New $B \to D^* l \nu$ at Zero Recoil

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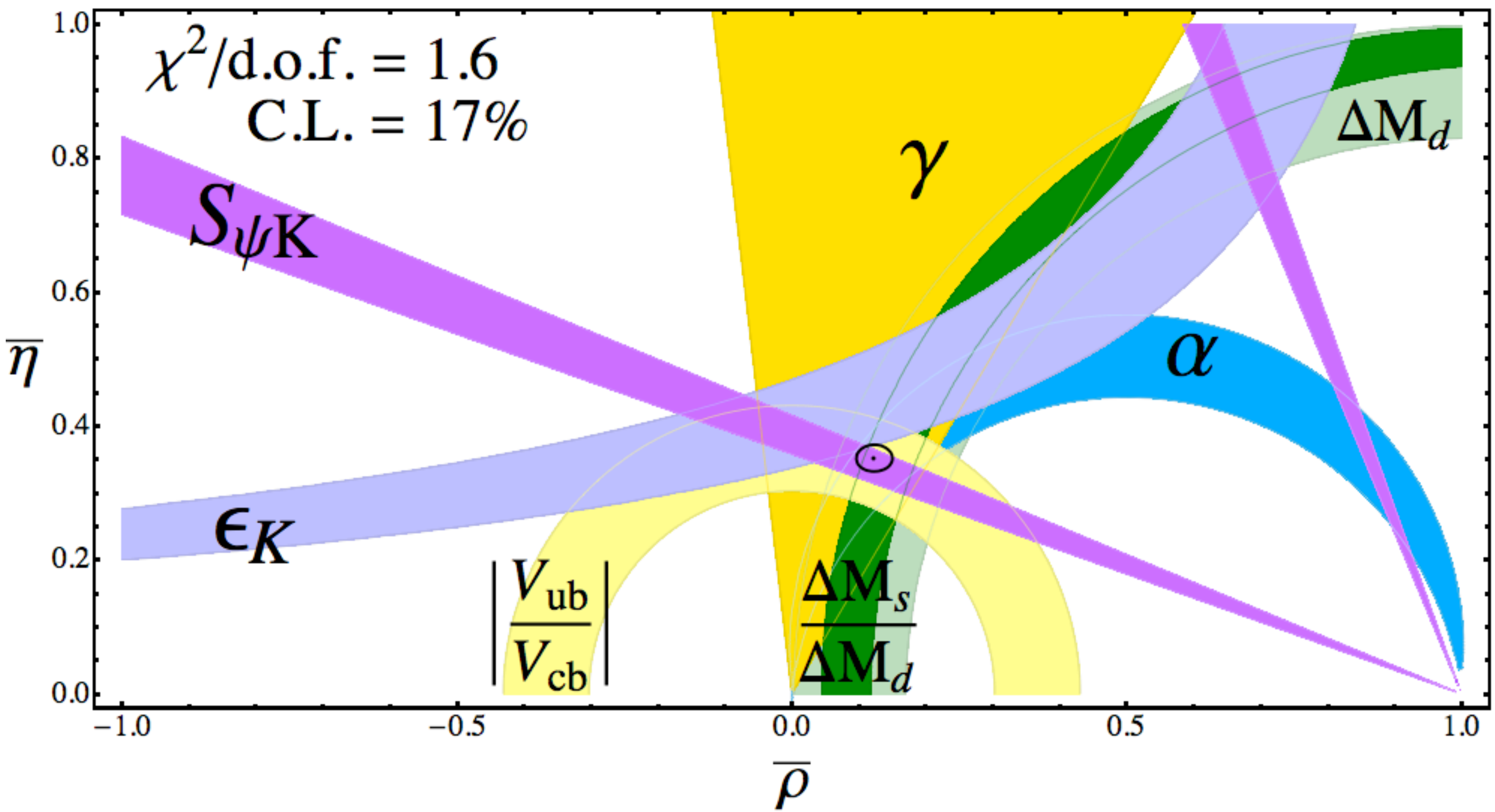
Motivation: Detemine $|V_{cb}|$ of the CKM Matrix

Cabibbo, *PRL* **10** (1963) 531;
Kobayashi, Maskawa, *Prog. Theor. Phys.* **49** (1973) 652

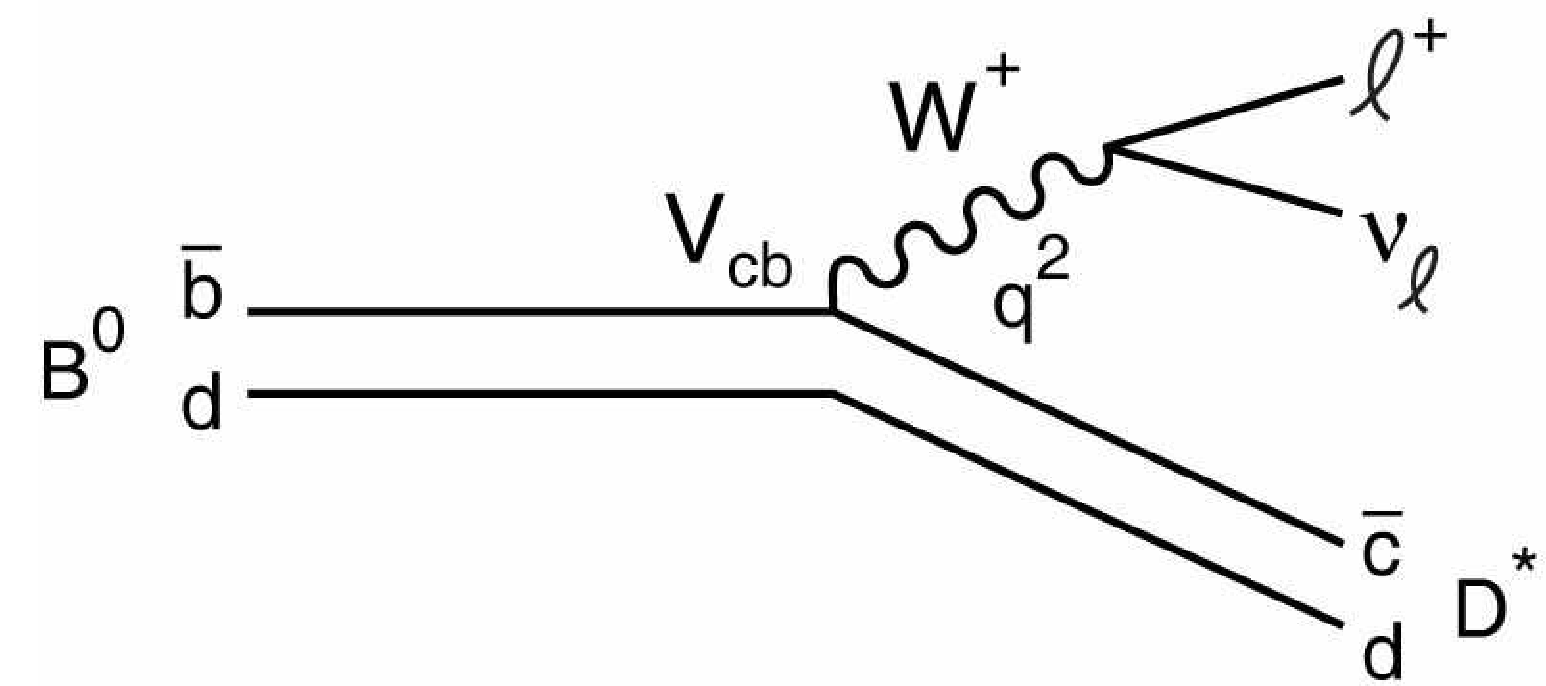
$|V_{cb}|$ Normalizes the whole Unitarity Triangle

c.f., Laiho, Lunghi, Van de Water, arXiv:0910.2928



Semileptonic Form Factors

- Kinematics: $q^2 = M_B^2 + M_{D^*}^2 - 2wM_BM_{D^*}$, $w = v_B \cdot v_{D^*}$:



$$\frac{\langle D|\mathcal{V}^\mu|B\rangle}{\sqrt{m_B m_D}} = (v_B + v_D)^\mu h_+(w) + (v_B - v_D)^\mu h_-(w),$$

$$\frac{\langle D^*_\alpha|\mathcal{V}^\mu|B\rangle}{\sqrt{m_B m_{D^*}}} = \varepsilon^{\mu\nu\rho\sigma} v_B^\nu v_{D^*}^\rho \varepsilon_\alpha^{*\sigma} h_V(w),$$

$$\frac{\langle D^*_\alpha|\mathcal{A}^\mu|B\rangle}{\sqrt{m_B m_{D^*}}} = i\varepsilon_\alpha^{*\nu}\{(1+w)g^{\nu\mu}h_{A_1}(w) - v_B^\nu[v_B^\mu h_{A_2}(w) + v_{D^*}^\mu h_{A_3}(w)]\},$$

$$\frac{d\Gamma(B \to D\ell\nu)}{dw} = \frac{G_F^2}{48\pi^3} m_D^3 (m_B + m_D)^2 (w^2-1)^{3/2} |V_{cb}|^2 |\mathcal{G}(w)|^2$$

$$\mathcal{G}(w) = h_+(w) - \frac{m_B - m_D}{m_B + m_D} h_-(w) \propto f_+(q^2)$$

$B \to D^* l\nu$ at Zero Recoil, $w \to 1$:

$$\frac{1}{\sqrt{w^2-1}}\frac{d\Gamma}{dw} = \frac{G_F^2}{4\pi^3} m_{D^*}^3 (m_B - m_{D^*})^2 |V_{cb}|^2 \chi(w) |\mathcal{F}(w)|^2$$

$$\chi(w) = \frac{w+1}{12}\left(5w+1-\frac{8w(w-1)m_B m_{D^*}}{(m_B-m_{D^*})^2}\right) \to 1$$

$$\mathcal{F}(w) = h_{A_1}(w)\frac{1+w}{2}\sqrt{\frac{H_0^2(w)+H_+^2(w)+H_-^2(w)}{3\chi(w)}} \to h_{A_1}(1)$$

$$H_0(w) = \frac{m_B w - m_{D^*} - m_B(w-1)R_2(w)}{m_B - m_{D^*}} \to 1$$

$$H_\pm(w) = t(w)\left[1 \mp \sqrt{(w-1)/(w+1)}R_1(w)\right] \to 1$$

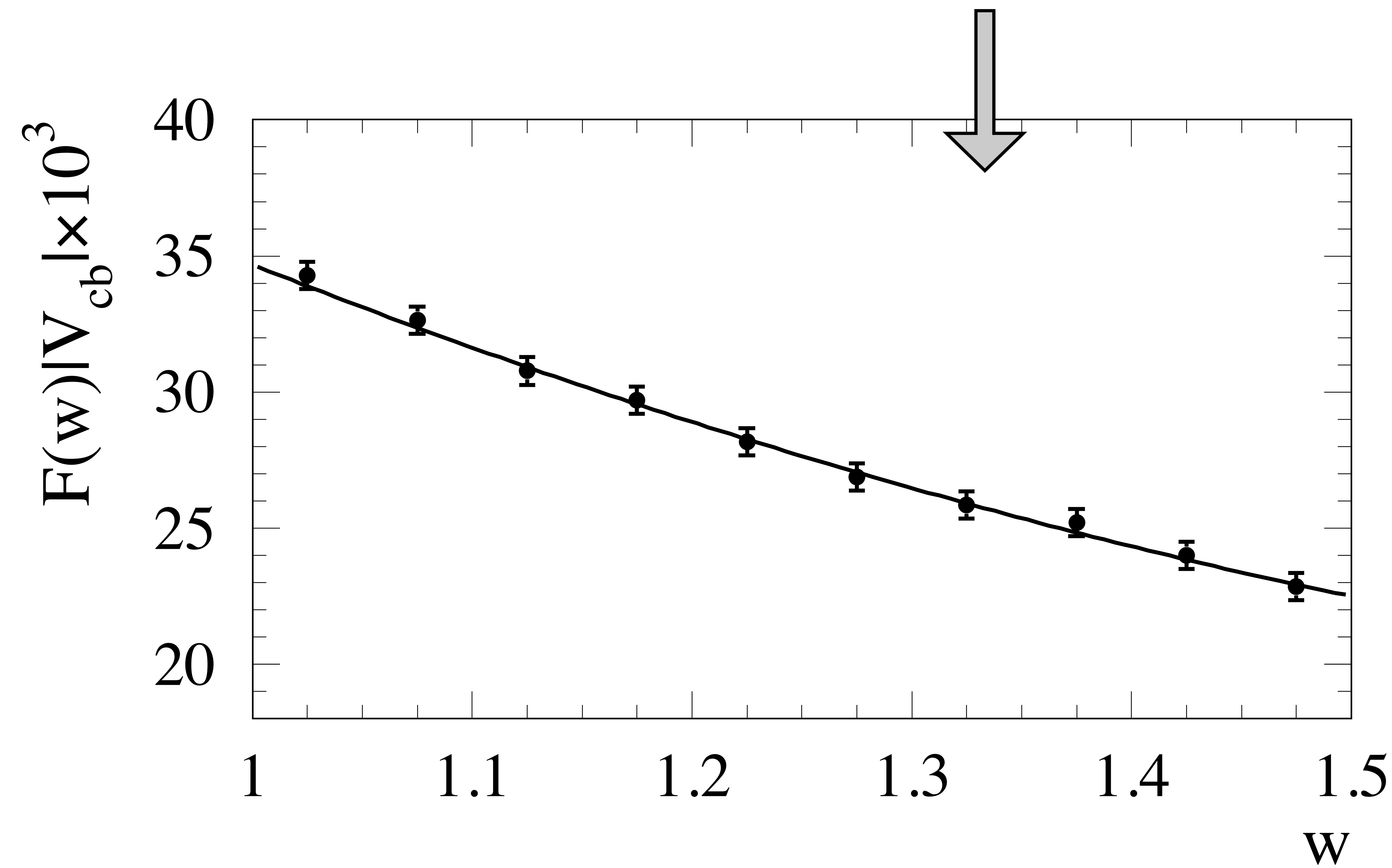
$$t^2(w) = [m_B^2 + m_{D^*}^2 - 2w m_B m_{D^*}]/(m_B - m_{D^*})^2 \to 1$$

$$R_1(w) = h_V(w)/h_{A_1}(w)$$

$$R_2(w) = [m_B h_{A_3}(w) + m_{D^*} h_{A_2}(w)]/m_B h_{A_1}(w)$$

Advantages of Zero Recoil

- Simpler: one number to compute, not four functions: take shape from experiment.
- More powerful HQS:
    - Luke's theorem, $1/m_Q^2$;
    - control errors.
- Nonzero recoil has $1/m_Q$:
    - *e.g.*, larger discretization errors.
- In the end, of course, adopt strategy that minimizes error in $|V_{cb}|$.



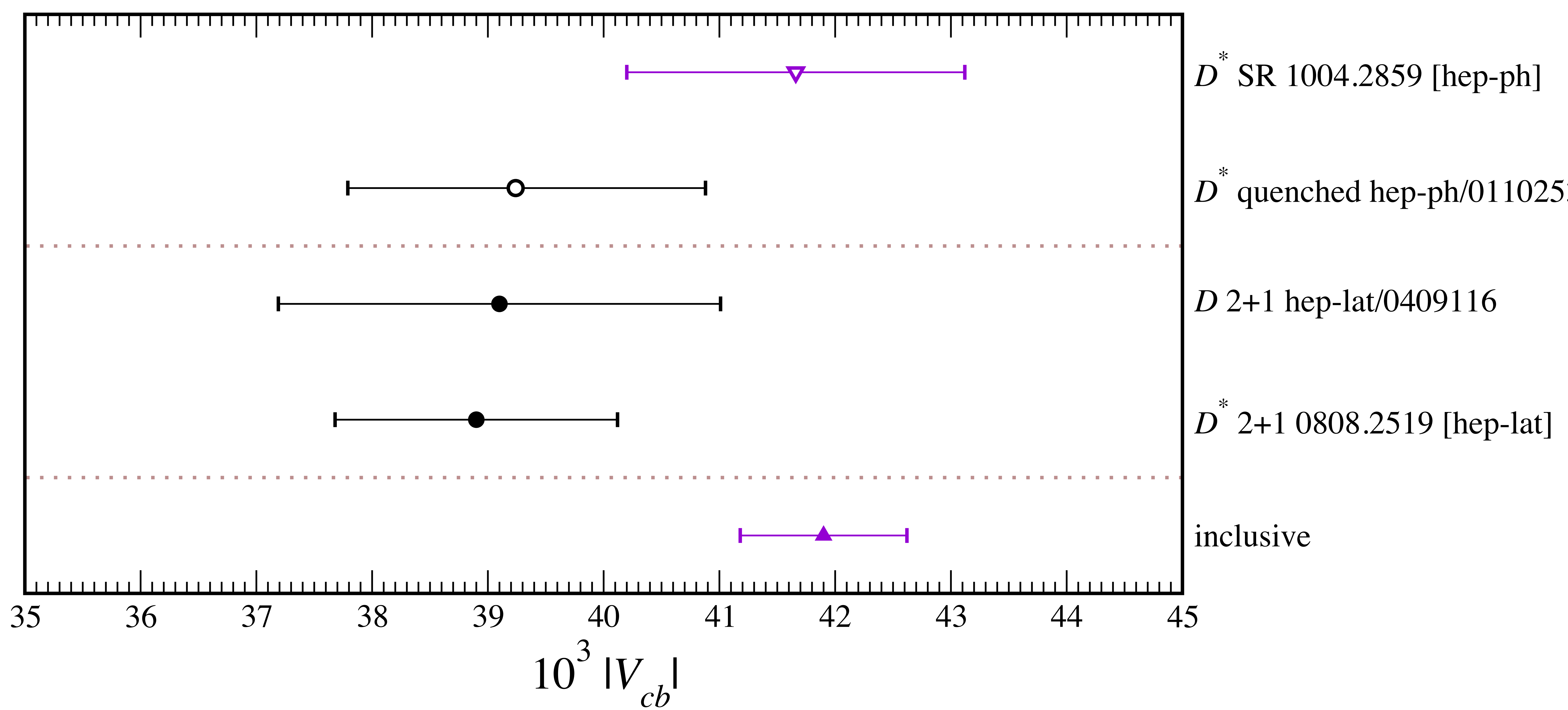
BaBar

History

- Quenched calculation: [Hashimoto *et al.*, hep-ph/0110253]
  - $\mathcal{F}(1) = 0.913^{+0.024}_{-0.017} \pm 0.016^{+0.003}_{-0.014}{}^{+0.000}_{-0.016}{}^{+0.006}_{-0.014}$ stats, match, a, χPT, quenching
  - became default value used by experiments to determine $|V_{cb}|$, until
- Unquenched (2+1 staggered sea quarks): [*alia et* Laiho *et al.*, arXiv:0808.2519]
  - $\mathcal{F}(1) = 0.921 \pm 0.013 \pm 0.008 \pm 0.008 \pm 0.014 \pm 0.006 \pm 0.003 \pm 0.004$
    stats $g_{DD^*\pi}$ χPT disc. $\varkappa_{b,c}$ match u_0
  - leading to (latest HFAG) $|V_{cb}| = (38.9 \pm 0.7_{\text{expt}} \pm 1.0_{\text{LQCD}}) \times 10^{-3}$ [Kowalewski, FPCP]
- Methods for nonzero recoil, with different matching/discretization strategies [Tantalo *et al.*].

Tension

- Determinations of $|V_{cb}|$ via *exclusive* (+ LQCD) and *inclusive* (+ OPE & pQCD) decays agree poorly (~2). Using HGAF update for FPCP (2010.05.26) [Kowalewski]:



Tension Tamers

- Exclusive:
  - firm up existing lattice-QCD calculations (this talk); cross-check from other groups;
  - re-examine extrapolation $w \to 1$;
  - determine $|V_{\mathrm{cb}}|$ at $w \neq 1$.
- Inclusive: higher-order corrections being computed.

Ingredients

- Gluon fields from MILC ensembles:
  - Lüscher-Weisz improved action with g^2N_c corrections but not g^2n_f—$\mathrm{O}(\alpha_\mathrm{s}^{\text{“2”}}a^2)$, $\mathrm{O}(a^4)$;
  - 2+1 flavors of sea quarks: rooted asqtad determinant—$\mathrm{O}(\alpha_\mathrm{s}a^2)$, $\mathrm{O}(a^4)$ “small” ⇐ Fat7.
- Light spectator quark: asqtad action—$\mathrm{O}(\alpha_\mathrm{s}a^2)$, $\mathrm{O}(a^4)$ “small” ⇐ Fat7.
- Heavy quarks: Sheikholeslami-Wohlert (aka clover) action with Fermilab interpretation:
  - discretization effects $\mathrm{O}(\alpha_\mathrm{s}a^2b_{\Sigma\cdot B}^{[1]}(ma))$, $\mathrm{O}(\alpha_\mathrm{s}a^2d_1^{[1]}(ma))$, $\mathrm{O}(a^2b_i^{[0]}(ma))$;
  - functions $b_i^{[0]}(ma)$ derived from HQET matching (see below).

MILC asqtad ensembles

a (fm)	lattice	# confs	(am_l, am_s)	am_q	κ_b	κ_c	c_{SW}
≈ 0.15	$16^3 \times 48$	596	(0.0290, 0.0484)	{0.0484, 0.0068,			
medium	$16^3 \times 48$	640	(0.0194, 0.0484)	0.0453, 0.0421,			
coarse	$16^3 \times 48$	631	(0.0097, 0.0484)	0.0290, **0.0194**,	0.0781	0.1218	1.570
	$20^3 \times 48$	603	(0.0048, 0.0484)	*0.0097*, 0.0048}			
≈ 0.12	$20^3 \times 64$	2052	(0.02, 0.05)	{0.05, 0.03,	0.0918	0.1259	1.525
coarse	$20^3 \times 64$	2259	(0.01, 0.05)	0.0415, 0.0349,	0.0901	0.1254	1.531
	$20^3 \times 64$	2110	(0.007, 0.05)	***0.02***, *0.01,*	0.0901	0.1254	1.530
	$24^3 \times 64$	2099	(0.005, 0.05)	*0.007*, *0.005*}	0.0901	0.1254	1.530
≈ 0.09	$28^3 \times 96$	1996	(0.0124, 0.031)	{0.031, 0.261,	0.0982	0.1277	1.473
fine	$28^3 \times 96$	1946	(0.0062, 0.031)	0.0093,	0.0979	0.1276	1.476
	$32^3 \times 96$	983	(0.00465, 0.031)	***0.0124***, *0.0062*	0.0977	0.1275	1.476
	$40^3 \times 96$	1015	(0.0031, 0.031)	0.0047, *0.0031*}	0.0976	0.1275	1.478
≈ 0.06	$48^3 \times 144$	668	(0.0072, 0.018)	{0.0188, 0.0160,	0.1052	0.1296	1.4287
superfine	$48^3 \times 144$	668	(0.0036, 0.018)	0.0054,	0.1052	0.1296	1.4287
	$56^3 \times 144$	800	(0.0025, 0.018)	***0.0072***, *0.0036*			
	$64^3 \times 144$	826	(0.0018, 0.018)	0.0025, 0.0018}			
≈ 0.045 ultrafine	$64^3 \times 192$	860	(0.0028, 0.014)	0.014, 0.0028			

Scope of analysis

- This update encompasses the ensembles highlighted in red:
  - mass and decay correlators for all $am_q = am_l$ aka “full QCD” or “unitary” (in *italics*);
  - also for all $am_q = 0.4am_s$ (in **bold**).;
  - hence 2 + 7 + 5 + 3 (partially-quenched) correlators at $am_q = 0.15, 0.12, 0.09, 0.06$ fm.
- Bare quark mass (aka $\varkappa$) determined from spin-averaged kinetic meson mass:
  - improving strategies with twisted b.c. and, eventually, better sources.
- Tree-level tadpole improved $c_{\rm SW} = 1/u_0^3$, where $u_0^4 = \langle \text{plaquette} \rangle$.

Correlators and Ratios of Correlators

- Our objective is

$$\mathcal{R}_{A_1} = \frac{\langle D^* | \bar{c}\gamma_j\gamma_5 b | \overline{B}\rangle \langle \overline{B} | \bar{b}\gamma_j\gamma_5 c | D^*\rangle}{\langle D^* | \bar{c}\gamma_4 c | D^*\rangle \langle \overline{B} | \bar{b}\gamma_4 b | \overline{B}\rangle} = |h_{A_1}(1)|^2$$

- We define 3-point correlations functions:

$$\begin{aligned}
C^{B\to D^*}(t_i,t_s,t_f) &= \sum_{\mathbf{x},\mathbf{y}} \langle 0| O_{D^*}(\mathbf{x},t_f)\overline{\Psi}_c\gamma_j\gamma_5\Psi_b(\mathbf{y},t_s)O_B^\dagger(\mathbf{0},t_i)|0\rangle, \\
C^{B\to B}(t_i,t_s,t_f) &= \sum_{\mathbf{x},\mathbf{y}} \langle 0| O_B(\mathbf{x},t_f)\overline{\Psi}_b\gamma_4\Psi_b(\mathbf{y},t_s)O_B^\dagger(\mathbf{0},t_i)|0\rangle, \\
C^{D^*\to D^*}(t_i,t_s,t_f) &= \sum_{\mathbf{x},\mathbf{y}} \langle 0| O_{D^*}(\mathbf{x},t_f)\overline{\Psi}_c\gamma_4\Psi_c(\mathbf{y},t_s)O_{D^*}^\dagger(\mathbf{0},t_i)|0\rangle.
\end{aligned}$$

- So look for plateau in

matching ρ_A

$$R_{A_1}(t) = \frac{C^{B\to D^*}(0,t,T)C^{D^*\to B}(0,t,T)}{C^{D^*\to D^*}(0,t,T)C^{B\to B}(0,t,T)} = \rho_A^{-2}\mathcal{R}_{A_1}$$

Oscillating states:

- A staggered correlator couples to opposite-parity states with $(-1)^t$:

$$C^{X\to Y}(0,t,T) = \sum_{k=0}\sum_{\ell=0}(-1)^{kt}(-1)^{\ell(T-t)}A_{\ell k}e^{-m_X^{(k)}t}e^{-m_Y^{(\ell)}(T-t)}$$
$$= A_{00}^{X\to Y}e^{-m_X t-m_Y(T-t)} + (-1)^{T-t}A_{01}^{X\to Y}e^{-m_X t-m'_Y(T-t)}$$
$$+\ (-1)^t A_{10}^{X\to Y}e^{-m'_X t-m_Y(T-t)} + (-1)^T A_{11}^{X\to Y}e^{-m'_X t-m'_Y(T-t)} + \ldots$$

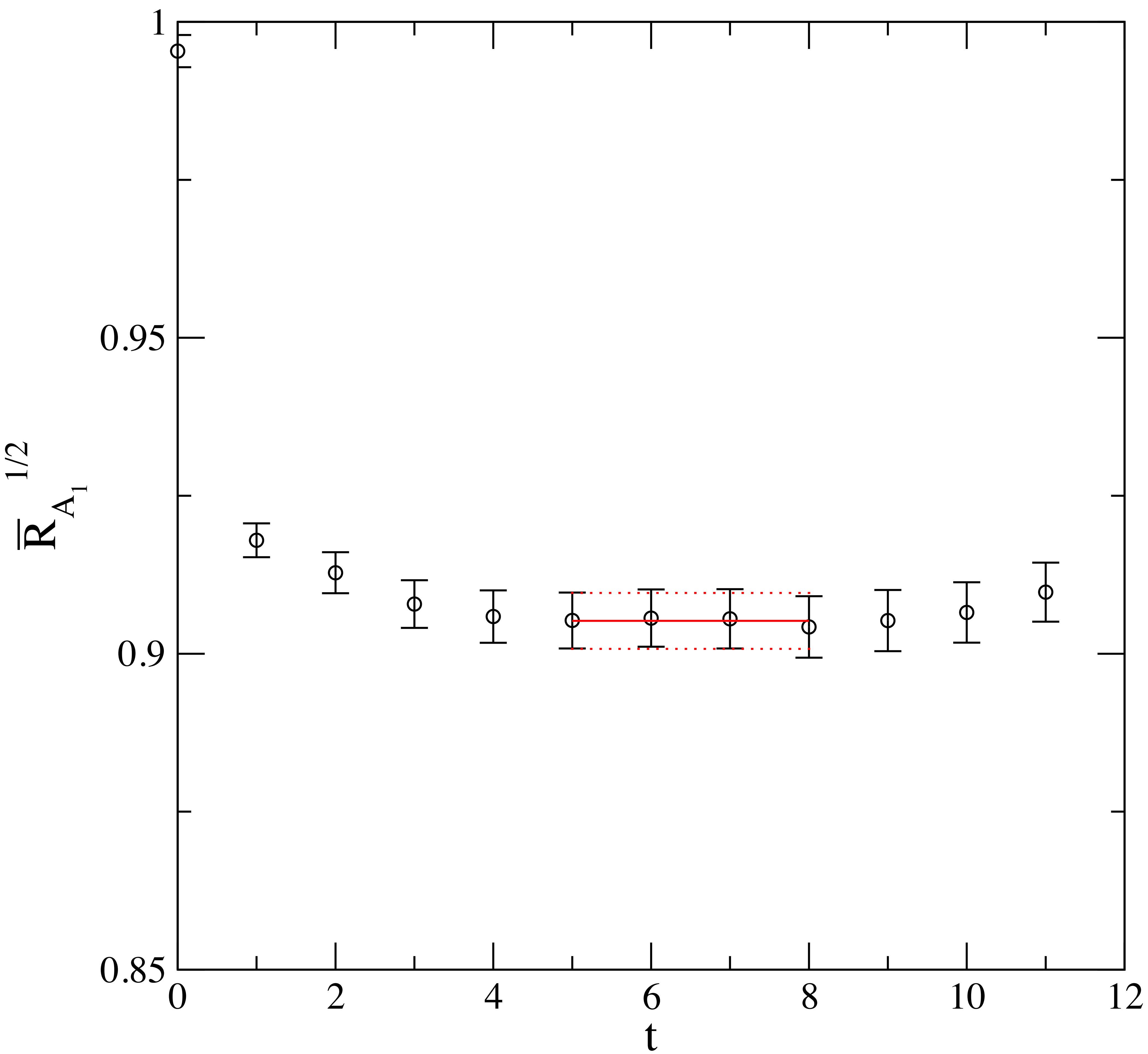
- Last term is wrong-parity–to–wrong-parity transition, and doesn't oscillate in t.

- *Does* oscillate in T, so control by computing $C^{X\to Y}(0, t, T)$ and $C^{X\to Y}(0, t, T+1)$:

$$\overline{R}_{A_1}(0,t,T) = \tfrac{1}{2}R_{A_1}(0,t,T) + \tfrac{1}{4}R_{A_1}(0,t,T+1) + \tfrac{1}{4}R_{A_1}(0,t+1,T+1)$$

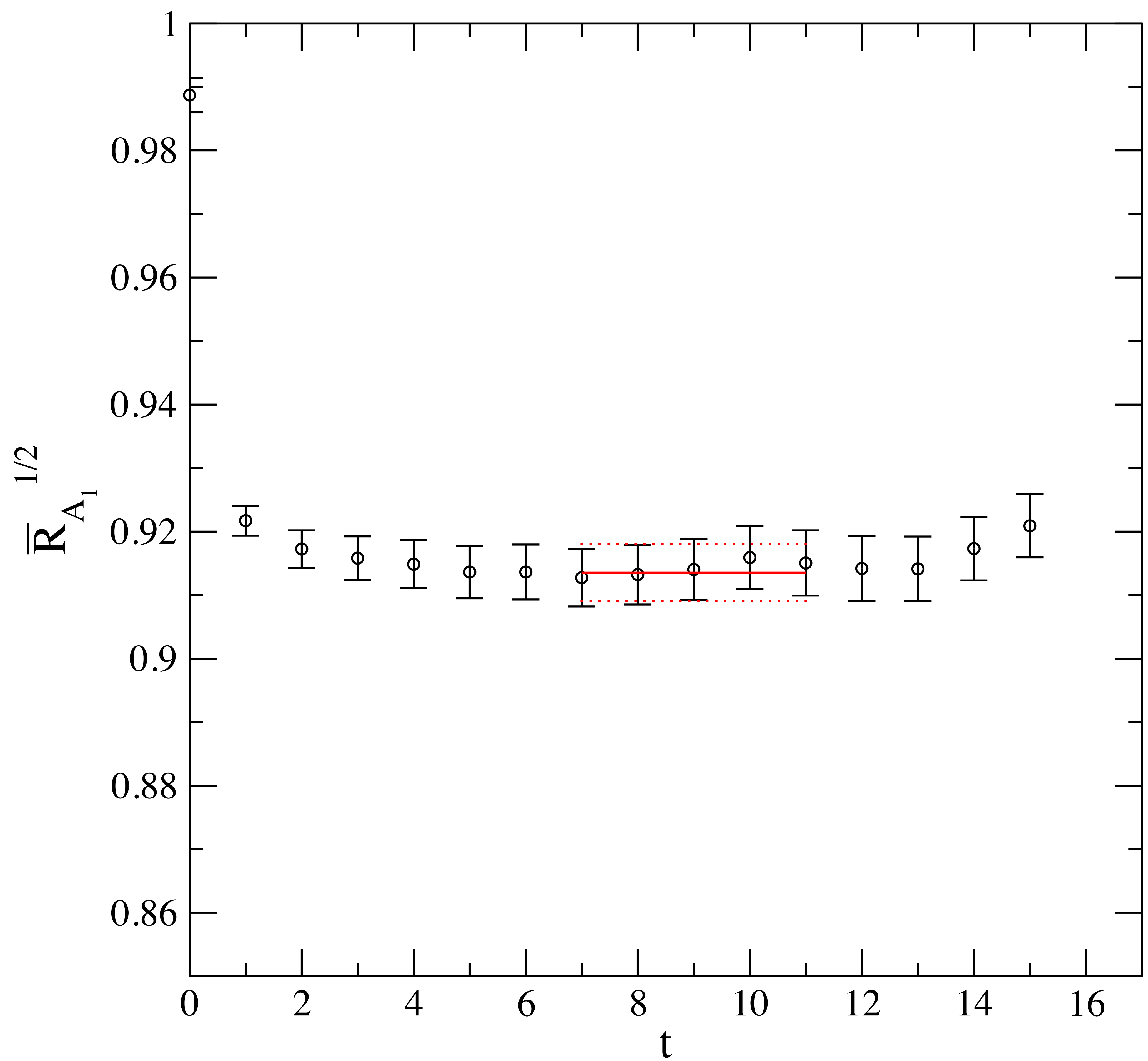
Plateau on a Coarse Ensemble

$(am_l, am_s) = (0.01, 0.05)$



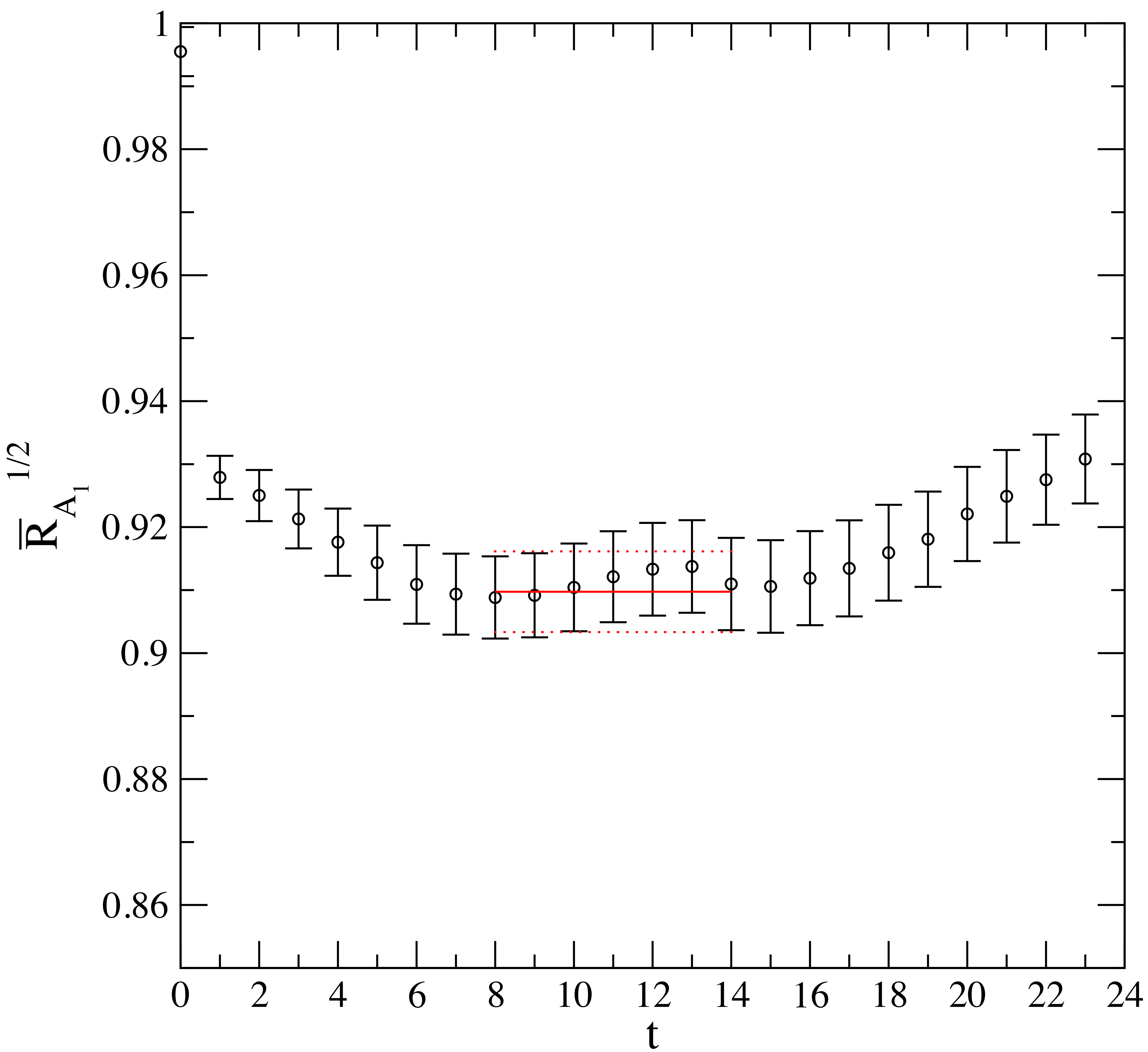
Plateau on a Fine Ensemble

$(am_l, am_s) = (0.0062, 0.031)$



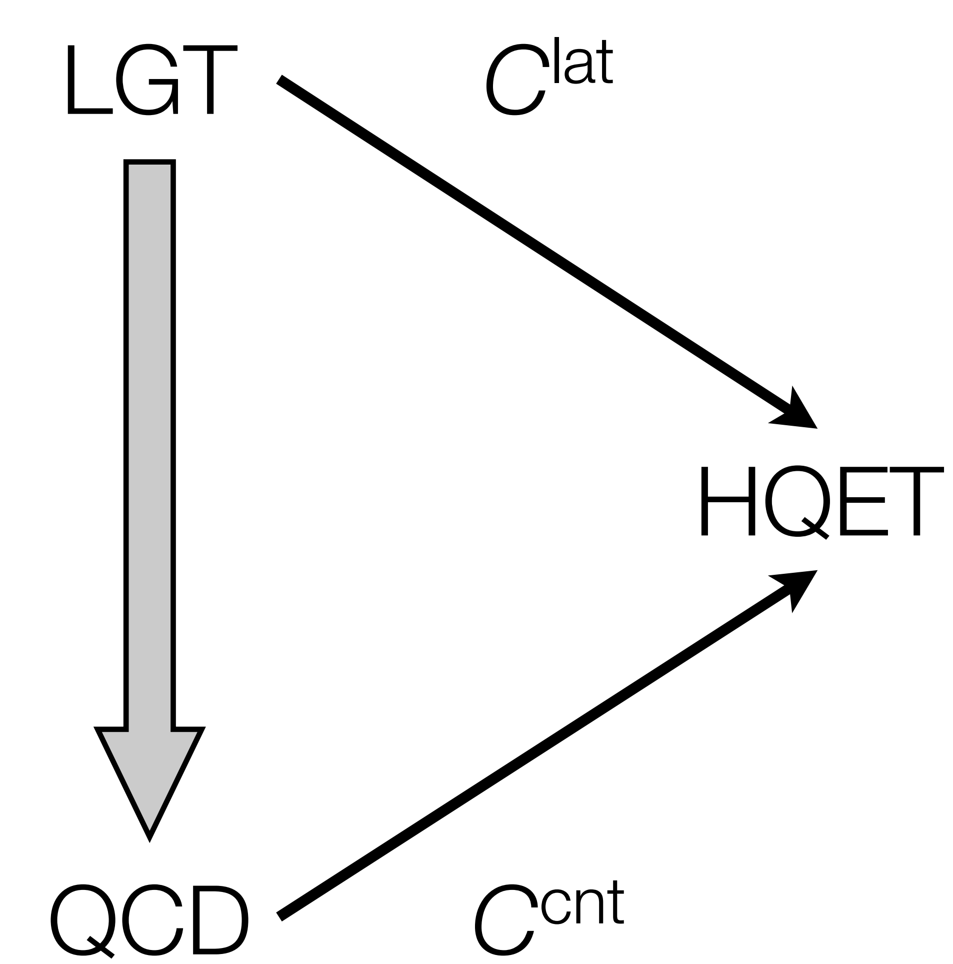
Plateau on a Superfine Ensemble

$(am_l, am_s) = (0.0036, 0.018)$



Matching and Discretization via HQET

- Heavy-light hadrons can be described by heavy-quark effective theory.
- Founded on basic dynamics and emerging symmetries.
- LGT has the same basic dynamics and symmetries, so an HQET description exists here too.
- Relating HQET for two underlying theories (LGT & QCD) yields
  - theory of cutoff effects;
  - definition of matching factors;
  - relationships between observables.



- From HQET matching, non-zero recoil [J. Harada et al., hep-lat/0112045]:

$$
\begin{aligned}
\langle D^*|\varepsilon\cdot A_{\text{lat}}|B\rangle = & \ \bar{C}^{\text{lat}}_{A_\perp}\langle D^{(0)}_{v'}|\bar{c}_{v'}\not{\varepsilon}\gamma_5 b_v|B^{(0)}_v\rangle - \sum_{i\in S_\perp}\bar{B}^{\text{lat}}_{Ai}\langle D^{(0)}_{v'}|\varepsilon\cdot\bar{Q}_{Ai}|B^{(0)}_v\rangle \\
& - C^{\text{lat}}_{2c}\bar{C}^{\text{lat}}_{A_\perp}\int d^4x\langle D^{(0)}_{v'}|T\, O_{2c}(x)\bar{c}_{v'}\not{\varepsilon}\gamma_5 b_v|B^{(0)}_v\rangle^\star \\
& - C^{\text{lat}}_{\mathcal{B}c}\bar{C}^{\text{lat}}{}_{A_\perp}\int d^4x\langle D^{(0)}_{v'}|T\, O_{\mathcal{B}c}(x)\bar{c}_{v'}\not{\varepsilon}\gamma_5 b_v|B^{(0)}_v\rangle^\star \\
& - C^{\text{lat}}_{2b}\bar{C}^{\text{lat}}_{A_\perp}\int d^4x\langle D^{(0)}_{v'}|T\, \bar{c}_{v'}\not{\varepsilon}\gamma_5 b_v O_{2b}(x)|B^{(0)}_v\rangle^\star \\
& - C^{\text{lat}}_{\mathcal{B}b}\bar{C}^{\text{lat}}_{A_\perp}\int d^4x\langle D^{(0)}_{v'}|T\, \bar{c}_{v'}\not{\varepsilon}\gamma_5 b_v O_{\mathcal{B}b}(x)|B^{(0)}_v\rangle^\star \\
& + aK_{\sigma\cdot F}C^{\text{lat}}_{A_\perp}\int d^4x\langle D^{(0)}_{v'}|T\, \bar{c}_{v'}\not{\varepsilon}\gamma_5 b_v\bar{q}i\sigma F q(x)|B^{(0)}_v\rangle^\star + \mathrm{O}(\Lambda^2 a^2 b(ma))
\end{aligned}
$$

but as $v' \to v$, all corrections vanish.

- Luke's theorem.

- From (tree-level) HQET matching, zero recoil [ASK, hep-lat/00020085]:

$$\delta h_{A_1}(1) = \left(\frac{1}{8m_{3c}^2} - \frac{1}{8m_{D_\perp^2 c}^2} + \frac{1}{8m_{3b}^2} - \frac{1}{8m_{D_\perp^2 b}^2} \right) \mu_\pi^2 + \left(\frac{1}{8m_{3c}^2} - \frac{1}{8m_{sBc}^2} - \frac{3}{8m_{3b}^2} + \frac{3}{8m_{sBb}^2} \right) \frac{\mu_G^2}{3}$$

which cancel well for c ($m_c a < 1$) and to some extent for b.

- Previous work: conservative power counting with Λ = 500–700 MeV.

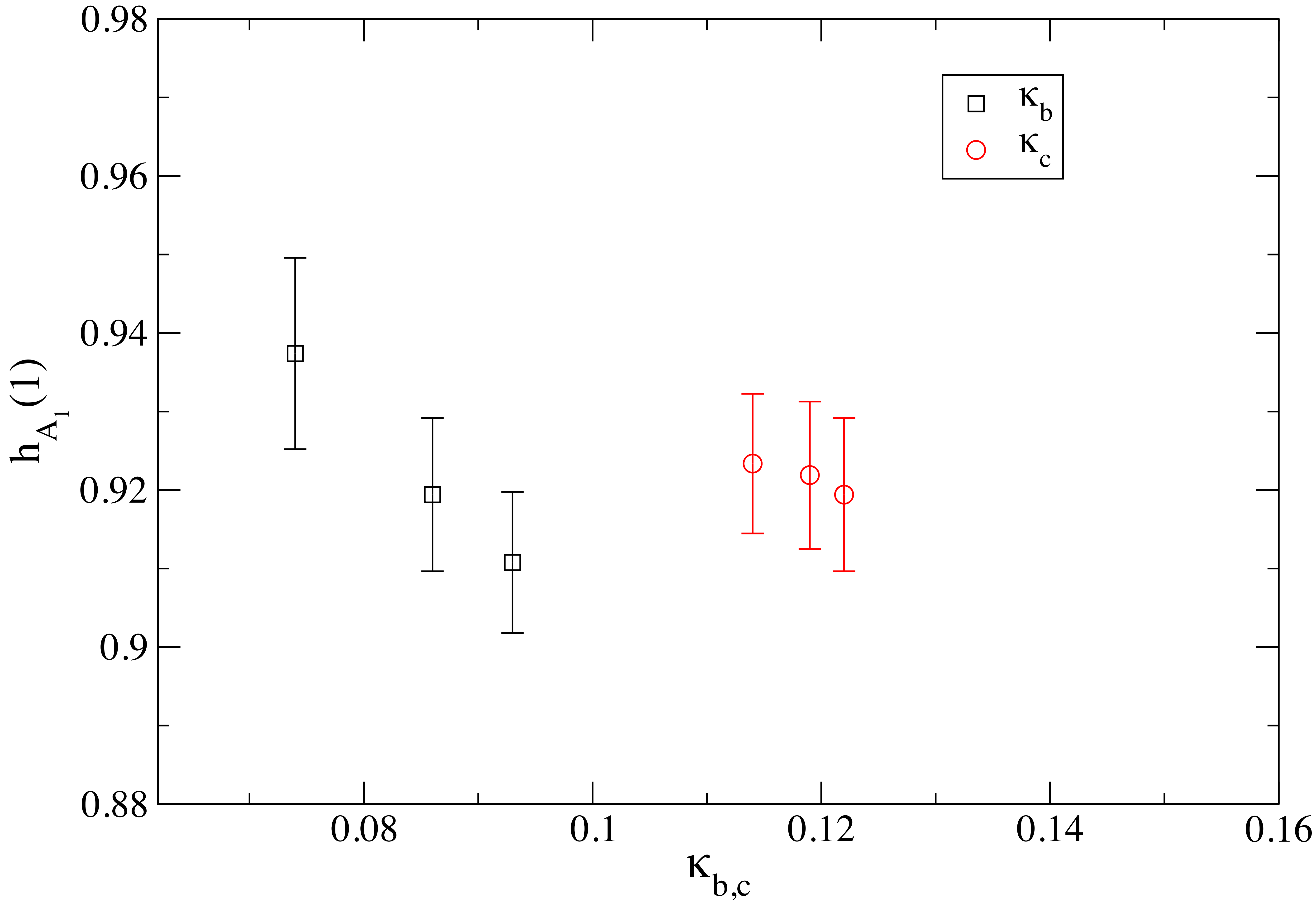
- Future work: use explicit formulae and experimental results or lattice data for $\mu_\pi{}^2$ and $\mu_G{}^2$.

- Remaining matching error is overall normalization, computed in one-loop PT w/ BLM α_s:

$$\rho_A^2 = \frac{Z_A^2}{Z_{V^{bb}} Z_{V^{cc}}}$$

Heavy-quark mass (aka $\varkappa$) tuning

coarse $(am_q, am_l, am_s) = (0.02, 0.02, 0.05)$



Chiral Extrapolation

- In arXiv:0808.2519 we introduce two intermediate quadruple ratios (ratios of double ratios) to disentangle chiral extrapolation from heavy-quark discretization errors.

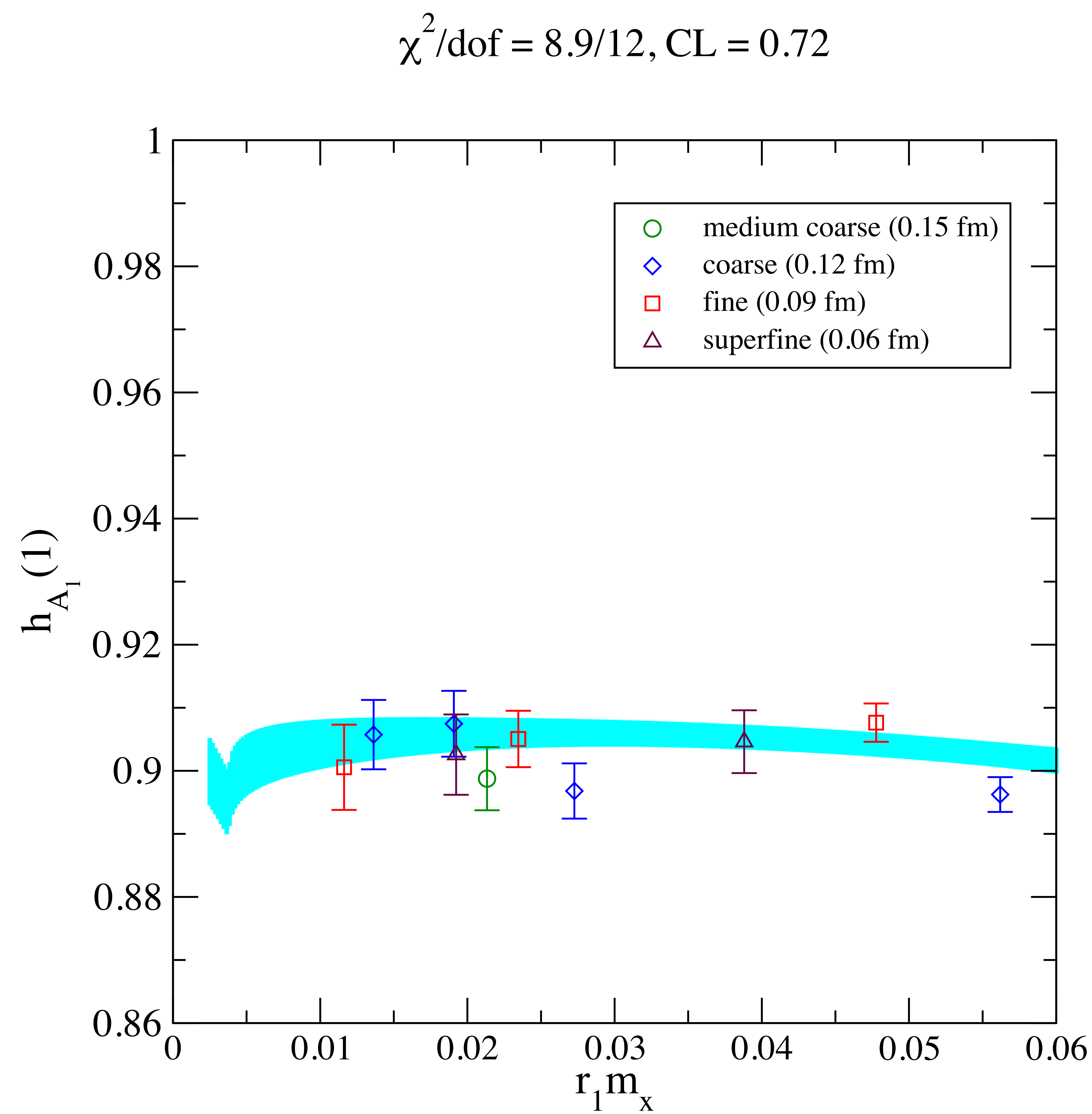
- Now we carry out the chiral extrapolation without the quadruple ratios, but with equivalent information in the fit.

- Partially-quenched staggered PT available from Laiho & Van de Water [hep-lat/0512007].

- Incorporates a cusp when pion is light enough for $D^* \to D\pi$ to be physical.

- Show only “full QCD” points on plot:

Chiral Extrapolation



New Preliminary Result

- With blinding factor (via ρ_A):

$$F_{\text{blind}}\,\mathcal{F}(1) = 0.8949 \pm 0.0051 \pm 0.0088 \pm 0.0072 \pm 0.0093 \pm 0.0050 \pm 0.0030$$

stats $\quad g_{DD^*\pi} \quad$ χPT $\quad$ disc. $\quad \varkappa_{b,c} \quad$ match

- Red errors still under study.

- Cannot determine $|V_{cb}|$ until blinding factor is revealed.

- To compare errors (F_F scales central value to agree with arXiv:0808.2519):

$$\mathcal{F}(1) = 0.921(13)(8)(8)(14)(6)(3)(4) \quad \text{[then]}$$
$$F_F\,\mathcal{F}(1) = 0.921(05)(9)(7)(10)(5)(3) \quad \text{[now]}$$

so improvements (lack thereof) make sense.

Outlook

- Further study of discretization effects in progress.
- Tuning of hopping parameters central to whole program; evolves gradually as more ensembles are analyzed; may need “new” study of kappa dependence.
- Hope to unblind for CKM 2010, September, Warwick, UK.
- Ultrafine ensemble designed to reduce normalization uncertainties.