Nucleon and $N^*(1535)$ Distribution Amplitudes

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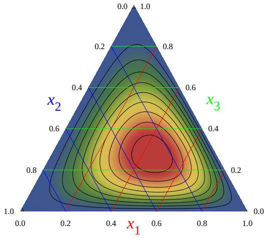
Motivation

- **Wanted:** Hadron wave function
- Experiment:
  - Form factor measurements at JLab (especially with the 12 GeV upgrade), FAIR, . . .
  - Light-Cone Sum Rules relate form factor data to distribution amplitudes
- Theory:
  - Direct calculation of distribution amplitudes from Lattice QCD



Outline

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Distribution Amplitudes in the Continuum

- In leading twist
  - with transverse momentum components integrated out
  - the nucleon wave function can be written as

$$|N,\uparrow\rangle = f_N \int \frac{[dx]\varphi(x_i)}{2\sqrt{24x_1x_2x_3}} \{|u^\uparrow(x_1)u^\downarrow(x_2)d^\uparrow(x_3)\rangle - |u^\uparrow(x_1)d^\downarrow(x_2)u^\uparrow(x_3)\rangle\}$$

  - where
    - x_i: longitudinal momentum fractions
    - $\int [dx] = \int_0^1 dx_1 dx_2 dx_3 \delta(1 - x_1 - x_2 - x_3)$
    - f_N: Leading-twist normalization constant, "Wave function at the Origin"
    - $\varphi(x_i)$: Nucleon Distribution Amplitude
- in next-to-leading twist
  - λ_1, λ_2: Next-to-leading twist normalization constants

Distribution Amplitudes on the Lattice

- On the lattice, calculate correlators of the form

$$\langle \mathcal{O}(x)_{\alpha\beta\gamma} \bar{\mathcal{N}}(y)_\tau \rangle$$

- where
  - $\mathcal{N}$ is a smeared nucleon interpolator
  - $\mathcal{O}$ is a local three-quark operator with up to two derivatives
- to determine moments of the distribution amplitude

$$\varphi^{lmn} = \int [dx] x_1^l x_2^m x_3^n \varphi(x_1, x_2, x_3)$$

- Limited to first and second moments ($l + m + n \leq 2$) due to
  - Mixing with lower dimensional operators for higher moments
  - Need for higher momenta $\rightarrow$ more noise

Overview of lattices

$N_f = 2$ clover Wilson fermions

κ	m_π / MeV	Size	# Configurations
$\beta = 5.29, a = 0.0753$ fm			
0.13590	627	$24^3 \times 48$	901
0.13620	407	$24^3 \times 48$	850
0.13632	282	$32^3 \times 64$	578 + more in progress
0.13632	271	$40^3 \times 64$	in progress
0.13640	170	$40^3 \times 64$	coming soon
$\beta = 5.40, a = 0.0672$ fm			
0.13610	648	$24^3 \times 48$	687
0.13625	558	$24^3 \times 48$	1180
0.13640	451	$24^3 \times 48$	1037
0.13660	233	$48^3 \times 64$	coming soon

[QCDSF collaboration]

New configurations are being generated on

Parity separation

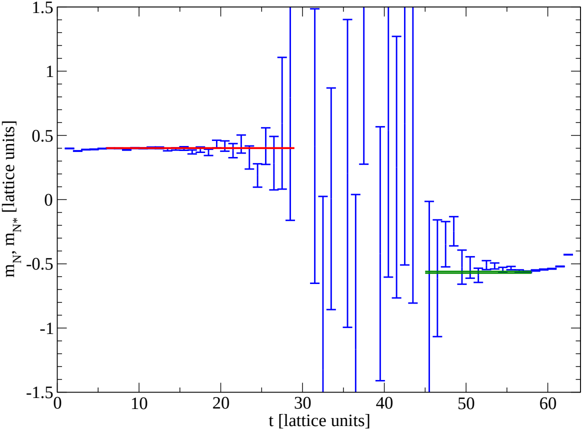
$32^3 \times 64$

$a = 0.0753$ fm
$a^{-1} = 2.620$ GeV

$m_\pi = 282(2)$ MeV

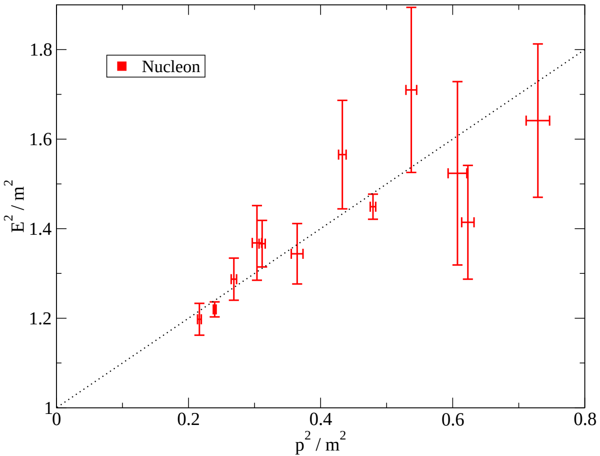
$m_\pi L = 3.44$

$$m_N = 1051(5)\ \text{MeV}$$
$$m_{N^*} = 1482(17)\ \text{MeV}$$



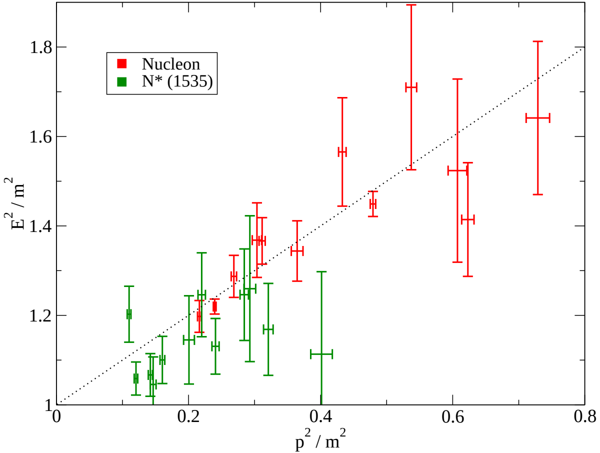
The generalized Lee-Leinweber parity projector, $\frac{1}{2}\Gamma\left(1 + \frac{m}{E}\gamma_4\right)$, where Γ is a suitable product of γ_i matrices, works well for all operator-momentum combinations that are used here

Dispersion Relation



- According to Einstein, $\frac{E^2}{m^2} = 1 + \frac{p^2}{m^2}$ (dotted line)
- All results preliminary, statistical errors only

Dispersion Relation



- According to Einstein, $\frac{E^2}{m^2} = 1 + \frac{p^2}{m^2}$ (dotted line)
- All results preliminary, statistical errors only

Wave functions at the origin

$32^3 \times 64$

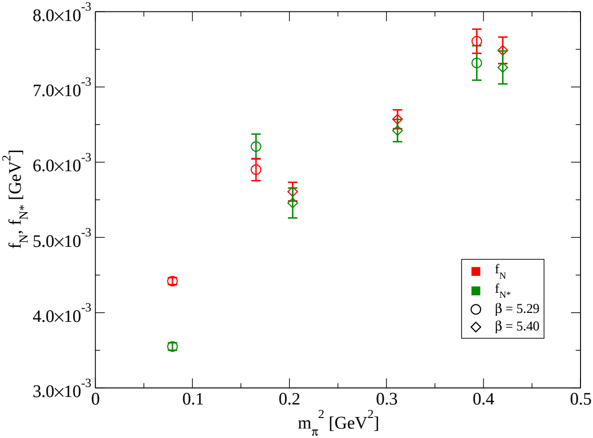
$a = 0.0753$ fm

$m_\pi = 282(2)$ MeV

$m_\pi L = 3.44$

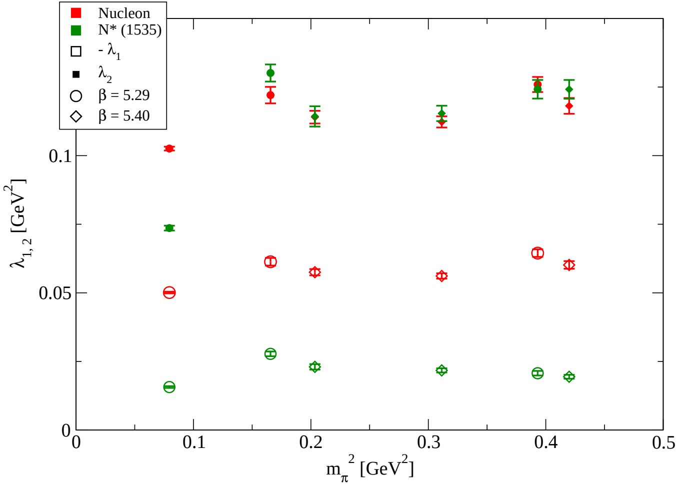
$f_N = 4.42(5) \cdot 10^{-3}\text{GeV}^2$

$f_{N^*} = 3.55(5) \cdot 10^{-3}\text{GeV}^2$



All results preliminary, statistical errors only, no chiral extrapolation

Next-to-leading twist Normalization Constants



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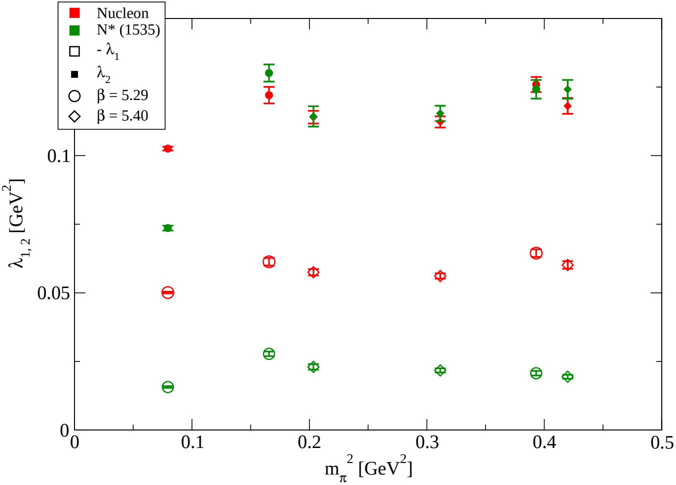
Next-to-leading twist Normalization Constants



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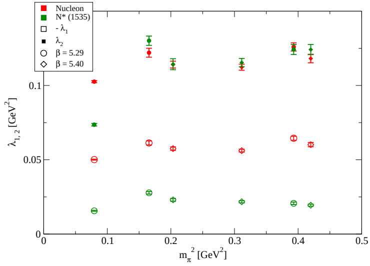
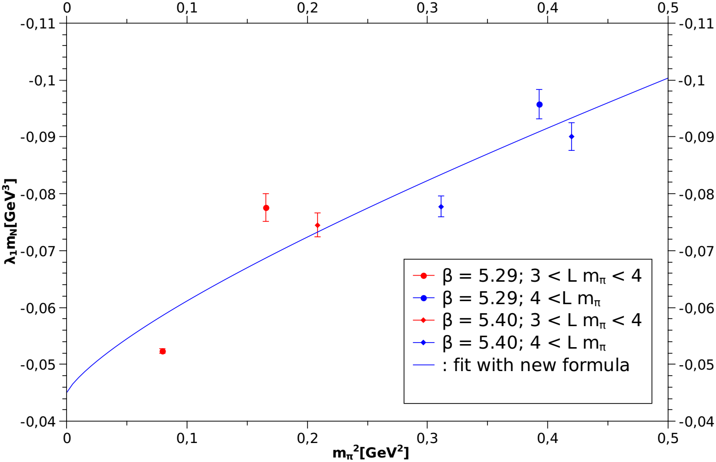
Leading-One-Loop Baryon χPT for λ_1

$$\lambda_1 = \frac{4}{m_N}\left[\alpha\left(1 + \frac{3}{64\pi^2 f^2}\left(1+3g_A^2\right) m_\pi^2 \ln\frac{\lambda^2}{m_\pi^2} - \frac{3g_A^2}{32\pi^2 f^2} m_\pi^2 + \right.\right.$$
$$+\frac{1}{64\pi^2 f^2}\frac{m_\pi^3}{M}\frac{(12g_A + 18g_A^2) - (3g_A + 6g_A^2)\frac{m_\pi^2}{M^2}}{\sqrt{1-\frac{m_\pi^2}{4M^2}}}\arccos\frac{m_\pi}{2M} +$$
$$\left.\left. + \frac{1}{64\pi^2 f^2}\frac{m_\pi^4}{M^2}\left(3g_A + 6g_A^2\right)\ln\frac{m_\pi^2}{M^2}\right) + \epsilon^{(r)}(\lambda) m_\pi^2\right]$$



- (Wein, Hemmert, Schäfer, in preparation)
- $\alpha, \epsilon^{(r)}$: free parameters
- $f = \sqrt{2} f_\pi$
- g_A: axial coupling constant
- λ: renormalization scale
- $M = 938\text{MeV}$

χPT-fit to λ_1 of the Nucleon



(thanks to Philipp Wein)

All results preliminary, statistical errors only, chiral extrapolation!

Wave functions and Distribution amplitudes

- Expand wave function in multiplicatively renormalizable terms (Braun, Manashov, Rohrwild):

$$
\begin{aligned}
\varphi(x_i;\mu^2) =& 120 x_1 x_2 x_3 \Big\{ 1 + c_{10}(x_1 - 2x_2 + x_3) L^{\frac{8}{3\beta_0}} \\
& + c_{11}(x_1 - x_3) L^{\frac{20}{9\beta_0}} + c_{20}\left[1 + 7(x_2 - 2x_1 x_3 - 2x_2^2)\right] L^{\frac{14}{3\beta_0}} \\
& + c_{21}\,(1 - 4x_2)\,(x_1 - x_3)\, L^{\frac{40}{9\beta_0}} \\
& + c_{22}\left[3 - 9x_2 + 8x_2^2 - 12 x_1 x_3\right] L^{\frac{32}{9\beta_0}} + \dots \Big\}
\end{aligned}
$$

- where $L \equiv \alpha_s(\mu)/\alpha_s(\mu_0)$
- c_{ij}: “shape parameters” are obtained from a constrained fit to φ^{lmn} with $l + m + n \leq i$ such that $\varphi^{lmn} = \varphi^{(l+1)mn} + \varphi^{l(m+1)n} + \varphi^{lm(n+1)}$ is fulfilled

Shape parameters

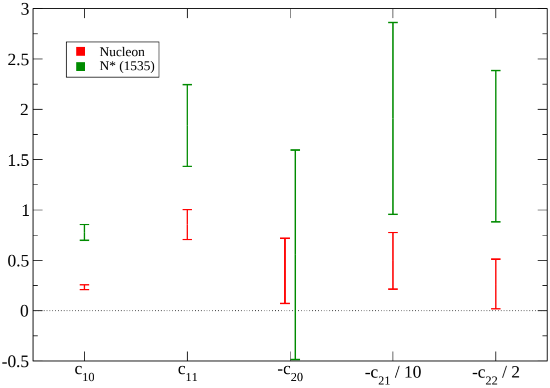
$32^3 \times 64$

$a = 0.0753$ fm

$m_\pi = 282(2)$ MeV

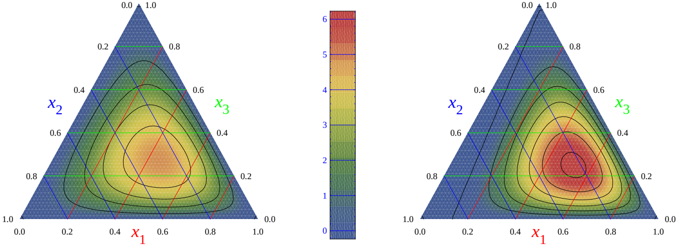
$m_\pi L = 3.44$

578×4 configurations



- c_{1j} are significantly different from zero, both for the nucleon and $N^*(1535)$
- All results preliminary, statistical errors only, no chiral extrapolation

Shape parameters (II)



Nucleon

$$c_{10}^{N} = 0.233 \pm 0.024\,,$$
$$c_{11}^{N} = 0.855 \pm 0.145\,,$$

$N^*(1535)$

$$c_{10}^{N^*} = 0.778 \pm 0.078$$
$$c_{11}^{N^*} = 1.84 \pm 0.41$$

- The distribution amplitude of the $N^*(1535)$ is much more asymmetric than that of the nucleon
- All results preliminary, statistical errors only, no chiral extrapolation

Conclusions and Outlook

- f_N, f_{N^*}, λ_1, λ_2 and the first two moments of the nucleon and $N^*(1535)$ distribution amplitudes have been calculated at several pion masses, the lowest being $m_\pi = 282\text{MeV}$
- Finer and larger volume lattices are still being investigated
- Chiral Perturbation Theory calculations for more quantities in progress
- Next big step: extension of this study to all octet and decuplet baryons