Static-Light Meson Potentials

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in collaboration with Gunnar Bali

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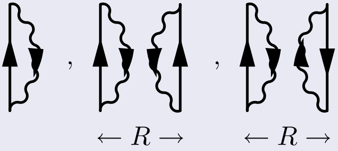
Outline

Procedure

Calculate potentials between two static-light mesons:



Fit the masses

For parallel static propagators determine:

$$V_{Q\bar{q}\bar{q}Q}(R) = M_{Q\bar{q}\bar{q}Q}(R) - (M_{Q\bar{q}} + M_{Q\bar{q}}). \tag{1}$$

Operators and quantum numbers I

For static light:

operator	O_h' **rep.**	J^P	$(\bar{Q}\ \Gamma\ q)_{meson}$
γ_5	G_1^+	$\frac{1}{2}^+$	$0^-, 1^-$
$\mathbb{1}$	G_1^-	$\frac{1}{2}^-$	$0^+, 1^+$
$\gamma_i \nabla_i$	G_1^-	$\frac{1}{2}^-$	$0^-, 1^-$
$(\gamma_1 \nabla_1 - \gamma_2 \nabla_2) + cycl.$	H^-	$\frac{3}{2}^-$	$1^+, 2^+$

Operators and quantum numbers II

For meson-meson potentials:

operator combinations	O_h $R=0$	SL parallel J^P	SL parallel $\Lambda_\eta^{\sigma_\nu}$	SL antiparallel J^{PC}	SL antiparallel $\Lambda_\eta^{\sigma_\nu}$
$\gamma_5 \otimes \gamma_5$	A_1^+	0^+	Σ_g^+	0^{++}	Σ_g^+
$\mathbb{1} \otimes \mathbb{1}$	A_1^+	0^+	Σ_g^+	0^{++}	Σ_g^+
$\gamma_5 \otimes \mathbb{1}$	A_1^-	0^-	Σ_u^-	0^{-+}	Σ_u^-
$\gamma_5 \otimes \gamma_i \nabla_i$	A_1^-	0^-	Σ_u^-	0^{-+}	Σ_u^-
$\gamma_5 \otimes (\gamma_1\nabla_1 - \gamma_2\nabla_2)$	T_1^-	1^-	Σ_u^+, Π_u	1^{--}	Σ_g^+, Π_g
$\gamma_i\nabla_i \otimes (\gamma_1\nabla_1 - \gamma_2\nabla_2)$	T_1^+	1^+	Σ_g^-, Π_g	1^{+-}	Σ_u^-, Π_u

$$R = 0 \longrightarrow J^{P(C)} \in O(3)(\otimes \mathcal{C}) \tag{2}$$

$$R > 0 \longrightarrow \Lambda_\eta^{\sigma_v} \in D_{\infty h};\ \eta = P(\cdot C) \tag{3}$$

Stochastic estimates

Masses are extracted from the time dependence of Euclidean two-point correlation functions:

$$\begin{aligned} C(t) &= \langle \mathcal{M}(\vec{y}, t+t_0)\, \mathcal{M}^\dagger(\vec{x}, t_0) \rangle, \\ \mathcal{M} &= \bar{Q}\, \mathcal{O}\, q. \end{aligned} \tag{4}$$

Stochastic estimator techniques:

$$\frac{1}{N} \sum_n |\eta\rangle\langle\eta| = \overline{|\eta\rangle\langle\eta|} = \mathbb{1} + \mathcal{O}(1/\sqrt{N}). \tag{5}$$

Solve the linear system

$$D|\chi^i\rangle = |\eta^i\rangle, \tag{6}$$

and substitute Eq.(5):

$$D^{-1} = \overline{|\chi\rangle\langle\eta|}. \tag{7}$$

Variational method

So our correlator reads:

$$C(t) = \frac{1}{N} \sum_n \eta^{(n)^\dagger}(t_0 + t)\, \mathcal{O}\, D_Q^{-1}(t|t_0)\, \mathcal{O}\, \chi^{(n)}(t_0), \tag{8}$$

$$D_Q^{-1}(t|t_0) = \frac{1+\gamma_4}{2} \prod_{k=t_0}^{t_0+t-1} U_4^\dagger(x + k\hat{4}). \tag{9}$$

To improve our data and to extract also excited states we use several different operators $\mathcal{M}_i, i = 1 \ldots N$ and compute all cross correlations

$$C(t)_{ij} = \langle \mathcal{M}(y,t)_i\, \mathcal{M}^\dagger(y,0)_j \rangle. \tag{10}$$

Solve the generalized eigenvalue problem and obtain the eigenvalues

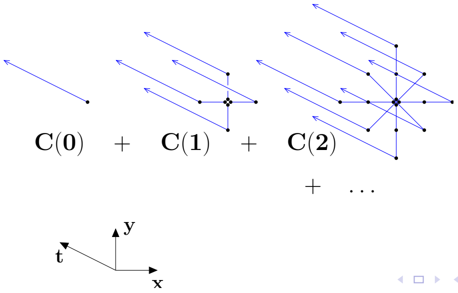
$$C(t)\overrightarrow{\nu}^{(k)} = \lambda^{(k)}(t) C(t_0) \overrightarrow{\nu}^{(k)}, \tag{11}$$

$$\lambda^{(k)}(t) \propto e^{-(t-t_0)M_k}[1 + O(e^{-(t-t_0)\Delta M_k})], \tag{12}$$

where M_k is the mass of the k-th state.

Noise reduction

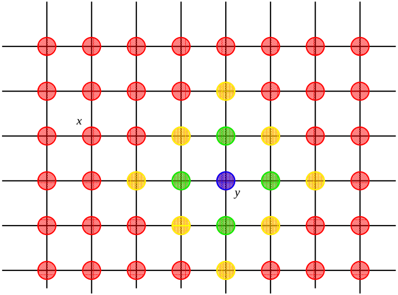
- Stout smearing → reduce static self-energy
- Gauss & APE smearing → improve ground state overlap of our operators
- Gauss smearing → generate operator basis
- Hopping Parameter Acceleration (HPA)



HPA

$$D = \mathbb{1} - \kappa H \tag{13}$$

$$D^{-1} = \sum_{j=0}^{\infty} (\kappa H)^j = \sum_{j=0}^{k-1} (\kappa H)^j + (\kappa H)^k D^{-1} \tag{14}$$

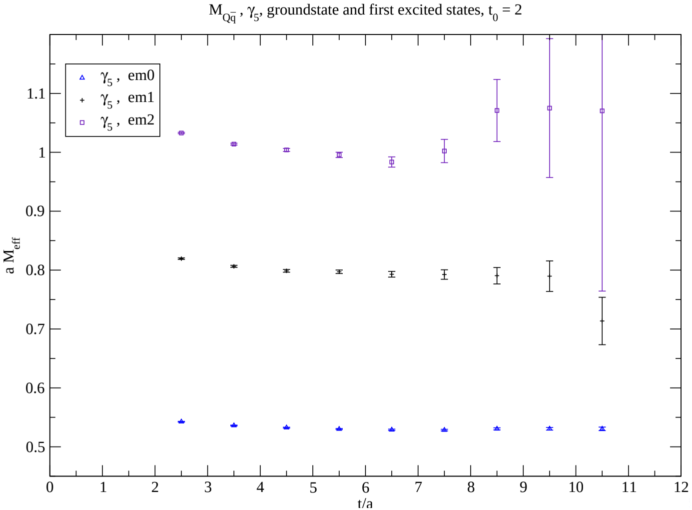


Technical details

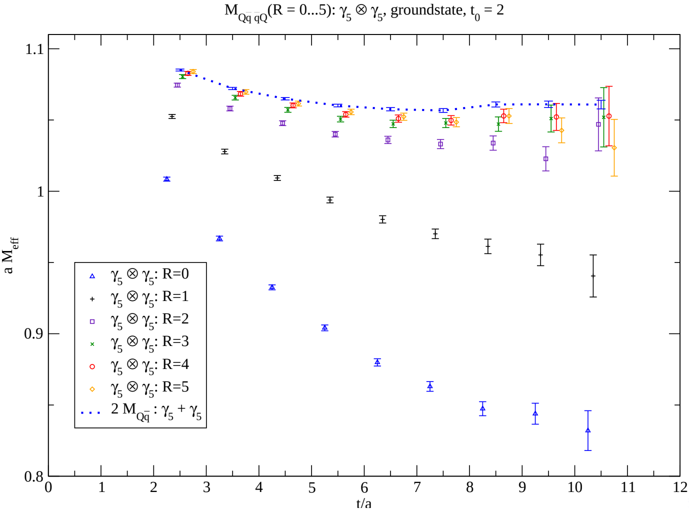
lattice size $L^3 \times T$	$16^3 \times 32$
β	5.29
c_{SW}	1.9192
$a\ [fm]$	0.084
$La\ [fm]$	1.34
$m_\pi\ [MeV]$	$781(3)$
κ	0.13550
# conf.	200
# estimates	300
smearing parameters:	
Stout	$N_{iter} = 1,\ \rho = \frac{1}{6}$
Gauss	$N_{iter} = 16, 50, 100,\ \kappa = 0.3$
APE	$N_{iter} = 15,\ \alpha = 2.5$

$$M_{\text{eff}}(t + 1/2) = \ln\left(C(t)/C(t+1)\right). \tag{15}$$

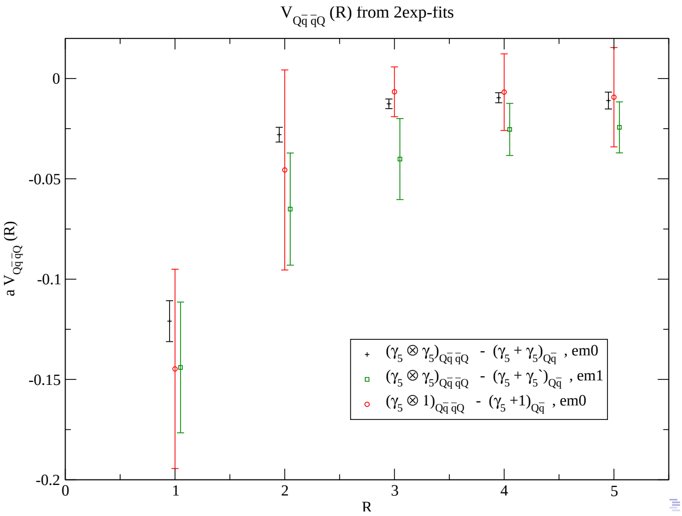
Effective masses: Static light



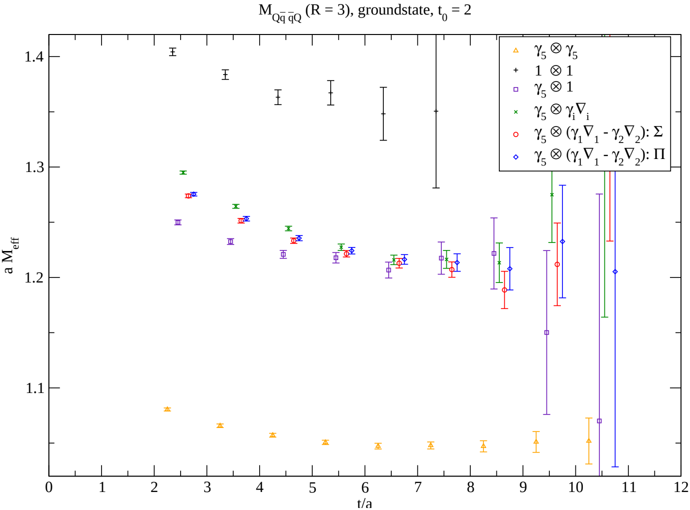
Effective masses: Meson potentials parallel I



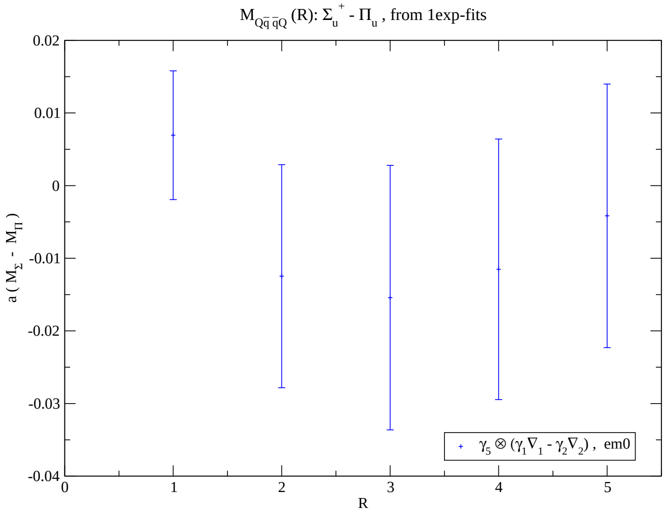
Meson potentials parallel: $V_{Q\bar{q}\bar{q}Q}(R)$



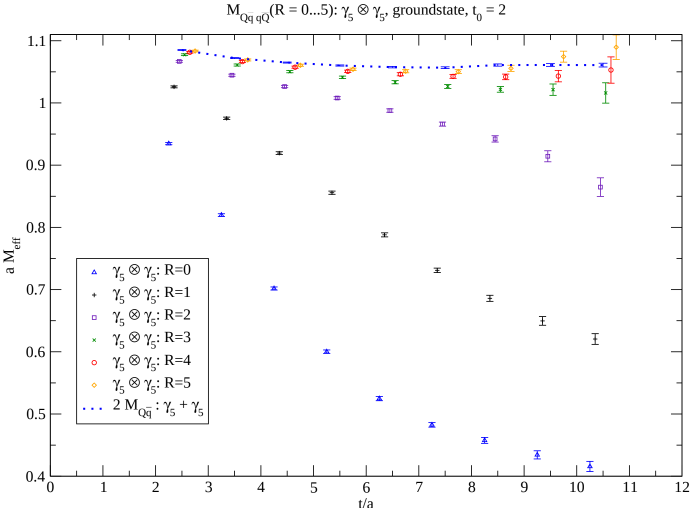
Effective masses: Meson potentials parallel II



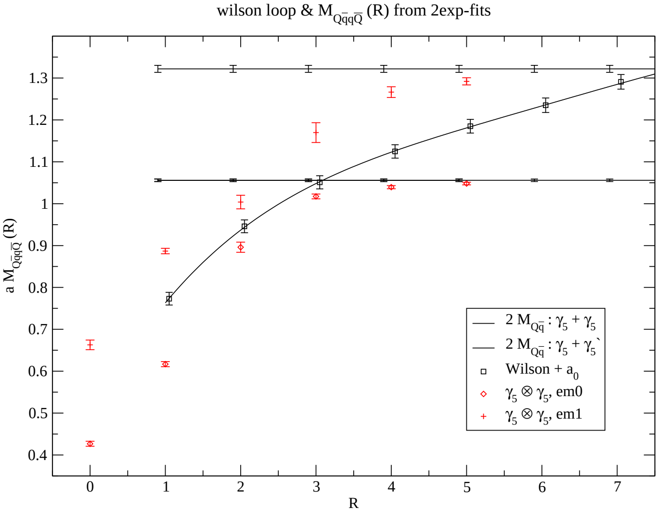
Meson potentials parallel: Mass splitting



Effective masses: Meson potentials antiparallel



Meson potentials: antiparallel



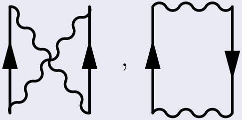
Summary & Outlook

Summary

- Attractive potential between two static light mesons for small distances
- Mass differences between Σ and Π states are smaller than $\approx 50\, MeV$
- $M_{Q\bar{q}\bar{q}Q}(R) \stackrel{R\to\infty}{\longrightarrow} 2\, M_{Q\bar{q}}$

Outlook

- Go to larger lattices
- Fit more operators
- Analyse crossing diagrams:



Thank you