Transfer Matrix for Partially Quenched QCD

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Motivation

Partial quenching is useful for Lattice QCD:

- Vary valence and sea quark masses independently
  (generalization: mixed actions)

- Contains full QCD (Bernard and MG, 1994); Low-energy constants are those of the real world (Sharpe and Shoresh, 2000)

⇒ Need Partially Quenched ChPT
but how do we know that PQChPT is the valid EFT for PQQCD?

PQQCD violates unitarity, is inherently euclidean

“Derivation” of ChPT

Weinberg: unitarity, causality, crossing symmetry, clustering,
Lorentz invariance

Leutwyler (1994): singles out clustering (and locality)

- vertices of EFT independent of the correlation function
- loop expansion

Sharpe & Shoresh (2001): PQQCD breaks chiral symmetry spontaneously

- because sea sector is identical to full QCD
- also Goldstone excitations in valence and ghost sectors (PQ symm.)

⇒ This talk: study of transfer matrix of PQQCD — in gauge-field background

Staggered ghosts

PQQCD contains sea quarks, valence quarks and ghost quarks with the same quark masses as the valence quarks, but bosonic statistics

Start with bosonic staggered quarks:

$$S = \sum_x \left\{ \frac{1}{2} \sum_\mu \eta_\mu(x) \left(\chi^\dagger(x) U_\mu(x) \chi(x+\mu) - \chi^\dagger(x+\mu) U_\mu^\dagger(x) \chi(x) \right) + m \chi^\dagger(x) \chi(x) \right\}$$

then $\int D\chi^\dagger D\chi \, \exp(-S)$ is convergent for $m > 0$

(more complicated for Wilson-like fermions (MG, Sharpe and Singleton, 2005))

Species doublers: Use two time-slice method for constructing transfer matrix (Smit, 1970's)

Identify $\chi(x) = \eta_4(x)\phi_1(x) + i\phi_2(x)$ and

for
$$t = 2k: \quad \phi_1(\vec{x},t) = \Phi_{1,k}(\vec{x}) \;, \quad \phi_2(\vec{x},t) = -\Phi_{2,k}(\vec{x})$$
$$t = 2k+1: \quad \phi_1(\vec{x},t) = \Pi_{2,k}(\vec{x}) \;, \quad \phi_2(\vec{x},t) = \Pi_{1,k}(\vec{x})$$

then $$Z(\mathcal{U}) = \mathrm{Tr}\left(\prod_{k=1}^{T/2} \hat{T}_k(\mathcal{U})\right)$$

$$\lim_{a_t \to 0} -\log \hat{T}_k(\mathcal{U})/(2a_t) = \hat{H}[\mathcal{U}(t)]$$

defines $\hat{H}[\mathcal{U}(t)] = \hat{H}_1 + i\hat{H}_2[\mathcal{U}(t)]$

$$\hat{H}_1 = \frac{1}{2}\, m \sum_{\vec{x}} \left(\hat{\Pi}_1^2(\vec{x}) + \hat{\Pi}_2^2(\vec{x}) + \hat{\Phi}_1^2(\vec{x}) + \hat{\Phi}_2^2(\vec{x})\right)$$ (mass term)

- $\hat{H}_{1,2}$ both hermitian, do not commute (cf. Lüscher and Weisz for improved gauge)
- transfer matrix not hermitian and not positive definite

Assume that $\hat{H}$ has complete set of left and right eigenstates $|L_\lambda\rangle$, $|R_\lambda\rangle$ (true for free theory)

then

$$2\langle R_\lambda|\hat{H}_1|R_\lambda\rangle = \langle R_\lambda|(\hat{H}[\mathcal{U}(t)] + \hat{H}^\dagger[\mathcal{U}(t)])|R_\lambda\rangle = (\lambda + \lambda^*)\langle R_\lambda|R_\lambda\rangle$$

$$\Rightarrow \qquad \mathrm{Re}\ \lambda = \frac{\langle R_\lambda|\hat{H}_1|R_\lambda\rangle}{\langle R_\lambda|R_\lambda\rangle} \geq 0$$

- expect that correlation functions in this theory decay exponentially with distance (in the free theory they do)
- extend this construction to the complete PQ theory
- prove that $|T| \leq 1$ rather than $\mathrm{Re}\ \hat{H} \geq 0$

Free case: in terms of canonical creation and annihilation operators

$$\hat{H} = \int_p \left\{ m \left(a_1^\dagger(\vec{p}) a_1(\vec{p}) + a_2^\dagger(\vec{p}) a_2(\vec{p}) \right) + i \sum_j \sin(p_j) \left(a_1(\vec{p}) \alpha^j a_2(\vec{p}) - a_2^\dagger(\vec{p}) \alpha^j a_1^\dagger(\vec{p}) \right) \right\}$$

The α^j are 8 x 8 Dirac matrices (2^3 doublers, times two from time direction), and $\sum_j \sin(p_j)\alpha^j$ can be diagonalized with eigenvalues ,

$$\pm i s(p) = \pm i \sqrt{\sum_i \sin^2 p_i}$$

making $\hat{H}$ a sum of terms of the form

$$h(\vec{p}) = m \left(a_1^\dagger(\vec{p}) a_1(\vec{p}) + a_2^\dagger(\vec{p}) a_2(\vec{p}) \right) \pm s(p) \left(a_1(\vec{p}) a_2(\vec{p}) - a_2^\dagger(\vec{p}) a_1^\dagger(\vec{p}) \right)$$

This can be diagonalized with a generalized Bogoliubov transformation:

$$
\begin{aligned}
b_1 &= \cos\theta \; a_1 - \sin\theta \; a_2^\dagger \\
b_2 &= \cos\theta \; a_2 - \sin\theta \; a_1^\dagger \\
\tilde{b}_1 &= \cos\theta \; a_1^\dagger + \sin\theta \; a_2 \\
\tilde{b}_2 &= \cos\theta \; a_2^\dagger + \sin\theta \; a_1
\end{aligned}
$$

with $\theta = \frac{1}{2}\tan^{-1}(s/m)$; this yields

$$
h = E(\tilde{b}_1 b_1 + \tilde{b}_2 b_2) + \text{constant} \;, \qquad E = \sqrt{m^2 + s^2}
$$

The b_i and $\tilde{b}_i$ are annihilation and creation operators, and there is indeed a complete set of of left- and right-eigenstates in the free theory.

No surprise: determinant should cancel that of valence quarks

Correlation functions in the free theory:

$$\langle a_i(t) a_j^\dagger(0) \rangle = \delta_{ij} \; \frac{E+m}{2E} \; e^{-Et}$$
$$\langle a_i^\dagger(t) a_j(0) \rangle = -\delta_{ij} \; \frac{E-m}{2E} \; e^{-Et}$$
$$\langle a_i(t) a_j(0) \rangle = -\langle a_i^\dagger(t) a_j^\dagger(0) \rangle = \delta_{i+j,3} \; \frac{s}{2E} \; e^{-Et}$$

clearly show a violation of unitarity.

These correlation functions follow also directly from a path integral for this hamiltonian, without doing the Bogoliubov transformation first.

To do:

- extend our transfer matrix to a fully PQ theory: add quarks and gluons
- argue that the theory clusters, after integrating over the gauge fields
- does this imply validity of PQChPT à la Leutwyler?
- effective theory for *unequal* valence and ghost quark masses