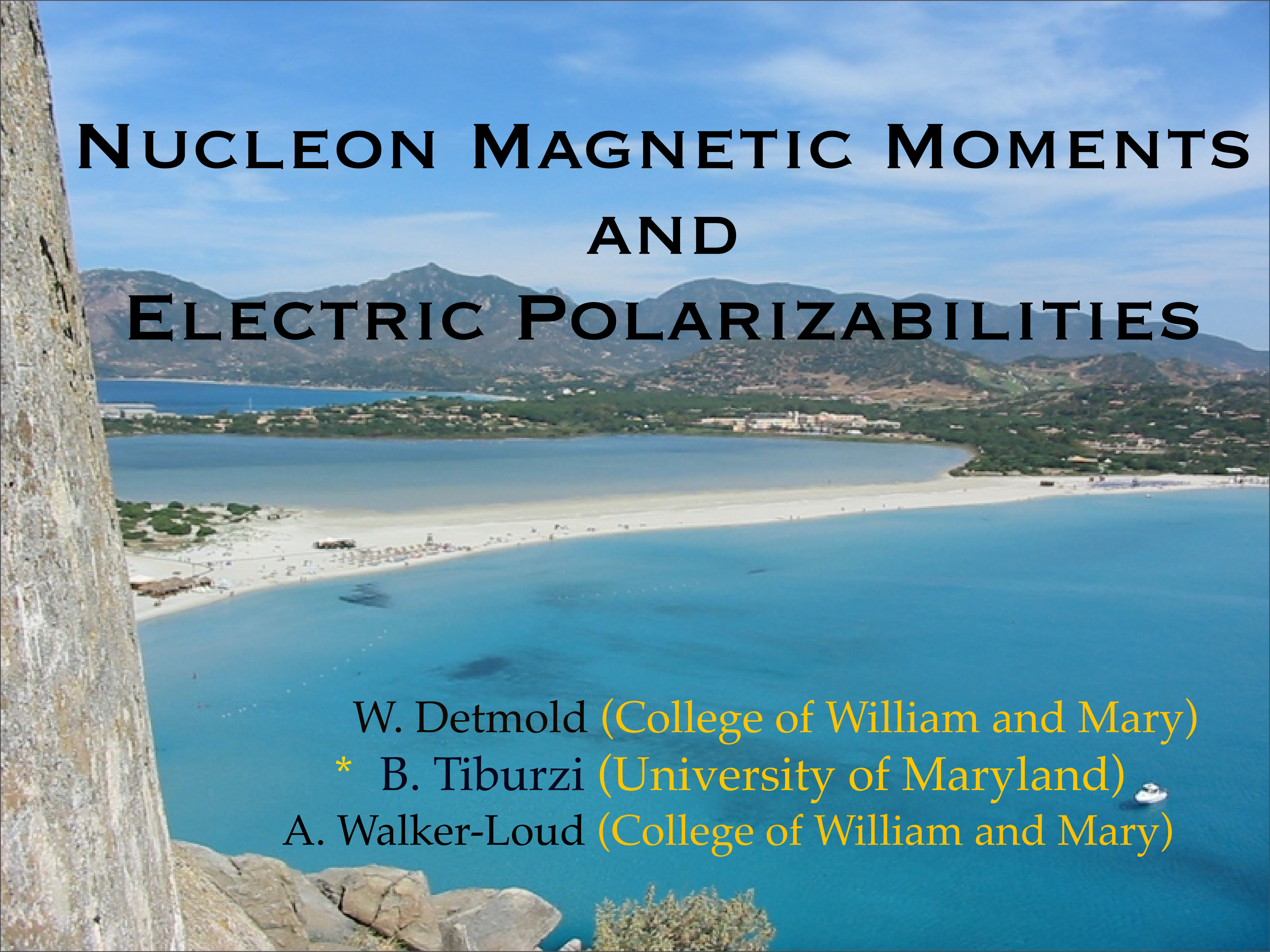
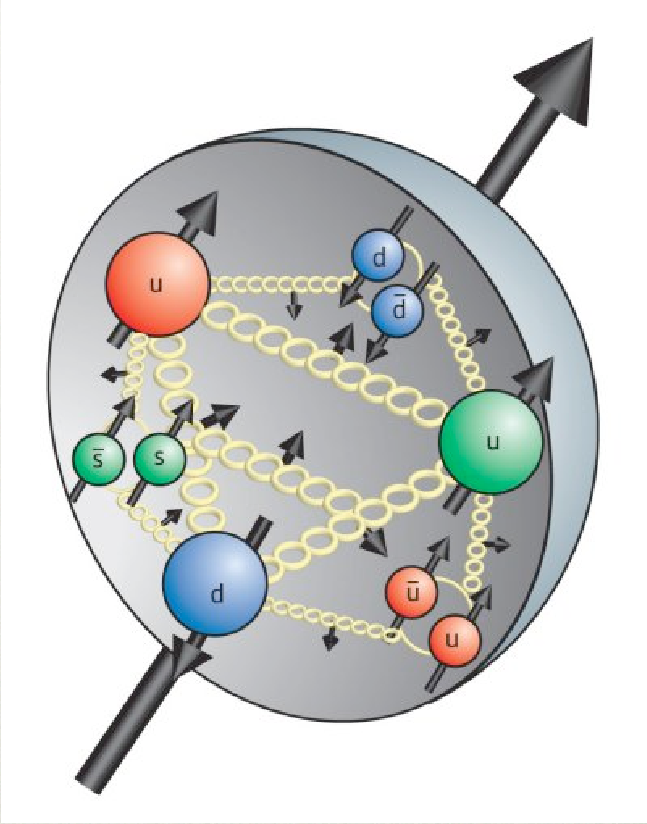


Nucleon Magnetic Moments and Electric Polarizabilities

W. Detmold (College of William and Mary)
* B. Tiburzi (University of Maryland)
A. Walker-Loud (College of William and Mary)

Overview

- Nucleon Physics:
  Spinor particles
  Electric fields
- Lattice Results:
  Electric polarizabilities
  Magnetic moments
  Neutron & Proton



W. Detmold, B. Tiburzi, A. Walker-Loud, Phys.Rev.D79:094505,2009
Phys.Rev.D81:054502,2010

Nucleon
in
Electric Fields

Neutron in Electric Fields

Interactions restricted by symmetries: discrete, gauge, Lorentz, ...

$$\overline{N} F_{\mu\nu} F^{\mu\nu} N$$

$$\partial^{\mu} \overline{N} F_{\{\mu\rho} F^{\rho}{}_{\nu\}} \partial^{\nu} N$$

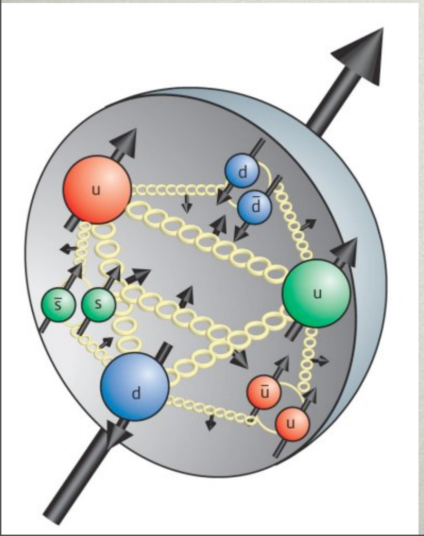
$$H_N = -\frac{1}{2} \alpha_E \vec{E}^{\,2}$$

Non-minimal couplings

$$\overline{N} \sigma_{\mu\nu} F^{\mu\nu} N$$

Fundamental Properties

$$H_N = -\mu \, \vec{E} \cdot \vec{K}$$



Electric Dipole Moment

$$\langle N(\vec{v}) | \, \vec{d} \, | N(\vec{v}) \rangle = -\alpha_E \vec{E} + \mu \, \langle \vec{\sigma} \rangle \times \vec{v}$$

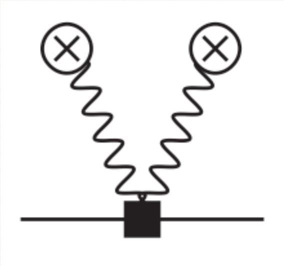
Induced

Motional

$$\vec{B} = \vec{v} \times \vec{E}$$

Neutron Energy Shift

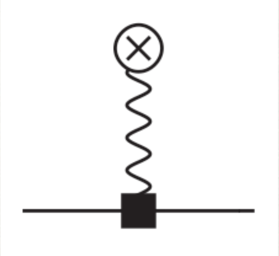
Electric Polarizability:



$$\Delta E = -\frac{1}{2}\alpha_E \vec{E}^{\,2}$$

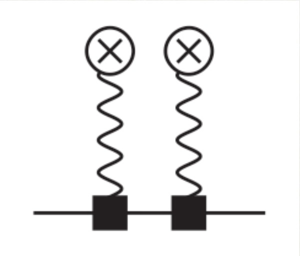
Motional EDM:

$$\vec{\mu}\cdot(\vec{v}\times\vec{E}\,)$$



$$\Delta E\big(\vec{v}=0\big) = 0$$

$$\vec{\mu}\cdot(\vec{v}\times\vec{E}\,)\frac{1}{0-\frac{1}{2}M\vec{v}^{\,2}}\vec{\mu}\cdot(\vec{v}\times\vec{E}\,)$$



$$\Delta E = \frac{\mu^2}{2M}\vec{E}^{\,2}$$

Unpolarized Correlators: Need Born subtraction

Energy shift to second order:

$$\Delta E = -\frac{1}{2}\Big(\alpha_E - \frac{\mu^2}{M}\Big)\vec{E}^{\,2}$$

A Solution: Boost Projection

Magnetic Field	$\sigma_{\mu\nu}F^{\mu\nu} = \vec{S}\cdot\vec{B}$	Electric Field	$\sigma_{\mu\nu}F^{\mu\nu} = \vec{K}\cdot\vec{E}$
Spin Projection	$\Sigma_\pm = \frac{1}{2}(1 \pm S_3)$	Boost Projection	$\mathcal{P}_\pm = \frac{1}{2}(1 \pm K_3)$

Simultaneous measurements $\texttt{Tr}\left[\mathcal{P}_\pm G(t)\right] = Z\big(M \pm \mu E\big)e^{-itE_{\text{eff}}}$

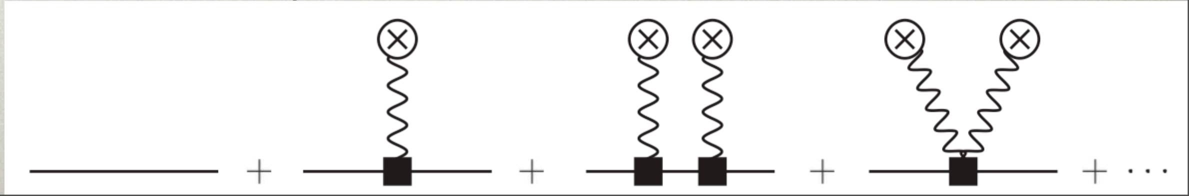
Separate Magnetic Moment from Electric Polarizability

$$E_{\text{eff}} = M - \frac{1}{2}\left(\alpha_E - \frac{\mu^2}{M}\right)E^2$$

Proton Born terms: anomalous magnetic moment and charge

Non-Born: electric polarizability

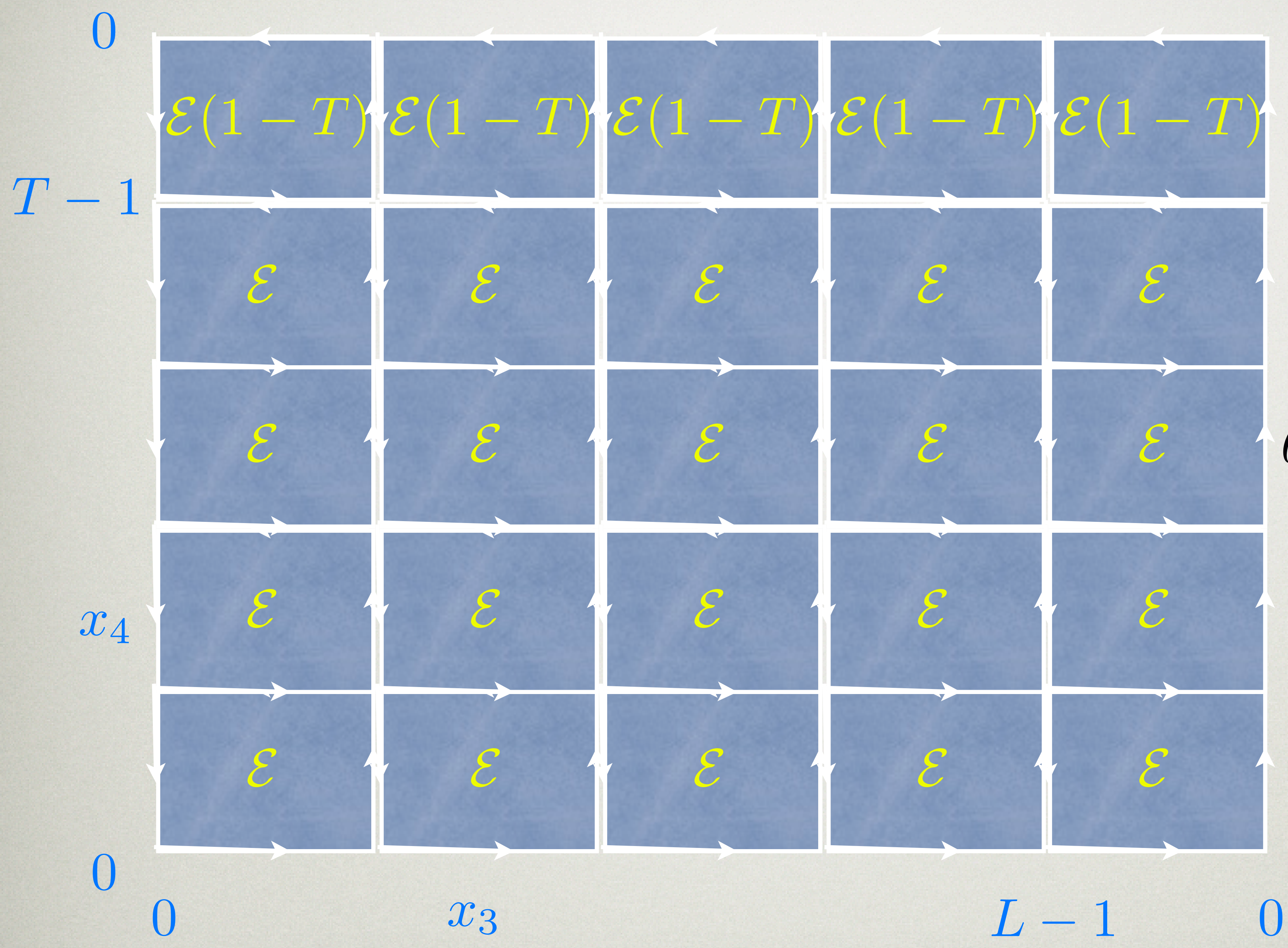
$\texttt{Tr}\left[\mathcal{P}_\pm G_Q(t)\right]$



Lattice QCD
in
Electric Fields

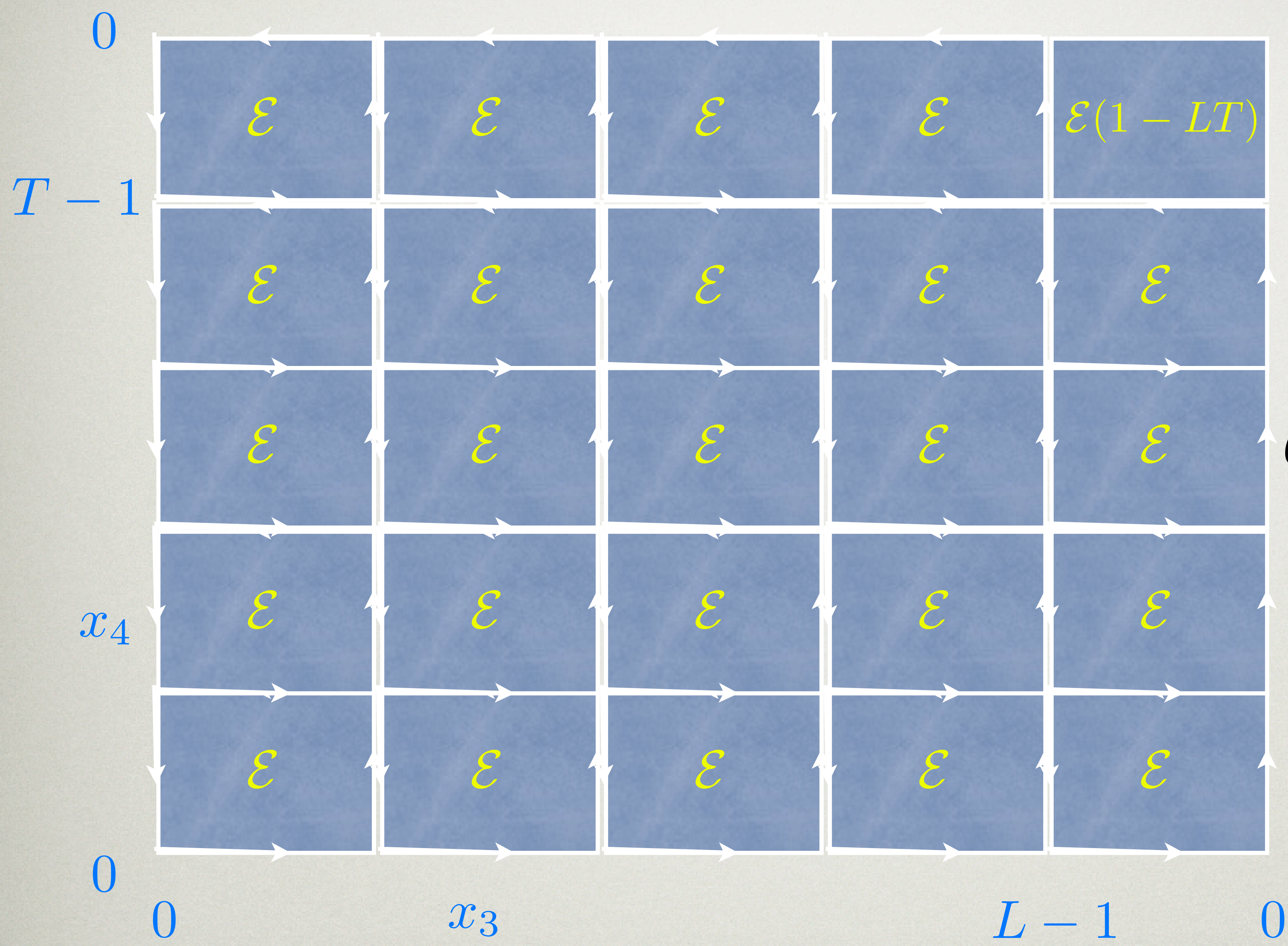
ELECTRIC FIELD ON A EUCLIDEAN LATTICE

$U^{\mathrm{cl}}_\mu(x) = \exp(-iq\mathcal{E}x_4\delta_{\mu 3})$



$\theta_{34}(x) = e^{iqF_{34}}$

Electric Field on a Euclidean Lattice $U_\mu^{\rm cl}(x) = \exp(-iq\mathcal{E}x_4\delta_{\mu 3})$



$\theta_{34}(x) = e^{iqF_{34}}$

$U_\mu^\perp(x) = \exp(iq\mathcal{E}\,T\,x_3\,\delta_{\mu 4}\,\delta_{x_4\,T-1})$

$$q\,\mathcal{E} = \frac{2\pi n}{TL}$$

Simulation Details

Anisotropic Clover Lattices (courtesy of Hadron Spectrum Collaboration)

N_s	N_t	$a_t m_l$	$a_t m_s$	m_π	m_K
20	128	−0.0840	−0.0743	390 MeV	546 MeV

	$n = 0$	$n = \pm 1$	$n = \pm 2$	$n = \pm 3$	$n = \pm 4$
$\lvert e a_t a_s \mathcal{E} \rvert$	0.00000	0.00736	0.01472	0.02209	0.02945
$N_{\rm src} \times N_{\rm cfg}$	20×200	20×200	10×200	10×200	10×200

$78,000$ inversions $\xi_{\rm aniso} = 3.5$ temporally shorter lattices

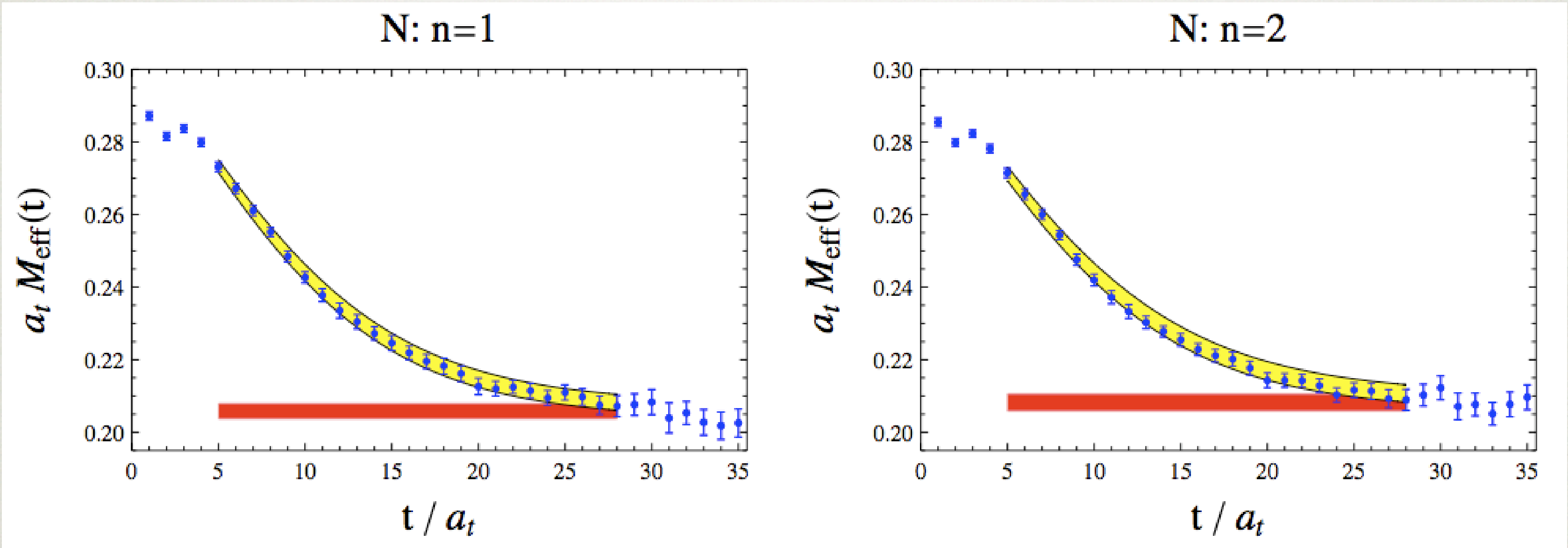
Electric field post multiplied: proof of principle, isovector differences

I: Unpolarized Neutron

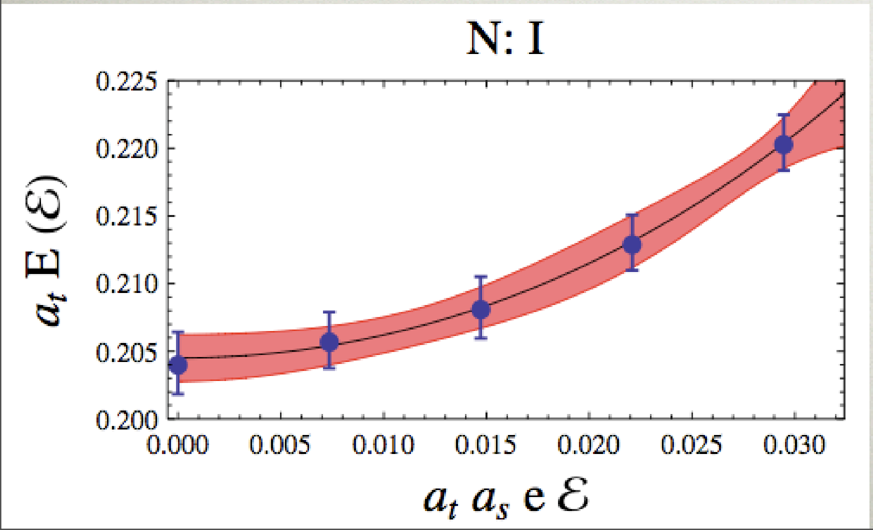
II: Boost Projected Neutron

III: Boost Projected Proton

I: Unpolarized Neutron



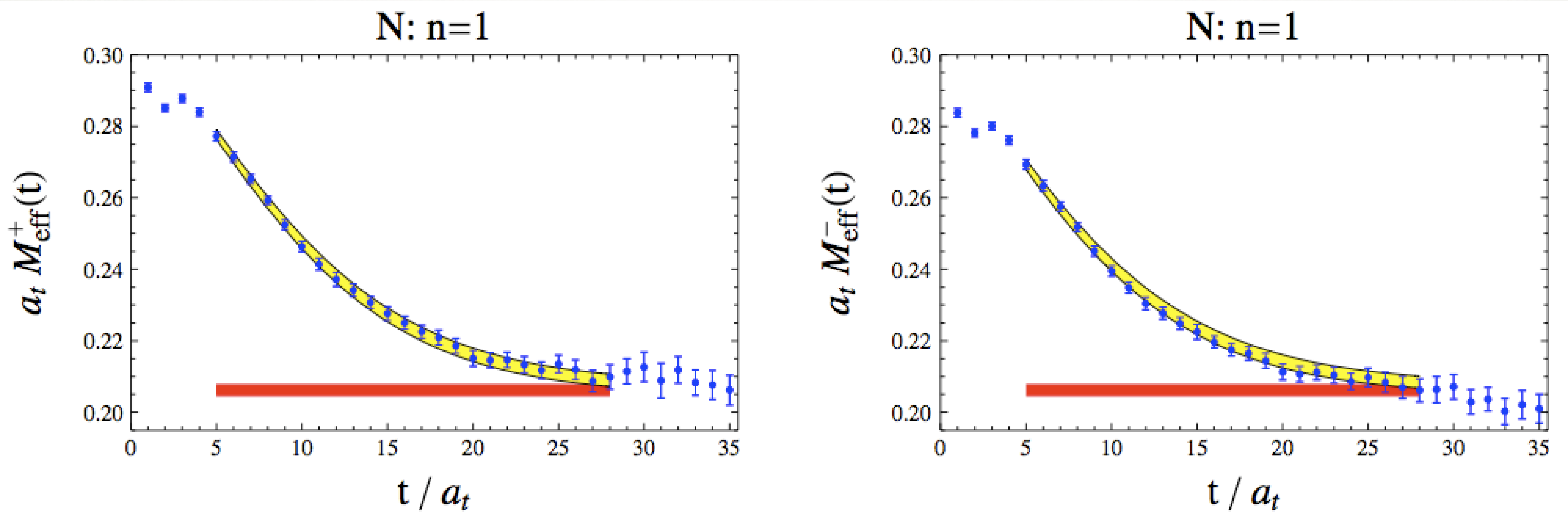
Two-state fits remove excited state contamination



$$E_{\rm eff} = M + \mathcal{A}_E \mathcal{E}^2 + \ldots$$

$$\begin{aligned} \mathcal{A}_E &= \alpha_E - \frac{\mu^2}{M} \\ &= 1.3(9)(1)(1)10^{-4}\,\texttt{fm}^3 \end{aligned}$$

II: Boost Projected Neutron



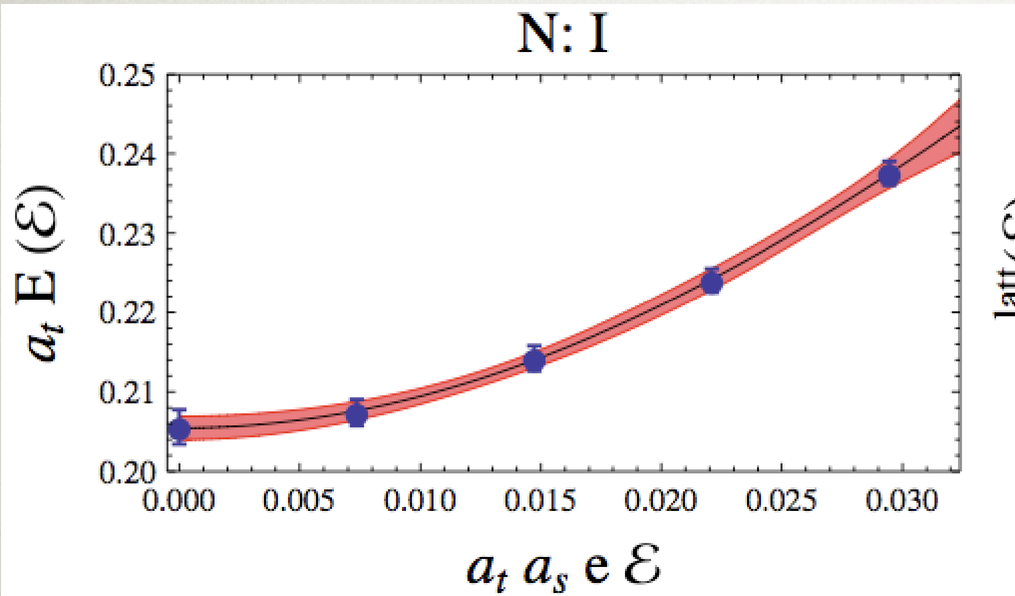
Two, simultaneous, two-state fits: amplitude & exponential

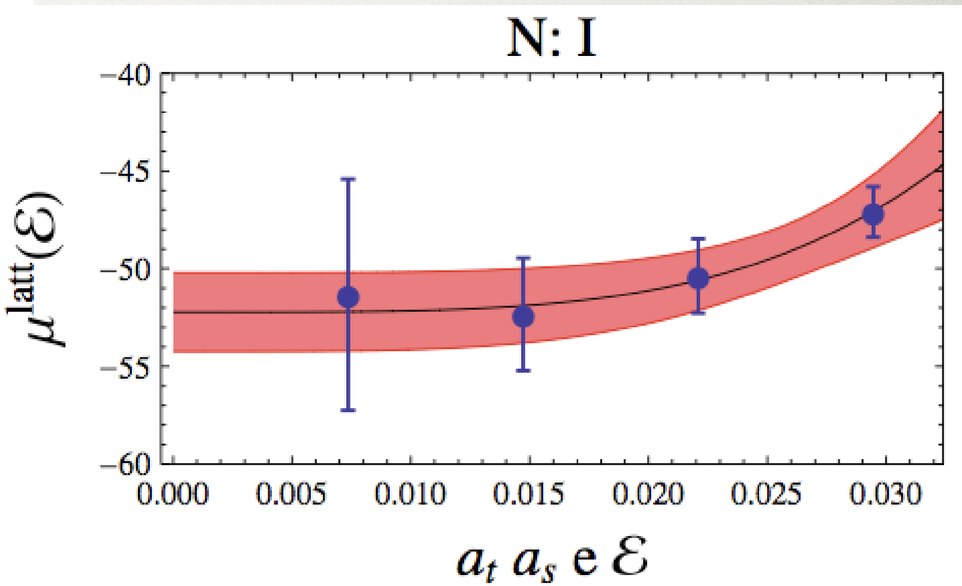
$$Z_{\pm} = Z\,(1 \pm \mu\,\mathcal{E})$$

$$E_{\rm eff} = M + \mathcal{A}_E \mathcal{E}^2 + \ldots$$

$$\mathcal{A}_E = \alpha_E - \frac{\mu^2}{M}$$

II: Boost Projected Neutron





Two, simultaneous, two-state fits: amplitude & exponential

$$Z_\pm = Z\,(1 \pm \mu\,\mathcal{E}) \qquad E_{\rm eff} = M + \mathcal{A}_E \mathcal{E}^2 + \ldots$$

$$\mathcal{A}_E = \alpha_E - \frac{\mu^2}{M}$$

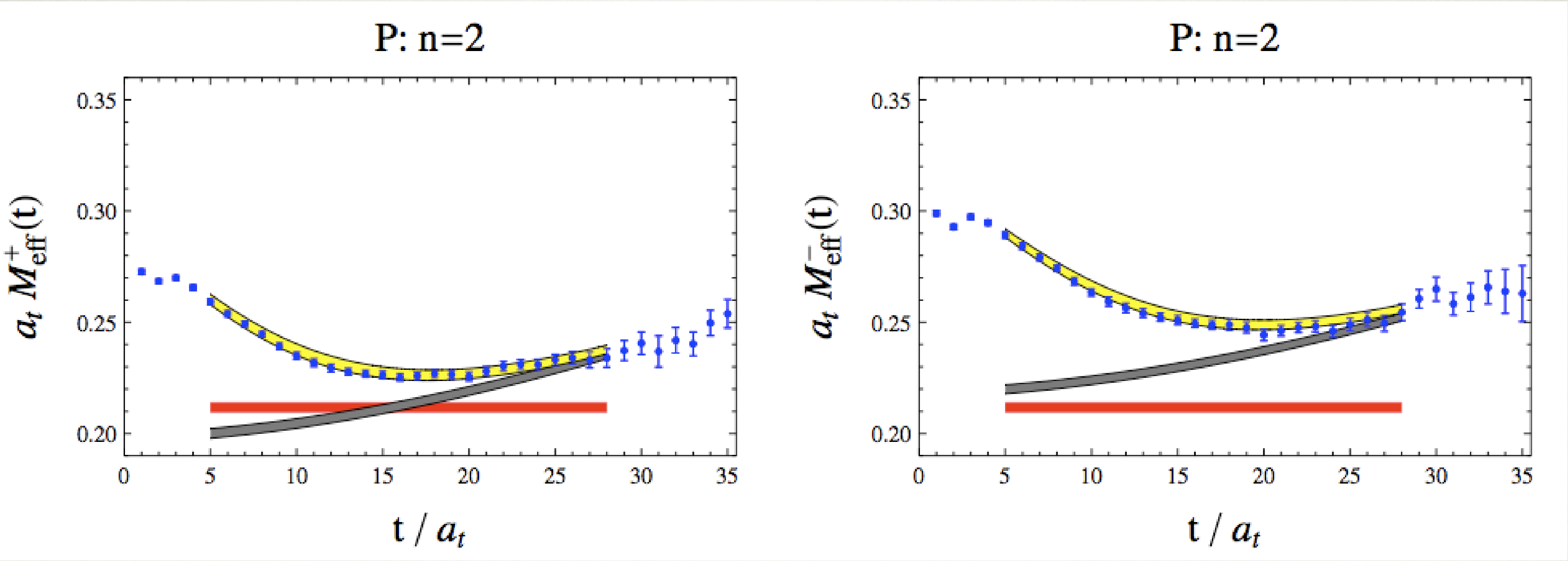
$$= 1.3(7)(2)(1)10^{-4}\,\texttt{fm}^3$$

$$\mu = -1.63(10)(4)(5)[\mu_N]$$

$$\alpha_E = 3.3(1.5)(2)(3)10^{-4}\,\texttt{fm}^3$$

consistent with unpolarized

III: Boost Projected Proton



Two, simultaneous, two-state fits: complicated propagator function

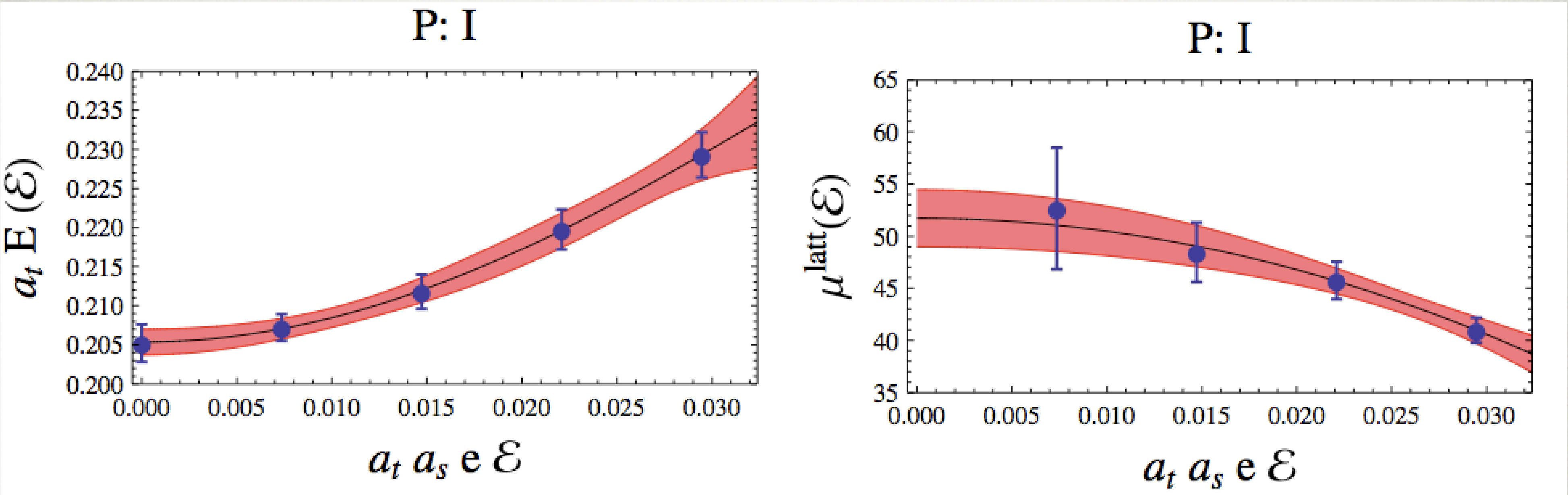
$$Z_\pm = Z\,(1 \pm \tilde{\mu}\,\mathcal{E})$$

$$E_{\rm eff} = M + \mathcal{A}_E \mathcal{E}^2 + \ldots$$

$$\mathcal{A}_E = \alpha_E - \frac{\tilde{\mu}^2}{M}$$

Anomalous moment enters

III: Boost Projected Proton



Two, simultaneous, two-state fits: complicated propagator function

$$Z_\pm = Z\,(1 \pm \tilde{\mu}\,\mathcal{E}) \qquad E_{\rm eff} = M + \mathcal{A}_E \mathcal{E}^2 + \ldots$$

$$\mathcal{A}_E = \alpha_E - \frac{\tilde{\mu}^2}{M}$$

$$\mu = +2.63(13)(1)(4)[\mu_N]$$

$$\alpha_E \quad = \quad 2.4(1.9)(3)(2)10^{-4}\,\texttt{fm}^3$$

Anomalous moment enters

Outlook

Outlook

Shown how to extract nucleon Magnetic Moments & Electric Polarizabilities

Periodicity eliminates electric field Gradient & Eliminates possibility of boundary critical Phenomena

Exactly as many refinements possible as flavors of Gelato

Usual flavors:

pion mass, sea quark charges, . . .

Exotic Flavors:

Charged particles suffer gauge defect @ boundary. . .
--- But calculable
Exp. Volume Corrections from external field holonomy. . .
--- But calculable in EFT framework