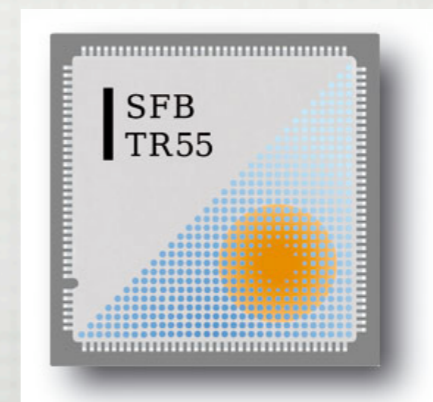
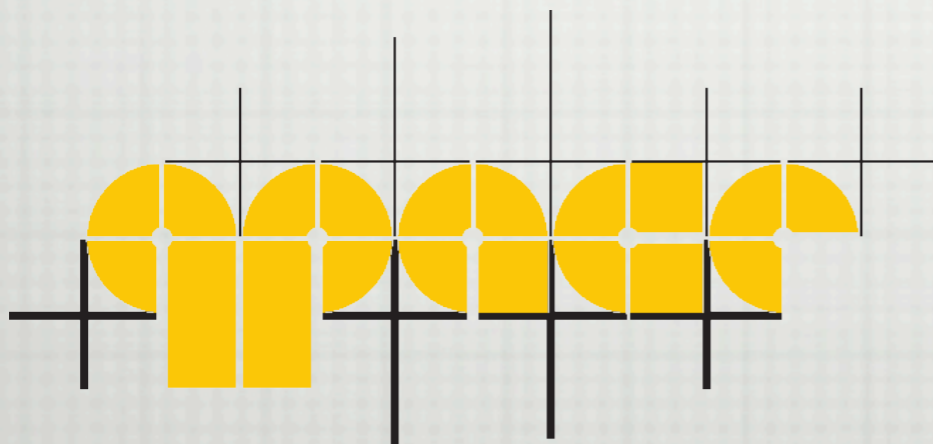


# SU(3) THERMODYNAMICS AN THE ONSET OF PERTURBATIVE BEHAVIOUR

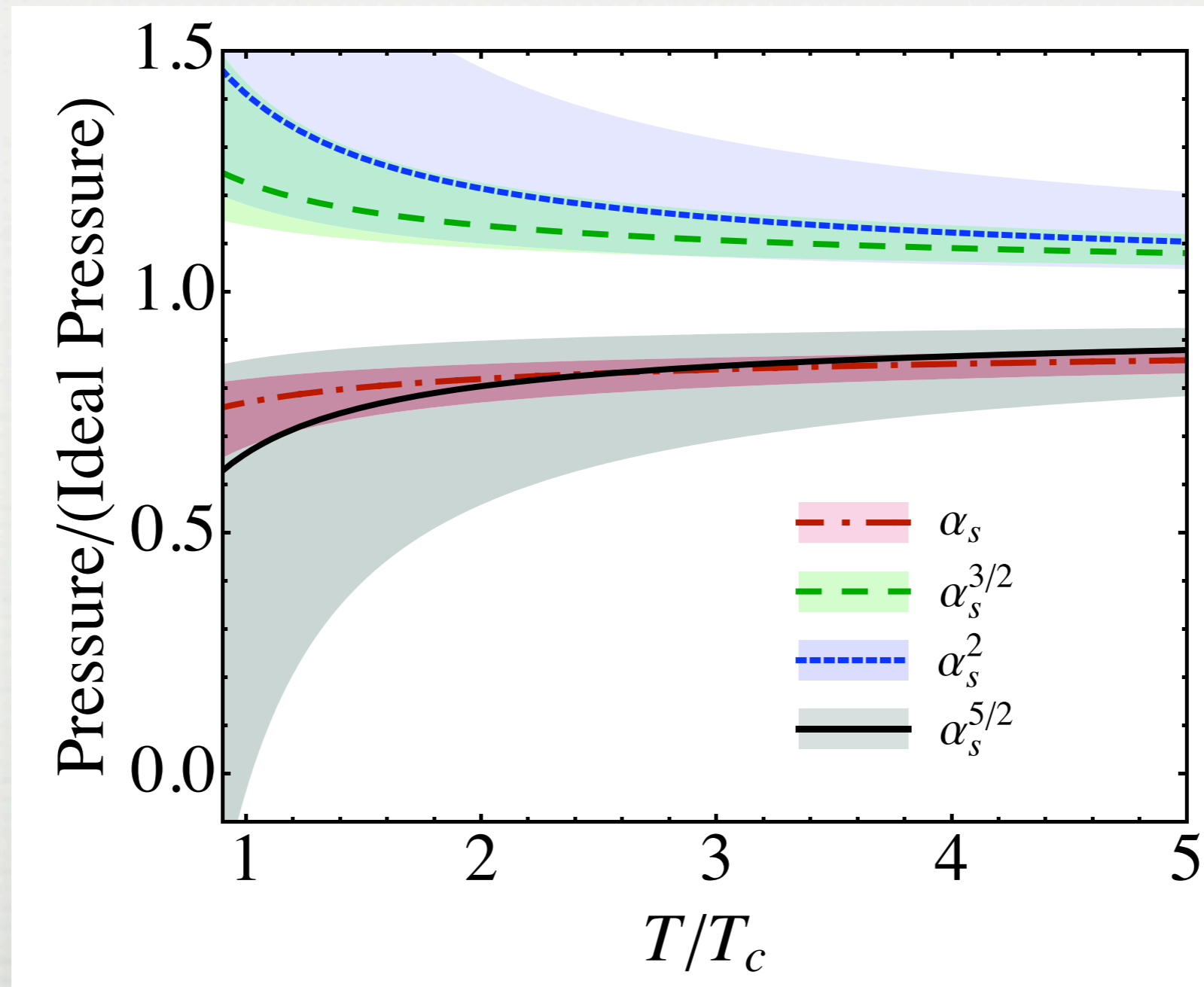
SZABOLCS BORSANYI

WUPPERTAL

G. ENDRÖDI, Z. FODOR, S. D. KATZ, K.K. SZABÓ



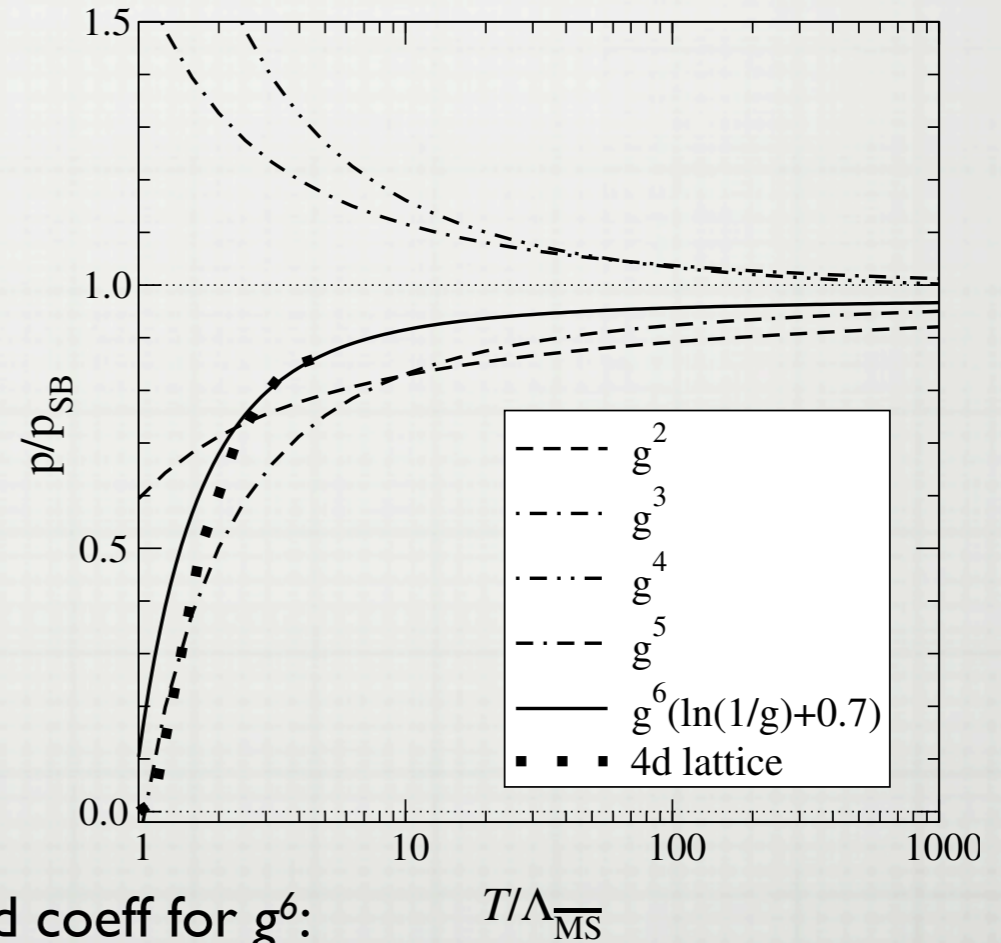
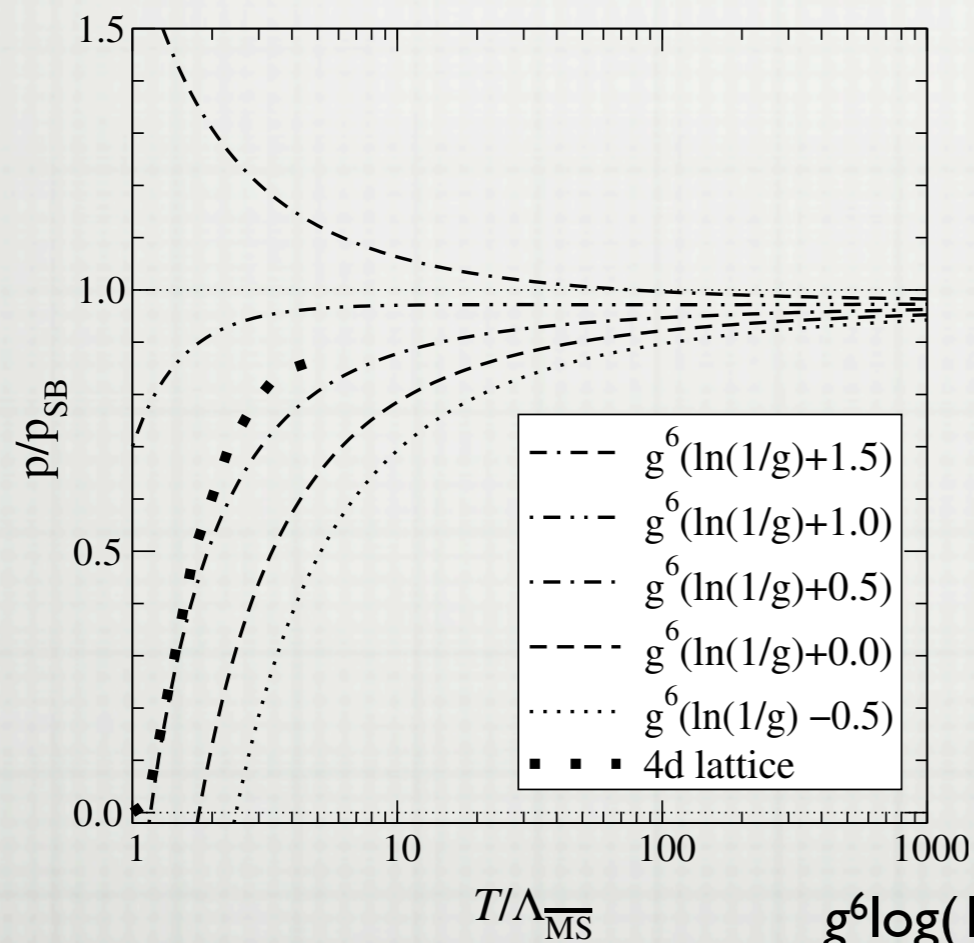
# PERTURBATION THEORY



(Plot: Andersen, Strickland, Su: 1005.1603)

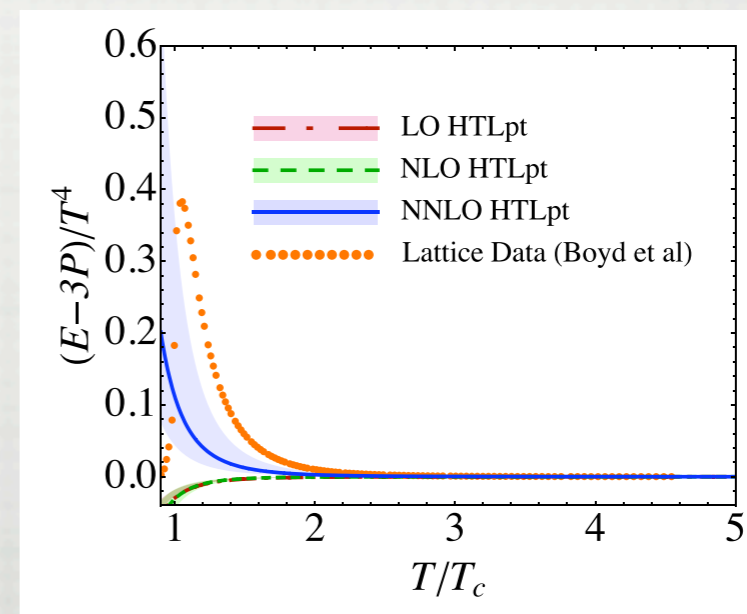
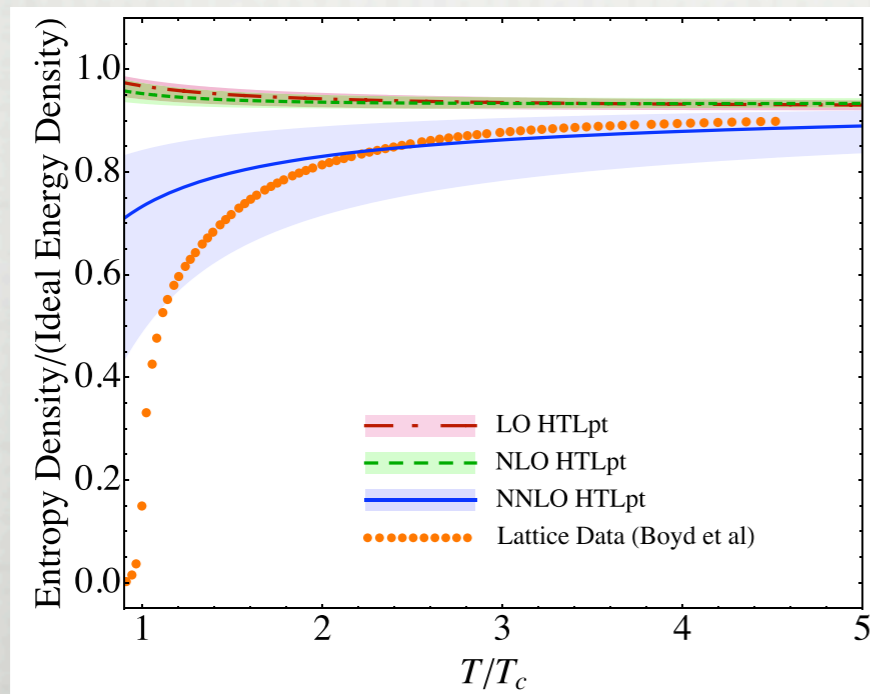
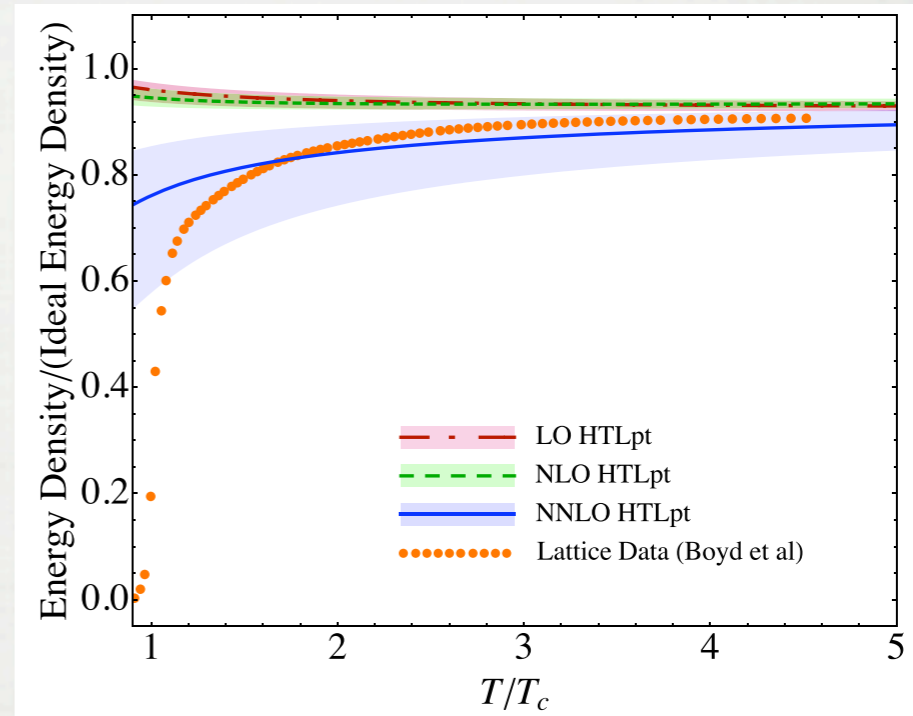
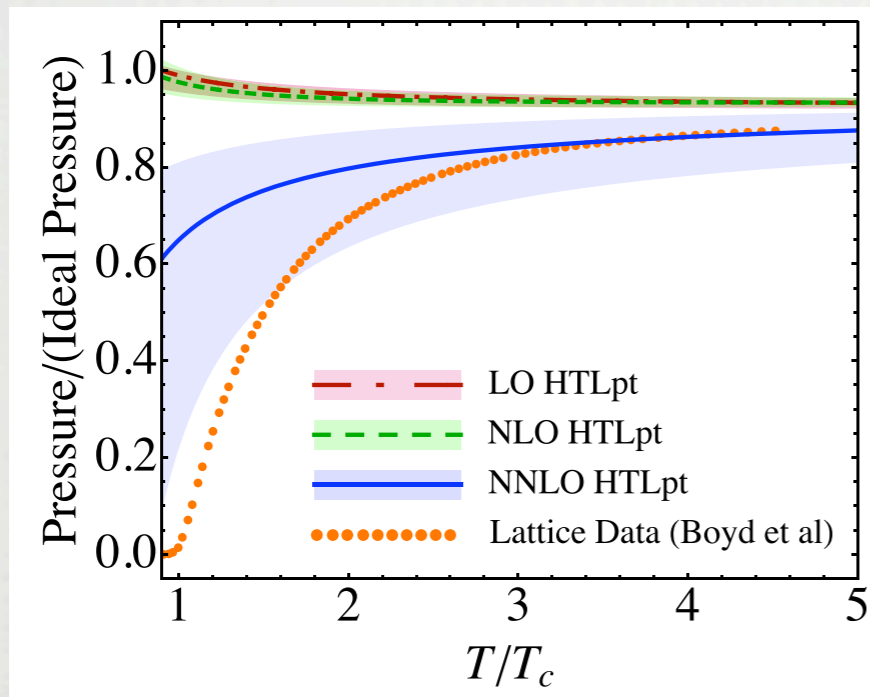
# PERTURBATION THEORY

PERTURBATIVE RESULTS ARE KNOWN UP TO  $O(g^6 \log(1/g))$   
 THE  $g^6$  COEFFICIENT IS FITTED TO '96 LATTICE DATA



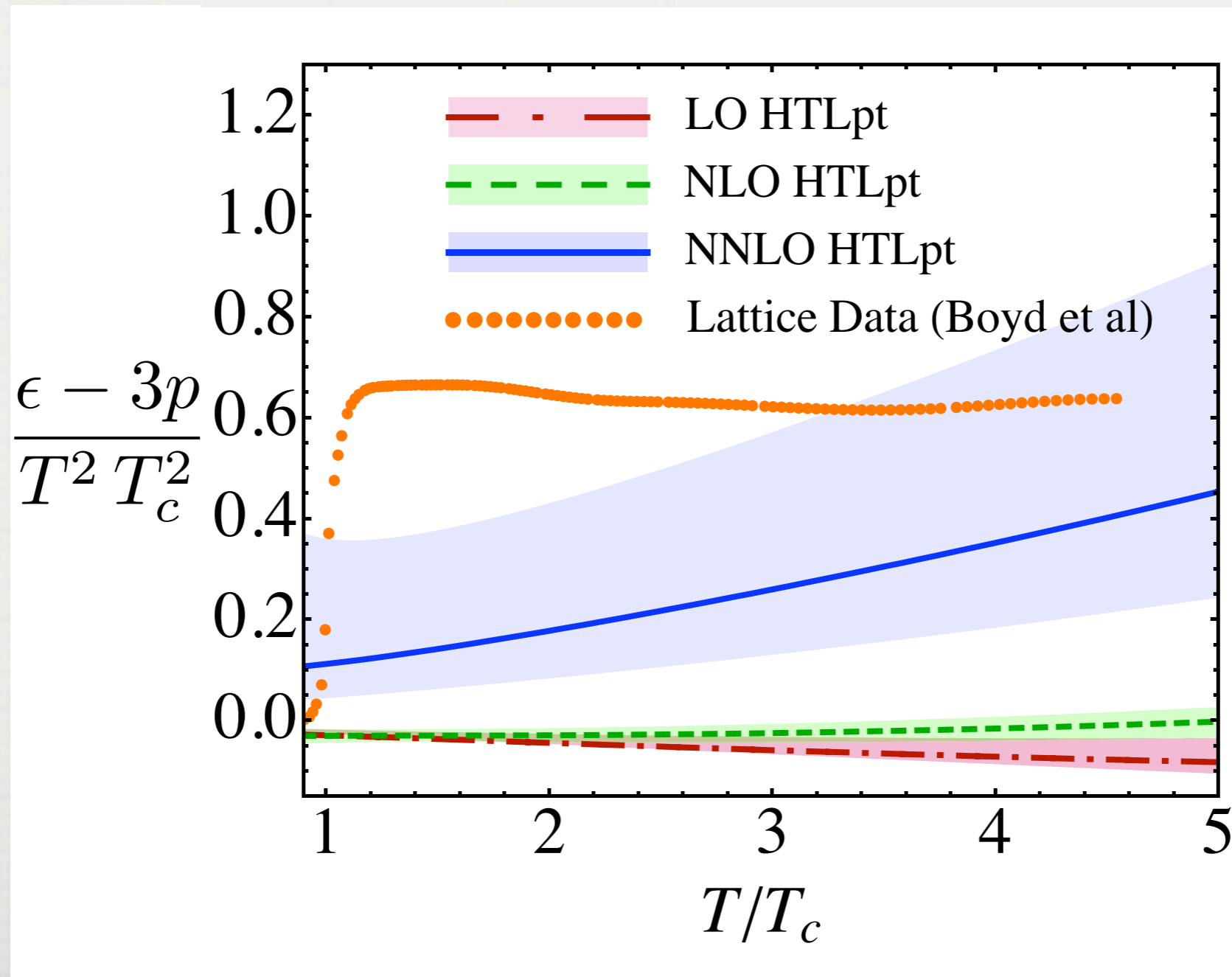
Kajantie, Laine, Rummukainen, Schroder: **Phys.Rev.D67:105008,2003**

# HARD THERMAL LOOPS



Andersen, Strickland, Su: 1005.1603

# RESCALED TRACE ANOMALY



Andersen, Strickland, Su: 1005.1603

GOAL:

LATTICE  $SU(3)$   
EQUATION OF STATE  
UP TO “PERTURBATIVE”  
TEMPERATURES

# SU(3) THERMODYNAMICS

---

- 1996 BOYD ET AL: EQUATION OF STATE  
 $T < 4.5 T_C$  (PLAQUETTE ACTION,  $N_T=8$ )
- 1998 BEINLICH ET AL: EQUATION OF STATE  
 $T < 3 T_C$  (SYMANZIK ACTION,  $N_T=4$ )
- 2007 GUPTA & KACZMAREK: POLYAKOV LOOP  
 $T < 24 T_C$  (PLAQUETTE ACTION,  $N_T=4$ )
- 2009 PANERO: EQUATION OF STATE  
 $0.8 T_c < T < 3.4 T_c$  (NO DEPENDENCE)
- 2010 DATTA & GUPTA: EQUATION OF STATE, LATENT HEAT  
 $0.8 T_c < T < 4 T_c$  (NO DEPENDENCE, CONT. LIM) [TALK LATER TODAY!]
- THIS WORK: EQUATION OF STATE + POLYAKOV LOOP  
 $0.8 T_C < T < 10000 T_C$  (TREE LEVEL SYMANZIK ACTION,  $N_T=5,6,8$ )

# CHALLENGES

---

- SCALE FUNCTION AT HIGH BETA

WHEN DOES THE 2-LOOP RUNNING START?

IMPROVED SCHEMES

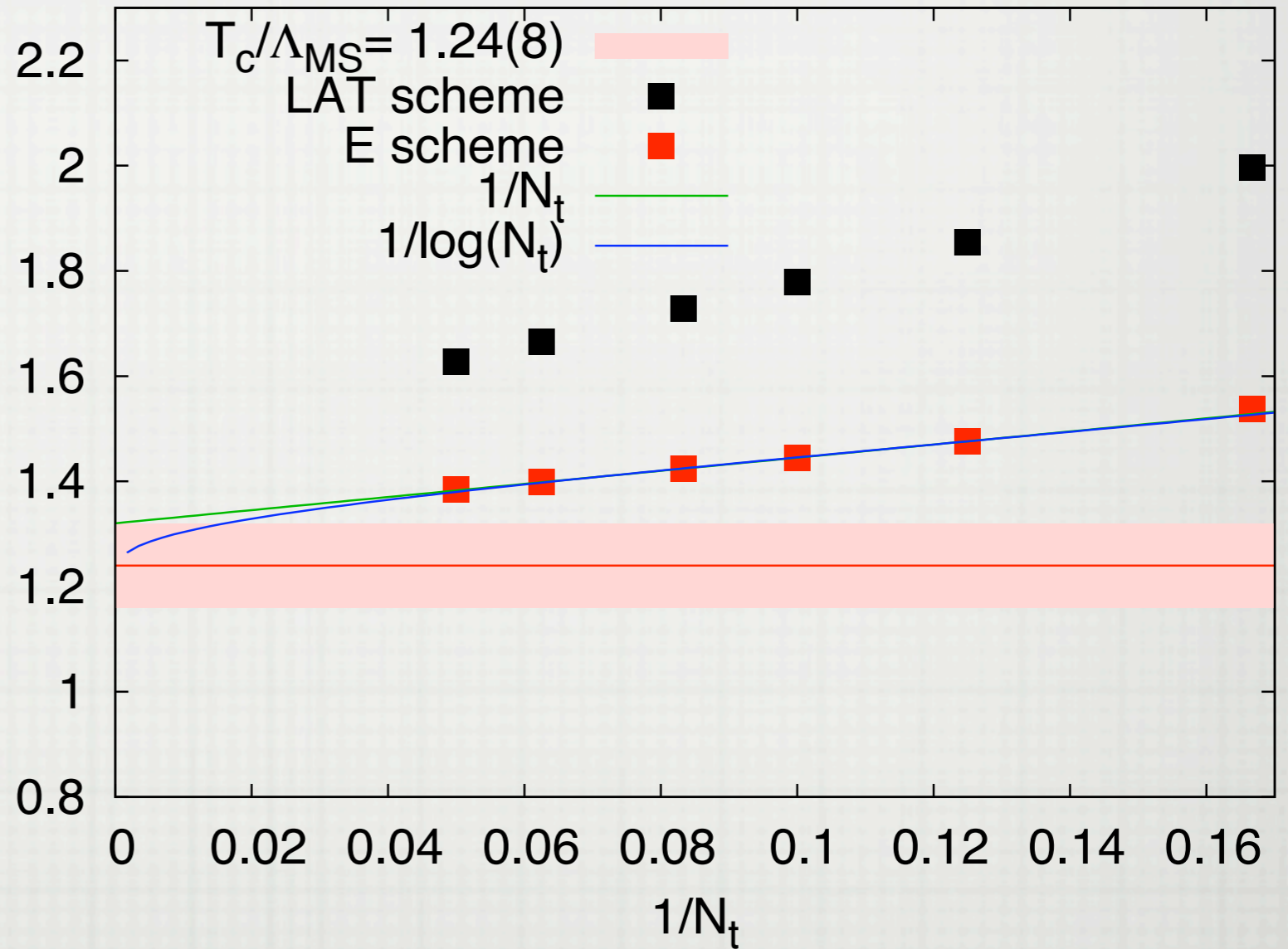


# ASYMPTOTIC SCALING

$N_\tau$	$\beta_c$ (Improved)
3	3.90812(7)
4	4.07252(13)
5	4.19963(14)
6	4.31466(24)

CELLA ET AL 1994

$T_c/\Lambda_{MS}$



OUR DATA:

$N_t$	$\beta_c$
8	4.5092(27)
10	4.6729(75)
12	4.811(10)
16	5.037(16)
20	5.217(30)

E SCHEME A'LA: BALI & SCHILLING 1992

SIMILAR WORK:

PLAQ ACTION/RO: GÖCKELER ET AL 2005

# CHALLENGES

---

- SCALE FUNCTION AT HIGH BETA

WHEN DOES THE 2-LOOP RUNNING START?

IMPROVED SCHEMES

- AUTOCORRELATION

DOES IT EXPLODE AS BETA GROWS?

FINITE T BOXES, MODERATE VOLUME, 100000 LONG STREAMS

# CHALLENGES

---

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DOES IT EXPLODE AS BETA GROWS?

FINITE T BOXES, MODERATE VOLUME, 100000 LONG STREAMS

- LARGE (ENOUGH) VOLUME

SMALL VOLUME MAY KILL NONPERTURBATIVE EFFECTS

VOLUME EFFECTS ARE STRONGLY SUPPRESSED (for EOS only)

BIG VOLUMES UNTIL  $T < 20T_c$

# CHALLENGES

---

- SCALE FUNCTION AT HIGH BETA

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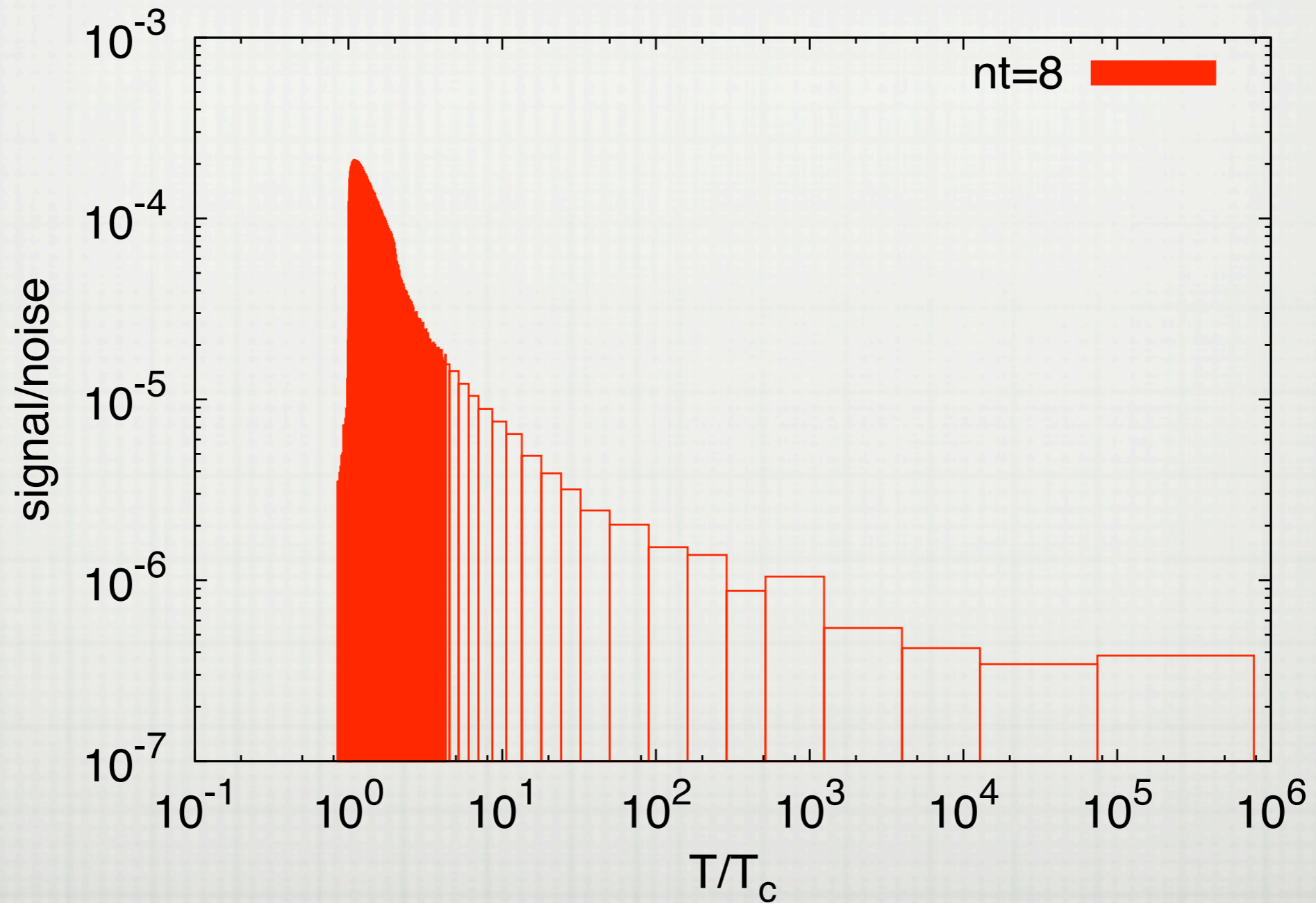
- STATISTICS ( $NT=5,6,8$ )

SIGNAL-TO-NOISE RATIO DROPS QUICKLY WITH TEMPERATURE

LARGE SCALE SU(3) PROJECT

QPACE

# EVEN SU(3) CAN BE EXPENSIVE



# STATISTICS

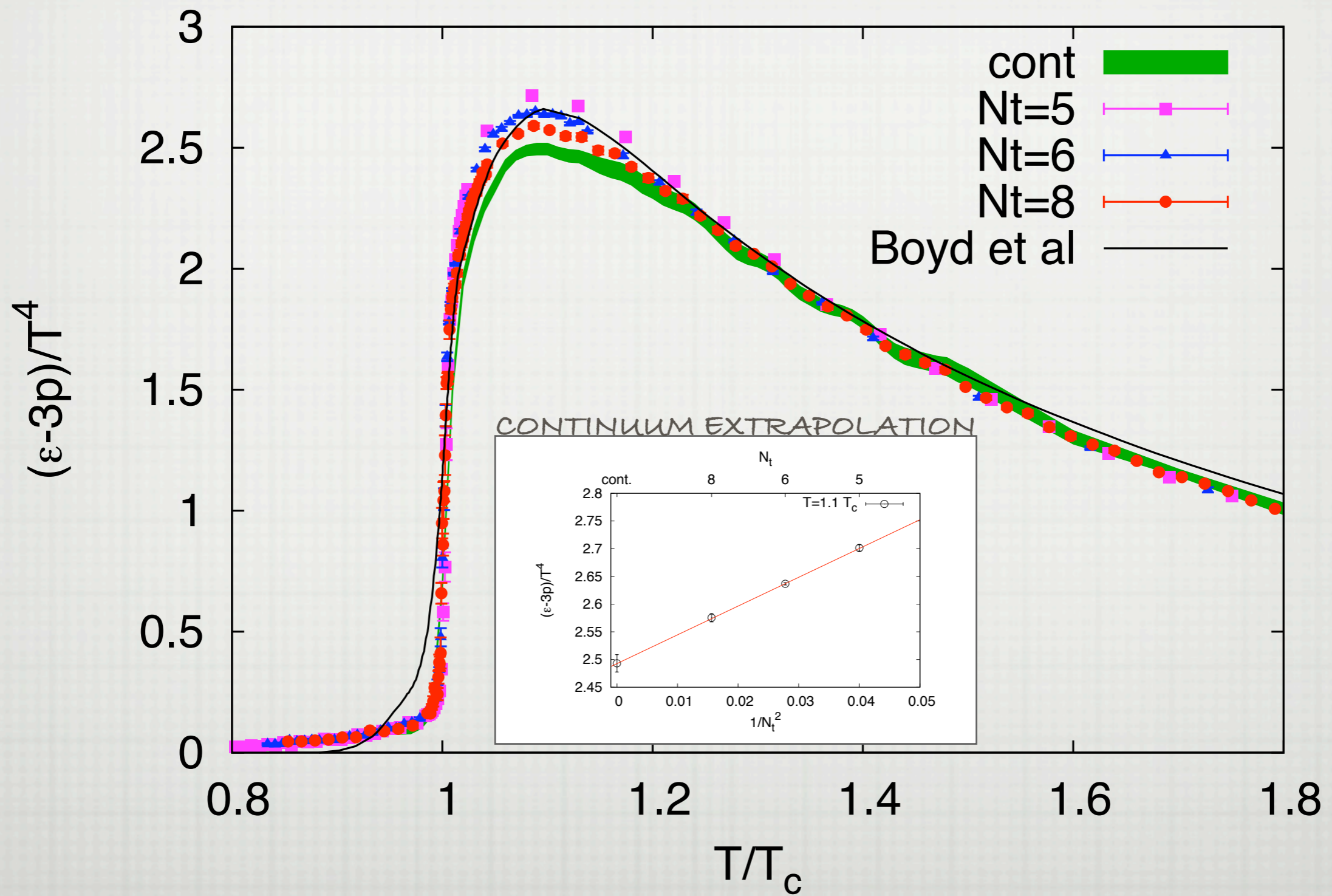
Lattice	Beta range	Temperature range [Tc]	No. of updates
$40^3 \times 5$	4.0 - 15.0	0.7 - 400000	182526873
$40^3 \times 10$			75644407
$48^3 \times 6$	4.2 - 15.0	0.83 - 330000	136243228
$48^3 \times 12$			63099000
$64^3 \times 8$	4.3 - 13.0	0.73 - 23650	89463241
$64^3 \times 16$			39061437

COMPARED TO THE UNQUENCHED (STAGGERED) EOS:

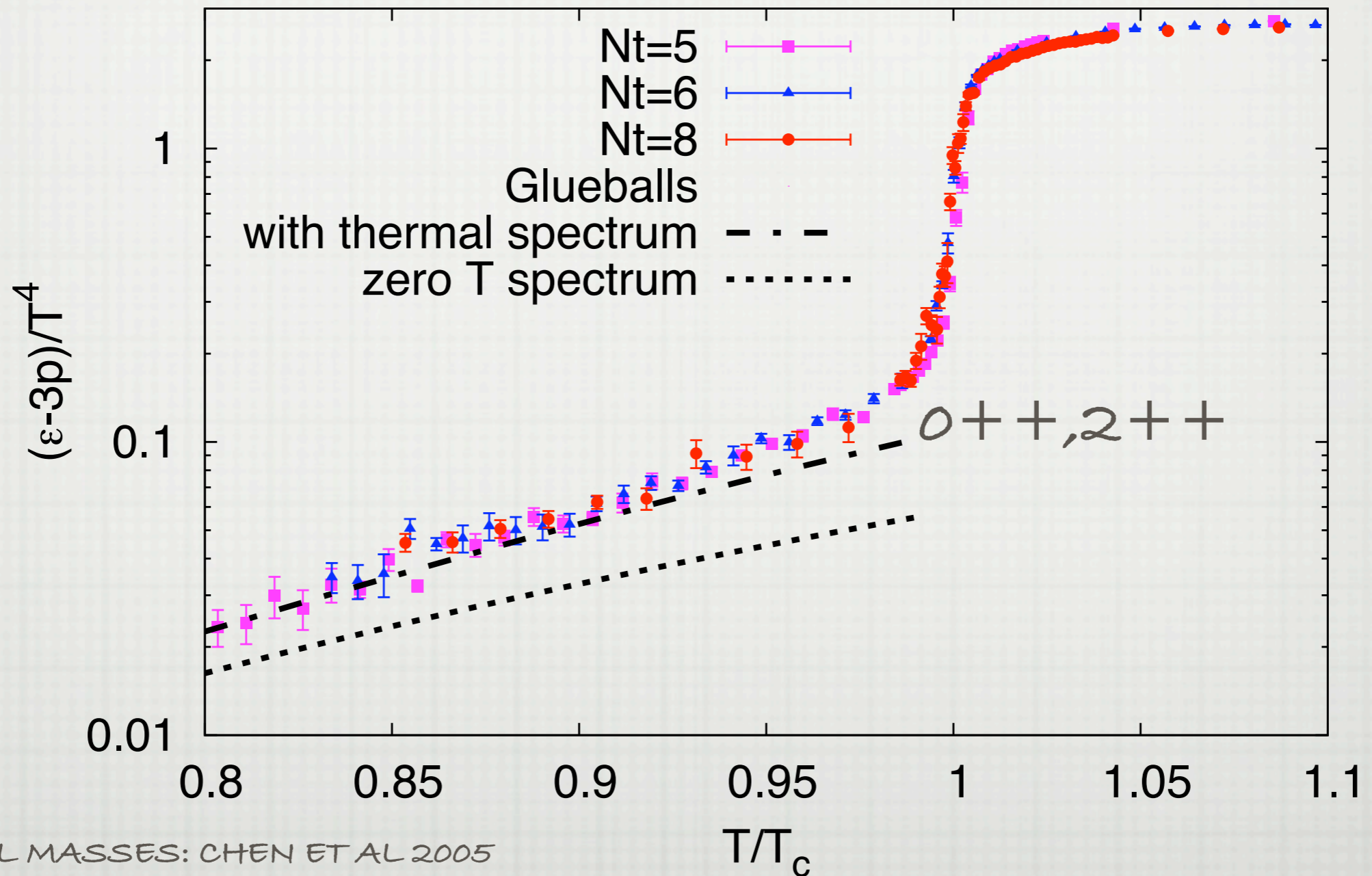
1000X CHEAPER UPDATE

10X MORE SIMULATION POINTS; 10 X SMALLER ERROR BARS

# TRANSITION REGION



# THERMAL GLUON GAS



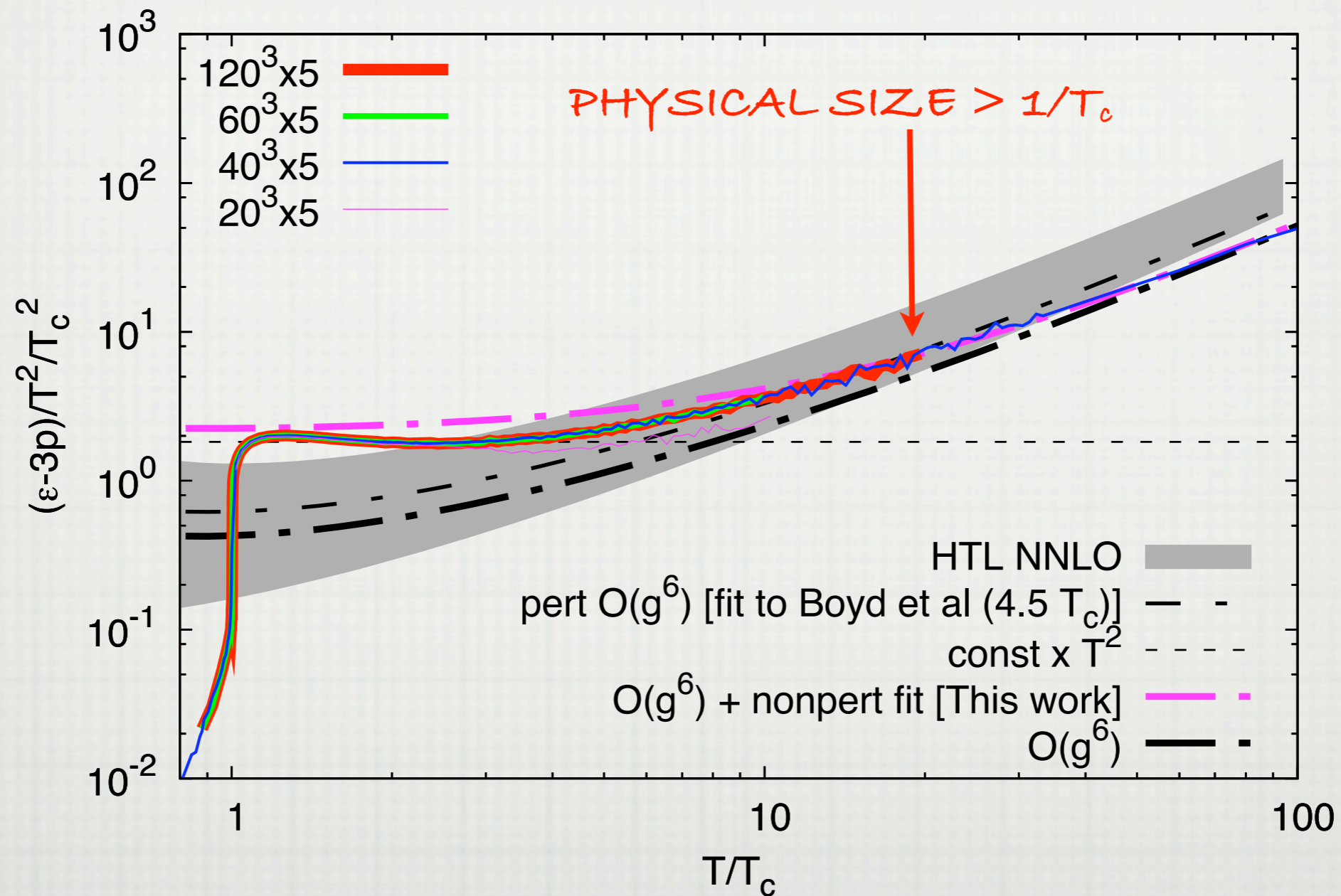
GLUEBALL MASSES: CHEN ET AL 2005

THERMAL GLUEBALL MASSES: ISHII ET AL 2002

SEE ALSO: BUISSERET, 0912.0678



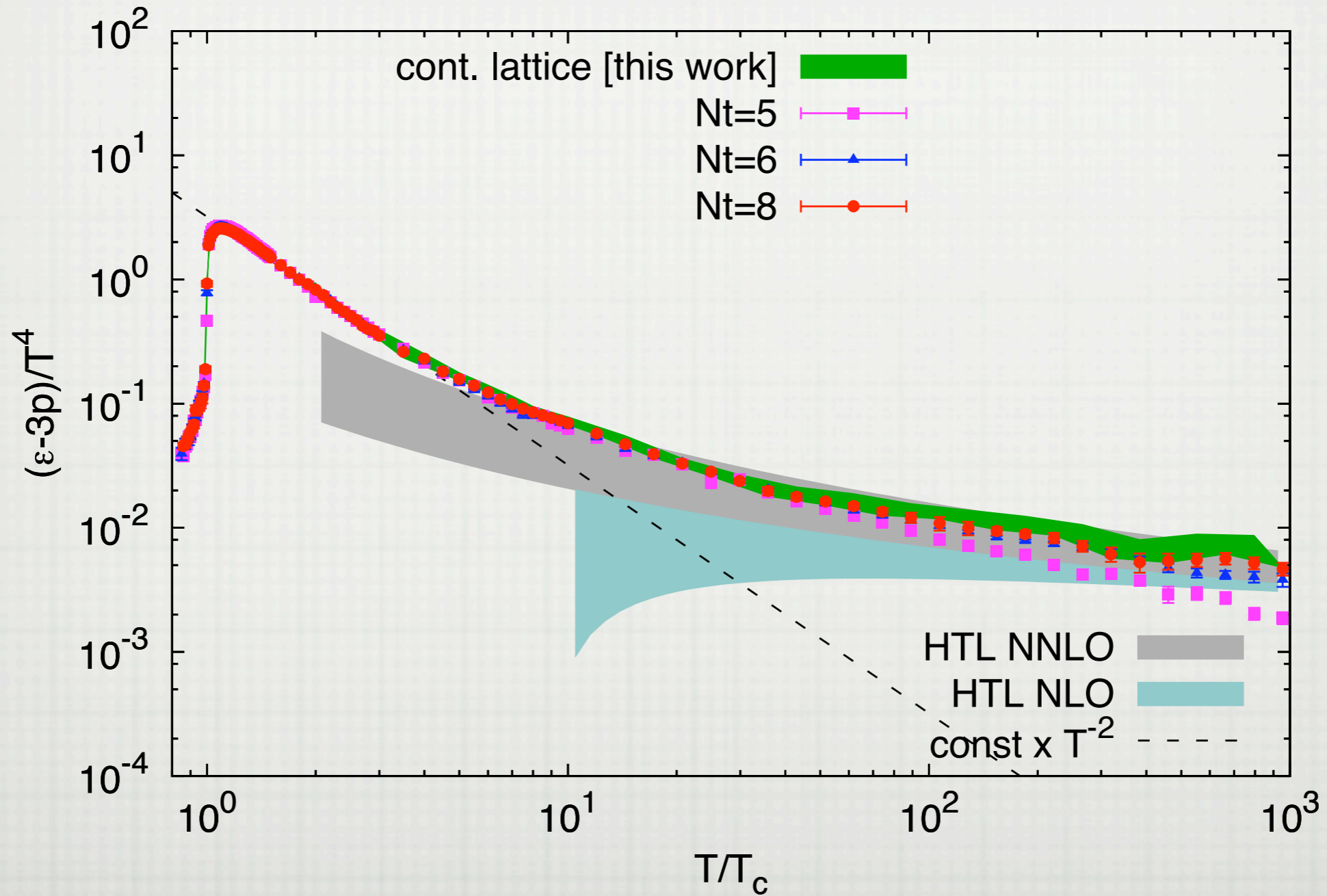
# ONSET OF PERTURBATIVE BEHAVIOUR



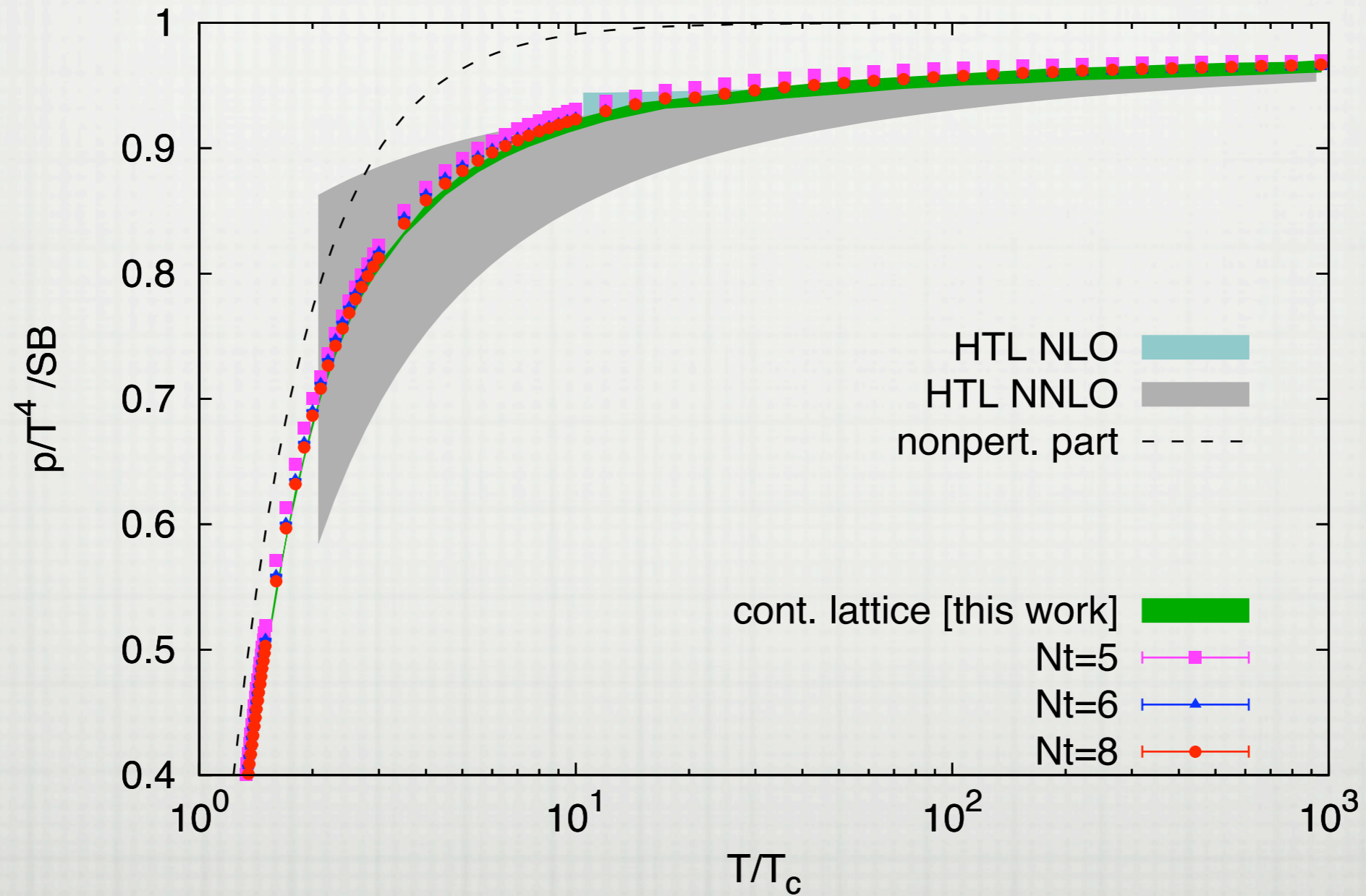
HTL NNLO: Andersen, Strickland, Su **Phys.Rev.Lett.**104:122003,2010

$O(g^6)$ : Kajantie, Laine, Rummukainen, Schroder: **Phys.Rev.D**67:105008,2003

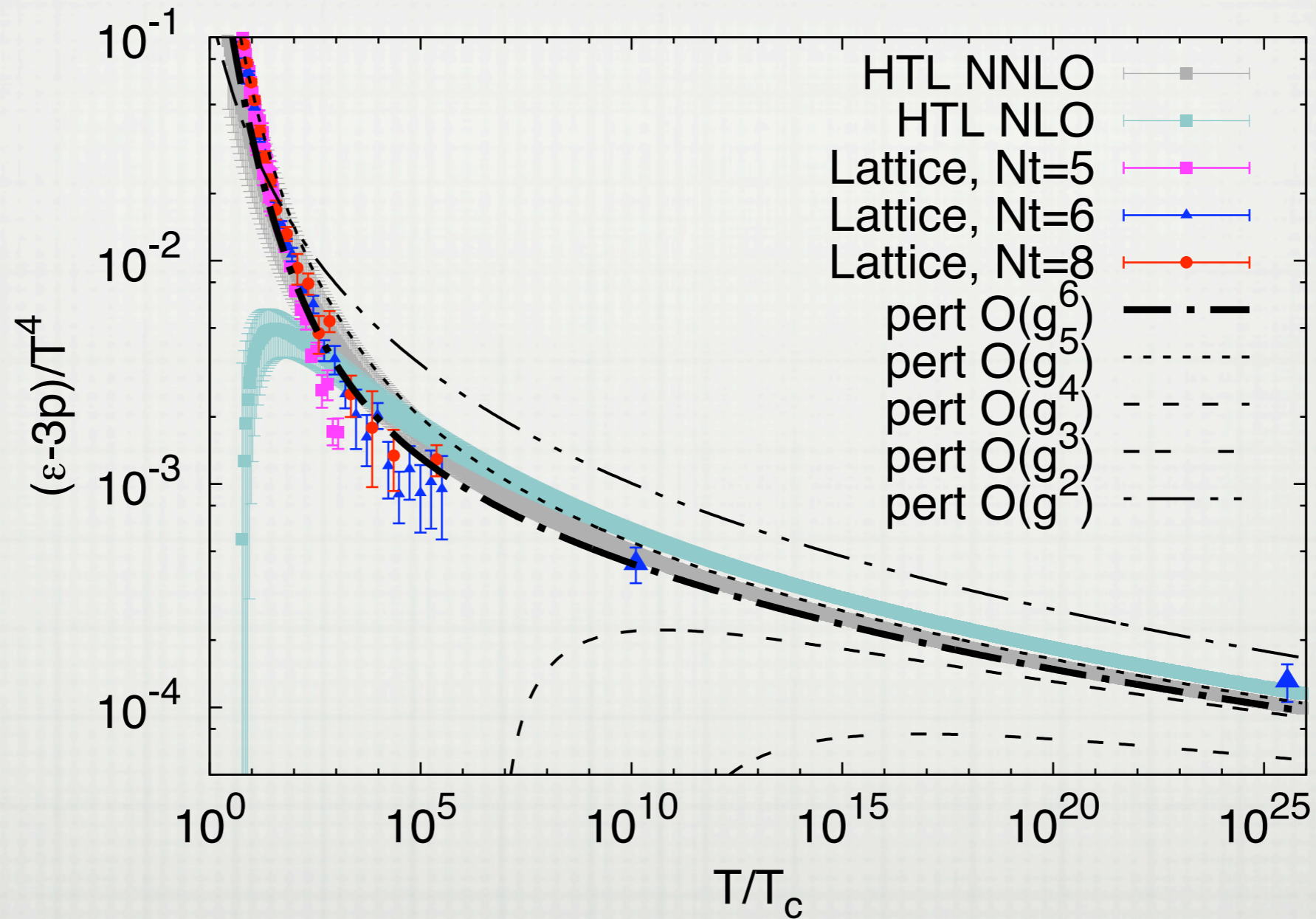
# HIGHER TEMPERATURES



# LATTICE VS THE PERTURBATIVE EOS



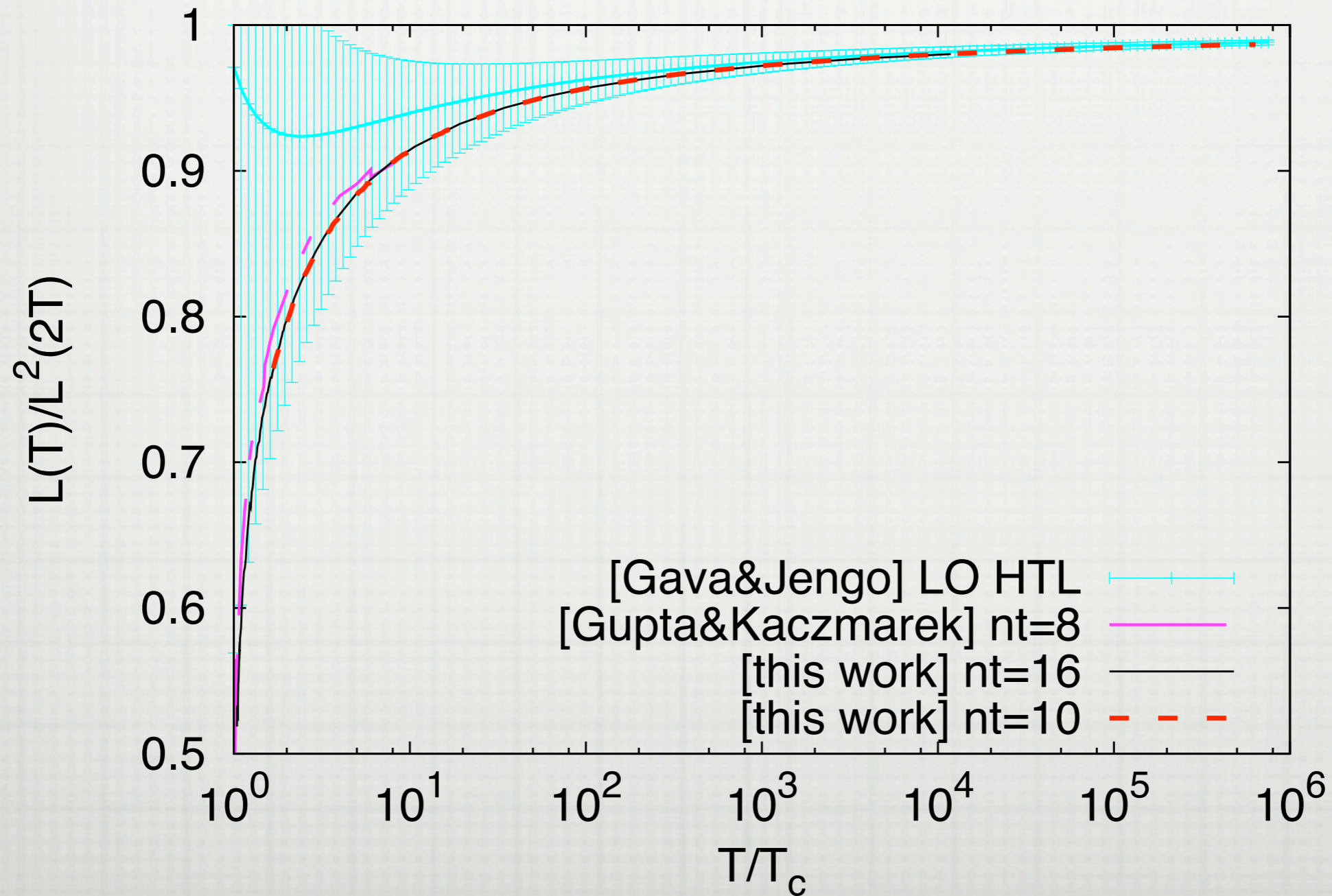
# EXTREME HIGH TEMPERATURES



SIMULATED IN "PERTURBATIVELY" SMALL VOLUMES,  
BUT THIS OBSERVABLE IS INSENSITIVE TO THAT.

# POLYAKOV LOOP

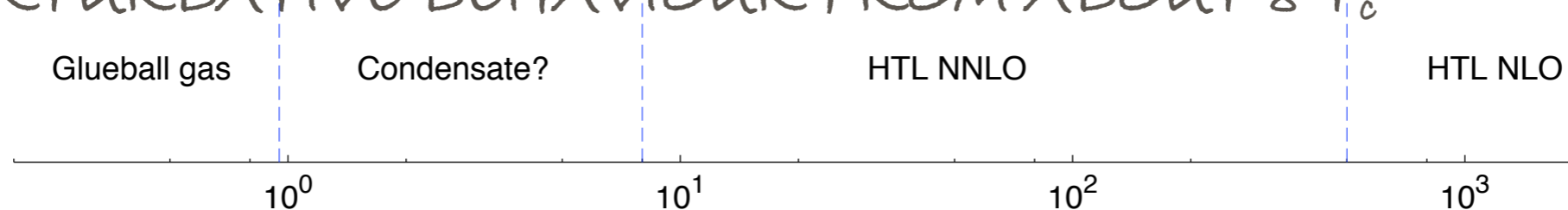
A RENORMALIZATION SCHEME INDEPENDENT COMBINATION:



# SUMMARY

- THERMODYNAMICS OF THE  $SU(3)$  THEORY HAS BEEN SOLVED UP TO THE TEMPERATURE WHERE ORDERS OF HTL PERTURBATION THEORY START TO AGREE.

- PERTURBATIVE BEHAVIOUR FROM ABOUT  $8 T_c$



- THE LOW TEMPERATURE REGION IS RESOLVED WITH PRECISION: THERMAL GLUEBALLS DOMINATE
- POLYAKOV LOOP RATIO HAS BEEN DETERMINED FOR ALL TEMPERATURES.

- REFINED ESTIMATE:  $T_c / \Lambda(\overline{MS}) = 1.24(8)$

SPARE SLIDES

# POLYAKOV LOOP

---

A RENORMALIZATION SCHEME INDEPENDENT COMBINATION:

$$L_r(T) = L_0(\beta) * Z(\beta)^{N_t}$$

$$L_r(2 * T) = L_0(\beta) * Z(\beta)^{N_t/2}$$

$$\frac{L_r(T)}{[L_r(2 * T)]^2} = \frac{L_0(\beta; N_t)}{[L_0(\beta; N_t/2)]^2}$$

THE RENORMALIZED POLYAKOV LOOP IS KNOWN TO LEADING ORDER IN HTL PERTURBATION THEORY.  
THIS COMBINATION, TOO, IS ACCESSIBLE.

SEE ALSO: GUPTA&KACZMAREK 2007



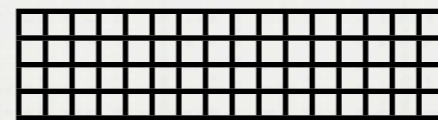
# LATTICE VOLUMES

STANDARD RUNS:  $40^3 \times 5$ ,  $48^3 \times 6$ ,  $64^3 \times 8$

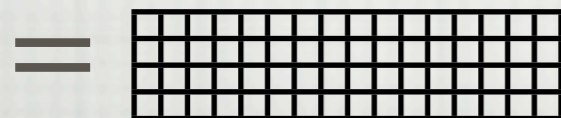
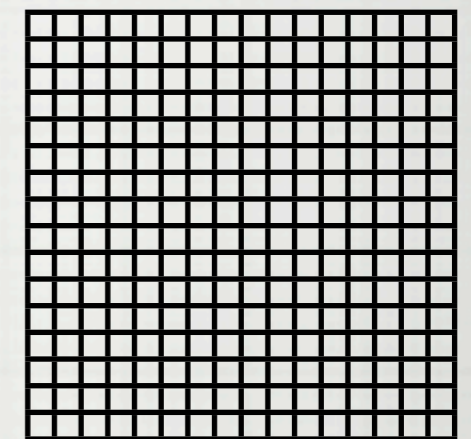
BIG VOLUMES:  $60^3 \times 5$ ,  $120^3 \times 5$

TREE LEVEL SYMANZIK ACTION

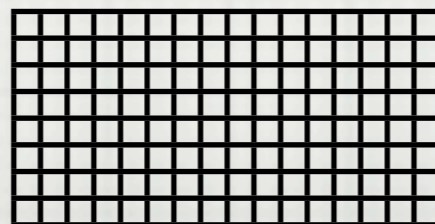
RENORAMALIZATION:



-



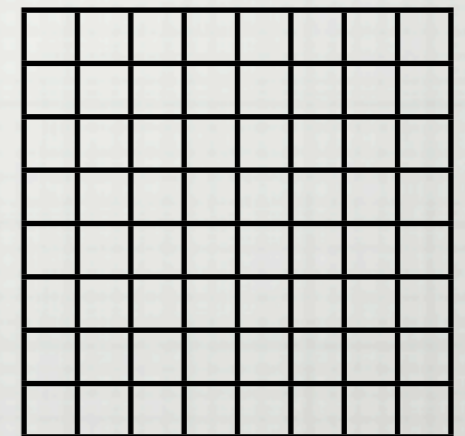
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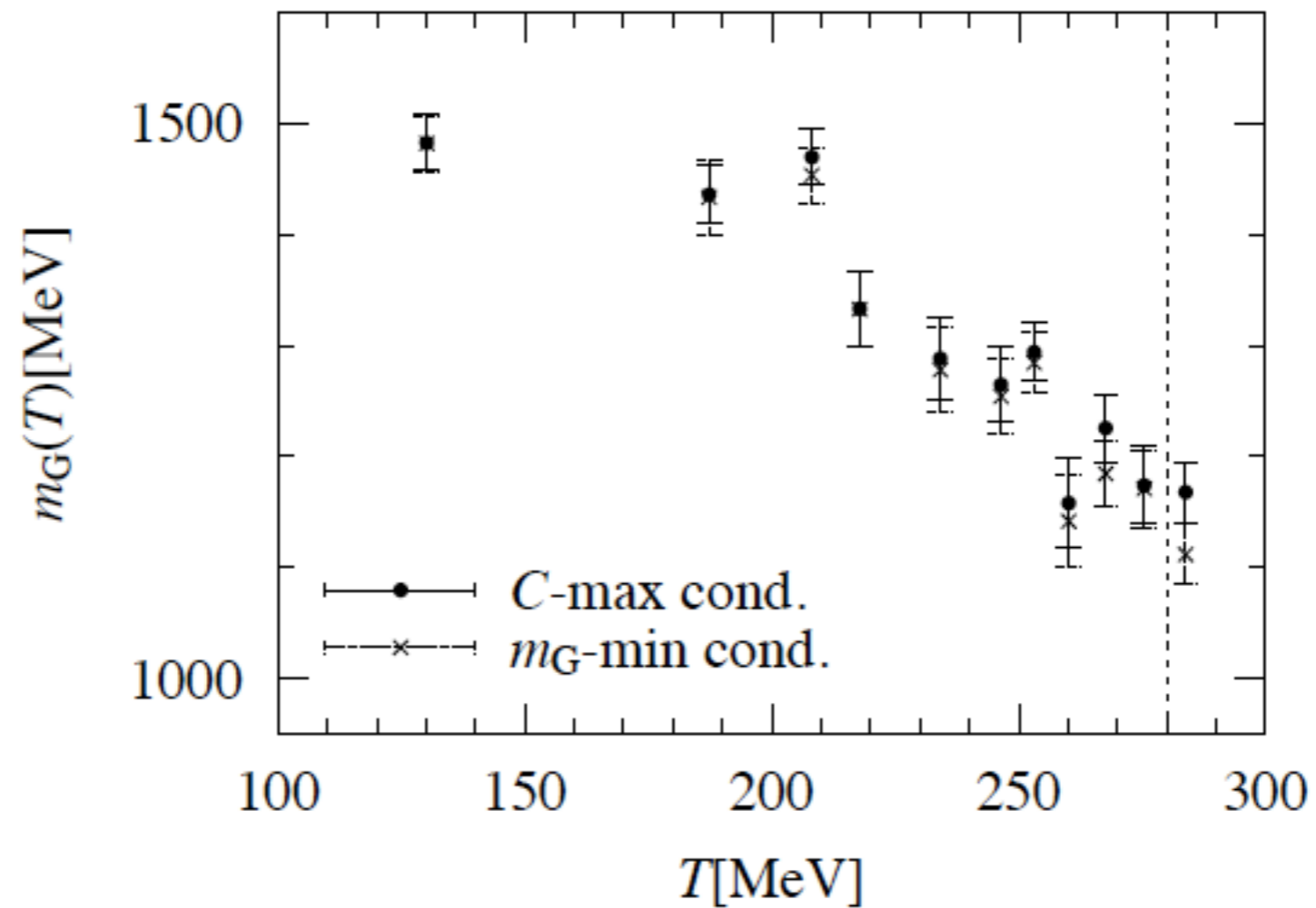


$$= \frac{\epsilon(T) - 3p(T)}{T^4} - \frac{\epsilon(T/2) - 3p(T/2)}{T^4}$$

COARSE LATTICE'S CONTRIBUTION IS SUPPRESSED BY 1/16

# THERMAL GLUEBALLS

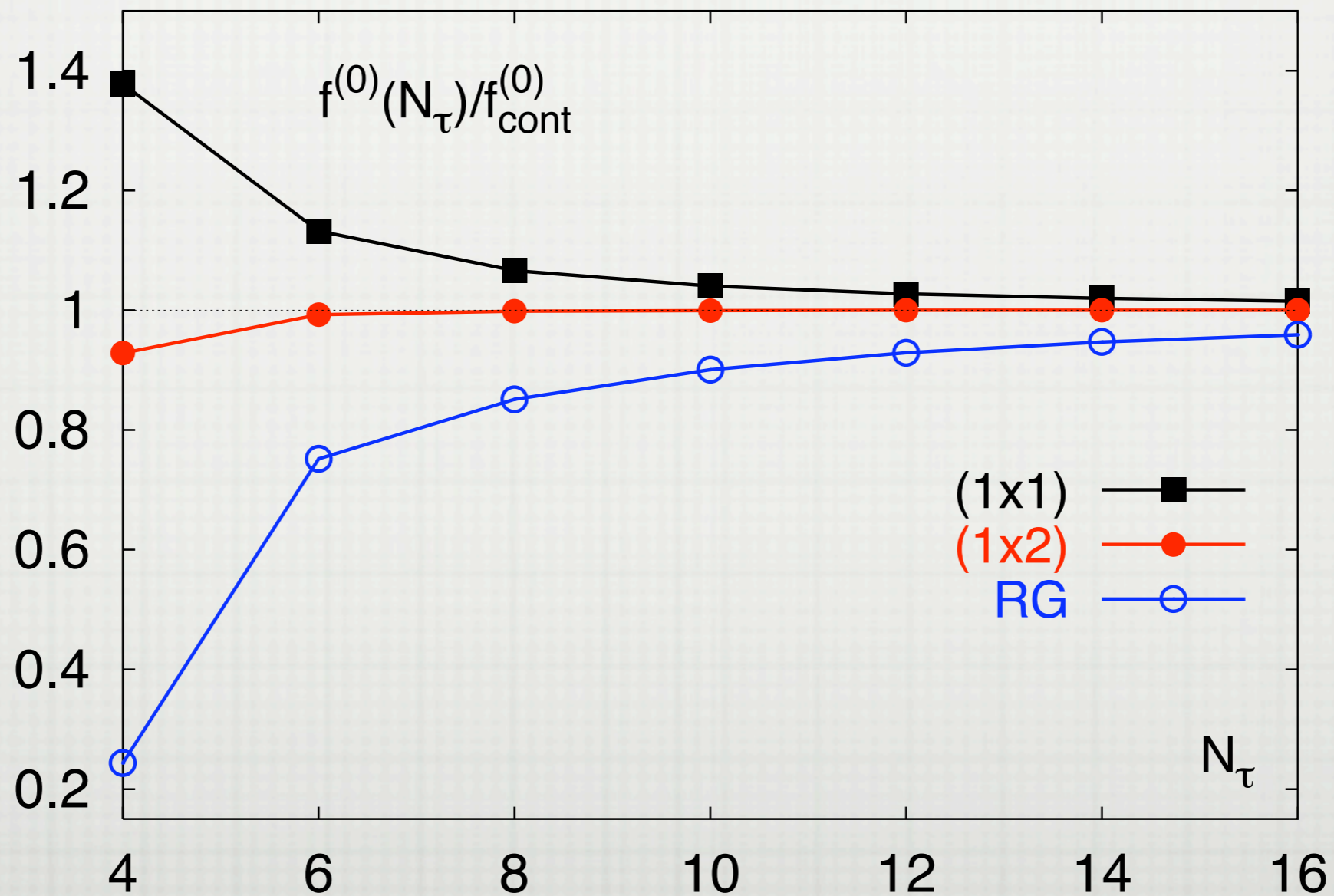
0++ GLUON MASS VS TEMPERATURE:



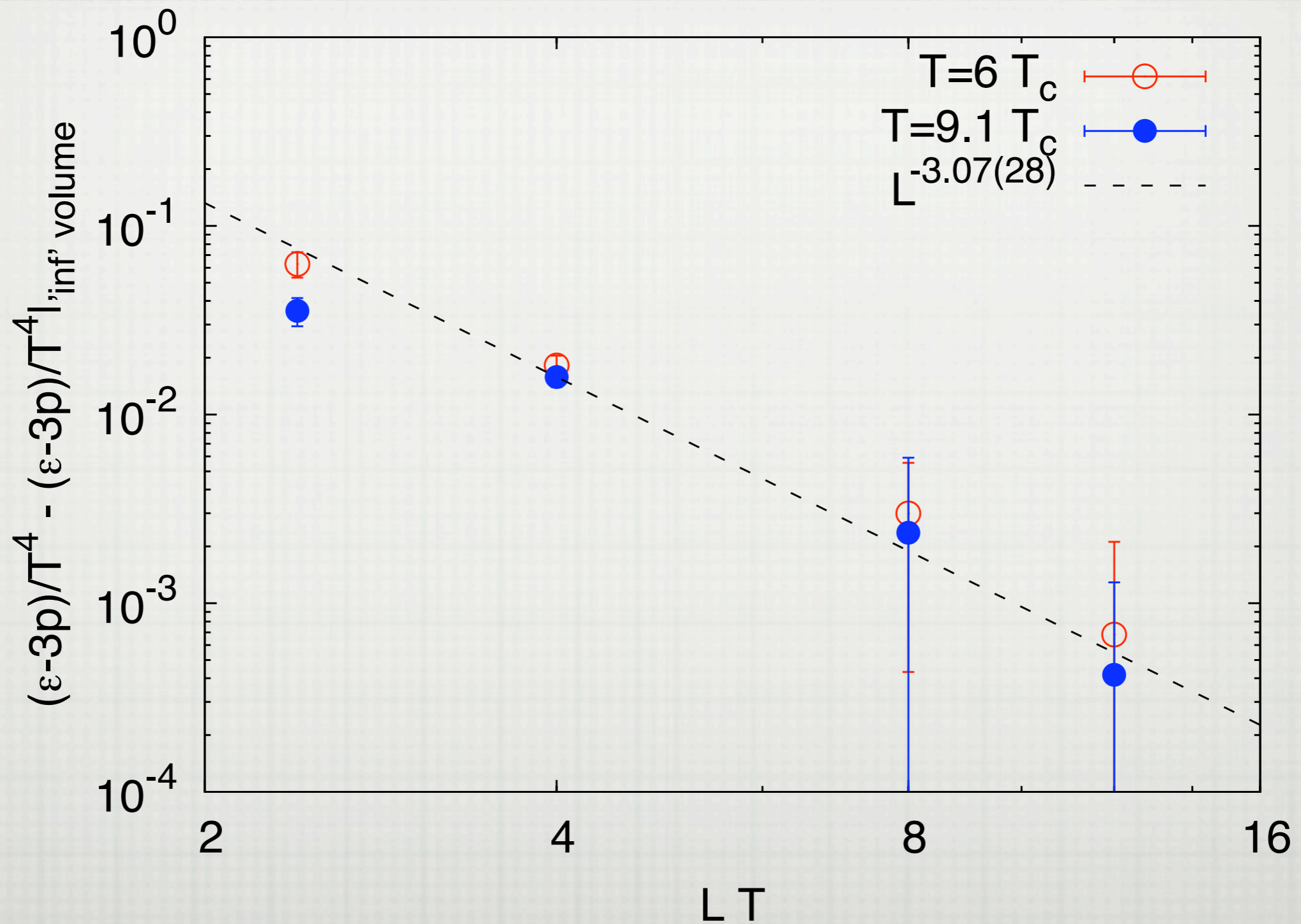
THERMAL GLUEBALL MASSES: ISHII ET AL 2002

# IMPROVED ACTIONS

FREE ENERGY AT INFINITE TEMPERATURE



# SCALING WITH VOLUME



# UNQUENCHED ENTROPY

