Analysis of LQCD results on the baryon masses in chiral perturbation theory

J. Martin-Camalich[1] @ Lattice 2010

in collaboration with

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LQCD & χPT

- **ChPT**: Effective field theory of QCD in the low-energy limit
  (Weinberg, Gasser and Leutwyler, ...)
  - Exploits the global symmetries of the QCD Lagrangian and ground state
  - QFT perturbative expansion respect with $\Lambda_{ChSB} \simeq 1$ GeV (m_q and $p \simeq 0$)
  - Two types of terms
    - **Analytical** Not constrained by symmetries (**LECs**)
    - **Non-analytical** Consequences of chiral symmetry breaking
- The LQCD and χPT friendship provides a model-independent framework to non-perturbative QCD
  - χPT helps to reduce systematical LQCD uncertainties
    - Extrapolation to the physical point
    - Finite volume corrections
    - Lattice artifacts
  - LQCD is a source of information to fix the LECs of χPT
- χPT can correlate seemingly uncorrelated observables

Far-reaching phenomenological applications!

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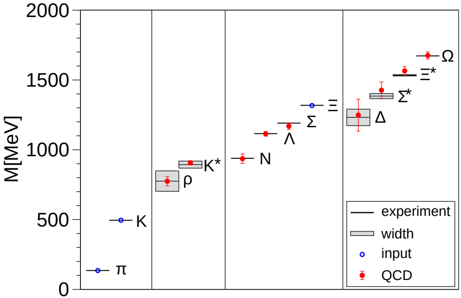
$N_f = 2 + 1$ LQCD simulations on the baryon spectrum



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- $N_f = 2 + 1$ dynamical simulations
- Multiple lattice spacings
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- Various strange quark masses
- Chiral regime $m_{PS} \geq 190$ MeV
  Phenomenological extrapolation

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  LHPC (2008), PACS-CS (2008,2009), HSC (2008), ...
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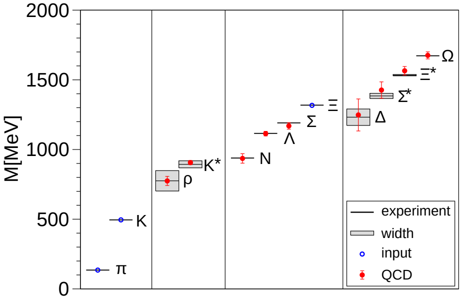

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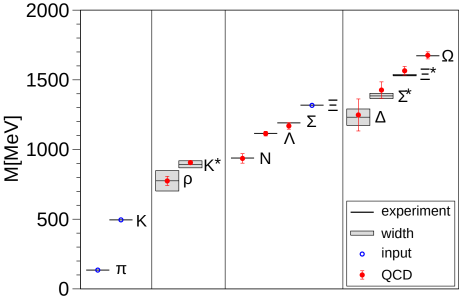

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$SU(3)_F$ Lorentz covariant BχPT in the EOMS scheme

1. Employs a Lorentz covariant baryon chiral Lagrangian
   - Consistent power-counting in d-reg: **EOMS scheme** Fuchs et al., PRD'03
     $\overline{MS}$ is extended to absorb the breaking pieces into a finite set of LECs
2. $\epsilon \gtrsim \delta = M_D - M_B \rightarrow$ Decuplet included as relativistic Rarita-Schwinger fields
   - Spurious DOF of the spin-3/2 field are filtered Pascalutsa, PLB'01
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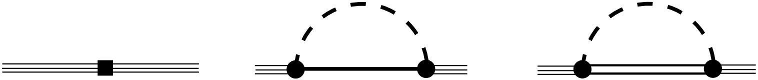
Baryon masses in χPT: Beyond Gell-Mann Okubo formulas



- At tree-level: **LO** contribution

$$\mathcal{L}_B^{(2)} = b_0 \langle \chi_+ \rangle \langle \bar{B} B \rangle + b_D \langle \bar{B} \{ \chi_+, B \} \rangle + b_F \langle \bar{B} [\chi_+, B] \rangle,$$

$$\mathcal{L}_D^{(2)} = \frac{\gamma_0}{6} \bar{T}_\mu^{abc} g^{\mu\nu} T_\nu^{abc} \langle \chi_+ \rangle + \frac{\gamma_D}{2} \bar{T}_\mu^{abc} g^{\mu\nu} \left(\chi_+, T_\nu \right)^{abc},$$

SU(3)-breaking $\delta M_B \sim m_q$ leads to the Gell-Mann-Okubo mass formulas:

$$3M_\Lambda + M_\Sigma - 2(M_N + M_\Xi) = 0$$

$$M_{\Sigma^*} - M_\Delta = M_{\Xi^*} - M_{\Sigma^*} = M_{\Omega^-} - M_{\Xi^-}$$

- Loops provide the **NLO** contribution
  SU(3)-breaking beyond GMO formulas!

Low-lying baryon masses: Experimental data

	M_N	M_Λ	M_Σ	M_Ξ	M_{B0}^{eff}	b_D	b_F
GMO	942(2)	1115(1)	1188(4)	1325(3)	1192(5)	0.060(4)	-0.213(2)
HB	939(2)	1116(1)	1195(4)	1315(3)	2422(5)	0.412(4)	-0.781(2)
Cov.	941(2)	1116(1)	1190(4)	1322(3)	1840(5)	0.199(4)	-0.530(2)
Expt.	940(2)	1116(1)	1193(5)	1318(4)		—	

- Masses in MeV, LECs in GeV^{-1}; errors of Expt. describes isospin splittings
- **LO** and **NLO** in HB and Cov. approaches describe the splittings very well
  - Fit of **3** parameters to **4** data points: $M_{B0}^{eff} = M_{B0} - b_0(4m_K^2 + 2m_\pi^2)$, b_D, b_f
  - This is despite of the **LARGE NLO** loop-corrections (∼hundreds MeV)
  - The $SU(3)_F$-structure of the loops is not accidental, Jenkins et al.,PRD'10
- Similar pattern for the decuplet masses: Fit of **2** parameters to **4** data points

$$M_{D0}^{eff} = 1519(2)\ \text{MeV}, \qquad t_D = -0.694(2)\ \text{GeV}^{-1}$$

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- Allows to disentangle the LECs b_0 and M_{B0} (t_0 and M_{D0})
  - Extraction of b_0 and t_0 $\longrightarrow$ **Prediction** of σ_π and σ_s terms
- **Test covariant approach as a framework to interpret LQCD**

- **lQCD calculation:** PACS-CS (Aoki et al., PRD'08)
  - Contains more points close to the χ-limit
  - One of the results almost on the physical point $m_\pi = 156$ MeV
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- **Strategy:** Fit LECs comparing $M_B^{(3)}(m_{\pi,i}, M_{K,i})$ in EOMS to $M_B^{LQCD}(i)$
  - Masses in physical units obtained using the lattice spacing a
  - Choose any point (i) where $m_{\pi,i}$ and $m_{K,i} \lesssim 600$ MeV$\rightarrow$ 3 or 2 per baryon
  - Fit of M_{B0}, b_0, b_D, b_F, M_{D0}, t_0, t_D (7 LECs) to
    - ⋆ **24 PACS-CS points**
    - ⋆ **16 LHPC points**
  - Fit of the octet and decuplet masses connected through octet-decuplet loops
  - Two kind of fits: WITHOUT (χ^2) and WITH ($\bar{\chi}^2$) Expt. values
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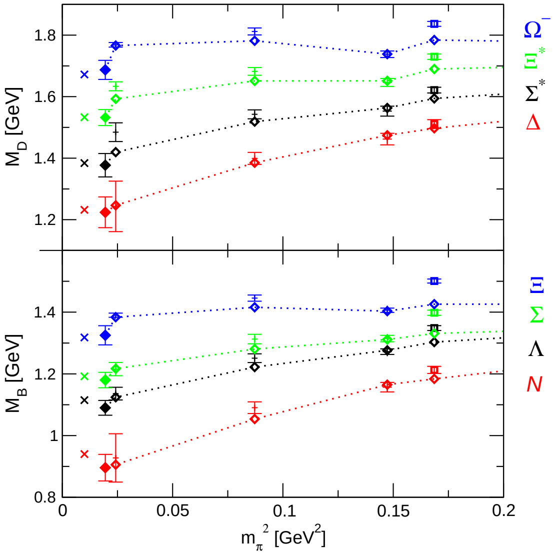
Chiral extrapolation of PACS-CS: Numerics

	GMO	HB	Covariant	Expt.
M_N	971(22)	764(21)	893(19)(39)	940(2)
M_Λ	1115(21)	1042(20)	1088(20)(14)	1116(1)
M_Σ	1165(23)	1210(22)	1178(24)(7)	1193(5)
M_Ξ	1283(22)	1392(21)	1322(24)(20)	1318(4)
M_Δ	1319(28)	1264(22)	1222(24)(49)	1232(2)
M_{Σ^*}	1433(27)	1466(22)	1376(24)(29)	1385(4)
M_{Ξ^*}	1547(27)	1622(23)	1531(25)(8)	1533(4)
M_{Ω^-}	1661(27)	1733(25)	1686(28)(13)	1672(1)
$\chi^2_{\mathrm{d.o.f.}}$	0.63	9.2	2.1	
$\bar{\chi}^2_{\mathrm{d.o.f.}}$	4.2	36.6	2.8	

- Covariant BχPT$^{(3)}$ gives a **good extrapolation** of baryon masses $\chi^2 \sim 2$.
  - Describes properly $m_\phi \lesssim 600$ MeV
  - Sizable **non-analytical** effect still below $m \leq 156$ MeV!
  - LO and NLO SU(3)-breaking chiral corrections show a good convergence



- **Bars:** PACS-CS fitted
- **Boxes:** PACS-CS no fitted
- **Diamonds:** $\chi PT^{(3)}$
- **Filled diamond:** Extrap.
  Slightly shifted to the right
- **Crosses:** experiment

Chiral extrapolation: Values of the LECs

- We compare the LECs obtained fitting Expt. **OR** LQCD results

  - **Octet**

	M_{B0} [GeV]	b_0 [GeV^{-1}]	M_{B0}^{eff} [GeV]	b_D [GeV^{-1}]	b_F [GeV^{-1}]
Expt.	-	-	1.840(5)	0.199(4)	$-0.530(2)$
PACS-CS	0.756(32)	$-0.978(38)$	1.76(7)	0.190(24)	$-0.519(19)$
LHPC	0.780(31)	$-1.044(45)$	1.85(8)	0.236(24)	$-0.523(21)$

  - **Decuplet**

	M_{D0} [GeV]	t_0 [GeV^{-1}]	M_{D0}^{eff} [GeV]	t_D [GeV^{-1}]
Expt.	-	-	1.519(2)	$-0.694(2)$
PACS-CS	954(37)	$-1.05(8)$	1.49(8)	$-0.682(20)$
LHPC	944(42)	$-1.28(8)$	1.60(8)	$-0.609(14)$

- LECs are consistent with each other!

Chiral extrapolation of PACS-CS: σ-terms

- The σ-terms can be obtained via the **Hellman-Feynman theorem**

$$\sigma_{\pi B} = m_{u,d} \frac{\partial M_B}{\partial m_{u,d}} \quad , \quad \sigma_{sB} = m_s \frac{\partial M_B}{\partial m_s}$$

- **Preliminary predictions!**
  - **Octet**

	N	Λ	Σ	Ξ
σ_π PACS-CS	59(2)(17)	39(1)(10)	26(2)(5)	13(2)(1)
σ_π LHPC	61(2)(21)	41(2)(13)	25(2)(7)	14(2)(3)
σ_s PACS-CS	−7(23)(25)	123(26)(35)	157(27)(44)	264(31)(50)
σ_s LHPC	−4(20)(25)	103(19)(8)	164(23)(16)	234(23)(21)

- The σ terms are important for super-simetric dark matter searches
  Ellis et al., PRD'08, Giedt et al., PRL'09
- Our results favor small values for σ_s and relatively large for σ_π
- **More accurate prediction is in progress!**

Summary and Outlook

- **Conclusions**
  - $SU(3)_F$-BχPT describes the quark mass dependence of LQCD results on the baryon masses and provides a reliable extrapolation to the physical point
    - ★ **Non-analytic** contributions are important even from $m_\pi \simeq 156$ MeV
    - ★ Valid up-to $m_\phi \lesssim 600$ MeV
    - ★ Important cancellations occur between different orders in the HB expansion **Highlights the importance of Lorentz Invariance in $SU(3)_F$ approach!**
  - Extraction of LECs from LQCD **consistent** with phenomenology
  - Phenomenological applications: **Prediction** of the σ-terms
- **Outlook**
  - Simultaneous analysis of current and forthcoming world LQCD "data"
    - ★ Inclusion of the meson-baryon couplings in the fits
    - ★ Accurate prediction of σ-terms
    - ★ *Universality* of the actions under $SU(3)_F$-BχPT
  - Extrapolation of baryon structure
    - ★ Electromagnetic structure
    - ★ Axial couplings
    - ★ ...

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