_d + m_u} = \frac{m_d - m_u}{m_d + m_u} = 0.67 \pm 0.03 \pm \text{ sys.} \pm 0.11$$

$$\frac{\delta}{m_d + m_u} = 0.40 \pm 0.04 \qquad \text{MILC}$$

$$\frac{\delta}{m_d + m_u} = \frac{m_d - m_u}{m_d + m_u} = 0.67 \pm 0.03 \pm \text{ sys.} \pm 0.11$$

$$\frac{\delta}{m_d + m_u} = 0.40 \pm 0.04 \qquad \text{MILC}$$

Conclusions

- demonstrated novel method for precisely determining $2\delta = m_d - m_u$ utilizing baryon spectrum
- understanding hadron electromagnetic self-energy is most important systematic:
  $\longrightarrow$ investigating use of modern structure functions to determine elastic and inelastic self-energies C. Carlson, W. Detmold, AWL
- use of anisotropic clover configurations limiting - will not be a second lattice spacing in near future - challenging to determine $\mathcal{Z}_m$ or m_{crit}
- lattice determination of hyperon form factors (structure functions) will allow for an estimation of electromagnetic hyperon self-energies Zanotti, J. Mon. 15:50