A novel method for computing hadronic correlation functions in lattice QCD spectroscopy

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The Hadron Spectrum Collaboration

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- University of Washington H.W. Lin.

Outline

- Background and Motivation
- Distillation
- A new algorithm
- Tests of the new method
- Summary

Motivation

- Long-term aim: a systematic determination of hadron masses, resonance energies and widths.
- Current focus is the extraction of stationary-state energies on a number of lattice volumes.
- Promising results to date in the isovector meson, kaon, and baryon sectors using single-particle operators. C. Thomas, S. Wallace.
- The next step is to include multi-hadron operators and flavor-singlet meson channels.

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  Required for multi-hadrons and isoscalar mesons.

Multi-hadrons

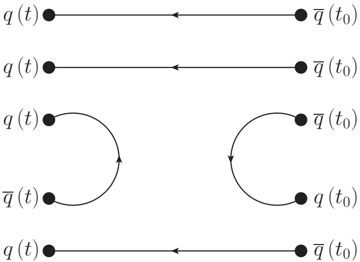
- Stationary-state energy levels extracted from variational analyses of hadronic correlator matrices.
- The reliable measurement of a particular energy depends on the accurate determination of all lower-lying levels in the same symmetry channel.
- Approximate multi-hadron energy levels are given by

$$E_{AB} = \sqrt{m_A^2 + \mathbf{p}^2} + \sqrt{m_B^2 + \mathbf{p}^2}, \qquad \mathbf{p} = \frac{2\pi\mathbf{n}}{L_s} \tag{1}$$

- At current quark masses and volumes, most of the accessible finite-box energy levels lie above multi-hadron thresholds.

Same-time quark lines

- Both multi-hadron and flavor-singlet meson correlators receive contributions from quark lines which begin and end on a single time slice.
- For example, $\langle\Omega|B(-p,t)\,M(p,t)\,\overline{B}(-p,t_0)\,\overline{M}(p,t_0)\,|\Omega\rangle$ may involve



- Need a means of efficiently evaluating such correlation functions.

Distillation

- [M. Peardon et al, 2009]
- In a hadronic correlation function, M^{-1} is sandwiched between quark-field smearing operators: $S\left(t\right)M^{-1}\left(t,t_0\right)S\left(t_0\right)$, where, for Jacobi smearing,

$$\left(S\psi\right)\left(x\right)=\left(1+\frac{\sigma_s}{4n_\sigma}\tilde{\triangle}\right)^{n_\sigma}\psi\left(x\right) \tag{2}$$

- Define a modified smearing operator by writing S as a spectral sum, then truncating the sum to exclude the high-momentum modes S was designed to suppress

$$S\to S'=\sum_{i=1}^{N_{\mathrm{cutoff}}}\lambda_i|v_i\rangle\langle v_i|, \qquad S|v_k\rangle=\lambda_k|v_k\rangle \tag{3}$$

Distillation continued

- The smeared quark line factorises

$$SM^{-1}S \longrightarrow \lambda|v\rangle\langle v|M^{-1}|v\rangle\langle v|\lambda \tag{4}$$

- Relevant quark-field information is encoded in a set of perambulators

$$\langle v_j\,(t)\,|M^{-1}\,(t,t_0)\,|v_i\,(t_0)\rangle, \qquad i,j = 1,..,N_{\mathrm{cutoff}} \tag{5}$$

- For a high-enough level of quark smearing (i.e., N_{cutoff} low enough), the full set of perambulators can be computed, and any hadron correlation function can be evaluated exactly.

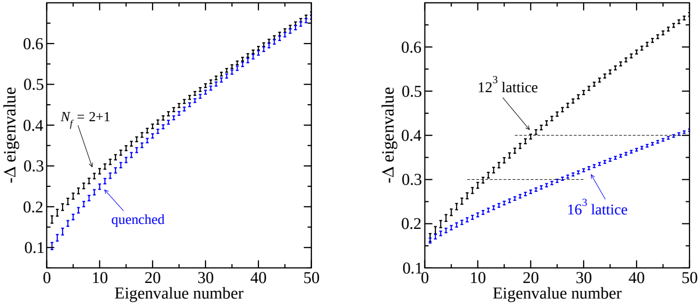
LapH quark-field smearing

- The choice of smearing operator, S, is not unique, and a particularly simple definition gives the Laplacian-Heaviside, or LapH smearing scheme:

$$S = \sum_i \Theta\left(\lambda_i - \lambda_{\mathrm{cutoff}}\right) |v_i\rangle\langle v_i|, \qquad \tilde{\triangle}|v_k\rangle = \lambda_k |v_k\rangle,\ \lambda_k < 0 \tag{6}$$

$$\tilde{\triangle}\phi\left(x\right) = \sum_{i=1}^{3} \tilde{U}_i\left(x\right)\phi\left(x+\hat{\imath}\right) + \tilde{U}_i^{\dagger}\left(x-\hat{\imath}\right)\phi\left(x-\hat{\imath}\right) - \phi\left(x\right) \tag{7}$$

- For fixed $\lambda_{\mathrm{cutoff}}$, the number of eigenmodes in S has a mild pion mass dependence, but increases linearly with the spatial lattice volume.



- Exact evaluation becomes prohibitively expensive on large volumes.

Stochastic LapH

- Use stochastic estimation to mitigate the volume dependence:

$$S(t)\,M^{-1}S(t_0) = S(t)\,M^{-1}|v\rangle E\left(\rho\rho^*\right)\langle v(t_0)\,| \tag{8}$$

- The noise vector components $\rho_{i\alpha}(t)$, with time, spin, and eigenmode indices, satisfy

$$E\left(\rho_{i\alpha}(t)\right) = 0, \qquad E\left(\rho_{i\alpha}(t)\,\rho^*_{j\beta}(t')\right) = \delta_{ij}\delta_{\alpha\beta}\delta_{tt'} \tag{9}$$

- In practice, the stochastic estimator is useful only when combined with variance reduction techniques.

Dilution

- Use Z_n noise.
- Partition the noise vector indices (i.e., time, spin and eigenmode indices) into disjoint sets.
- To each set $\mathcal{D}$, assign a projection operator acting on the noise vectors $P^{[\mathcal{D}]}$ with components

$$\begin{aligned} P^{[\mathcal{D}]}_{ij\alpha\beta}\left(t,t'\right) &= 1, \quad \text{if}\left(t,i,\alpha\right) \text{and} \left(t',j,\beta\right) \in \mathcal{D} \\ P^{[\mathcal{D}]}_{ij\alpha\beta}\left(t,t'\right) &= 0 \quad \text{otherwise} \end{aligned} \tag{10}$$

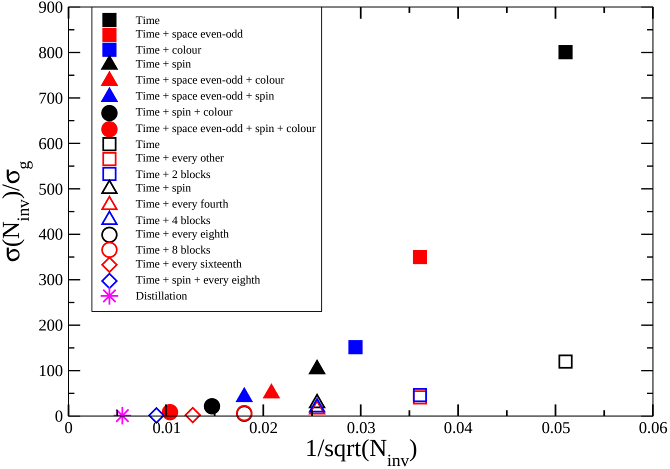
- Define $\rho^{[\mathcal{D}]} = P^{[\mathcal{D}]}\rho$, and substitute

$$E\left(\rho\rho^*\right) \longrightarrow \sum_{\mathcal{D}} E\left(\rho^{[\mathcal{D}]}\rho^{[\mathcal{D}]*}\right) \tag{11}$$

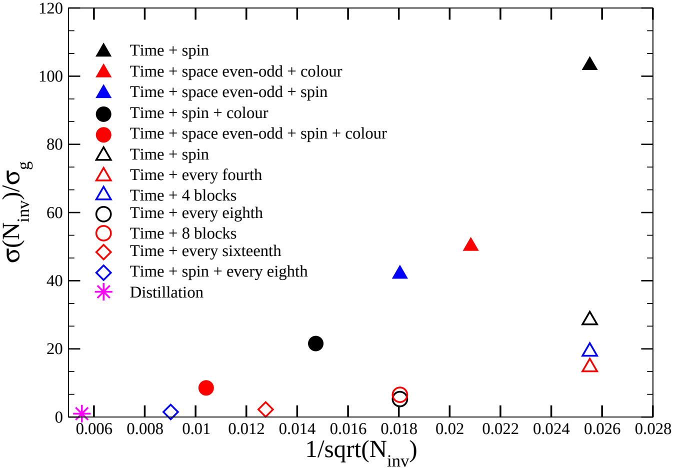
- Possible dilution schemes include full/interlaced time, interlaced/blocked eigenmodes, etc.

Comparison of the new method and standard dilution

Comparison of correlator signals for a triply-displaced nucleon operator at $t = 5$.

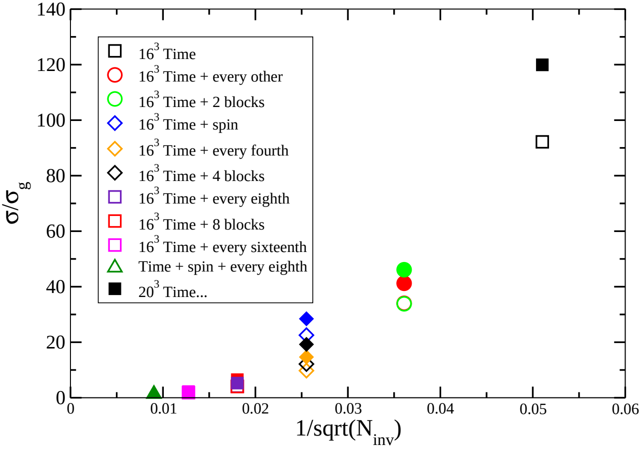


Focusing on the large N_{inv} region



Volume dependence of the new method

- 32 eigenvectors on the 16^3 lattice vs. 64 eigenvectors on the 24^3 lattice.



Summary

- The HSC spectroscopy program is driving the development of new hadron correlator algorithms.
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- The new stochastic algorithm has a much milder volume dependence, and significantly outperforms conventional dilution in comparisons involving connected single-particle correlators.
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- Thank you!