Exploration of the phase structure of $SU(N_c)$ lattice gauge theory with many Wilson fermions at strong coupling (PRD80('09)074508)

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Introduction

Perturbation theory allows for a non-trivial IR Fixed Point (IRFP)

Banks and Zaks, NPB196('82)

$$\beta(g) = -b_0 g^3 - b_1 g^5 + \cdots ,$$

$$b_0 = \frac{1}{(4\pi)^2}\left(\frac{11}{3}N_c - \frac{2}{3}N_f\right) , \; b_1 = \frac{1}{(4\pi)^4}\left[\frac{34}{3}N_c^2 - (\frac{13}{3}N_c - \frac{1}{N_c})N_f\right] .$$

- In $SU(3)$,
  - $N_f \leq 8 \rightarrow$ confinement
  - $9 \leq N_f \leq 16 \rightarrow$ conformal window
  - $17 \leq N_f \rightarrow$ free theory
- In $SU(2)$,
  - $N_f \leq 5 \rightarrow$ confinement
  - $6 \leq N_f \leq 10 \rightarrow$ conformal window
  - $11 \leq N_f \rightarrow$ free theory

Beyond perturbation theory $\Rightarrow$ Lattice Gauge Theory

Wilson fermion

$\rightarrow$ no $\chi-$sym. but Aoki-phase and Sharpe-Singleton scenario

Standard conjecture of the phase structure in the strong coupling limit for Wilson fermions

S. Aoki,
PRD30('84)2653, PRL57('86)3136, PTP.(Suppl)**122**('96)179.
$\rightarrow$ famous Aoki phase (parity-flavour broken phase)

The phase boundary at $\beta = 0$ in the calculation by strong coupling expansion and large N_c expansion;

$$\cosh(m_\pi) = 1 + \frac{(1-16\kappa^2)(1-4\kappa^2)}{8\kappa^2(1-6\kappa^2)} \,.$$

Then, the critical κ or $m_\pi = 0$ (and $m_q = 0$): $\kappa_c = \frac{1}{4}$

- $\kappa < \kappa_c \Rightarrow$ Confinement phase:
  $$m_\pi^\pm = m_\pi^0,\, m_\pi^2 \propto 2m_q (= \frac{1}{\kappa} - \frac{1}{\kappa_c})$$
- $\kappa > \kappa_c \Rightarrow$ Aoki phase:
  $$m_\pi^\pm \neq m_\pi^0,\, m_\pi^\pm = 0$$
  $$\langle \bar\psi \gamma_5 1 \psi \rangle = 0 \; \langle \bar\psi \gamma_5 \tau_3 \psi \rangle \neq 0$$ for N_f =even.

Our Motivation

Our work is motivated by the paper of Tsukuba,
Y. Iwasaki *et al*, PRD69('04), PRL69('92).

♣ Their result of $SU(3)$

- $N_f \leq 6 \rightarrow$ confinement
- $7 \leq N_f \leq 16 \rightarrow$ conformal window
- $17 \leq N_f \rightarrow$ free theory

♠ Their prediction of $SU(2)$

- $N_f \leq 2 \rightarrow$ confinement
- $3 \leq N_f < \cdots \rightarrow$ conformal window

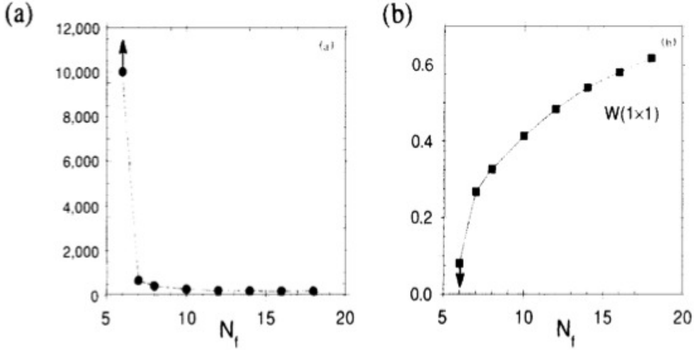


FIG. 2. Results for $N_f = 18-6$. (a) Number of iterations needed for the quark matrix inversion by CG. (b) $W(1\times 1)$.

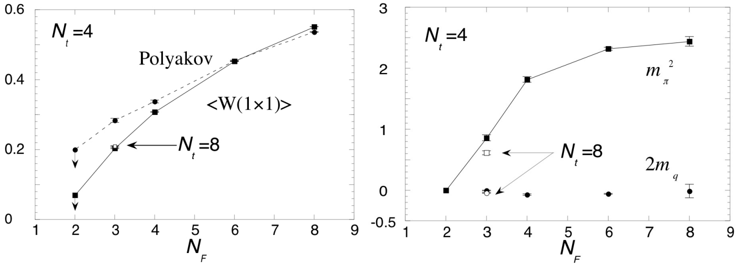
Y. Iwasaki *et al*, PRL69('92).

$N_f = 6$ in $SU(3)$ $\rightarrow$ They didn't compute m_π itself.

Instead, they monitored N_{CG} in MD of R-algorithm.

$N_{CG} = O(10^4)$ in thermalizing $\Rightarrow$ signal of massless pion!!

About $SU(2)$ at $\beta = 0$ and $\kappa = 0.25$, as a function of N_f



Left: Plaquette and Polyakov loop. Right: m_π^2 and m_q at $\kappa = 0.25$.

Y. Iwasaki *et al*, PRD69('04).
$N_f = 2 \rightarrow$ They monitored N_{CG} in MD of R-algorithm.
$N_{CG} = O(10^4)$ in thermalizing, not for thermalized ensemble.
Expectation: $m_\pi = 0$ at $\kappa = 0.25$ for $N_f = 2$ in $SU(2)$.

Target of our job

- We simulate the case of many flavours in $SU(2)$, to study m_π^2 and m_q vs. κ.
- We also study $N_f = 6$ at $\kappa = 0.25$ in $SU(3)$: $m_\pi = 0$ or $\neq 0$.
  $\rightarrow$ for the reference of the $N_f = 2$ case in $SU(2)$

Lattice actions

♠ $S = S_G + \sum_{f=1}^{N_f} S_W^f$.
♠ The Wilson gauge action:
♠ The Wilson fermion action (in the degenerated case):
♠ The partition function: $Z = \int [dU_\mu(x)] \left(\det(D_W^\dagger D_W)\right)^{\frac{N_f}{2}} \exp(S_G)$
where D_W is the kernel of the fermion action
$S_W = \bar{\psi}(x) D_W(x,y) \psi(y)$.

♣ Simulation $\rightarrow$ standard HMC only for the even number of the flavours with $\Delta\tau \cdot N_{MD} = 1$.

Simulation details

♣ $\beta = 0.0$ and 2.0

♣ Lattice size:

$6 \times 6 \times 12 \times 12$ at $\beta = 0.0$ in $SU(2)$ and $SU(3)$

$8 \times 8 \times 16 \times 16$ at $\beta = 2.0$

For the check of the finite size effect; $8^2 \times 16^2$ and $12^2 \times 24^2$ ($12^3 \times 24$) for some flavours.

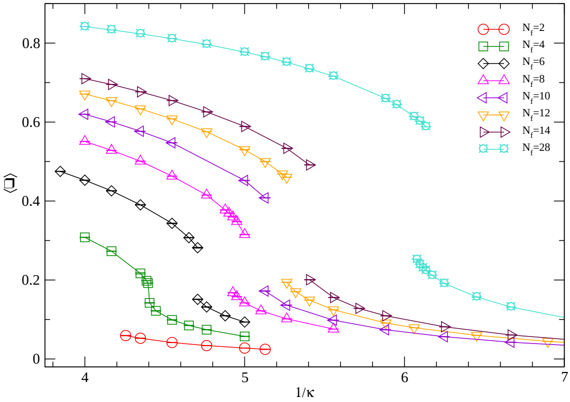
♣ Periodic boundary condition on $N_t \geq N_s$ setup

♣ After thermalizing, we compute the observables of 50∼100 trajectories with 4∼5 interval.

♣ Observables:

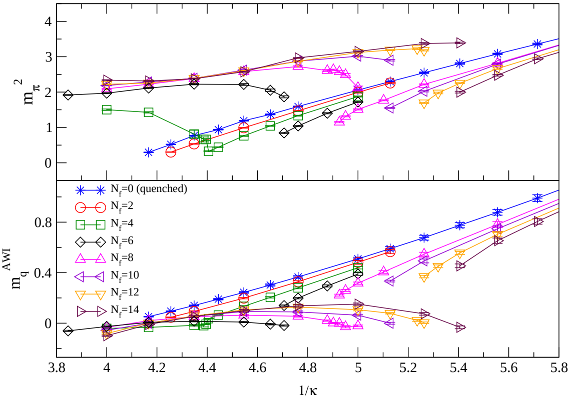
m_π^2, m_ρ, plaquette value, the axial-Ward-Takahashi identity quark mass ($m_q^{\mathrm{AWI}} = \frac{\nabla_4 \langle \sum_{\vec{x}} A_4(\vec{x},t) P(0) \rangle}{2 \langle \sum_{\vec{x}} P(\vec{x},t) P(0) \rangle}$), Polyakov loop, Creutz ratio, the condensate (or the propagator norm), the lowest eigenvalue, $m_q^{AWI}(t)$ and $\langle S(t) S(0) \rangle$ vs. t.

Plaquette value of $SU(2)$ at $\beta = 0$ for various flavours



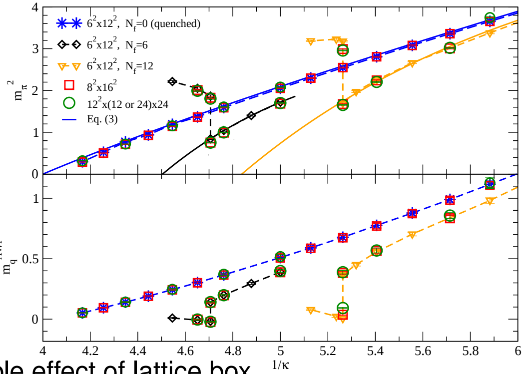
- For $N_f \geq 6$, 2-state signal (Hysteresis, meta-stability)
- For $N_f = 4$, no 2-state signal. (The $N_f = 2$ case is inconclusive.)

m_π^2 and m_q^{AWI} vs. $1/\kappa$ (Close-up of small $1/\kappa$ region)



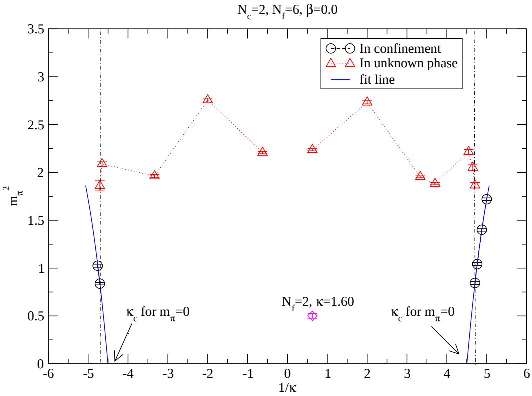
- m_π^2(unknown)$>m_\pi^2$(confine); m_q^{AWI}(unknown)$<m_q^{AWI}$(confine)
- No negative quark mass $\to$ not Sharpe-Singleton scenario (??)
  ($m_\pi^2 \neq f(1/\kappa)$: similar behaviour with Iwasaki's data.)
- For $N_f > 0$, m_π^2 and m_q depend on N_f. $\to$ opposed to Aoki's.

Lattice size effect of m_π^2 and m_q^{AWI} in $N_f = 0, 6$ and 12



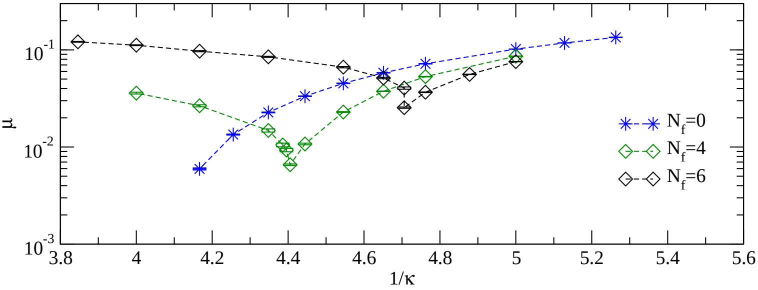
- No sizable effect of lattice box.
- $N_f = 0$ case equals to the Aoki's prediction ($\kappa_c = 0.25$).
  $N_f > 0$ case deviates from Aoki's prediction of m_π^2 and m_q.
- κ_c belongs to the massive pion phase, if it exists.
  no κ_c where $m_\pi^2 = 0$ and $m_q^{AWI} = 0$!!

Phase in $SU(2)$ with $N_f = 6$ data of the negative κ



- Symmetric for $\frac{1}{\kappa} = 0$ $(\kappa = \infty)$
- We don't find the massless pion phase. $\Rightarrow$ no Aoki phase?
- The existence of κ_c, namely $m_\pi = 0$ $(m_q = 0)$, is not trivial.
  $\rightarrow$ The extrapolation to $m_\pi = 0$ is not valid.

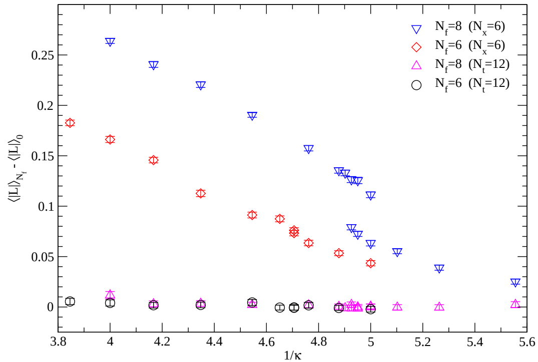
The lowest Eigenvalue: $\mu = \sqrt{\lambda_0(H_W^2)}$ vs. $1/\kappa$



Not small EV in the high-plaquette phase. $\Downarrow$

No Aoki phase?? $\leftarrow \langle\bar{\psi}\gamma_5\tau_3\psi\rangle$ from Banks-Casher relation

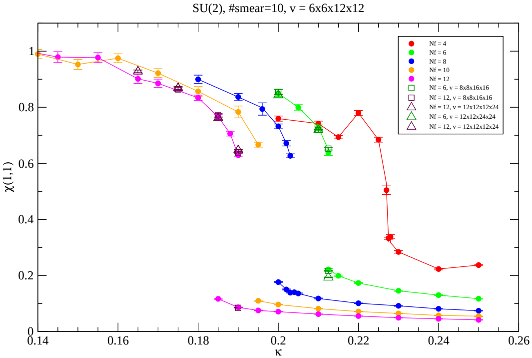
Polyakov loop: $\langle |L| \rangle_{N_f} - \langle |L| \rangle_{N_f=0}$



In the large extent, Polyakov loop is consistent with that in $N_f = 0$

$\rightarrow$ not deconfinement phase (??)

Creutz ratio, $\chi(1,1)$ vs. κ



In the massive pion phase, $\chi(1,1)$ is small and stable.

$\Rightarrow$ String tensionless in the massive pion phase??

m_ρ (and m_π/m_ρ) vs. $1/\kappa$, $m_q^{AWI}(t)$ vs. t,
The propagator norm: $\mathcal{N} = (2\kappa)^2 \sum_{\vec{x},t} \left\langle P(\vec{x},t)P(\vec{0},0) \right\rangle \sim \frac{1}{m_\pi^2}$

m_ρ (and m_π/m_ρ) vs. $1/\kappa$, $m_q^{AWI}(t)$ vs. t, The propagator norm: $\mathcal{N} = (2\kappa)^2 \sum_{\vec{x},t} \left\langle P(\vec{x},t)P(\vec{0},0) \right\rangle \sim \frac{1}{m_\pi^2}$

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$\langle S(t)S(0)\rangle$ vs. t

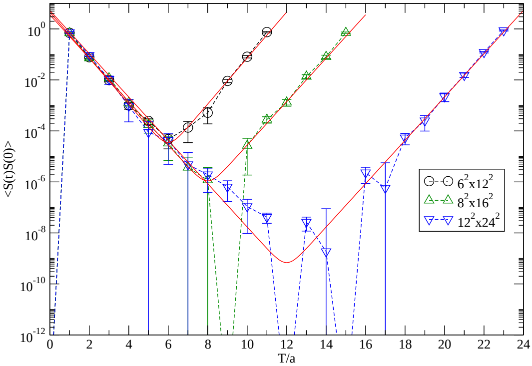


Figure: Size dependence of $\langle S(t)S(0)\rangle$ vs. T/a in the unknown-phase for $N_f = 6$ at $\kappa_f = 0.2125$. The fit is done by cosh-function.

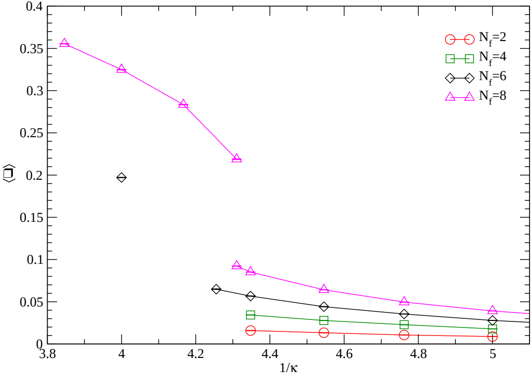
Why is the signal clear?

Why does $\langle S(t)S(0)\rangle$ show the good cosh-fit?

Result of $SU(3)$ case

- How is the case of $N_f = 6$ (at $\kappa = 0.25$)?
- Check of $SU(3)$ case (plaquette value)
  We re-compute it by our code and by MILC code.
- Comparison with Iwasaki's data
  $\longrightarrow$ We will find the discrepancy from their conclusion.

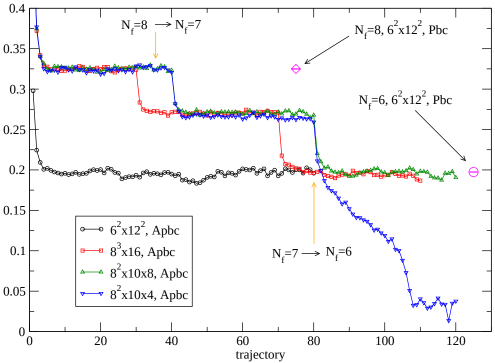
Plaquette values of $SU(3)$



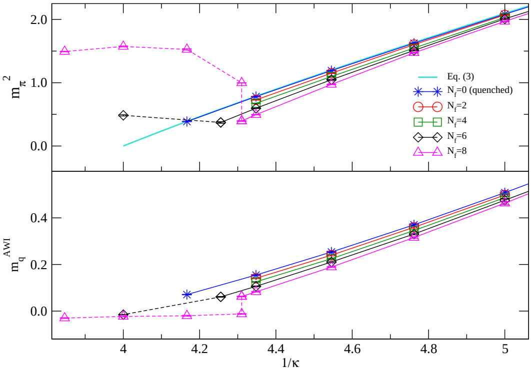
- 2-state signal at $N_f = 8$
- Not small value in $N_f = 6 \rightarrow$ opposed to Iwasaki's result
  Why?

Plaquette history by using our code and MILC code

- Our code: HMC with the periodic boundary (Pbc in Fig.)
  MILC code: R-algorithm with the anti-periodic boundary (Apbc)
- cold start of $N_f = 8 \to N_f = 7 \to N_f = 6$
- Consistent result (not small value),
  except of the $N_f = 6$ case on $8^2 \times 10 \times 4$.



m_π^2 and m_q^{AWI} vs. $1/\kappa$



- 2-state signal at $N_f = 8$.
  $\kappa_c \neq 0.25$ for giving $m_\pi = 0$ and $m_q^{AWI} = 0$, if there is κ_c.
  no region of $m_\pi = 0$ and $m_q^{AWI} = 0$
- In $N_f = 6$, $m_\pi^2 > 0$ $\rightarrow$ opposed to Iwasaki's data
  No problem! Our data is obtained in thermalized conf. Iwasaki's data is not (and is in $N_t = 4 < N_s$).

Summary

We explored the phase structure of $SU(2)$ and $SU(3)$ lattice gauge theories with many Wilson fermions at $\beta = 0$.

- 2-phases. $\kappa_c \neq 0.25$ if it exists. No massless pion.
  Deviation from the Aoki's result in the dynamical case.
- Conjecture: In large N_f Wilson, no ordinary Aoki phase, no Sharpe-Singleton, no deconfinement.
  The $N_f = 2$ case in $SU(2)$ needs further investigation.
  perhaps using twisted mass Wilson fermions
- $N_f = 6$ in $SU(3)$ is not in the confinement region.
  (different result from Iwasaki's conclusion)
  Then, N_f^c is 5 or less.
  Or, to obtain N_f^c at the strong coupling is difficult.

Discussion: What is the unknown phase?

♠ **List of phases:**
Deconfinement phase?
Sharpe-Singleton-Bitar scenario?
Coulomb phase?
Higgs(NJL-BCS) phase?
Strong-Weak transition? (due to $\beta_{eff} = \beta + c_1 N_f \kappa^4 + \ldots$)
ordinary Aoki phase? or alternative Aoki phase?
$\ldots$

♠ Advantage of $\beta = 0$ is to be independent of lattice gauge action. Only the fermion effect to the vacuum can be seen.
♠ To make clear the phases of Wilson fermions, we intend to investigate of $N_f = 2$(and larger) in $SU(2)$ with twisted mass term ($\mu\gamma_5\tau_3$) and with the limit of $\mu \to 0$ at $\beta = 0$.
$\Longrightarrow m_\pi = 0$ or $\neq 0$? $\langle\bar{\psi}\gamma_5\tau_3\psi\rangle =$? $\Longrightarrow$ In progress

Discussion: What is the unknown phase?

♠ Our finding in PRD80('09)074508
presented in this talk (Plaq., m_π^2 and m_q behaviours)
$\rightarrow$ Indication of universal phenomena **as**
(1st order) Bulk phase transition at strong coupling
$\Leftarrow$ the appearance of high- and low-plaquette phase

- Y. Iwasaki *et al*, PRL69('92)21, PRD69('04)014507.
  $\rightarrow$ deconfining
- JLQCD (S. Aoki *et al*), PRD72('05)054510.
  $N_f = 3$ Clover fermion $\rightarrow$ unphysical phase
- F.R. Brown *et al*, PRD46('92)5655.
  $\rightarrow N_f = 8$ staggered fermions
- T. DeGrand *et al*, arXiv:1006.0707 [hep-lat].
  $\rightarrow$ symmetric repr. of Wilson with SF

• *c.f.* F. Farchioni *et al*, Eur.Phys.J.C39('05)421.
$\rightarrow$ metastability in $N_f = 2$ tmQCD at weak coupling

Additional data

At $\beta = 2$

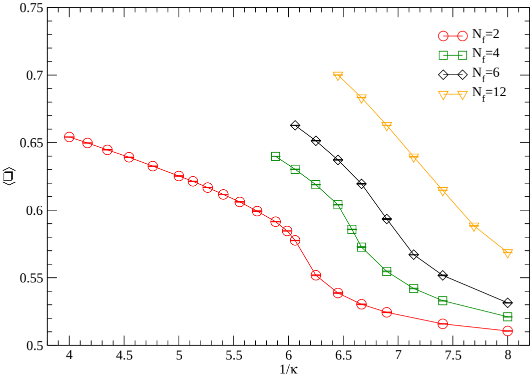


Figure: Plaquette value as a function of $1/\kappa$ at $\beta = 2$ in $SU(2)$.
no 1st order transition

At $\beta = 2$

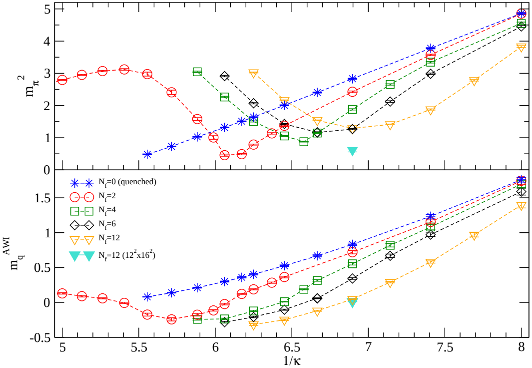


Figure: m_π^2 and m_q^{AWI} as a function of $1/\kappa$ at $\beta = 2$ in $SU(2)$.
There seems to be the confinement phase.
The m_q^{AWI} crosses the zero smoothly.

At $\beta = 2$

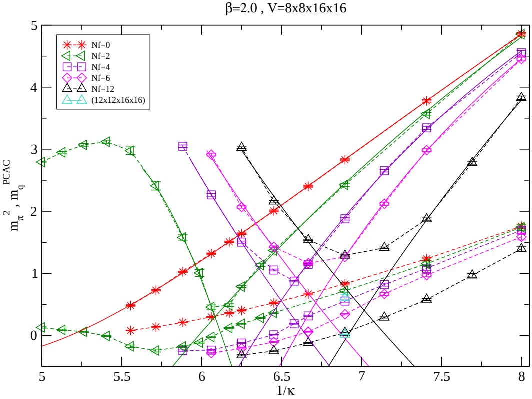
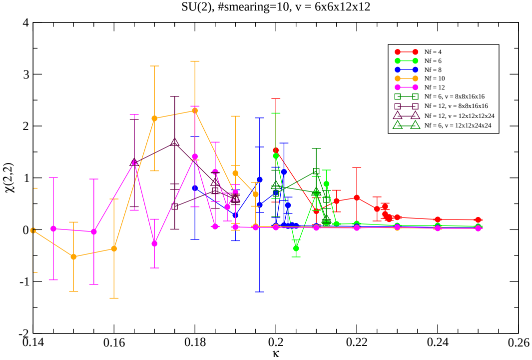


Figure: the fit result of m_π^2 at $\beta = 2$ in $SU(2)$.

The fitting in both side for the minimum point is very well.

Creutz ratio $\chi(2,2)$ in $SU(2)$ at $\beta = 0$



In the massive pion phase, $\chi(2,2)$ is small and stable.

In $SU(2)$ at $\beta = 0$: $N_{\rm CG}(\tau) = \frac{1}{N_{\rm MD}} \sum_{i=1}^{N_{\rm MD}} n_i^{\rm CG}(\tau)$

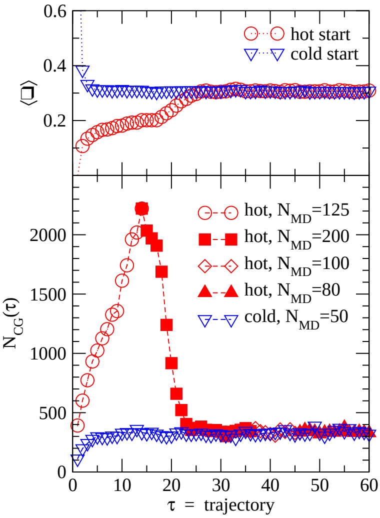


Figure: The plaquette and $N_{CG}(\tau)$ versus trajectory index τ in $SU(2)$ on $8^2 \times 16^2$ at $\beta = 0$ and $\kappa = 0.215$ with $N_f = 6$.

In $SU(3)$ at $\beta = 0$: $N_{\mathrm{CG}}(\tau) = \frac{1}{N_{\mathrm{MD}}} \sum_{i=1}^{N_{\mathrm{MD}}} n_i^{\mathrm{CG}}(\tau)$

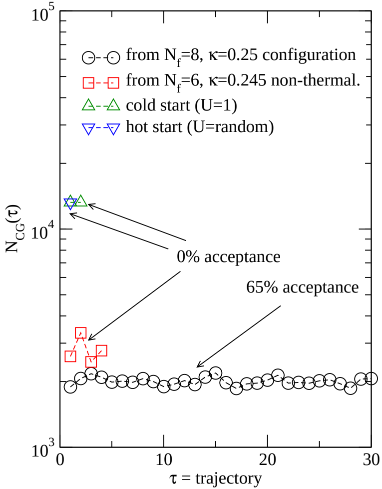
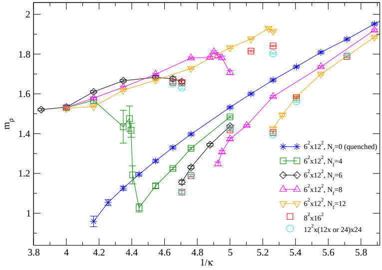
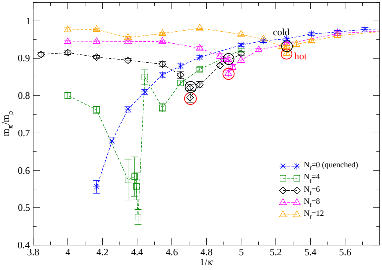


Figure: $N_{CG}(\tau)$ versus τ in $SU(3)$ with $N_f = 6$ on $6^2 \times 12^2$ at $\beta = 0$, $\kappa = 1/4$ and $N_{MD} = 200$.

m_ρ and m_π/m_ρ





2-state signal in m_ρ and m_π/m_ρ

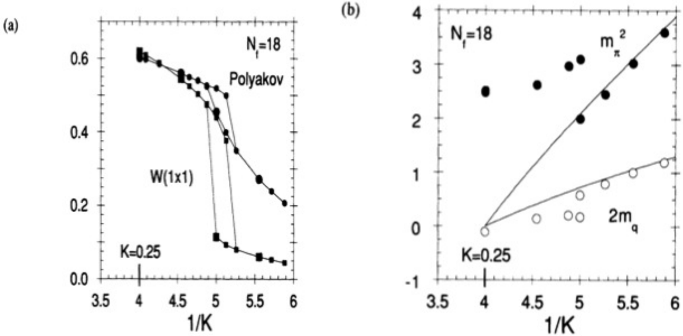


FIG. 1. Results for $N_f=18$ at $\beta=0.0$ on the $T=4$ lattice with $\Delta\tau=0.01$. (a) $W(1\times1)$ (squares) and Polyakov line (circles). The large symbols are for long runs. (b) m_π^2 (solid circles) and $2m_q$ (open circles).

Y. Iwasaki *et al*, PRL69('92). : $N_f=18$ in $SU(3)$

In confinement phase, the data obey to Aoki's line.

Polyakov loop >0 for $1/\kappa<1/\kappa_d \rightarrow$ Deconfinement phase

$m_\pi>0$ for $1/\kappa<1/\kappa_d$.

No negative quark mass ($m_\pi^2 \neq 2Bm_q$)

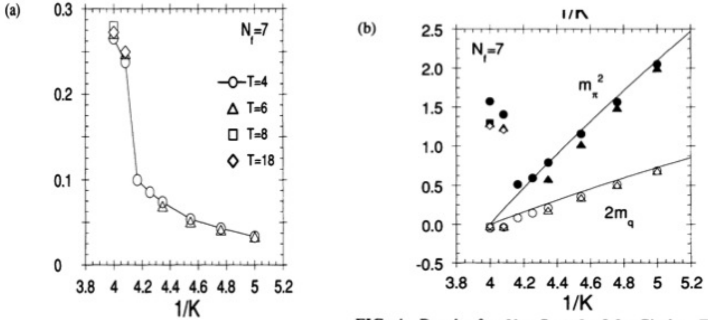


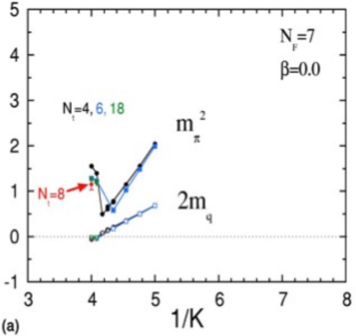
FIG. 4. Results for $N_f = 7$ at $\beta = 0.0$: Circles, $T = 4$; triangles, $T = 6$; squares, $T = 8$; and diamonds, $T = 18$. (a) $W(1\times1)$. (b) m_π^2 and $2m_q$.

Y. Iwasaki *et al*, PRL69('92).

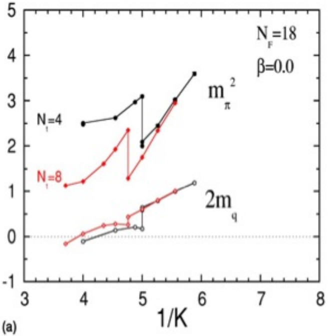
$N_f = 7$ in $SU(3)$ $\rightarrow$ same with $N_f = 18$ case

In the large N_t, it seems the data deviates from the line.

They didn't comment about it.

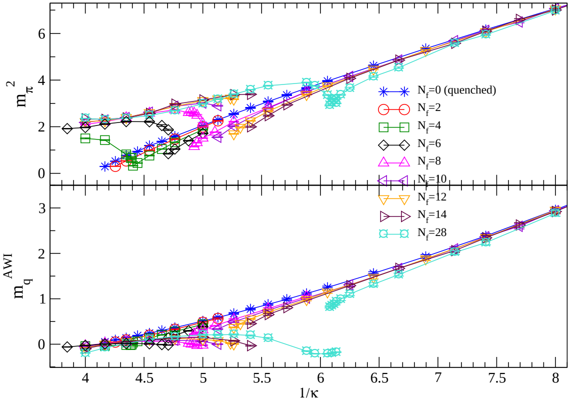


(a)



(a)

m_π^2 and m_q^{AWI} vs. $1/\kappa$ (in the wide region of $1/\kappa$)



- In large $1/\kappa$, m_π^2 and m_q^{AWI} are almost independent of N_f.
  $\rightarrow$ The heavy mode does not affect the vacuum.

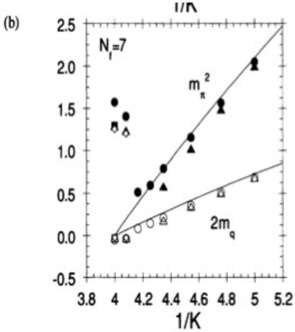


FIG. 4. Results for $N_f=7$ at $\beta=0.0$: Circles, $T=4$; triangles, $T=6$; squares, $T=8$; and diamonds, $T=18$. (a) $W(1\times1)$. (b) m_π^2 and $2m_q$.

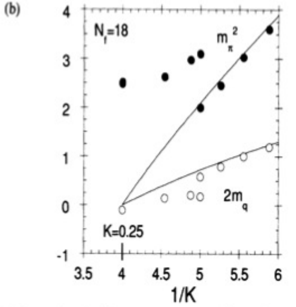


FIG. 1. Results for $N_f=18$ at $\beta=0.0$ on the $T=4$ lattice with $\Delta\tau=0.01$. (a) $W(1\times1)$ (squares) and Polyakov line (circles). The large symbols are for long runs. (b) m_π^2 (solid circles) and $2m_q$ (open circles).

Y. Iwasaki *et al*, : $N_f=7,18$ in $SU(3)$

In confinement phase, the data obey to Aoki's line.

Polyakov loop >0 for $1/\kappa < 1/\kappa_d \rightarrow$ Deconfinement phase

$m_\pi > 0$ for $1/\kappa < 1/\kappa_d$.

No negative quark mass ($m_\pi^2 \neq 2Bm_q$)

$m_q^{AWI}(T/a)$ vs. T/a for $N_f = 12$ at $\kappa = 0.190$ on $6^2 \times 12^2$, $8^2 \times 16^2$ and $12^3 \times 24$

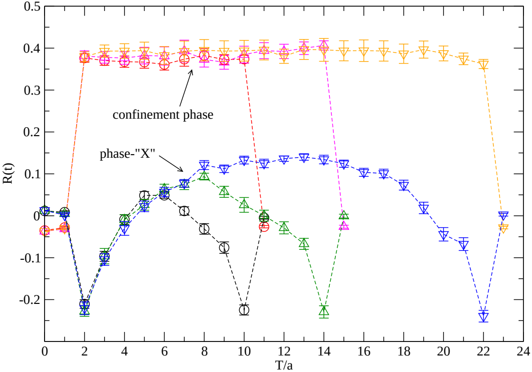
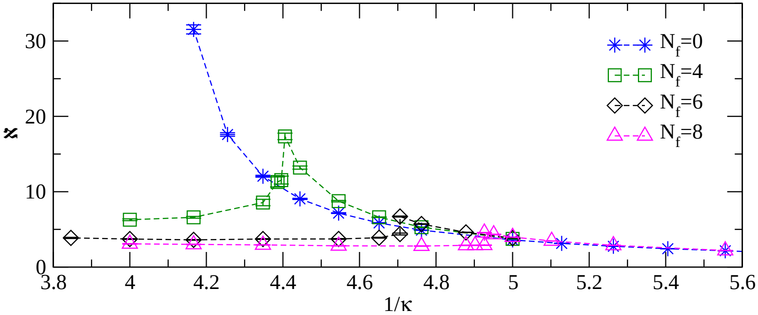


Figure: $m_q^{AWI}(t)$ for $N_f = 12$ at $\kappa = 0.190$ in high- and low-plaq. phase .

- No plateau in the high-plaquette phase (the massive pion phase)
- No negative quark mass ($m_\pi^2 \neq 2Bm_q$) $\Rightarrow$ not Sharpe-Singleton scenario ??

The propagator norm: $\mathcal{N} = (2\kappa)^2 \sum_{\vec{x},t} \left\langle P(\vec{x},t)P(\vec{0},0) \right\rangle \sim \frac{1}{m_\pi^2}$



In the confinemnt phase, there is $\frac{1}{m_q}$ pole.

In the unknown phase, however, there is not the pole. $\rightarrow$ Why?