Excited States of the Nucleon in 2+1 flavour QCD

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CSSM Lattice Collaboration

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Outline

Roper Resonance

- *Roper resonance* (P_{11}) is the first positive parity excited state of the nucleon
- Observed in 1960's from πN scattering
- The resonance is interesting due to its low mass (1440 MeV) relative to the nearest negative-parity (S_{11}) resonance (1535 MeV).
- In a constituent quark model, the Roper state is ≈100 MeV *above* the S_{11} (1535 MeV) state.
- The Roper state appeared very high in all previous lattice simulations using the variational method.

- Two point correlation function:

$$G_{ij}(t,\vec{p}) = \sum_{\vec{x}} e^{-i\vec{p}.\vec{x}} \langle \Omega | T\{\chi_i(x)\bar{\chi}_j(0)\} | \Omega \rangle.$$

- Inserting completeness

$$\sum_{B,\vec{p'},s} |B,\vec{p'},s\rangle\langle B,\vec{p'},s| = I$$

- Then

$$G_{ij}(t,\vec{p}) = \sum_{B^+} \lambda_{B^+}\bar{\lambda}_{B^+} e^{-E_{B^+}t} \frac{\gamma.p_{B^+} + M_{B^+}}{2E_{B^+}} + \sum_{B^-} \lambda_{B^-}\bar{\lambda}_{B^-} e^{-E_{B^-}t} \frac{\gamma.p_{B^-} - M_{B^-}}{2E_{B^-}}$$

- $\lambda_{B^\pm}$, $\bar{\lambda}_{B^\pm}$ are the couplings of $\chi(0)$ and $\bar{\chi}(0)$ with $|B^\pm\rangle$ defined by

$$\langle\Omega|\chi(0)|B^+,\vec{p},s\rangle = \lambda_{B^+}\sqrt{\frac{M_{B^+}}{E_{B^+}}}u_{B^+}(\vec{p},s),$$

$$\langle B^+,\vec{p},s|\bar{\chi}(0)|\Omega\rangle = \bar{\lambda}_{B^+}\sqrt{\frac{M_{B^+}}{E_{B^+}}}\bar{u}_{B^+}(\vec{p},s),$$

and for the negative parity states,

$$\langle\Omega|\chi(0)|B^-,\vec{p},s\rangle = \lambda_{B^-}\sqrt{\frac{M_{B^-}}{E_{B^-}}}\gamma_5 u_{B^-}(\vec{p},s),$$

$$\langle B^-,\vec{p},s|\bar{\chi}(0)|\Omega\rangle = -\bar{\lambda}_{B^-}\sqrt{\frac{M_{B^-}}{E_{B^-}}}\bar{u}_{B^-}(\vec{p},s)\gamma_5.$$

- At $\vec{p} = 0$

$$G_{ij}^{\pm}(t,\vec{0}) = \mathrm{Tr}_{\mathrm{sp}}[\Gamma_{\pm} G_{ij}(t,\vec{0})] = \sum_{B^{\pm}} \lambda_i^{\pm} \bar{\lambda}_j^{\pm} e^{-M_{B^{\pm}} t}.$$

- Parity projection operator,

$$\Gamma_{\pm} = \frac{1}{2}(1 \pm \gamma_0).$$

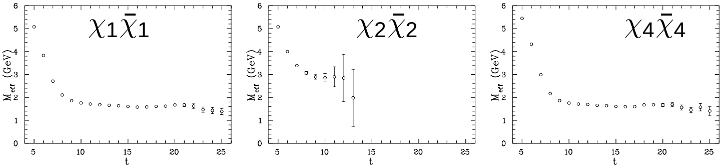
- And

$$G_{ij}^{\pm}(t,\vec{0}) \stackrel{t\to\infty}{=} \lambda_{i0}^{\pm} \bar{\lambda}_{j0}^{\pm} e^{-M_{0^{\pm}} t}.$$

Interpolators

- Consider

$$\chi_1(x) = \epsilon^{abc}(u^{Ta}(x)\, C\gamma_5\, d^b(x))\, u^c(x)\,,$$
$$\chi_2(x) = \epsilon^{abc}(u^{Ta}(x)\, C\, d^b(x))\, \gamma_5\, u^c(x)\,,$$
$$\chi_4(x) = \epsilon^{abc}(u^{Ta}(x)\, C\gamma_5\gamma_4\, d^b(x))\, u^c(x).$$



Variational Method

- Consider N interpolating fields, then

$$\bar{\phi}^{\alpha} = \sum_{i=1}^{N} u_i^{\alpha} \bar{\chi}_i ,$$

$$\phi^{\alpha} = \sum_{i=1}^{N} v_i^{\alpha} \chi_i ,$$

such that,

$$\langle B_\beta, p, s | \bar{\phi}^{\alpha} | \Omega \rangle = \delta_{\alpha\beta} \bar{z}^{\alpha} \bar{u}(\alpha, p, s),$$

$$\langle \Omega | \phi^{\alpha} | B_\beta, p, s \rangle = \delta_{\alpha\beta} z^{\alpha} u(\alpha, p, s),$$

- Then a two point correlation function matrix for $\vec{p} = 0$,

$$G_{ij}^{\pm}(t)u_j^{\alpha} = (\sum_{\vec{x}} \mathrm{Tr}_{\mathrm{sp}}\{\Gamma_{\pm}\langle\Omega|\chi_i\bar{\chi}_j|\Omega\rangle\})u_j^{\alpha}$$
$$= \lambda_i^{\alpha}\bar{z}^{\alpha}e^{-m_{\alpha}t}.$$

(no sum over α)

- t dependence only in the exponential term

- Then one can have a recurrence relation at time $(t_0 + \triangle t)$,

$$G_{ij}(t_0 + \triangle t)u_j^\alpha = e^{-m_\alpha \triangle t} G_{ij}(t_0)u_j^\alpha.$$

- Multiplying by $[G_{ij}(t_0)]^{-1}$ from left,

$$[(G(t_0))^{-1} G(t_0 + \triangle t)]_{ij} u_j^\alpha = c^\alpha u_i^\alpha,$$

- where $c^\alpha = e^{-m_\alpha \triangle t}$ is the eigenvalue.
- Similarly, it can also be solved for the left eigenvalue equation for v^α eigenvector,

$$v_i^\alpha [G(t_0 + \triangle t)(G(t_0))^{-1}]_{ij} = c^\alpha v_j^\alpha.$$

- The vectors u_j^α and v_i^α diagonalize the correlation matrix at time t_0 and $t_0 + \triangle t$ making the projected correlation function

$$v_i^\alpha G_{ij}(t) u_j^\beta = \delta^{\alpha\beta} z^\alpha \bar{z}^\beta e^{-m_\alpha t} .$$

- The projected correlator, is then analyzed to obtain masses of different states,

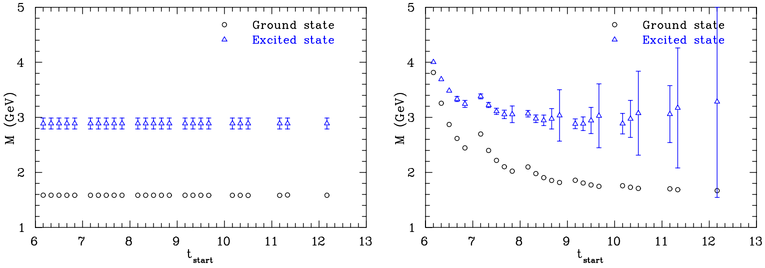
$$v_i^\alpha G_{ij}^{\pm}(t) u_j^\alpha \equiv G_{\pm}^\alpha ,$$

- We construct the effective mass

$$M_{\rm eff}^\alpha(t) = \ln\left(\frac{G_{\pm}^\alpha(t,\vec{0})}{G_{\pm}^\alpha(t+1,\vec{0})}\right) .$$

2×2 correlation matrix of $\chi_1\,\chi_2$ for a point source

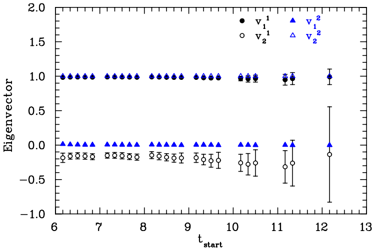
Projected Mass Vs Mass From Eigenvalue



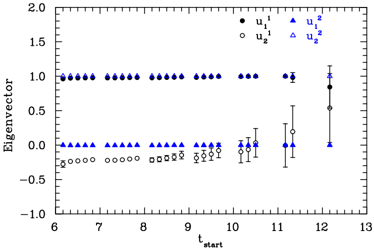
- $t_{\rm start} = t_0$ is shown in major tick marks
- $\triangle t$ is shown in minor tick marks

Eigenvectors - Point Source, for $\chi_1\chi_2$

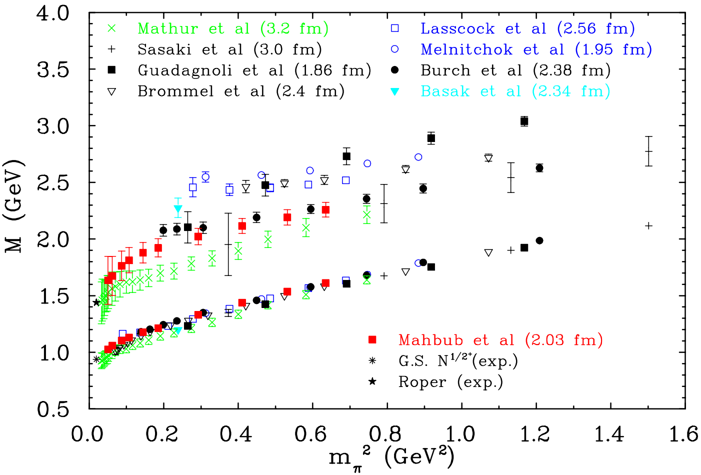
Left Eigenvectors



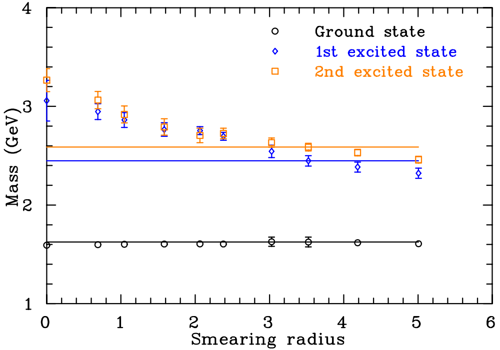
Right Eigenvectors



Roper state: Compilation of existing results in QQCD

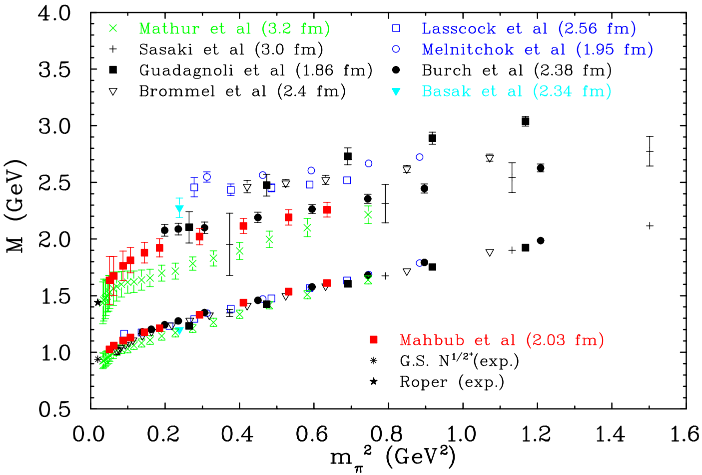


Smeared Source Problem



M. S. Mahbub, *et al.*, Phys. Rev. D **80**, 054507 (2009) [arXiv:0905.3616 [hep-lat]]

Roper state: Compilation of existing results in QQCD



Source Smearing

Correlation matrices are built from a variety of source and sink smearings.

$$\psi_i(x,t) = \sum_{x'} F(x,x')\psi_{i-1}(x',t),$$

where,

$$\begin{aligned} F(x,x') &= (1-\alpha)\delta_{x,x'} + \frac{\alpha}{6}\sum_{\mu=1}^{3}[U_\mu(x)\delta_{x',x+\hat{\mu}} \\ &\quad + U_\mu^\dagger(x-\hat{\mu})\delta_{x',x-\hat{\mu}}], \end{aligned}$$

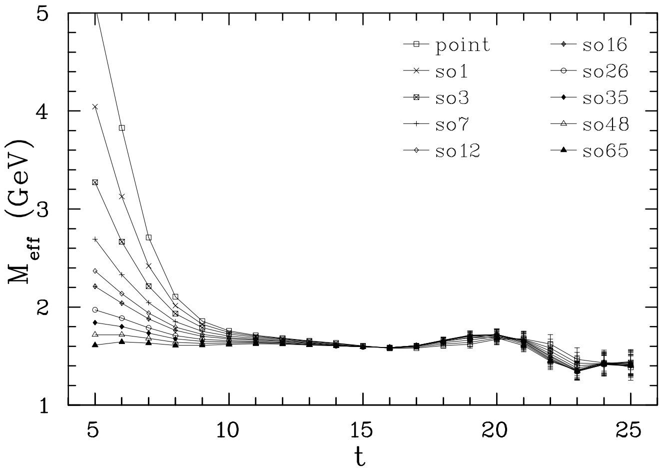
Fixing $\alpha = 0.7$, the procedure is repeated $N_{\rm sm}$ times.

4×4 bases of $\chi_1\bar{\chi}_1$

- Consider smeared–smeared correlation functions
- Variety of smearing sweeps used to form basis interpolators

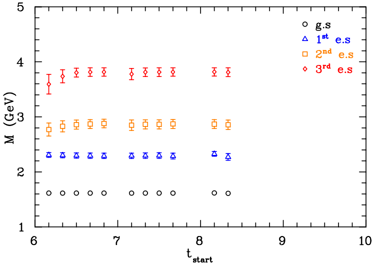
Sweeps $\rightarrow$	1	3	7	12	16	26	35	48
Basis No. $\downarrow$	Bases							
1	1	-	7	-	16	-	35	-
2	-	3	7	-	16	-	35	-
3	1	-	-	12	-	26	-	48
4	-	3	-	12	-	26	35	-
5	-	3	-	12	-	26	-	48
6	-	-	-	12	16	26	35	-
7	-	-	7	-	16	-	35	48

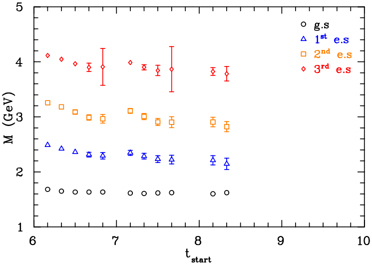
Smeared Source - Point Sink Correlators



4×4 correlation matrix for the 4^{th} basis (3, 12, 26, 35)

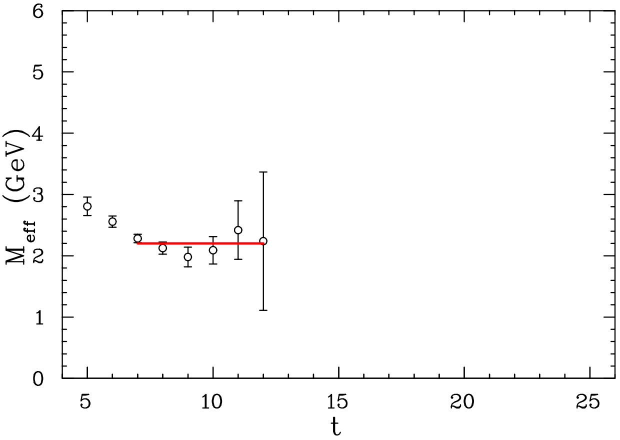
Projected Mass Vs Mass From Eigenvalue





- $t_{\text{start}} = t_0$ is shown in major tick marks
- $\triangle t$ is shown in minor tick marks

Effective Mass of Roper: 5$^{\text{th}}$ Basis

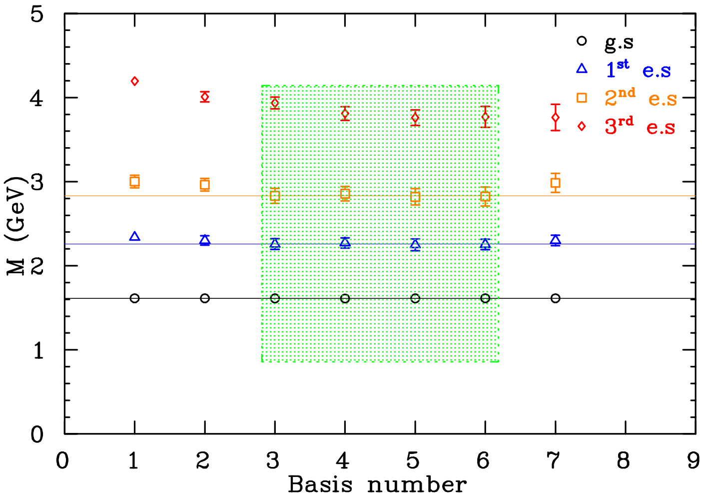


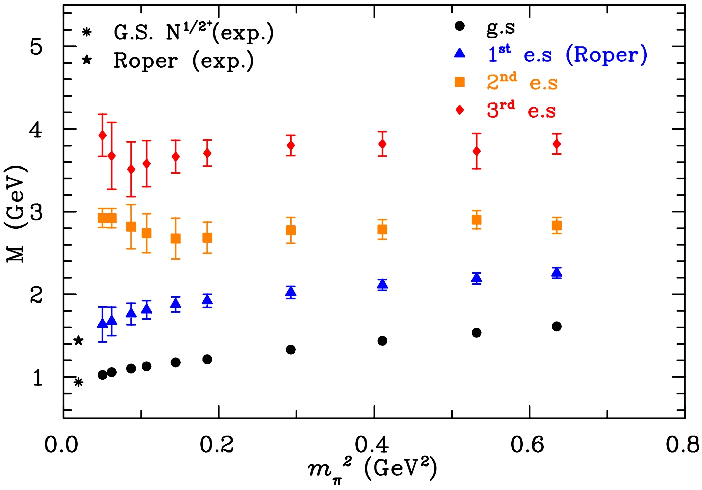
$\chi^2/dof = 0.51$

4×4 bases of $\chi_1\bar{\chi}_1$

Sweeps $\rightarrow$	1	3	7	12	16	26	35	48
Basis No. $\downarrow$	Bases							
1	1	-	7	-	16	-	35	-
2	-	3	7	-	16	-	35	-
3	1	-	-	12	-	26	-	48
4	-	3	-	12	-	26	35	-
5	-	3	-	12	-	26	-	48
6	-	-	-	12	16	26	35	-
7	-	-	7	-	16	-	35	48

Projected correlator masses from 4 × 4 analysis





Mahbub *et al.*, Phys. Lett. B **679**, 418 (2009), [arXiv:hep-lat/0906.5433].

6×6 bases of $\chi_1\bar{\chi}_1$

Sweeps $\rightarrow$	1	3	7	12	16	26	35	48
Basis No. $\downarrow$	Bases							
1	1	3	7	12	16	26	-	-
2	1	3	7	12	16	-	35	-
3	1	3	7	-	16	26	35	-
4	1	3	-	12	16	26	-	48
5	1	-	7	12	16	26	35	-
6	-	3	7	12	16	26	35	-

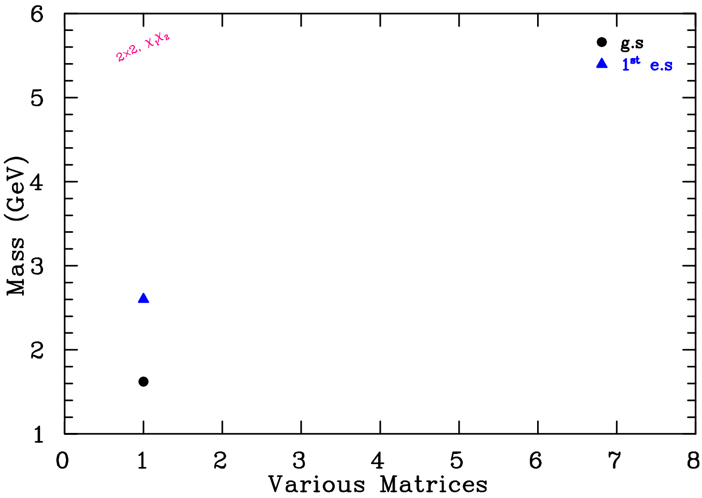
6×6 bases of $\chi_1 \, \chi_2$

Sweeps $\rightarrow$	1	3	7	12	16	26	35	48
Basis No. $\downarrow$	Bases							
1	1	-	-	-	16	-	-	48
2	-	3	-	12	-	26	-	-
3	-	3	-	-	16	-	-	48
4	-	-	7	-	16	-	35	-
5	-	-	-	12	16	26	-	-
6	-	-	-	-	16	26	35	-

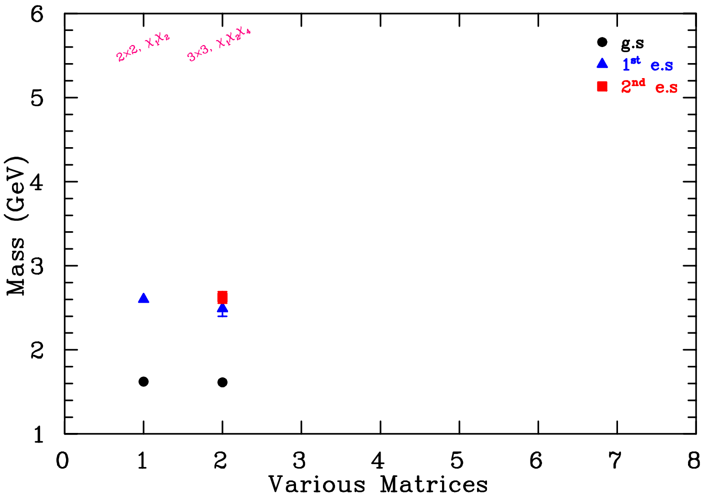
8×8 bases of $\chi_1\, \chi_2$

Sweeps $\rightarrow$	1	3	7	12	16	26	35	48
Basis No. $\downarrow$	Bases							
1	1	-	7	-	16	-	35	-
2	-	-	7	12	16	26	-	-
3	-	3	-	12	-	26	-	48
4	-	-	7	12	-	26	35	-
5	-	-	7	-	16	26	35	-
6	-	-	7	-	16	-	35	48
7	-	-	-	12	16	26	35	-

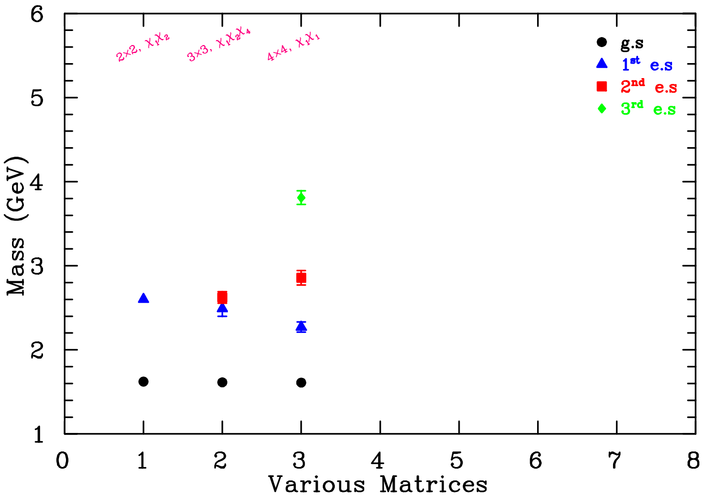
Review of excited “states”



Review of excited “states”



Review of excited "states"



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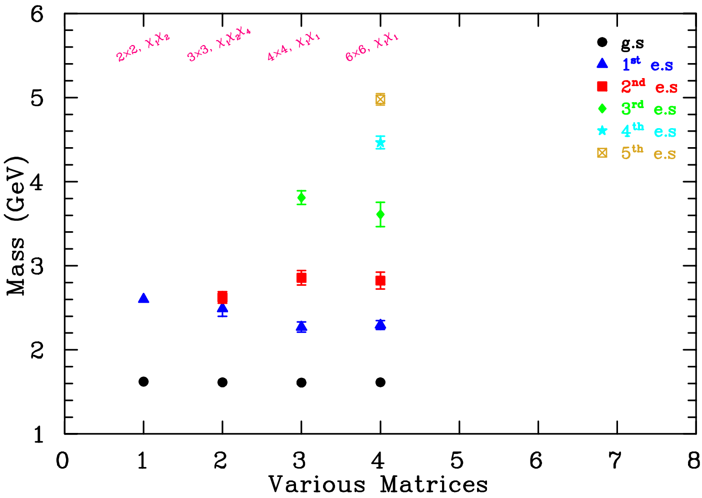
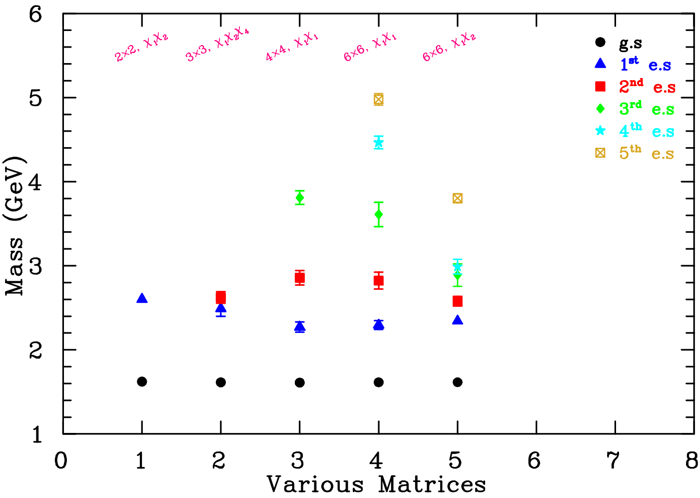


Review of excited “states”



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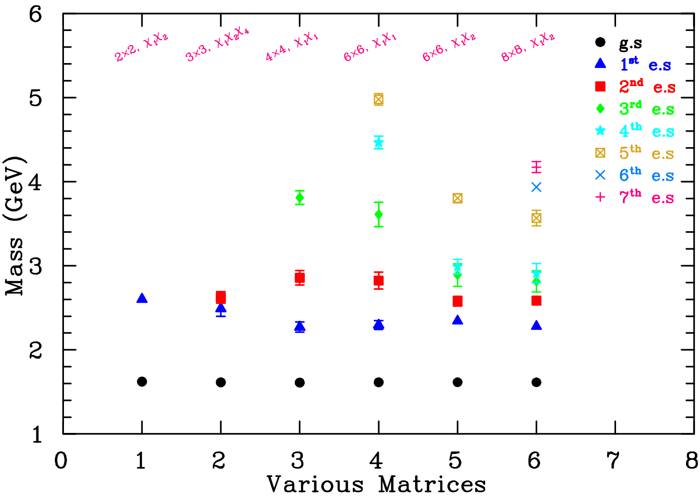
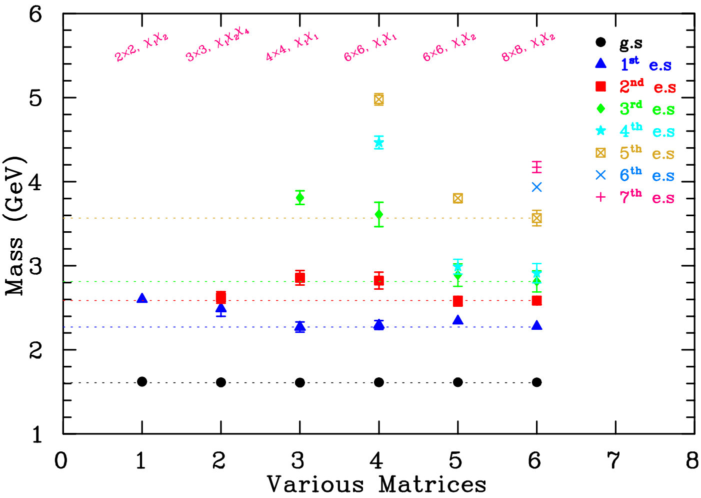
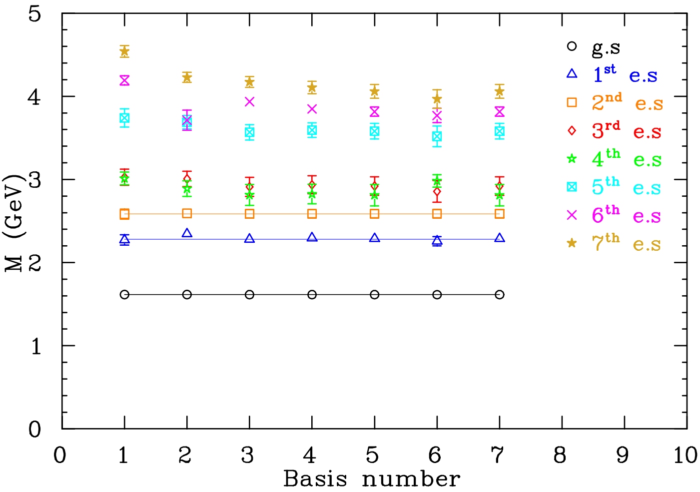


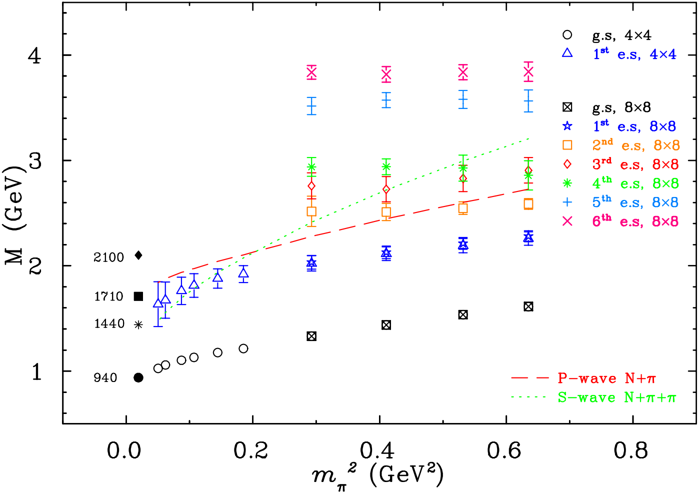
Review of excited “states”



Projected masses from 8 × 8 analysis of $\chi_1\,\chi_2$



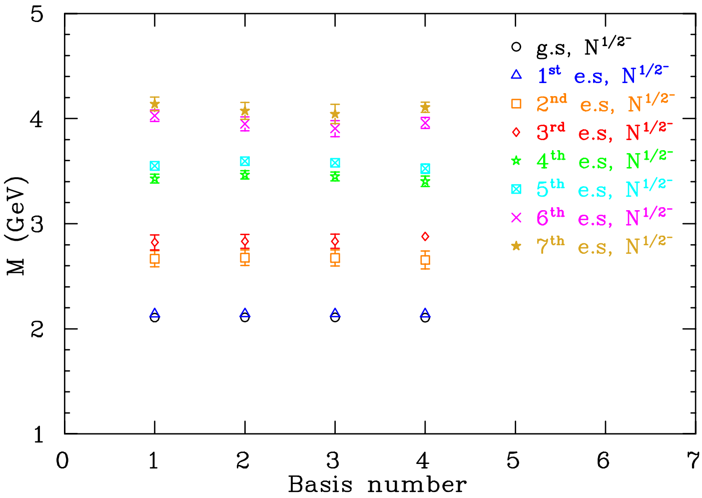
Positive Parity Results



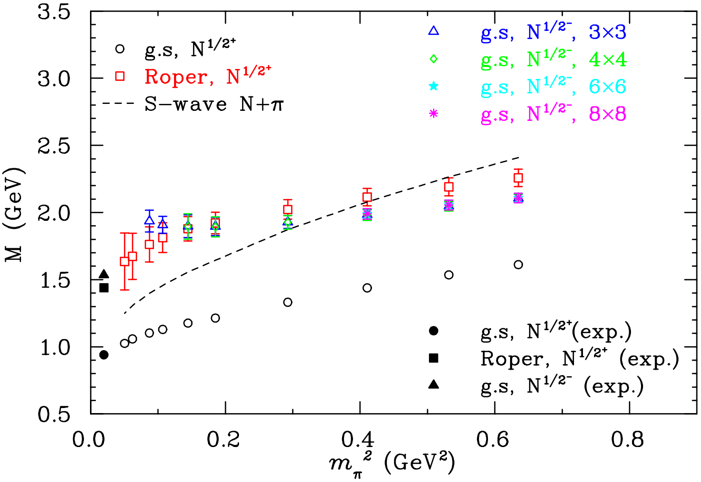
8×8 bases of $\chi_1\,\chi_2$ for $N1/2^-$ Analysis

Sweeps $\rightarrow$	1	3	7	12	16	26	35	48
Basis No. $\downarrow$	Bases							
1	-	3	-	12	-	26	-	48
2	-	-	7	12	-	26	35	-
3	-	-	7	-	16	26	35	-
4	-	-	7	-	16	-	35	48

Projected $N1/2^-$ masses from 8×8 bases



Roper and $N1/2^-$ states



PACS-CS lattice: Simulation details

PACS-CS Collaboration: S. Aoki, et al., Phys. Rev. **D79** (2009) 034503.

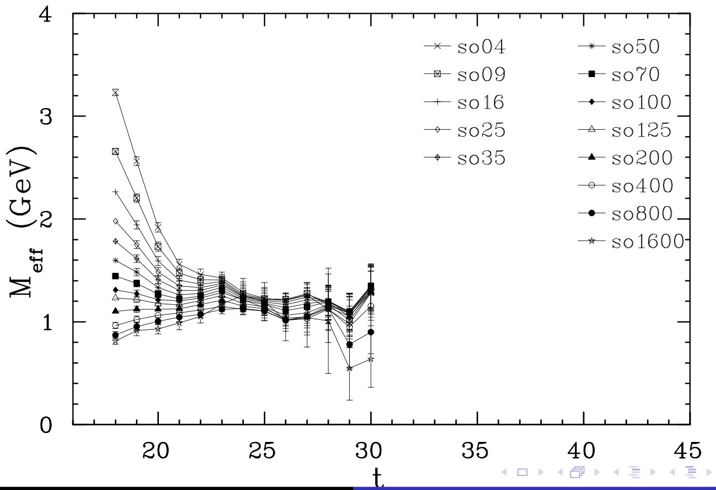
- Lattice volume: $32^3 \times 64$
- Non-perturbative $\mathcal{O}(a)$-improved Wilson quark action
- Iwasaki gauge action
- $2 + 1$ flavour dynamical-fermion QCD
- $\beta = 1.9$ providing $a = 0.0907$ fm
- $K_{ud} = \{ 0.13700, 0.13727, 0.13754, 0.13770, 0.13781 \}$
- $K_s = 0.13660$
- Lightest pion mass is 156 MeV.

4×4 bases of $\chi_1\bar{\chi}_1$

Sweeps →	16	25	35	50	70	100	125	200	400	800
Basis No. ↓						Bases				
1	16	-	35	-	70	100	-	-	-	-
2	16	-	35	-	70	-	125	-	-	-
3	16	-	35	-	-	100	-	200	-	-
4	16	-	35	-	-	100	-	-	400	-
5	16	-	-	50	-	100	125	-	-	-
6	16	-	-	50	-	100	-	200	-	-
7	16	-	-	50	-	-	125	-	-	800
8	-	25	-	50	-	100	-	200	-	-
9	-	25	-	50	-	100	-	-	400	-
10	-	-	35	-	70	-	125	-	400	-

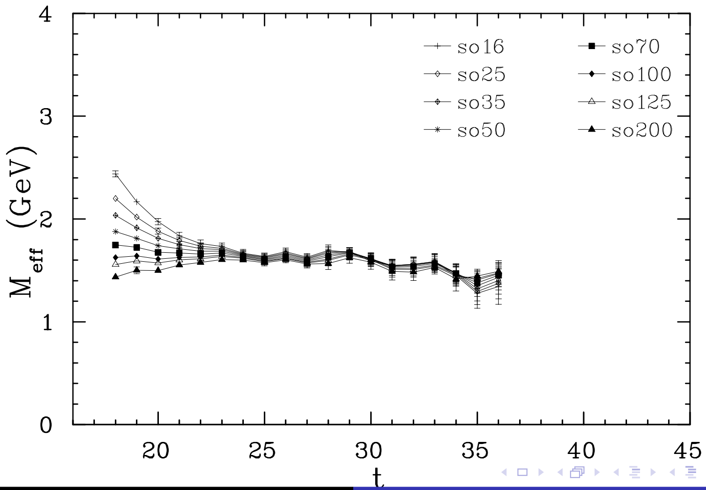
Smeared Source - Point Sink Effective Masses

For second lightest quark : 50 cfgs

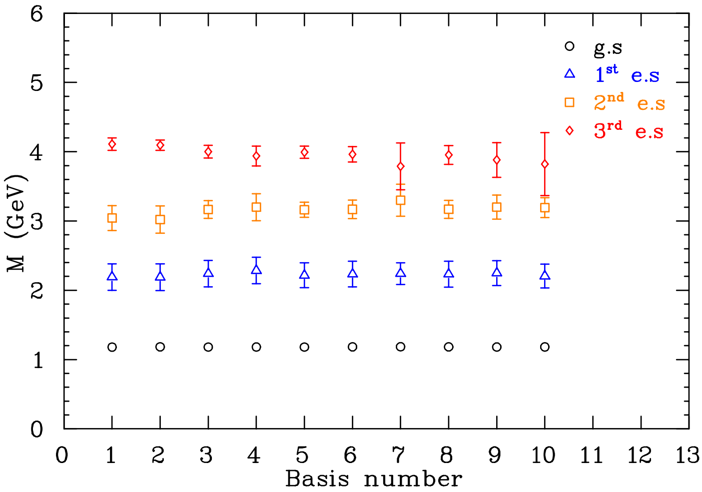


Smeared Source - Point Sink Effective Masses

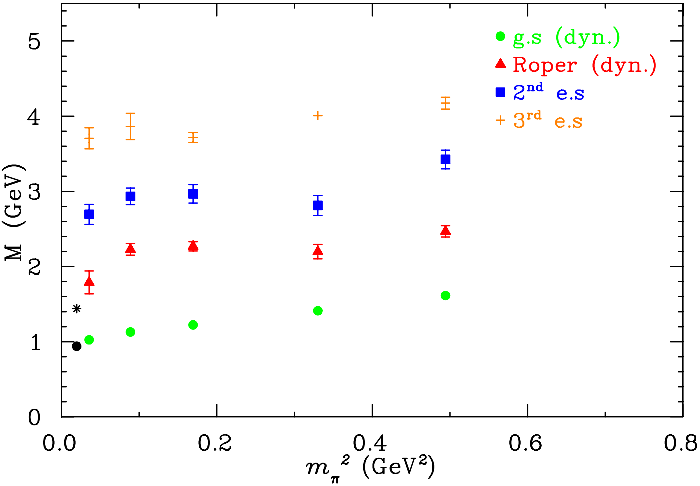
For the heaviest quark: 50 cfgs



For all 4×4 bases: $K_{ud} = 0.137700$

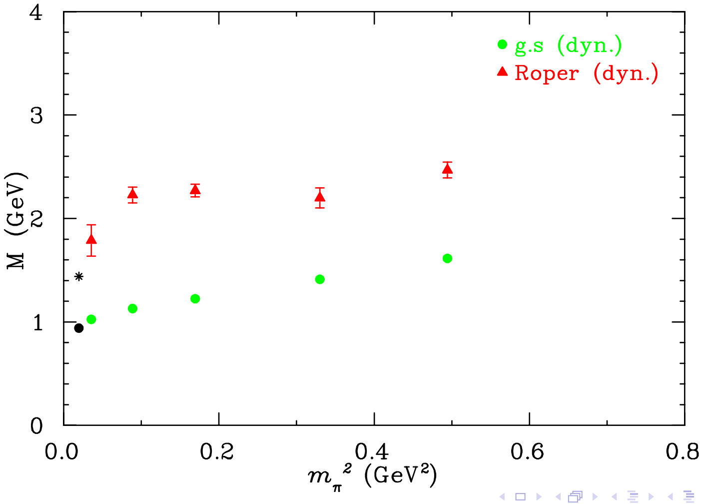


Even Parity Nucleon Spectrum in full QCD

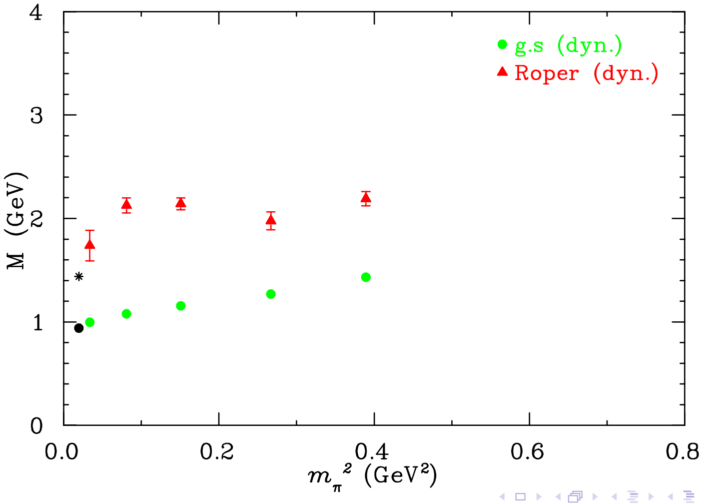


Configs: 296, 300, 200, 350, 200

Ground and Roper states (fixed lattice spacing)



Ground and Roper states (Sommer scale sets *a*)



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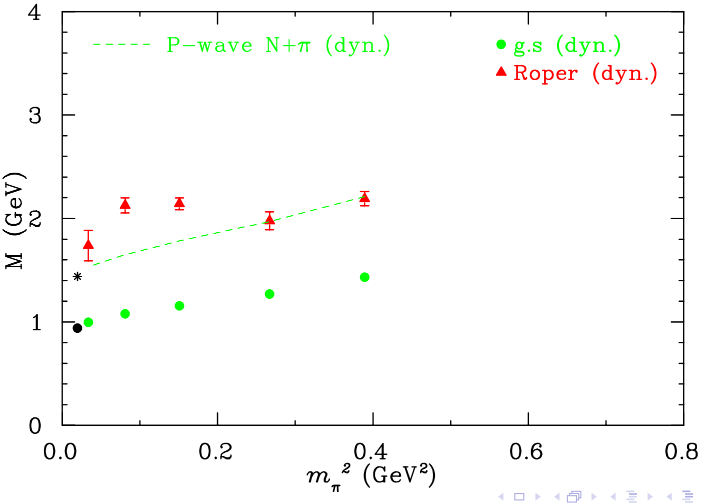

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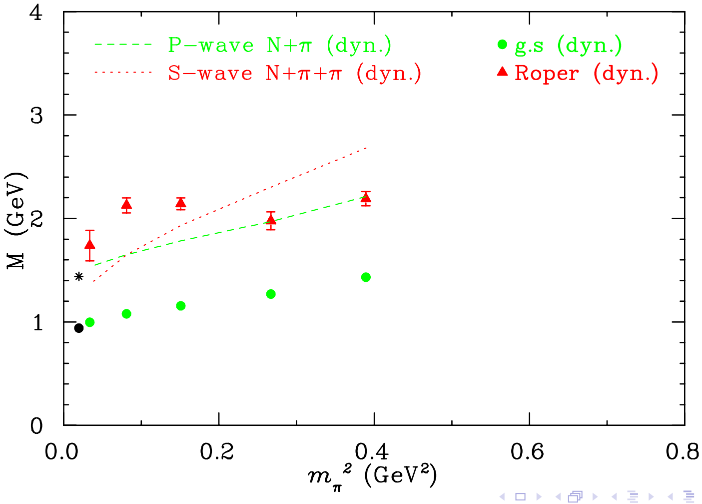
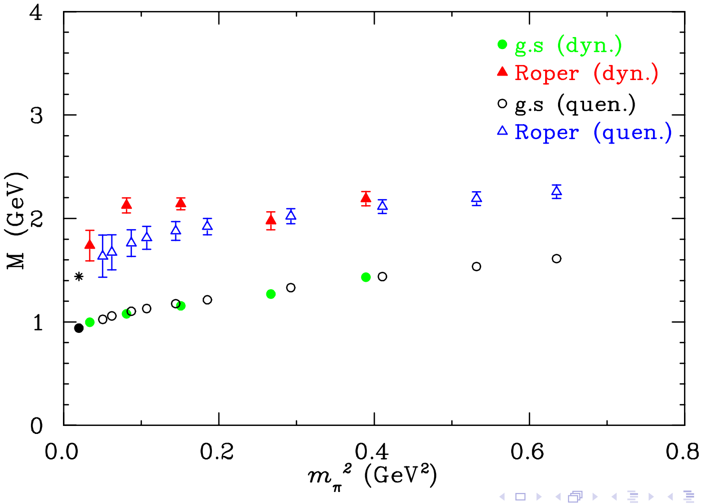
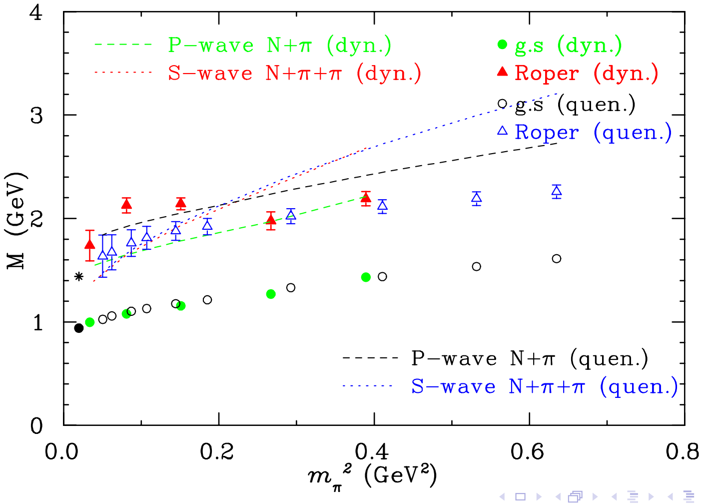


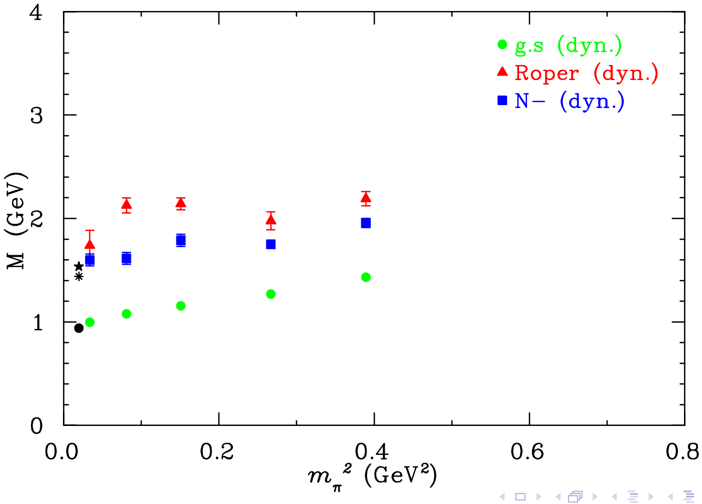
Quenched Vs Dynamical (Sommer scale)



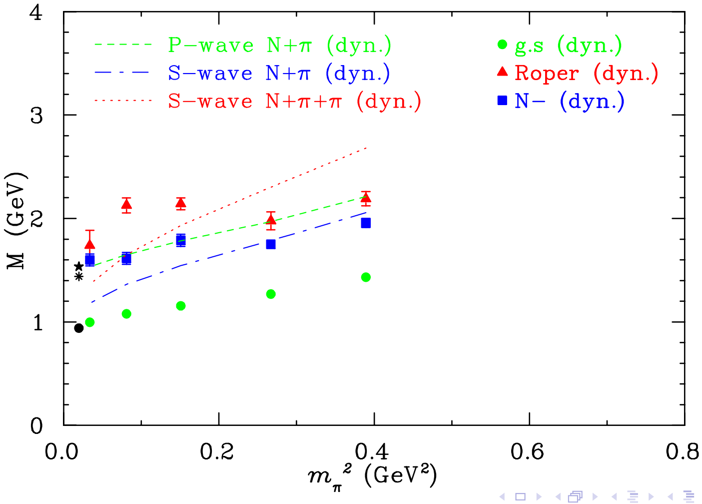
Quenched Vs Dynamical (Sommer scale)



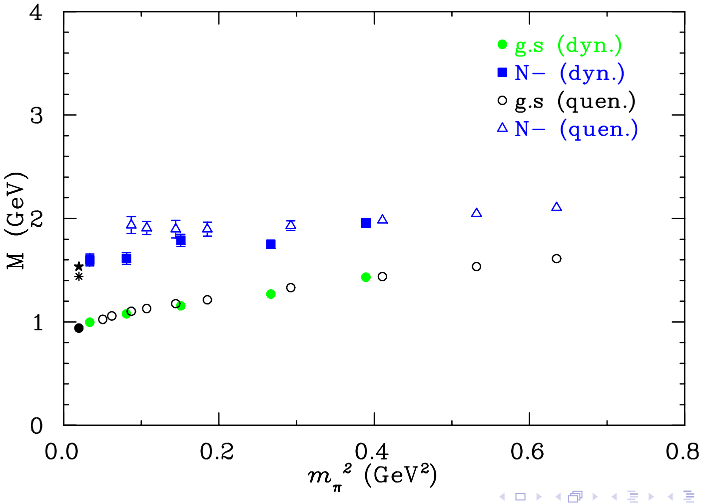
$N\frac{1}{2}^-$ (1535) (Sommer scale)



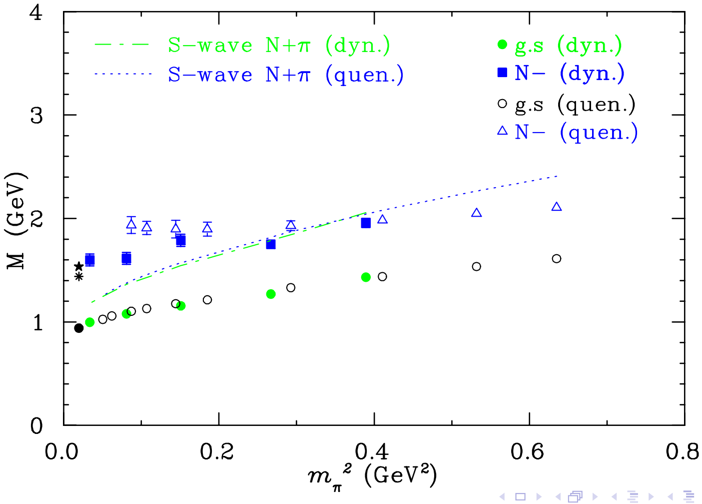
$N\frac{1}{2}^-$ (1535) (Sommer scale)



Quenched Vs Dynamical $N\frac{1}{2}^-$ (1535) (Sommer scale)



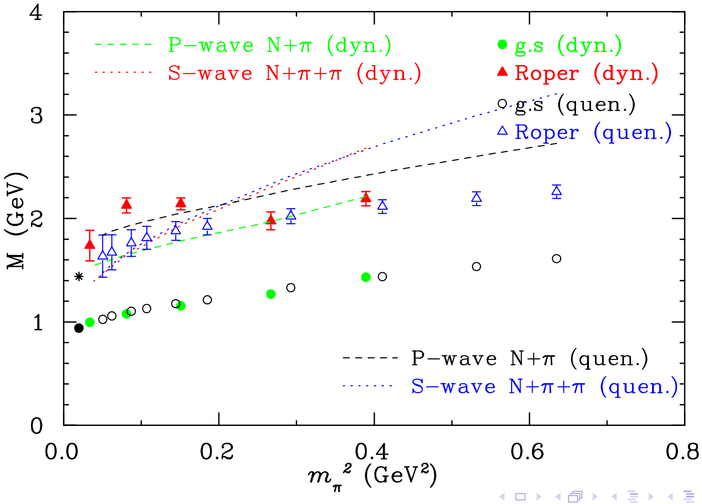
Quenched Vs Dynamical $N\frac{1}{2}^-$ (1535) (Sommer scale)



Summary

- Several fermion-source and -sink smearing levels have been used to construct correlation matrices.
- A variety of 4×4, 6×6, and 8×8 matrices were considered to demonstrate the independence of the eigenstate energies from the basis interpolators.
- A low-lying Roper state has been identified in both quenched and full QCD using this correlation-matrix based method.
- The approach to the chiral limit is significantly different.
- The two heaviest quark masses considered in the dynamical case provide states consistent with P-wave πN scattering states.

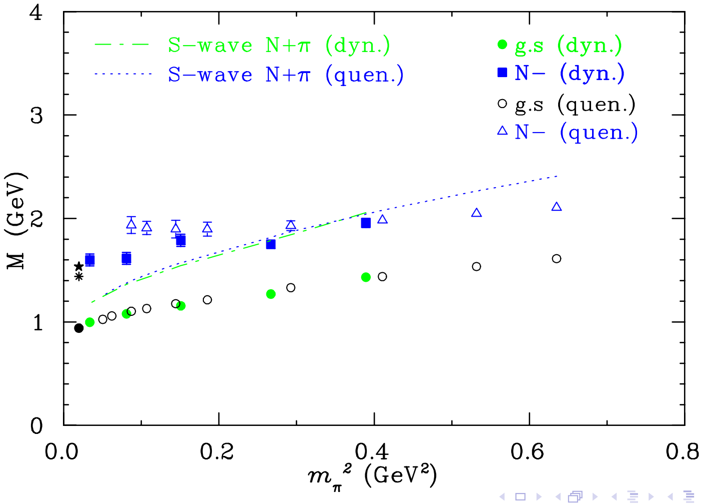
Quenched Vs Dynamical (Sommer scale)



Summary continued...

- The $N1/2^-$ results in quenched and dynamical QCD reveal significant differences in the approach to the physical point.
- A level crossing between the Roper and $N1/2^-$ states is observed in quenched QCD at $m_\pi \simeq 400$ MeV.
- A level crossing between the Roper and $N1/2^-$ states is anticipated in full QCD at $m_\pi \simeq 150$ MeV, just above the physical pion mass.
- The approach to the experimentally measured masses is encouraging.
- The effects of the finite volume and the role of scattering states remains to be resloved.

Quenched Vs Dynamical $N\frac{1}{2}^-$ (1535) (Sommer scale)



Future Plans

- Complete all quark masses at 400 configs (800 at lightest mass).
- Explore chiral curvature via chiral effective field theory.
- Extend to a comprehensive analysis of all baryons of interest.
- Complete determination of excited-state electromagnetic properties.