New results with colour-sextet quarks

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Introduction

Technicolor theories are QCD-like theories where the techni-pions play the rôle of the Higgs field, giving masses to the W and Z.

Extended Technicolor theories, where the fermion content is such that the coupling constant evolves very slowly – walks, – can avoid phenomenological problems.

QCD with $1\frac{28}{125} \le N_f < 3\frac{3}{10}$ flavours of massless colour-sextet quarks is expected to be either a Walking or a Conformal field theory.

For $N_f = 3$ conformal behaviour is expected. $N_f = 2$ could, a priori, exhibit either behaviour.

We use lattice QCD with staggered quarks to study these theories.

Our studies of the thermodynamics of the $N_f = 2$ theory on lattices with $N_t = 4, 6$ indicated that the (well-separated) deconfinement and chiral-symmetry restoration transitions move to weaker couplings as N_t increases from 4 to 6 suggesting that they are finite temperature transitions. If so, this would imply that this theory is QCD-like and 'walks'.

We are now extending these studies to $N_t = 8$. Again both transitions move to couplings weaker than those for $N_t = 6$. However, for the chiral transition, the decrease from $N_t = 6$ to $N_t = 8$ is far less than that from $N_t = 4$ to $N_t = 6$. This raises the possibility that this coupling will tend to a finite limit as $N_t \rightarrow \infty$. If so, the transition is a bulk transition, and the theory is conformal. Otherwise it remains a finite temperature transition, and the theory walks. It will be necessary to simulate even larger N_t to distinguish these two possibilities.

We are also performing finite temperature studies of the $N_f = 3$ theory on $N_t = 4, 6$ lattices. The behaviour of this theory appears very similar to that of the $N_f = 2$ theory, the deconfinement and chiral-symmetry restoration transitions are widely separated, and move to significantly weaker couplings as N_t increases from 4 to 6. The main difference is that all transitions occur at stronger couplings than for $N_f = 2$.

QCD with colour-sextet staggered quarks at finite T

We use the simplest (Wilson) gauge action:

$$S_g = \beta \sum_{\Box} \left[1 - \frac{1}{3} \mathrm{Re}(\mathrm{Tr} UUUU) \right]. \tag{1}$$

Formally, the unimproved staggered quark action action is:

$$S_f = \sum_{sites} \left[\sum_{f=1}^{N_f/4} \psi_f^\dagger [\not{D} + m] \psi_f \right], \tag{2}$$

where where $\not{D} = \sum_\mu \eta_\mu D_\mu$ with

$$D_\mu \psi(x) = \frac{1}{2} [U_\mu^{(6)}(x) \psi(x+\hat{\mu}) - U_\mu^{(6)\dagger}(x-\hat{\mu}) \psi(x-\hat{\mu})]. \tag{3}$$

We use the RHMC algorithm to simulate values of $N_f/4$ which are not integers.

For $N_f = 2$ we are extending our simulations to $16^3 \times 8$ lattices, with quark masses $m = 0.005, 0.01, 0.02$, for a selection of $\beta = 6/g^2$ in the range $5.5 \leq \beta \leq 7.4$ (At $m = 0.005$, and large βs we have started simulations on a $24^3 \times 8$ lattice).

For $N_f = 3$, we are simulating on $12^3 \times 4$ and $12^3 \times 6$ lattices, also with quark masses $m = 0.005, 0.01, 0.02$. Here we use a selection of βs in the range $5 \leq \beta \leq 7$.

Away from the deconfinement and complex-to-negative Wilson-Line transitions, we use 10,000-20,000 trajectory runs for each (β, m). Close to these transitions, we increase this to 50,000 trajectories.

$N_f = 2$ simulations with $N_t = 8$

Just above the deconfinement transition, the Wilson Line shows a strong 3-state signal, where it preferentially orients itself in a direction close to that of one of the cube roots of unity. We therefore bin our data into 3 bins according to whether the argument of the Wilson line is closest to 0, $2\pi/3$ or $-2\pi/3$, up to the point at which the complex Wilson line states disorder to a state with a negative Wilson line.

Figure 1 shows the Wilson Lines and chiral condensates ($\langle\bar{\psi}\psi\rangle$) as functions of β for each of the 3 quark masses, for our $N_f = 2$ runs on a $16^3 \times 8$ lattice. This ‘data’ is for the real positive Wilson Line bin.

The deconfinement transition is identified as the point at which the magnitude of the Wilson Line jumps from near zero to an appreciably larger value.

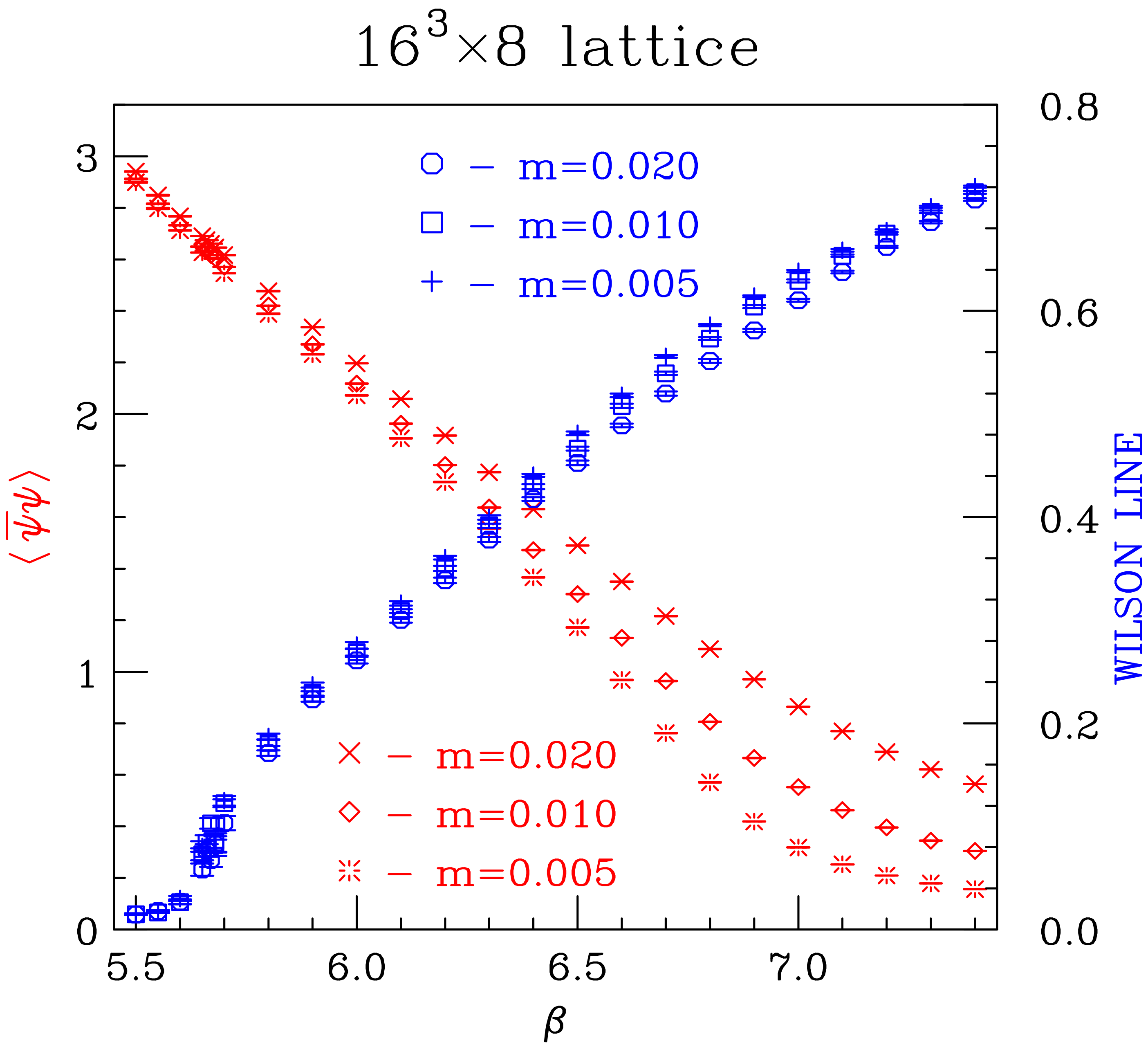

Figure 1: Wilson Lines and chiral condensates for the positive Wilson Line states, on a $16^3 \times 8$ lattice.

Figure 2 shows histograms of Wilson Line magnitudes close to the deconfinement transition for $m = 0.02$. From this we estimate that the transition β is $\beta_d = 5.665(10)$ at $m = 0.02$. Similarly for $m = 0.01$ we estimate $\beta_d = 5.660(10)$. We note that the chiral condensate remains finite until well above β_d, even in the chiral limit.

Figure 3 shows the disconnected chiral susceptibilities for each of the 3 quark masses. Since for each of the 2 lowest masses this peaks at $\beta \approx 6.7$, we estimate that for $m \to 0$ the chiral phase transition is at $\beta = \beta_\chi = 6.7(1)$.

Finally the states with complex Wilson Lines disorder into a state with a real negative Wilson Line at $6.7 \leq \beta \leq 6.9$, which is close to the chiral transition.

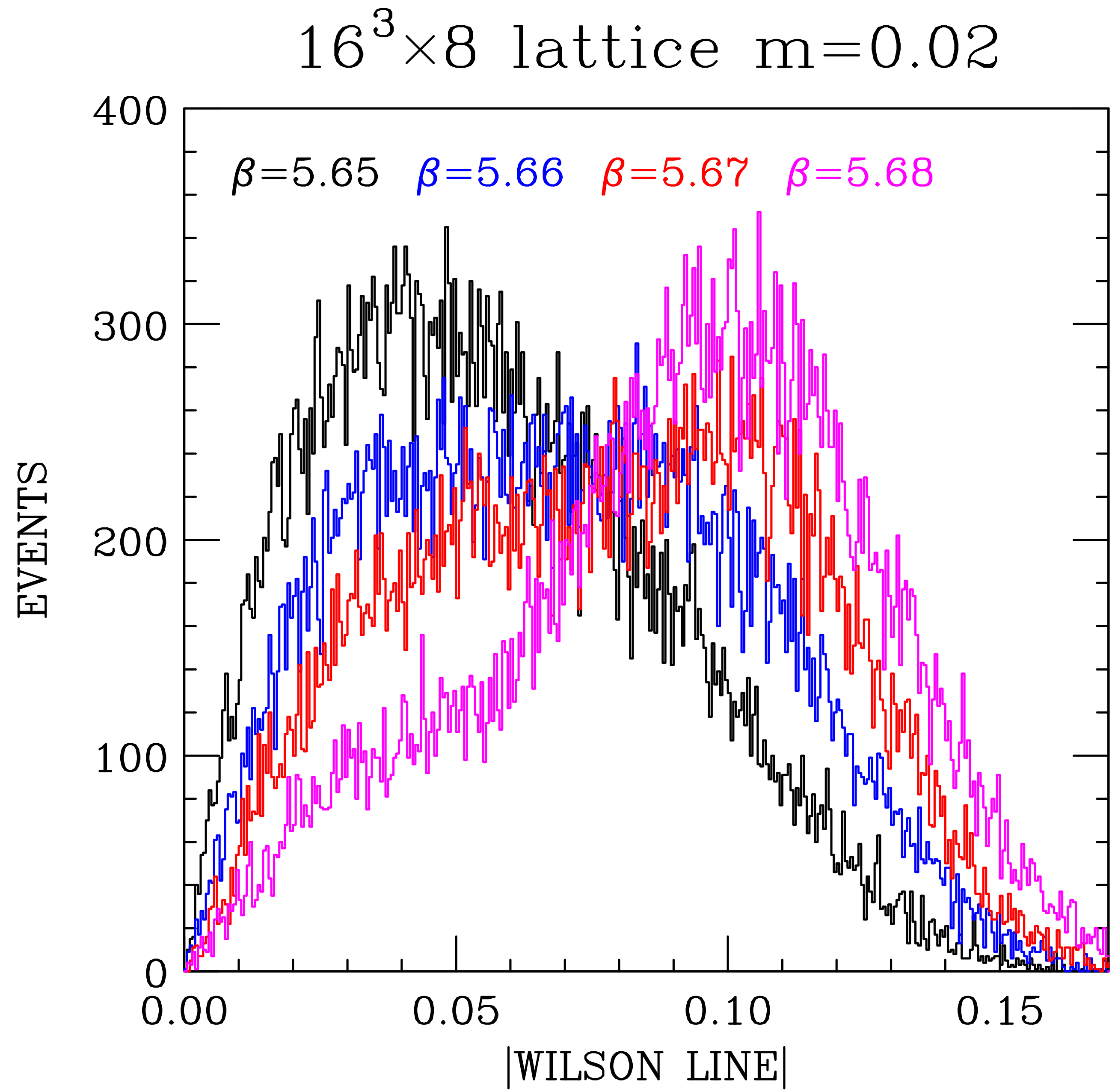

Figure 2: Histograms of magnitudes of the colour-triplet Wilson Lines close to β_d on a $16^3 \times 8$ lattice for $m = 0.02$.

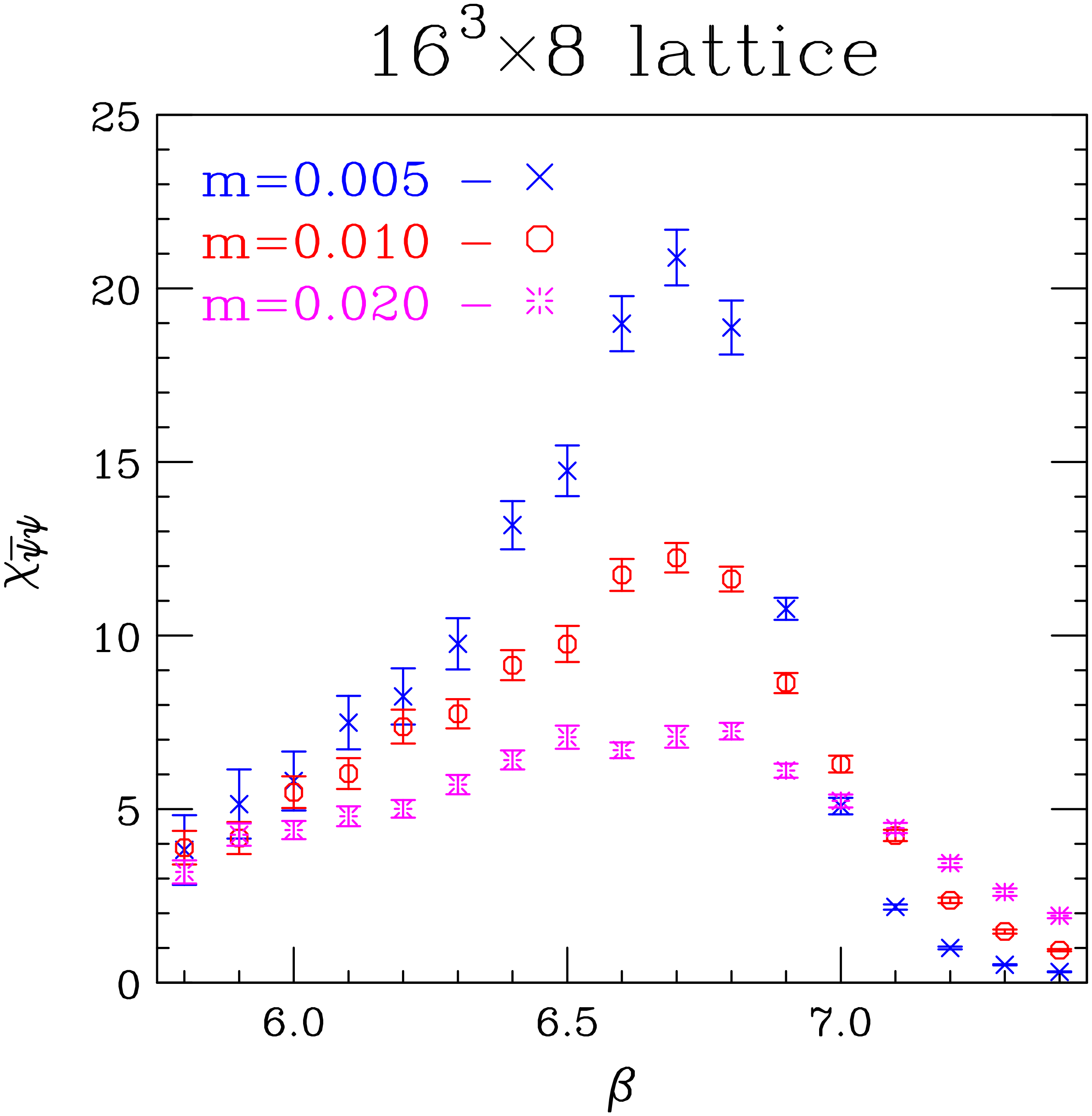

Figure 3: Chiral susceptibilities on a $16^3 \times 8$ lattice.

$N_f = 3$ simulations with $N_t = 4$ and $N_t = 6$

$N_t = 4$

We simulate lattice QCD with 3 flavours of colour-sextet quarks on $12^3 \times 4$ lattices, with quark masses $m = 0.005$, $m = 0.01$ and $m = 0.02$. βs are chosen covering the range $5 \leq \beta \leq 7$.

Figure 4 shows the Wilson Lines and chiral condensates for a series of runs starting from an ordered start ($U = 1$). Note that the deconfinement and chiral-symmetry restoration transitions appear far apart.

Figure 5 shows the time evolution of the Wilson Lines at $\beta = 5.29$ and $\beta = 5.3$ for $m = 0.02$. From this we conclude that $\beta_d(m = 0.02) = 5.295(5)$. Similarly we find that $\beta_d(m = 0.01) = 5.285(5)$.

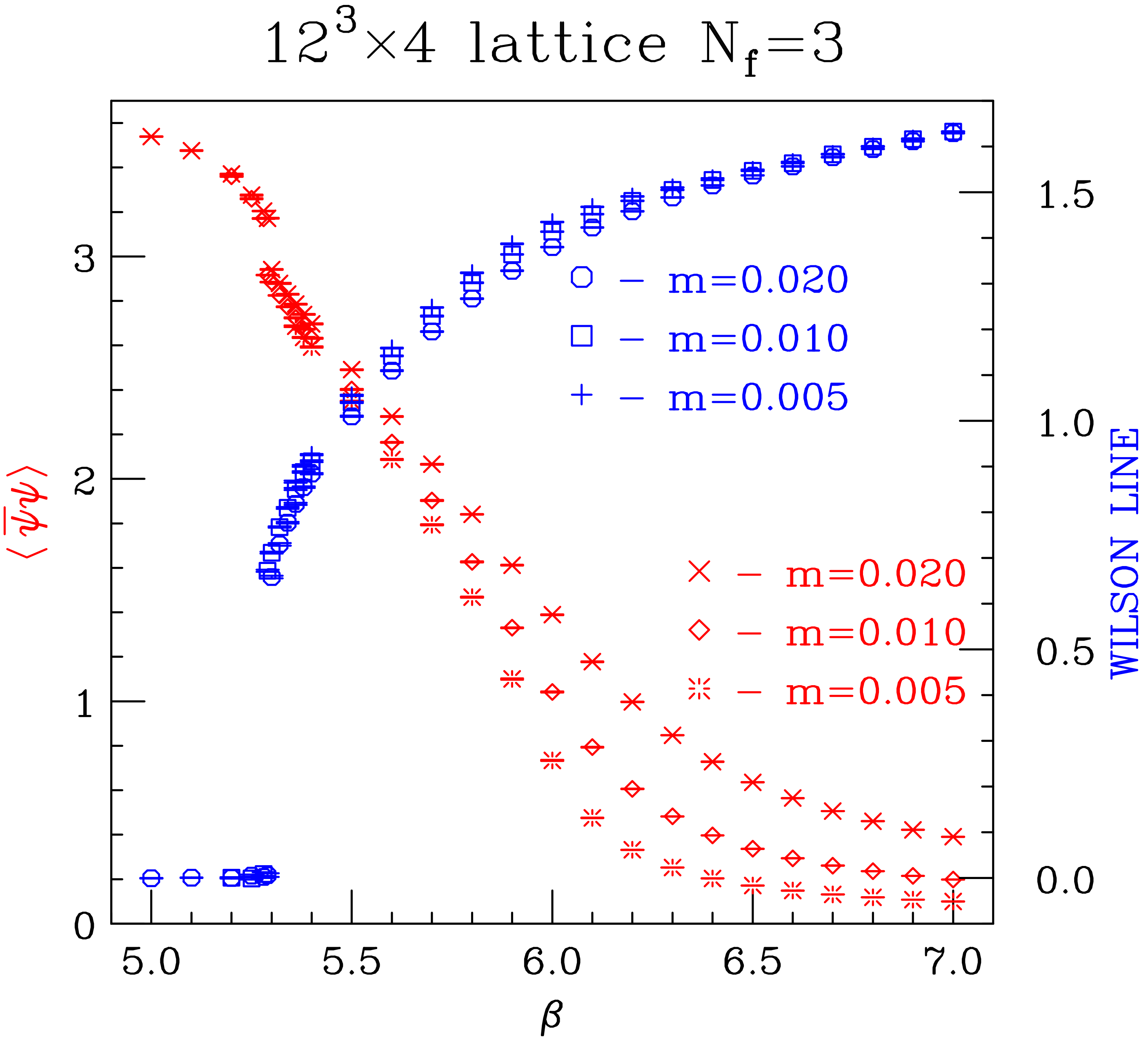

Figure 4: Wilson Line and chiral condensate for $N_f = 3$ on a $12^3 \times 4$ lattice.

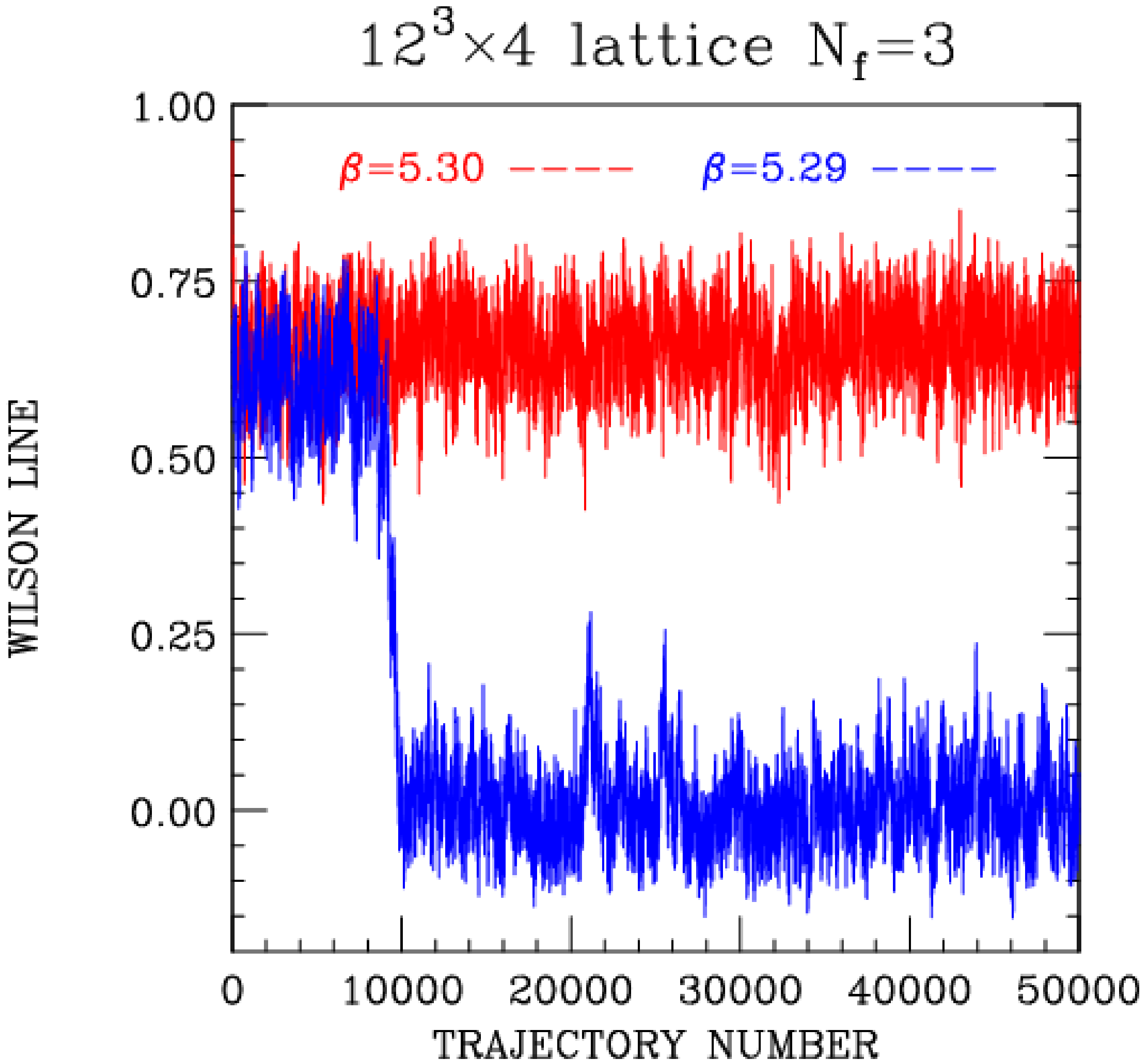

Figure 5: Time evolution of the Wilson Lines for $\beta = 5.29$ and $\beta = 5.3$ on a $12^3 \times 4$ lattice with $N_f = 3$, $m = 0.2$.

Figure 6 shows the disconnected susceptibility for the chiral condensate $\langle\bar{\psi}\psi\rangle$ for each of the 3 masses. From the peaks of these graphs, we estimate that the chiral phase transition at $m = 0$ is at $\beta = \beta_\chi = 6.0(1)$.

Finally we note that a state with a Wilson Line in a direction close to that of one of the complex cube roots of unity disorders into a state with a negative Wilson Line for some β in the range $5.5 < \beta < 5.6$.

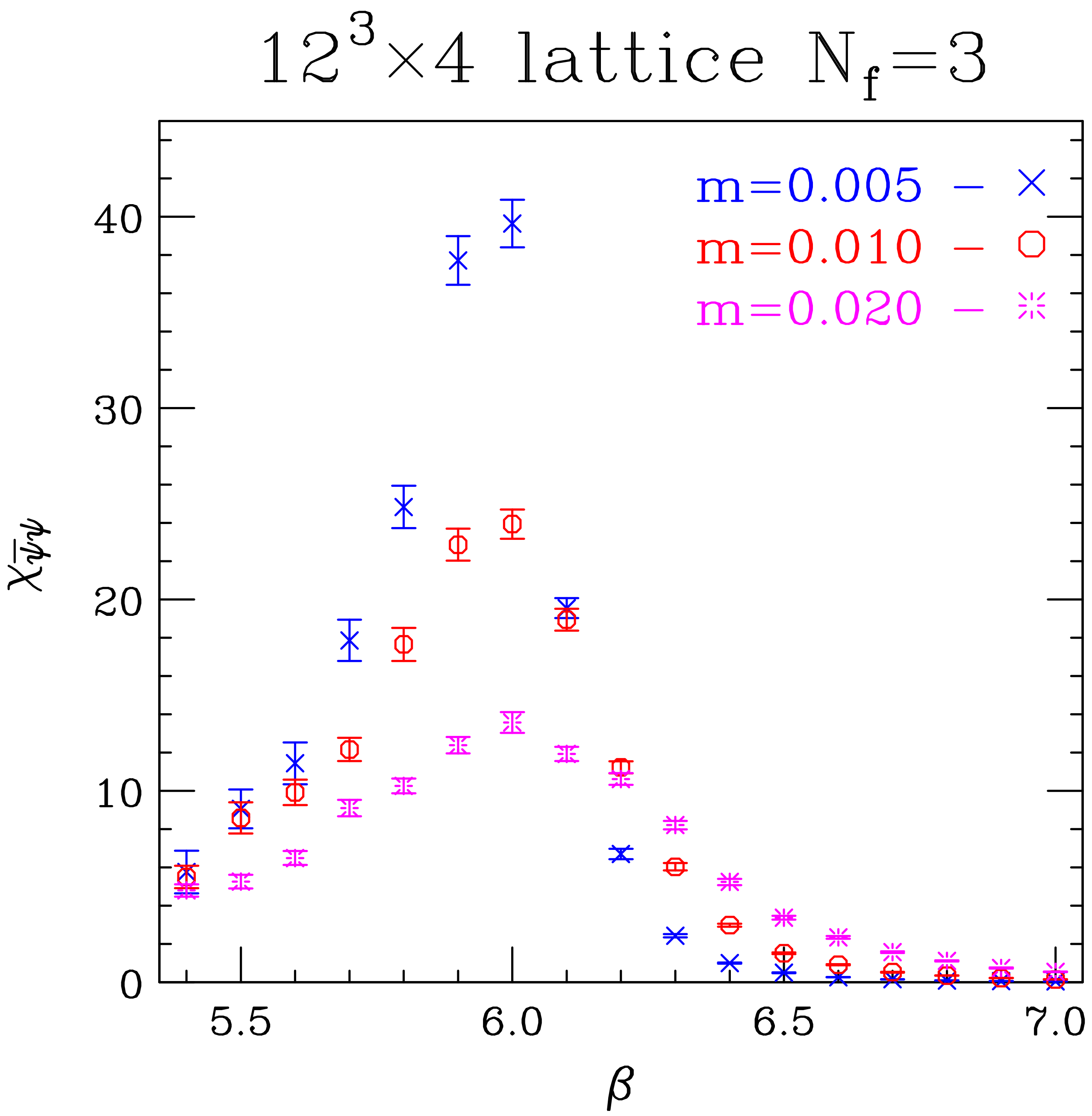

Figure 6: $N_f = 3$ chiral susceptibilities on a $12^3 \times 4$ lattice.

$N_t = 6$

We simulate lattice QCD with 3 flavours of colour-sextet quarks on $12^3 \times 6$ lattices, with quark masses $m = 0.005$, $m = 0.01$ and $m = 0.02$. βs are chosen covering the range $5.3 \leq \beta \leq 7.0$.

Above the deconfinement transition we see a clear 3-state signal, so we bin the 'data' appropriately. Figure 7 shows the Wilson Lines and chiral condensates for the states with real positive Wilson Lines for $N_f = 3$ on a $12^3 \times 6$ lattice, for all 3 masses. Again, the deconfinement and chiral-symmetry restoration transitions are well separated.

By looking at histograms of the magnitude of Wilson Lines we estimate that the deconfinement transition for $m = 0.2$ is at $\beta_d = 5.41(1)$. For $m = 0.1$ we estimate that $\beta_d = 5.395(5)$ (see figure 8).

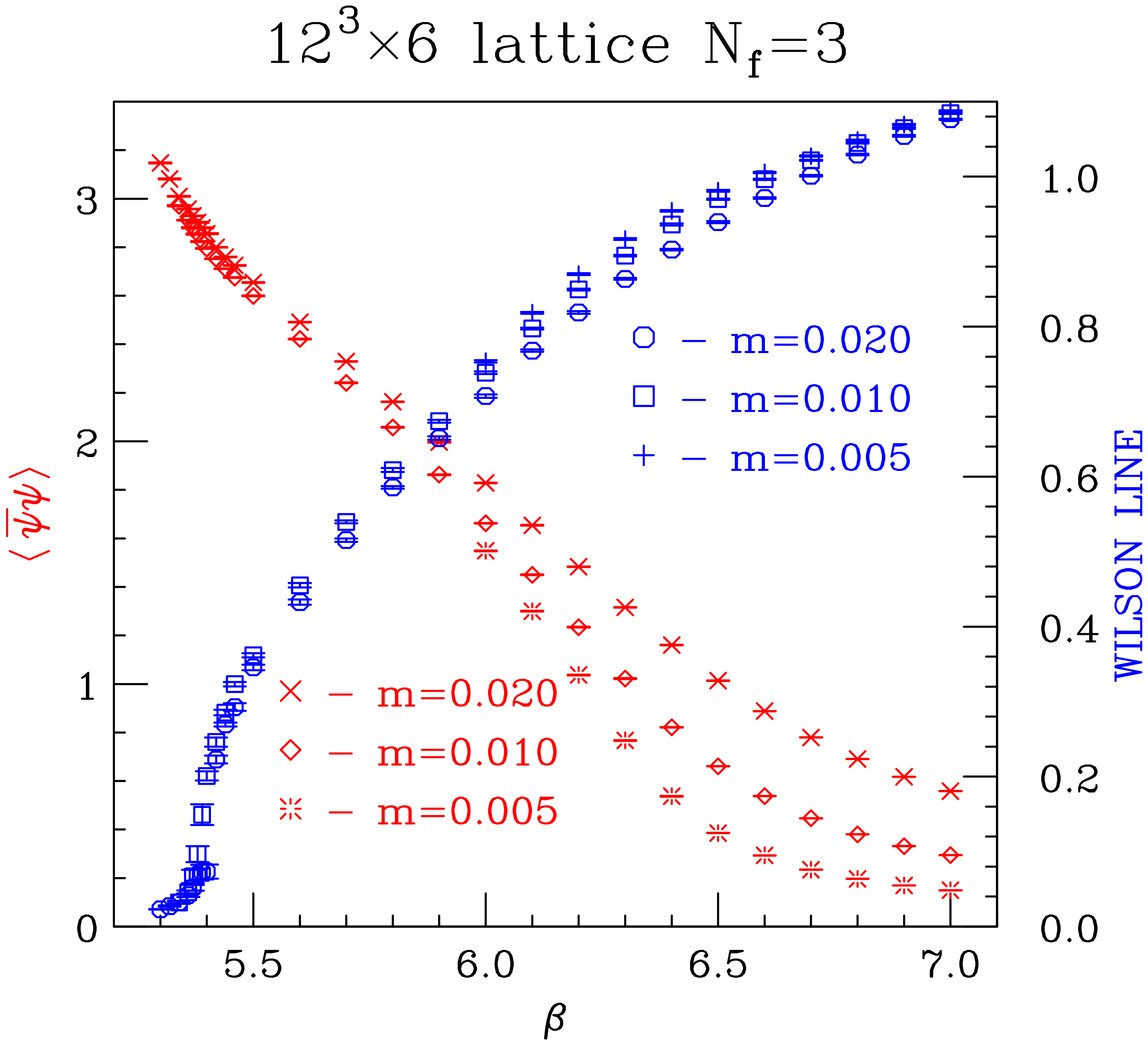

Figure 7: Wilson Lines and chiral condensates for states with real positive Wilson Lines for $N_f = 3$ on a $12^3 \times 6$ lattice.

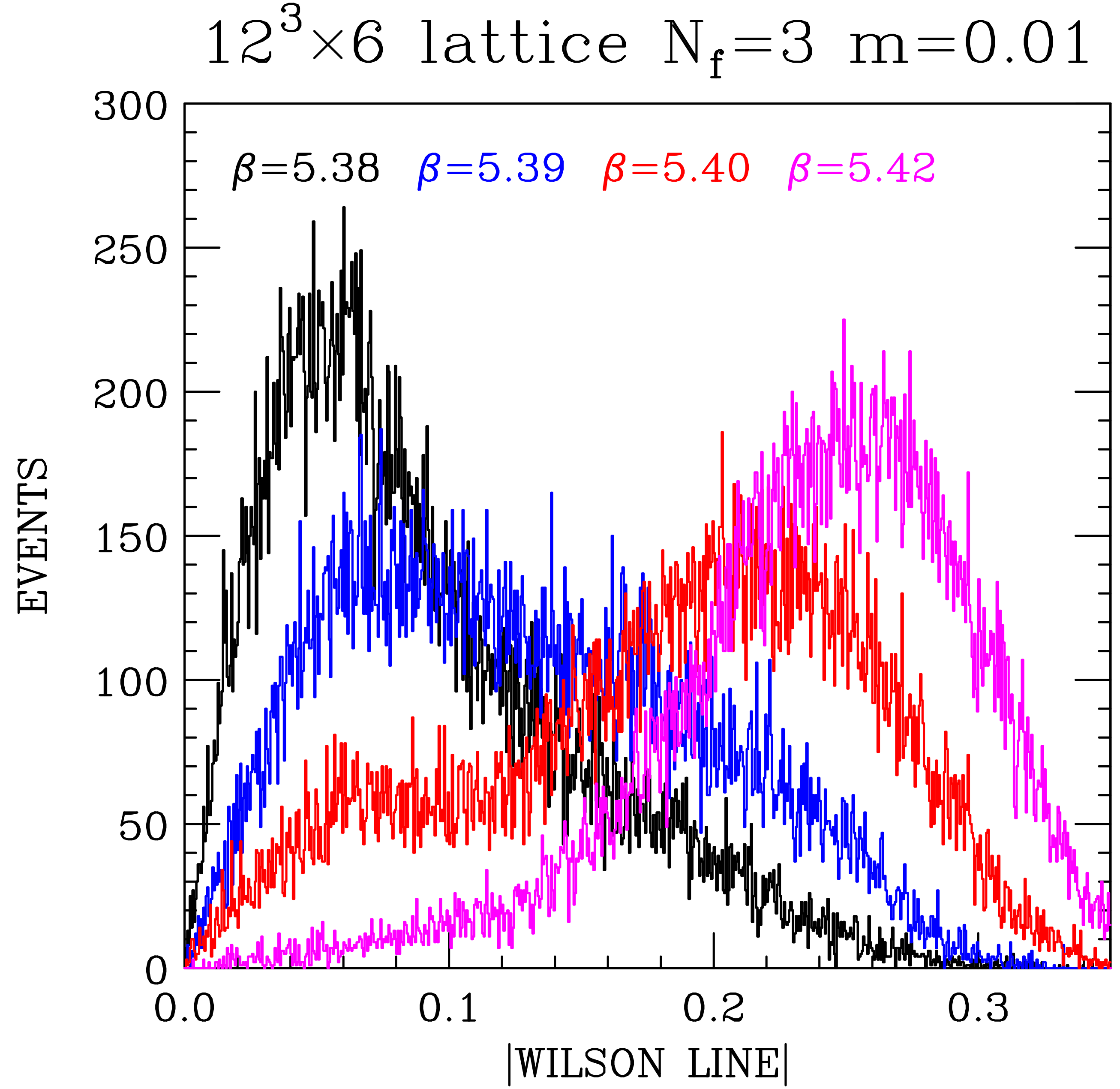

Figure 8: Histograms of magnitudes of Wilson Lines close to the deconfinement transition for $N_f = 3$, $m = 0.1$ on a $12^3 \times 6$ lattice.

In figure 9 we show the disconnected chiral susceptibilities for each of the 3 masses. From these we estimate that the chiral-symmetry restoring transition at $m = 0$ is at $\beta_\chi = 6.3(1)$.

There also exists a state with a negative Wilson Line at large β. For some β in the range $6.1 \leq \beta \leq 6.2$ this undergoes a transition to a state with its Wilson Line oriented in the direction of one of the cube roots of unity.

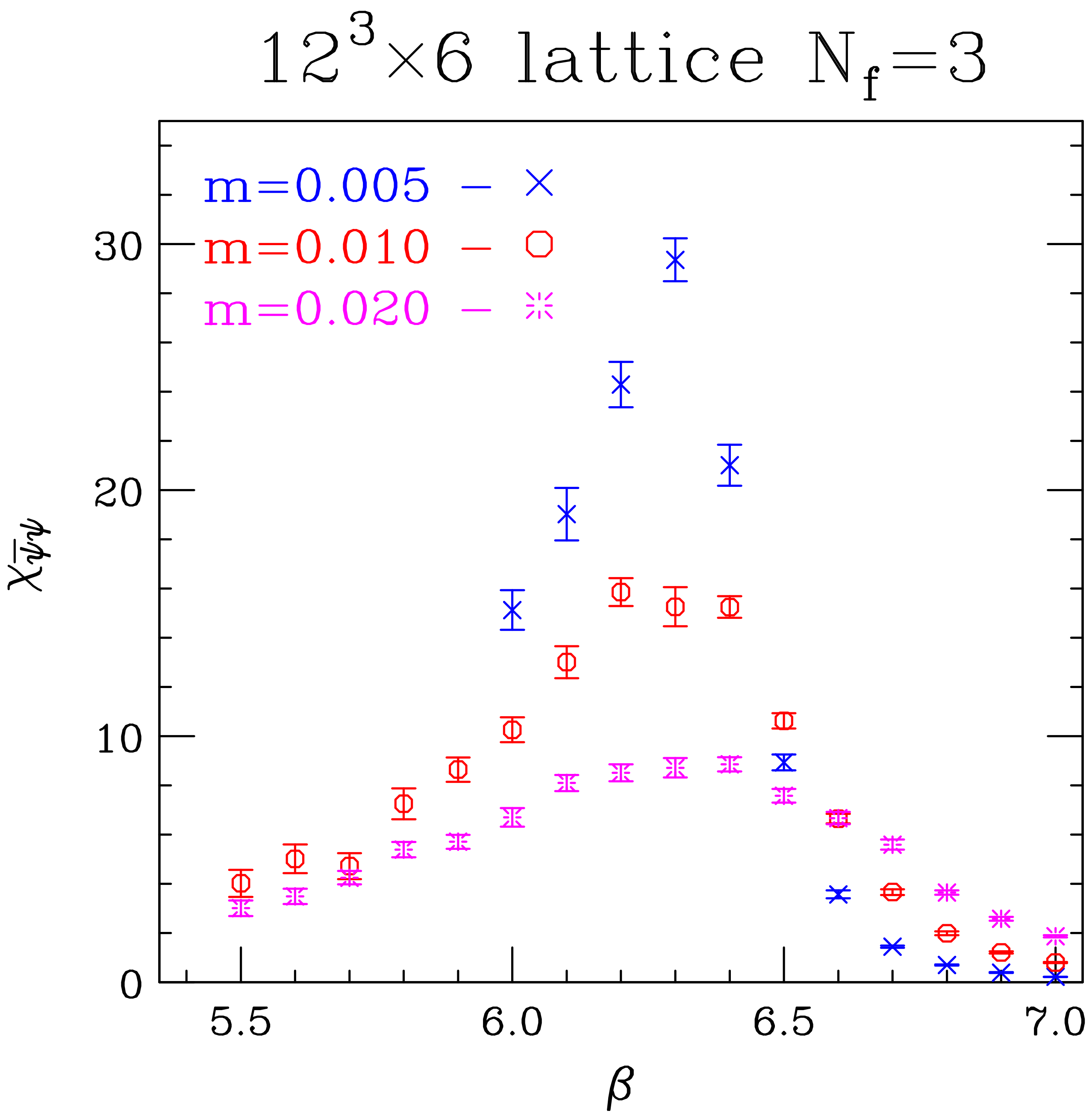

Figure 9: $N_f = 3$ chiral susceptibilities on a $12^3 \times 6$ lattice.

Discussion and Conclusions

- We are simulating the thermodynamics of lattice QCD with 2 and 3 flavours of staggered colour-sextet quarks, as models of walking/conformal Technicolor.

- We are extending our earlier $N_f = 2$ simulations with $N_t = 4, 6$ to $N_t = 8$. We again find widely separated deconfinement and chiral-symmetry restoration transitions. The β values for these transitions are compared with those at $N_t = 4, 6$ in the following table:

N_t	β_d	β_χ
4	5.40(1)	6.3(1)
6	5.54(1)	6.6(1)
8	5.66(1)	6.7(1)

- Because the increase in β_χ from $N_t = 6$ to $N_t = 8$ is so much smaller that that from $N_t = 4$ to $N_t = 6$, we will need to simulate at larger N_t in order to determine if β_χ continues to increase, indicating that it is a finite temperature transition, or if it approaches a limit and is thus a bulk transition.

In the first case the theory walks; in the second case it is conformal.

DeGrand, Shamir and Svetitsky have studied QCD with 2 sextet Wilson quarks. They also find it difficult to distinguish between walking and conformal behaviour. At finite temperature, they do not however see any separation between the deconfinement and chiral transitions.

We need to study the chiral transition on larger lattices and smaller quark masses to clarify its position and nature.

Above the deconfinement transition we see a 3 state signal, the remnant of the now-broken Z_3 centre symmetry. At a β comparable to β_χ the 2 states with complex Wilson Lines disorder into a state with a negative Wilson Line. The existence of states with Wilson Lines having phases $\pm 2\pi/3$ and π in addition to those with phase 0 is predicted by Machtey and Svetitsky and observed in their simulations with Wilson quarks.

We need to study the zero temperature properties of this theory – spectra, f_π, interquark potential(s)..., to better determine whether it is walking or conformal.

- We are also simulating QCD with 3 flavours of colour-sextet quarks on lattices with $N_t = 4$ and $N_t = 6$. Again we find well-separated deconfinement and chiral-symmetry restoration transitions. The following table shows the positions of these transitions:

N_t	β_d	β_χ
4	5.28(1)	6.0(1)
6	5.39(1)	6.3(1)

- These results look very similar to $N_f = 2$. Since we suspect that this $N_f = 3$ theory is conformal, this is further evidence that the large increase in β_χ from $N_t = 4$ to $N_t = 6$ does not represent asymptotic scaling.

- For both $N_f = 2$ and $N_f = 3$, the scaling seen between $N_t = 4$ and $N_t = 6$ probably represents quenched scaling appropriate at distances greater than that associated with chiral symmetry breaking.

The simulations reported here were performed on the Fusion Cluster at the LCRC at Argonne, the Franklin Cray XT4, the Hopper Cray XT5, and the Carver/Magellan Cluster at NERSC, and the Kraken Cray XT5 at NICS.

Appendix

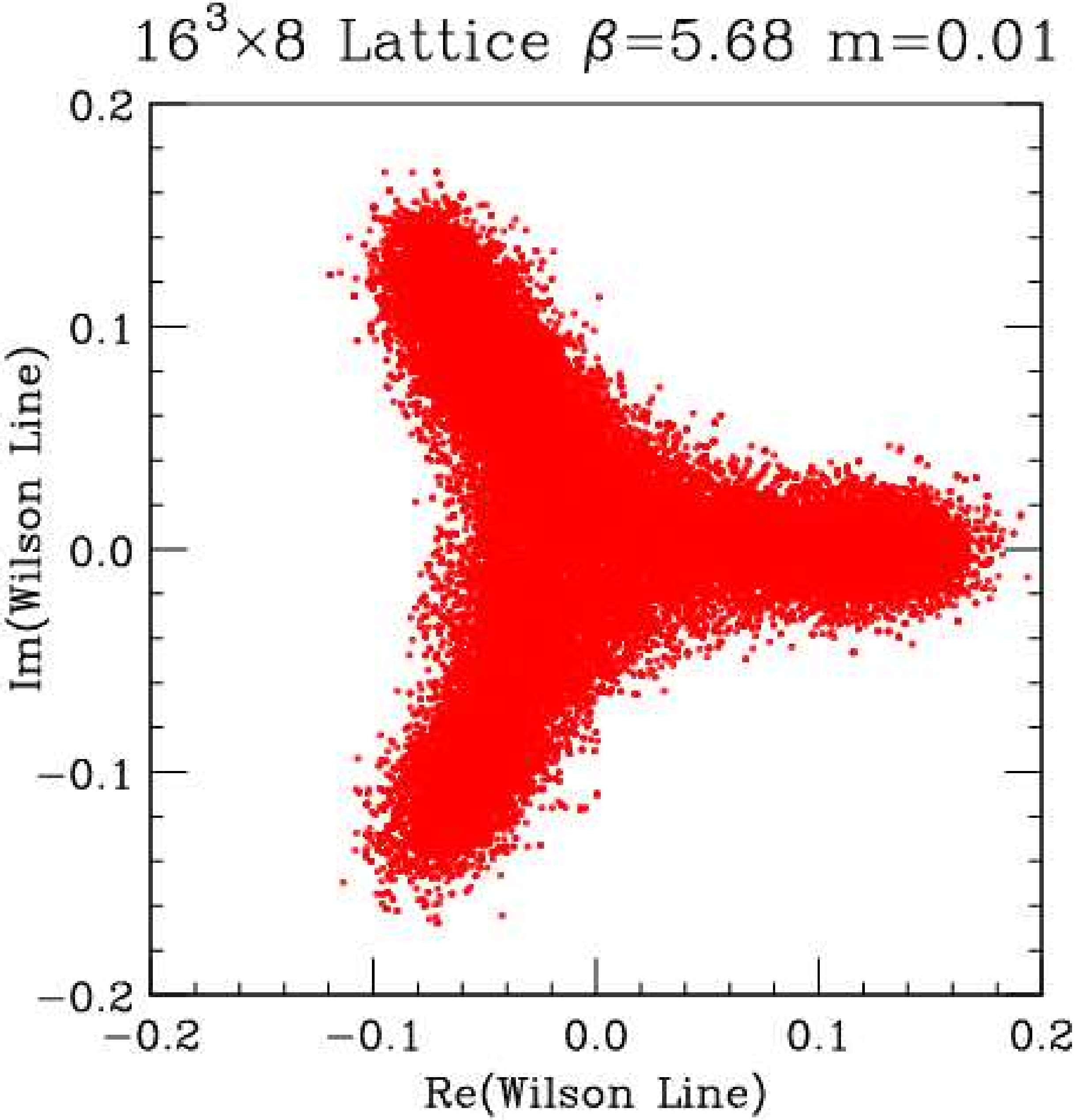

Figure 10: Scatterplot of Wilson Lines at $\beta = 5.68$, $m = 0.01$ (just above the deconfinement transition) on a $16^3 \times 8$ lattice with $N_f = 2$.