Wilson loops in very high order lattice perturbation theory

or

NNNNNNNNNNNNNNNNNNNNNNLO QCD

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Outline

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1. Introduction
2. Results
   - Perturbative coefficients $W_{NM}^{(n)}$
   - Perturbative model on finite lattices
   - Boosted perturbation theory
   - Gluon condensate
3. Summary

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- Lattice gauge theory provides a promising tool to calculate the non-perturbative gluon condensate $\langle \frac{\alpha}{\pi} G\, G \rangle$ from Wilson loops
- Study of the large order behaviour of perturbative series on the lattice (Factorial behaviour or (still) not???)
- Influence of the choice of the gauge action
- Investigation of Wilson loops of different sizes: $1 \times 1 \rightarrow N \times M$

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Numerical stochastic perturbation theory (NSPT)

- Standard diagrammatic approach in LPT is restricted essentially to two-loop
- Di Renzo et al. formulated the so-called Numerical Stochastic Perturbation Theory (NSPT)
- Especially suited for quantities without IR divergencies
- Our case: Wilson loops $W_{NM}(n^\star) = \sum_{n=0}^{n^\star} W_{NM}^{(n)} (g^2)^n$

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Computer implementation of NSPT

Computational framework:

- Quenched Wilson (plaquette) gauge action and tree-level improved Symanzik gauge action
- NSPT up to order $n = 20$ for Wilson loops W_{NM}
- Lattice sizes L^4 with $L = 4, 6, 8, 10, 12$ for plaquette gauge action
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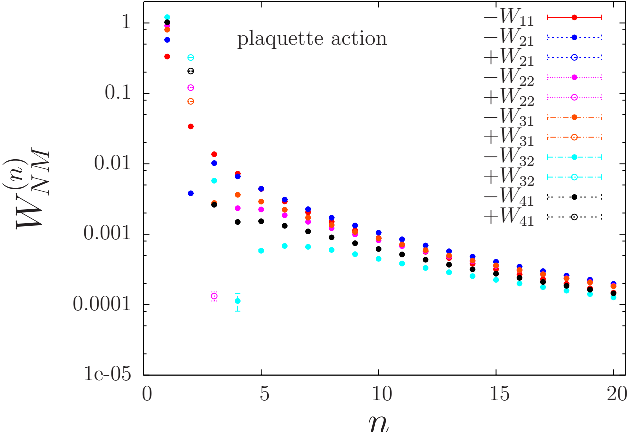
Perturbative coefficients: plaquette gauge action

For $L = 12$ we get for some moderate Wilson loop sizes and plaquette gauge action:



Perturbative coefficients: Symanzik gauge action

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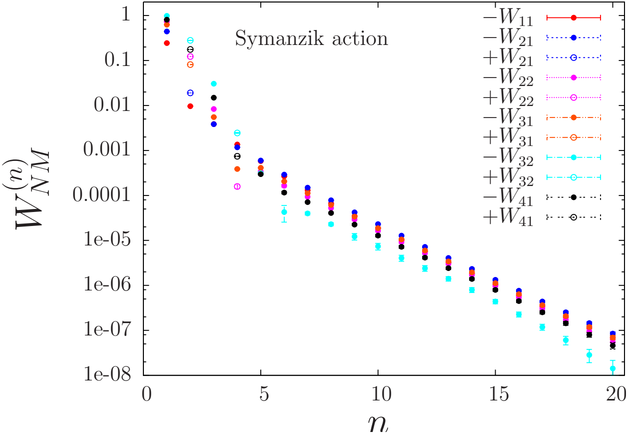

Comparison plaquette and Symanzik

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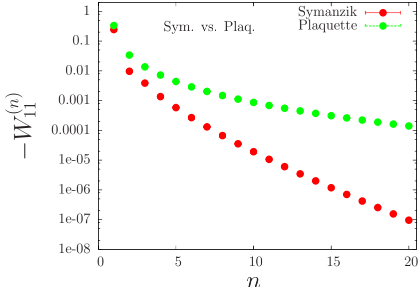

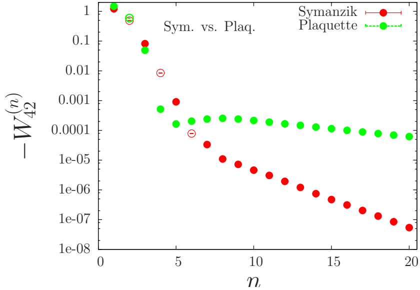

- Symanzik coefficients fluctuate (sign changes) more than their plaquette pendants
- Symanzik coefficients are considerably smaller than in the plaquette case
- $(\leftrightarrow)$ is *compensated* by larger coupling

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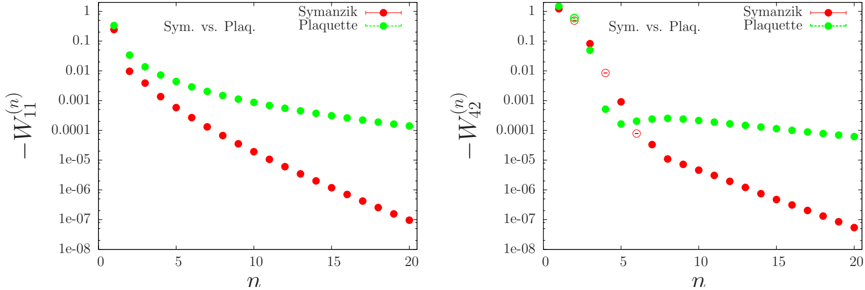

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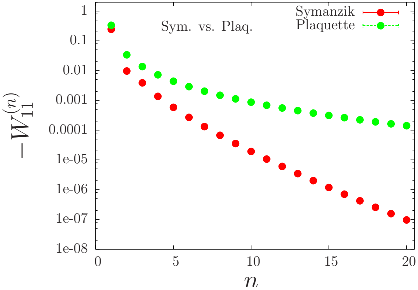

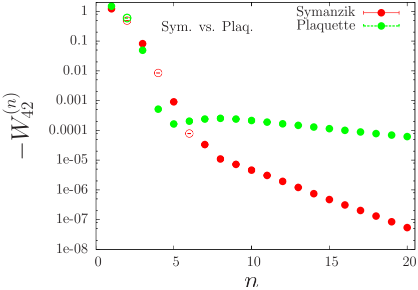

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Model ansatz

- Define coefficients c_n (for each (N, M)!) via $W(g^2) = \sum c_n\, g^{2n}$
- Ratio $r_n = c_n / c_{n-1}$ can be described surprisingly well by

$$r_n(u, q, t, s) = u\, \frac{n^2 + (s - q - 1)n + t}{n\,(n + s)} \quad (1)$$

- Convergence radius $g^2 < 1/u \rightarrow$ summable (hyperbolic function)
- Total sum $\rightarrow$ hypergeometric model
- (1) works well for moderate $N \times M$-loops
  plaquette:
  $1 \times 1, 2 \times 1, 3 \times 1, 4 \times 1 \;\; (n \geq 2), 2 \times 2, 3 \times 2 \;\; (n \geq 4)$
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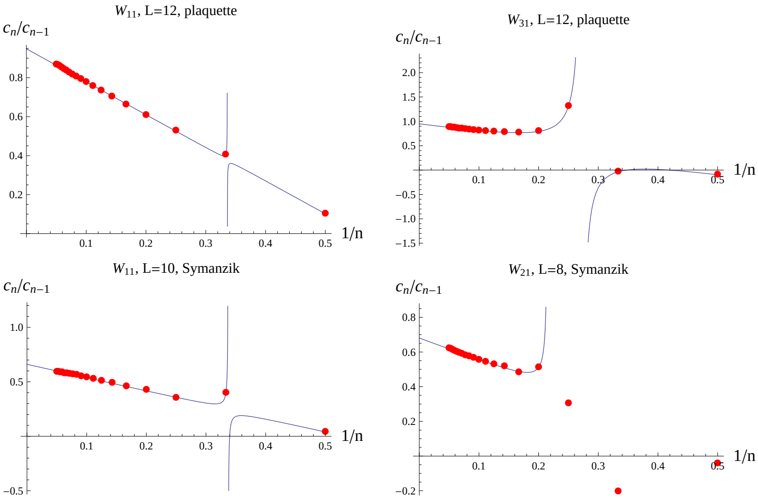
Example Domb-Sykes plots



Boosted perturbation theory

- Bare lattice coupling g^2 - bad expansion parameter
- Use instead $g_b^2 = g^2 / W_{11,pert}$ - boosted coupling
- Reordering of perturbative coefficients $c_n \to c_n^{(b)}$
- $\left.\begin{array}{l} g_b^2 > g^2 \\ |c_n^{(b)}| \ll |c_n| \end{array}\right\}$ improved convergence behaviour
- First successful application: Rakow (2005)
- Further advantage: no model assumption neccessary

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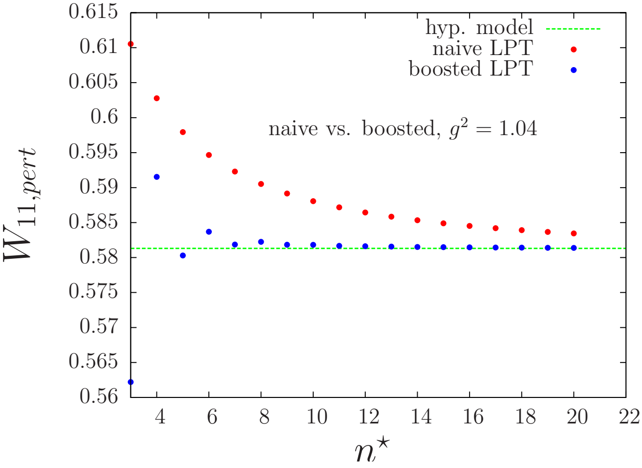
Example plot



$W_{11,pert}(n^\star)$: perturbative series summed up to $n^\star$

Chosen coupling g^2 at convergence limit

Gluon condensate ?

- $\langle GG \rangle$ as introduced by SVZ is an OPE quantitiy and has dimension $(\Lambda)^4$
- $\rightarrow$ on the lattice we would expect:

$$a^4 \langle GG \rangle \approx P_{PT}(n^\star) + \Delta_{n^\star}, \quad \Delta_{n^\star} \propto c_4 \, (a\Lambda)^4$$

($n^\star$: order of lattice perturbation theory)

- Speculations: $\Delta_{n^\star} \propto c_2 \, (a\Lambda)^2 + c_4 \, (a\Lambda)^4$
- Narison/Zakharov: $c_2(n^\star) \, (a\Lambda)^2$ is due to small $n^\star$, for large $n^\star$ they expect $\Delta_{n^\star} \propto c_4(n^\star) \, (a\Lambda)^4$

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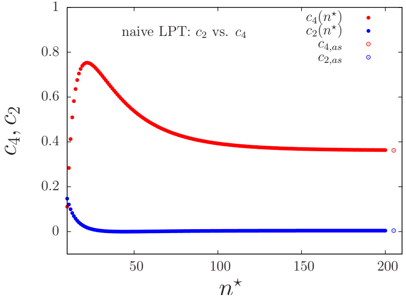
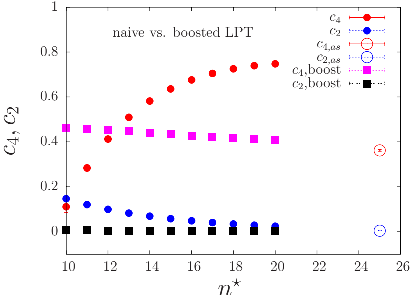
c_4 and c_2, plaquette action, L=12





- Naive LPT: $n^\star \leq 20$: NSPT data, $n^\star > 20$: hypergeometric model series expansion
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- Boosted LPT: only data for $n^\star \leq 20$
- Conclusion: boosted LPT approaches asymptotic value very early

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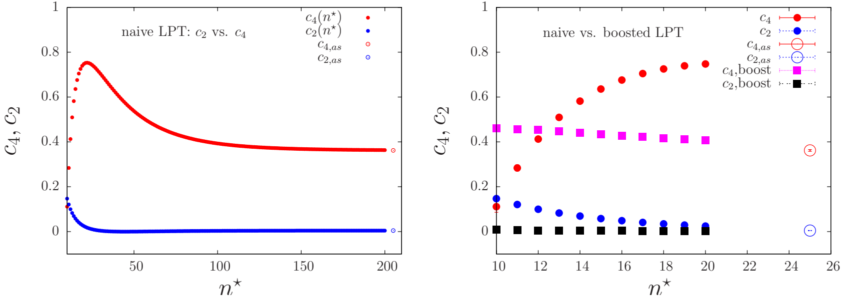

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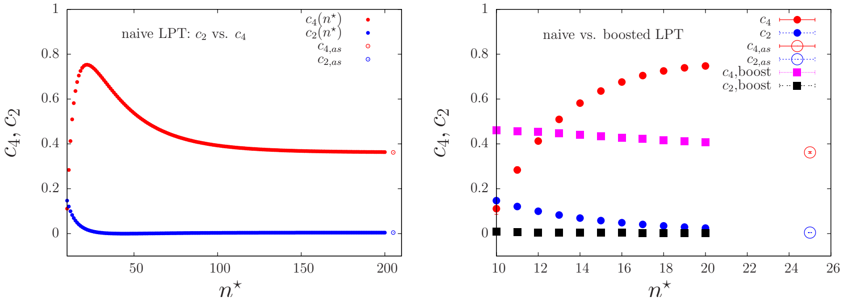

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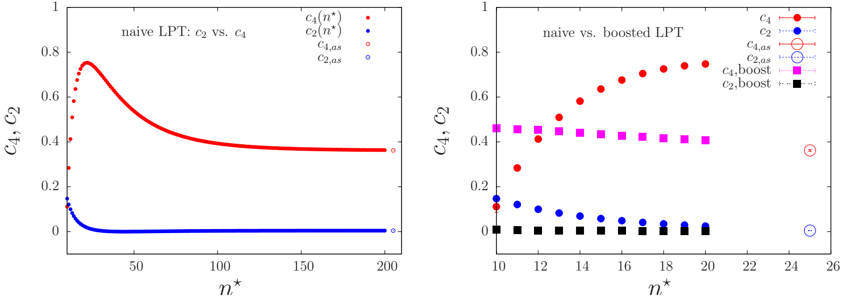

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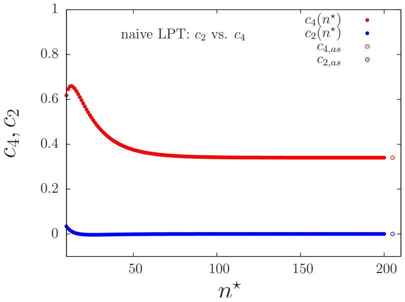

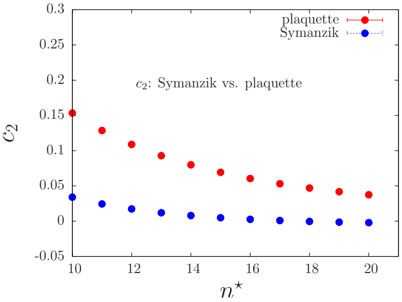

- $c_4(n^\star)$: Symanzik action (left: red) reaches asymptotiv value much erarlier than plaquette action
- $c_2(n^\star)$: Symanzik action (right: blue) reaches the (smaller) asymptotic value earlier than plaquette action (right: red)

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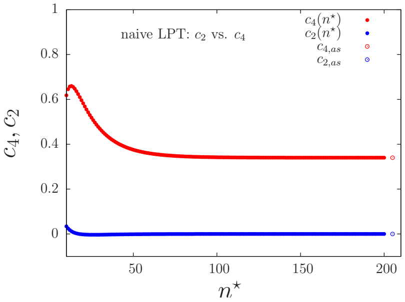

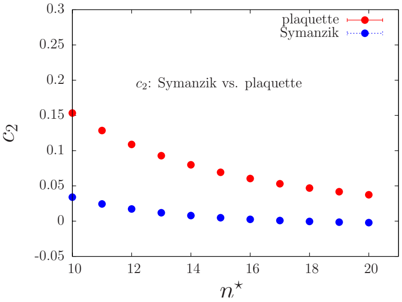

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Gluon condensate estimate(s)

Using the model function and/or the boosted perturbation theory we can estimate $\langle \alpha/\pi GG \rangle$ from plaquette ($P = W_{11}$)

$$a^4 \frac{\pi^2}{36} \left[\frac{b_0 g^2}{\beta(g)} \right] \langle \alpha/\pi GG \rangle = P_{MC} - P_{PT} = \Delta P$$

Systematic uncertainties:

- Choice of the action: plaquette/Symanzik
- Choice of β-range
- Naive vs. boosted perturbation theory
- Choice of size of Wilson loop - area law ansatz (?)

Gluon condensate estimate(s)

Using the model function and/or the boosted perturbation theory we can estimate $\langle \alpha/\pi GG \rangle$ from plaquette ($P = W_{11}$)

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Action	L^4	'Method'	W_{NM}	$\langle\alpha/\pi GG\rangle$ $[GeV^4]$
plaquette	8^4	naive LPT	W_{11}	0.037
plaquette	12^4	naive LPT	W_{11}	0.042
plaquette	12^4	boosted LPT	W_{11}	0.046
Symanzik	8^4	naive LPT	W_{11}	0.039
Symanzik	10^4	naive LPT	W_{11}	0.033

For larger Wilson loops the modified difference ansatz

$$\mathcal{S}_{NM}^2\langle\alpha/\pi GG\rangle \propto W_{NM,MC} - W_{NM,PT}$$

leads to **very small** $\langle\alpha/\pi GG\rangle$ with increasing loop area $\mathcal{S}_{NM}$

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??? Try a new ansatz ???

$$W_{NM,MC} = W_{NM,PT}\,(1 - const\,\mathcal{S}^2_{NM}\,\langle\alpha/\pi GG\rangle)$$

This gives

Action	L^4	W_{NM}	$\langle\alpha/\pi GG\rangle\ [GeV^4]$
plaquette	12^4	W_{21}	0.025
plaquette	12^4	W_{31}	0.024
plaquette	12^4	W_{22}	0.023
Symanzik	10^4	W_{21}	0.034
Symanzik	10^4	W_{31}	0.019
Symanzik	10^4	W_{22}	0.015

Summary

- Wilson loops of different sizes up to loop-order $n = 20$ for plaquette and Symanzik gauge actions
- No factorial behaviour of perturbative coefficients for both actions
- Symanzik action shows improved convergence behaviour
- Comparison: hypergeometric model vs. boosted perturbation theory successful
- Possible a^2-dependence of $\langle \alpha/\pi GG \rangle$ decreases significantly with loop order
- Estimates for $\langle \alpha/\pi GG \rangle$: W_{11} **-** consistent with former lattice results; **larger Wilson loops -** new ansatz could be needed

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