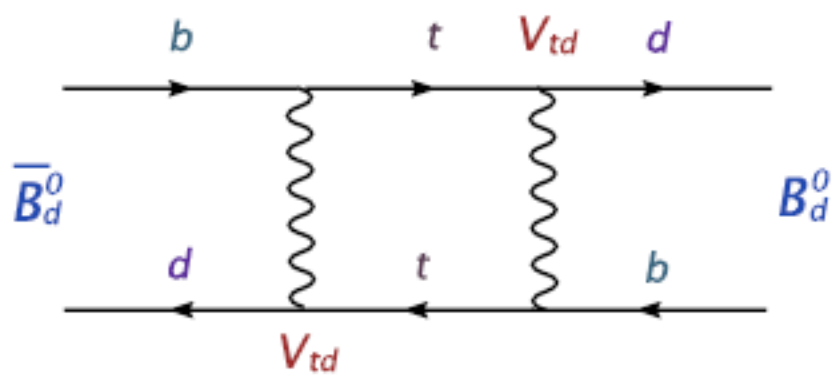
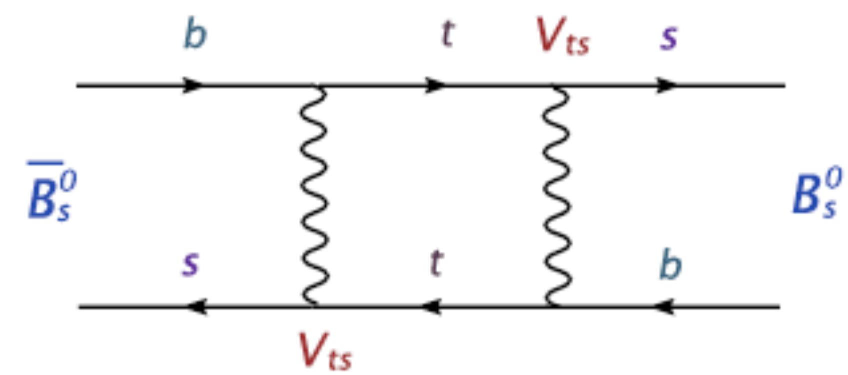
NEUTRAL B MESON MIXING WITH 2+1 FLAVOR DOMAIN-WALL LIGHT AND STATIC HEAVY QUARKS

Yasumichi Aoki (RIKEN BNL)

6/15/2010 @ Lattice 2010

B^0–$\overline{B^0}$ mixing in the Standard Model





$\Delta m_d = 0.507(5)\text{ps}^{-1}$ PDG $\Delta m_s = 17.77(12)\text{ps}^{-1}$ CDF

- B^0_q–$\overline{B^0_q}$ oscillation frequency ($q = d, s$)

$$\Delta m_q = (\text{known factor}) \cdot |V_{tb} V^*_{tq}|^2 \mathcal{M}_q,$$

- B^0_q–$\overline{B^0_q}$ mixing matrix elements

$$\mathcal{M}_q = \langle \bar{B}^0_q | \, [\bar{b}\gamma^\mu(1-\gamma_5)q][\bar{b}\gamma_\mu(1-\gamma_5)q] \, | B^0_q \rangle \, .$$

- $SU(3)_f$ breaking ratio:

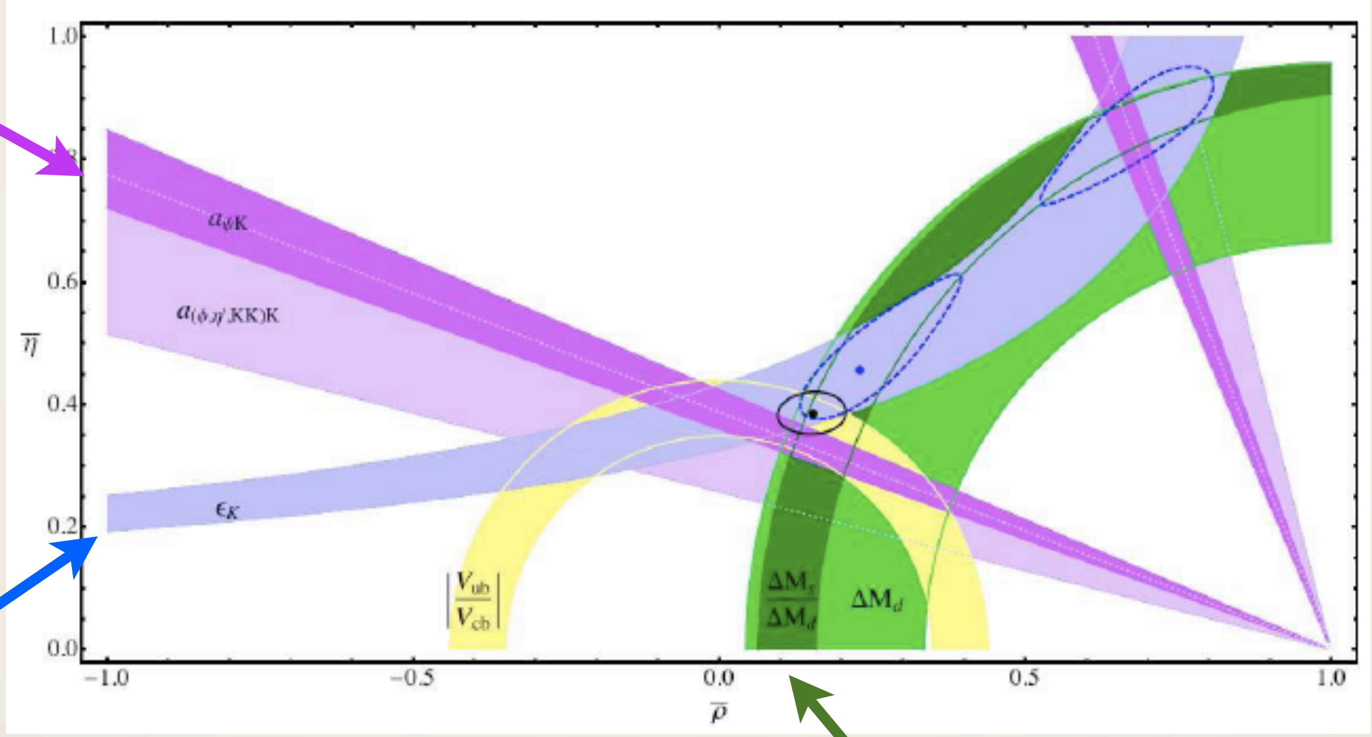
$$\xi \propto \sqrt{\frac{\mathcal{M}_s}{\mathcal{M}_d}} \to \left| \frac{V_{td}}{V_{ts}} \right| = \frac{1}{\xi} \frac{\Delta m_s}{\Delta m_d} \frac{m_{B_d}}{m_{B_s}}$$

  - Large fraction of statistical, systematic errors cancel in the ratio ξ.

B_K and CKM unitarity triangle

$B_0 \to \psi K_s$ (no need of Lattice QCD)

[Lunghi & Soni 08]



B_K determines ε_K band

ξ

- 2-3 tension → possible hint of new physics

unitarity triangle: 2 key quantities

- ξ from $B^0 - \overline{B^0}$ and $B_s^0 - \overline{B_s^0}$ mixing
- B_K from $K^0 - \overline{K^0}$ mixing
  - all induced through box diagram (1-loop)
  - enhanced by quark mass in the internal loop (Inami-Lim function)
  - high sensitivity to new physics
  - chiral symmetry is essential for B_K → Domain-wall fermions
  - chiral symmetry helps for determination of ξ

old and new works

- 1st feasibility study on 2+1 flavor DWF ensemble with static b quark

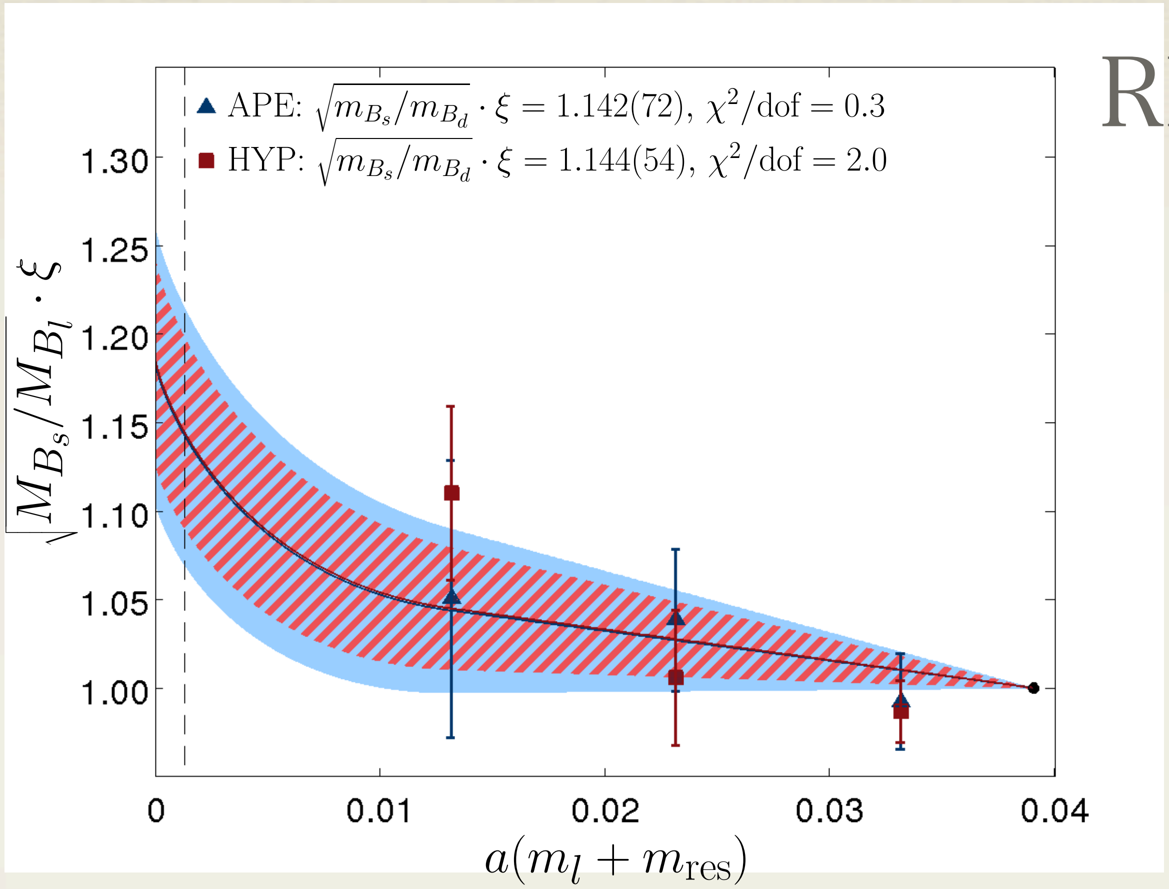
  “Neutral B-meson mixing from unquenched lattice QCD with domain-wall light quarks and static b-quarks”,

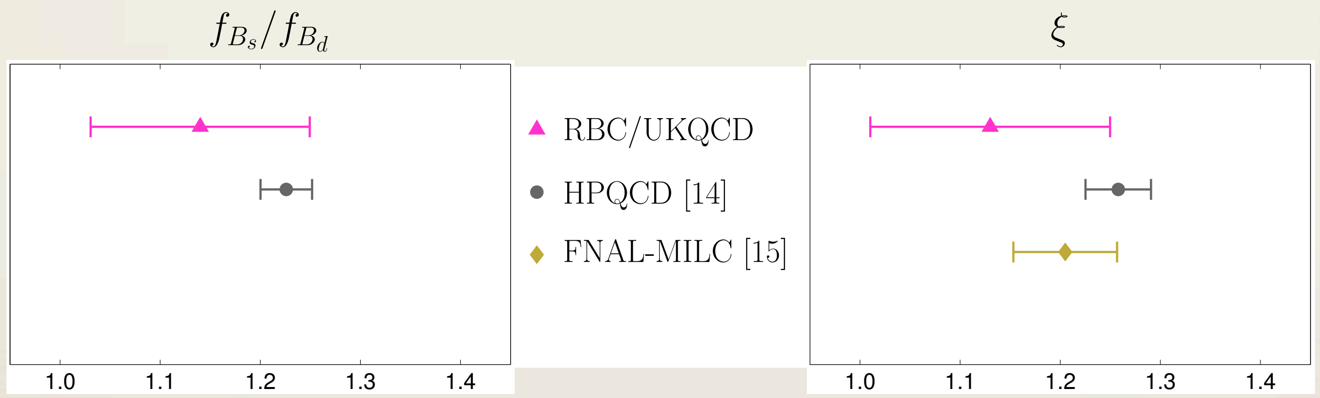
  RBC/UKQCD: C. Albertus, YA, P. A. Boyle, N. H. Christ, T. T. Dumitrescu, J. M. Flynn T. Ishikawa, T. Izubuch O. Loktik, C. T. Sachrajda, A. Soni, R. S. Van de Water, J. Wennekers, and O. Witzel

  - $V \sim (1.8\ \mathrm{fm})^3$, $a^{-1}=1.7$.
- 2nd study aiming precision (main topic of this talk)
  - mainly with a subset of US colleague of the 1st paper
  - RBC/UKQCD under USQCD collaboration project on QCDOC: 24^3x64
  - RBC/UKQCD under RIKEN RICC project: 32^3x64
  - both: $V \sim (2.7\ \mathrm{fm})^3$, $a^{-1}=1.7$ and 2.3 GeV.

ξ : Static approximation on b quark

RBC/UKQCD 2010





- our error is large compared to HPQCD or FNAL-MILC
- ours are feasibility studies
- significant improvements expected....

f_B and M_B new project

- b quark static limit [This work]
  - lighter m_q, larger V, with O(a) improvement
  - QCDOC(USQCD), RICC (RIKEN)
  - aiming ξ total error $< 5\%$
  - investigating non-perturbative renormalization
- b quark using lattice relativistic heavy quark [O. Witzel (previous talk)]
  - no $(m_b a)^n$ error, no $1/m_b$ error
  - $a \to 0$

error reduction strategy

uncertainty	f_{B_s}/f_{B_d} APE	f_{B_s}/f_{B_d} HYP	ξ APE	ξ HYP
statistics	8%	4%	6%	5%
chiral extrapolation	7%	7%	7%	7%
uncertainty in $g_{B^*B\pi}$	3%	3%	2%	2%
discretization error	3%	3%	4%	4%
renormalization factors	0%	0%	2%	2%
scale and quark mass uncertainties	1%	1%	1%	1%
finite volume error	1%	1%	1%	1%
$1/m_b$ corrections	2%	2%	2%	2%
total systematics	9%	9%	9%	9%

RBC/UKQCD 2010

- main errors for ξ
  - statistical
  - chiral extrapolation
  - discretization
  - source/sink optimization
  - smaller mass
  - 2nd (finer) lattice spacing

static approximation

- Eichten-Hill action
  - with smeared links (HYP) to enhance the S/N [Alpha collaboration]
- $1/m_b$ error (NLO in HQET): Λ/m_b~10%
  - SU(3) breaking ratio: $(m_s-m_d)/m_b$~2%

$$
\begin{aligned}
\xi &= 1+\frac{m_s-m_d}{\Lambda}F(m_d,m_s,m_b) \\
&= 1+\frac{m_s-m_d}{\Lambda}\left(c+c'\frac{\Lambda}{m_b}+\cdots\right) \\
&= 1+c\frac{m_s-m_d}{\Lambda}+c'\frac{m_s-m_d}{m_b}+\cdots
\end{aligned}
$$

method

- smeared source: beware of O(V) degeneracy of static quark
  (N.Christ, O.Loktik in Lattice 2007 by T. Dumitrescu)
  - #=V for wall source
  - #= δ V for smeared source
    - (δ V : the 3d volume where smeared source has a support)
- special ratio of correlation functions
- how to sample the volume
  - shifting the source position (in space-time) with trajectory
    - done in one ensemble (lightest), not in the other 2 (heavier)

tuning of smearing

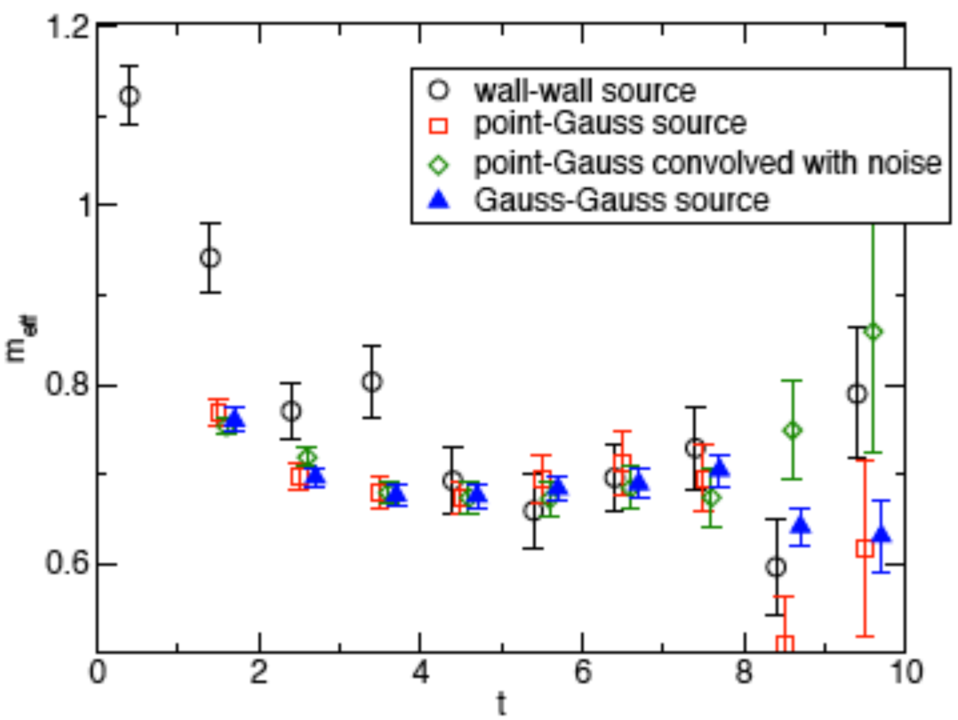
Test of source combination on small lattice

heavy	wall	point	wall	Gauss
light	wall	Gauss	Gauss wall	Gauss

Gauss smearing uses $\sigma_s = 2$ with Gaussian width $\sigma = \sigma_s/\sqrt{2}$

- quench Wilson $\beta = 5.7$ ($a = 0.2$ fm), $8^3 \times 16$ ($(1.6\text{ fm})^3$)
- domain wall fermion with $L_s = 4$, at about strange mass.

source-sink symmetric



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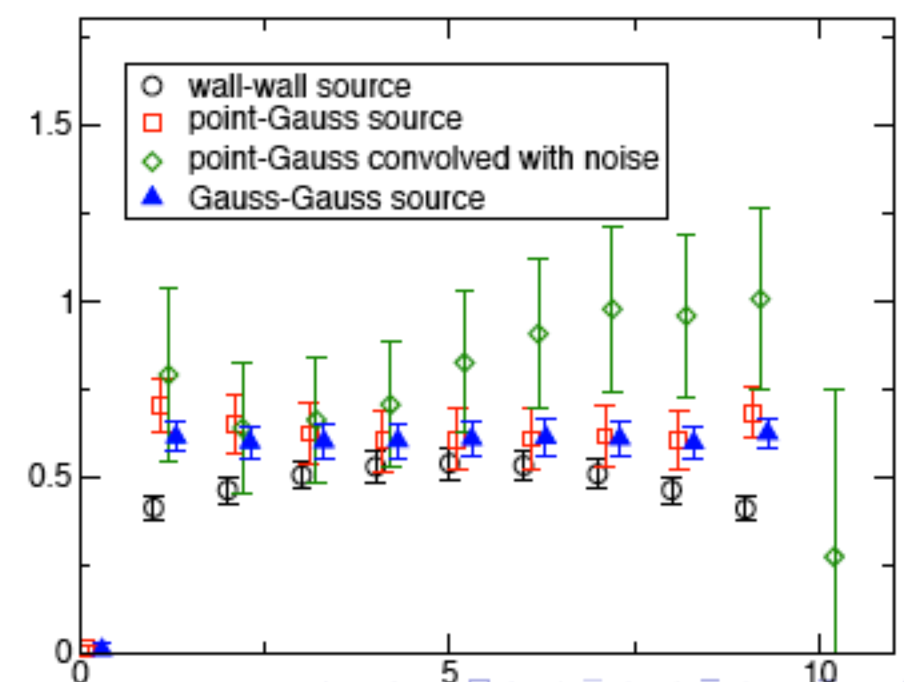
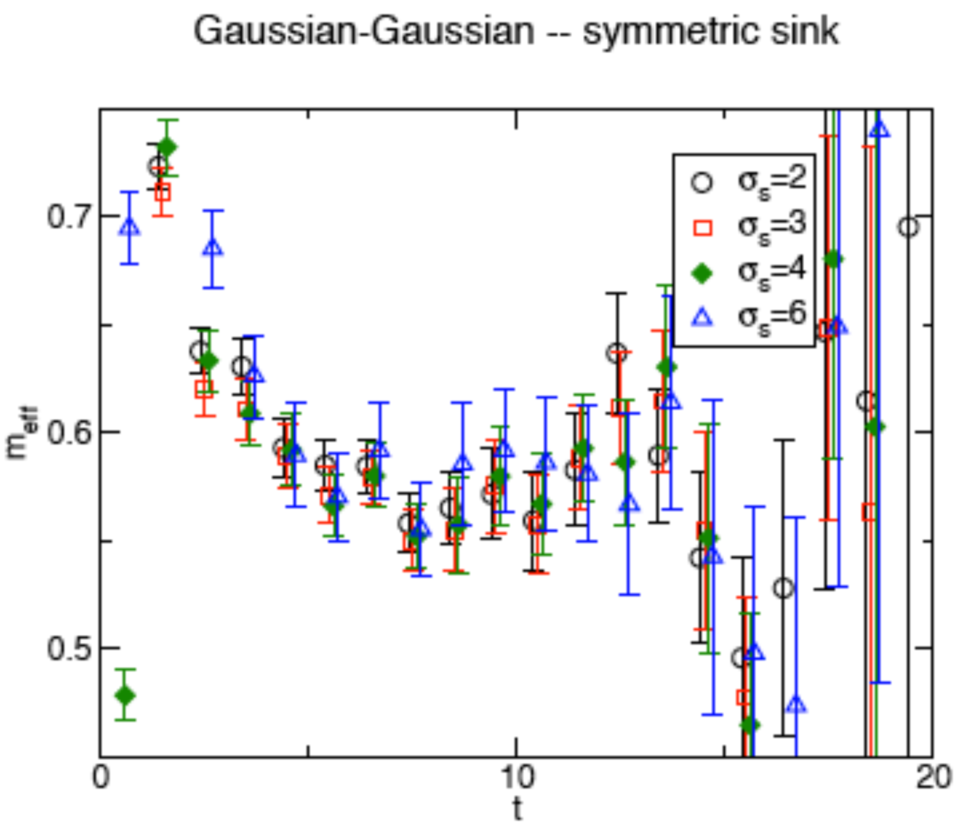
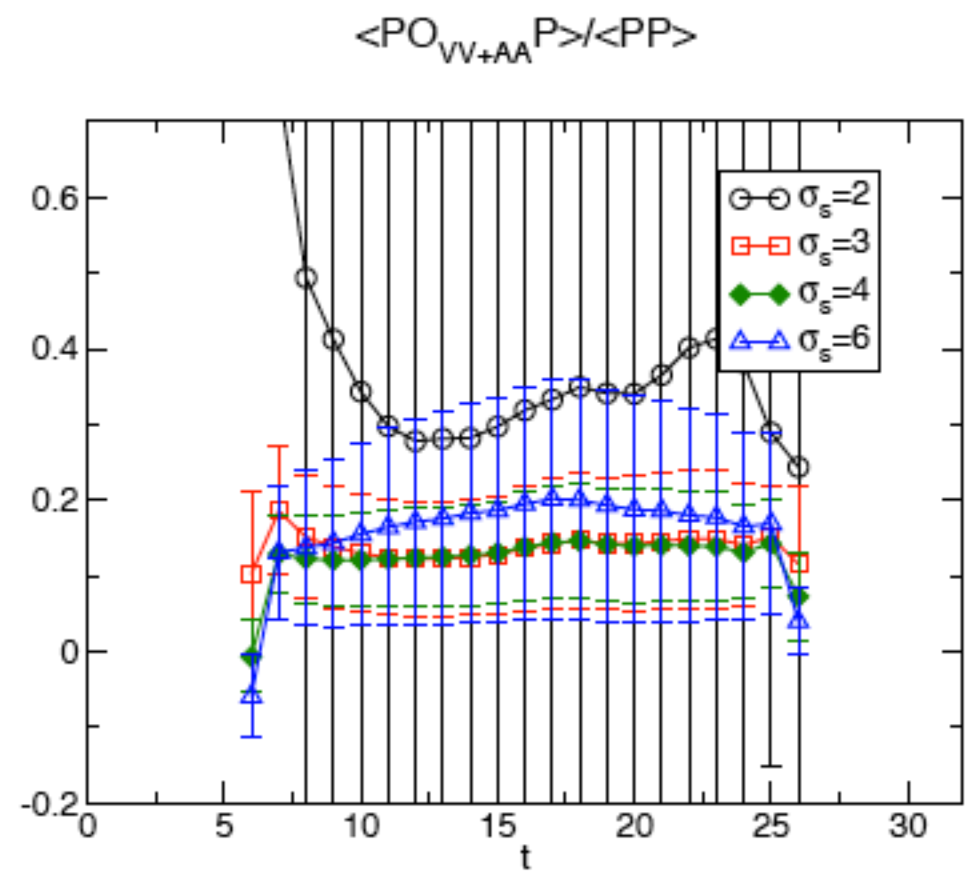


tuning of smearing

Finding optimal Gaussian width

- use smaller volume $16^3 \times 32$, with exactly same parameters as $24^3 \times 64$, except $L_s = 16 \to 4$, $m_f = 0.02$.



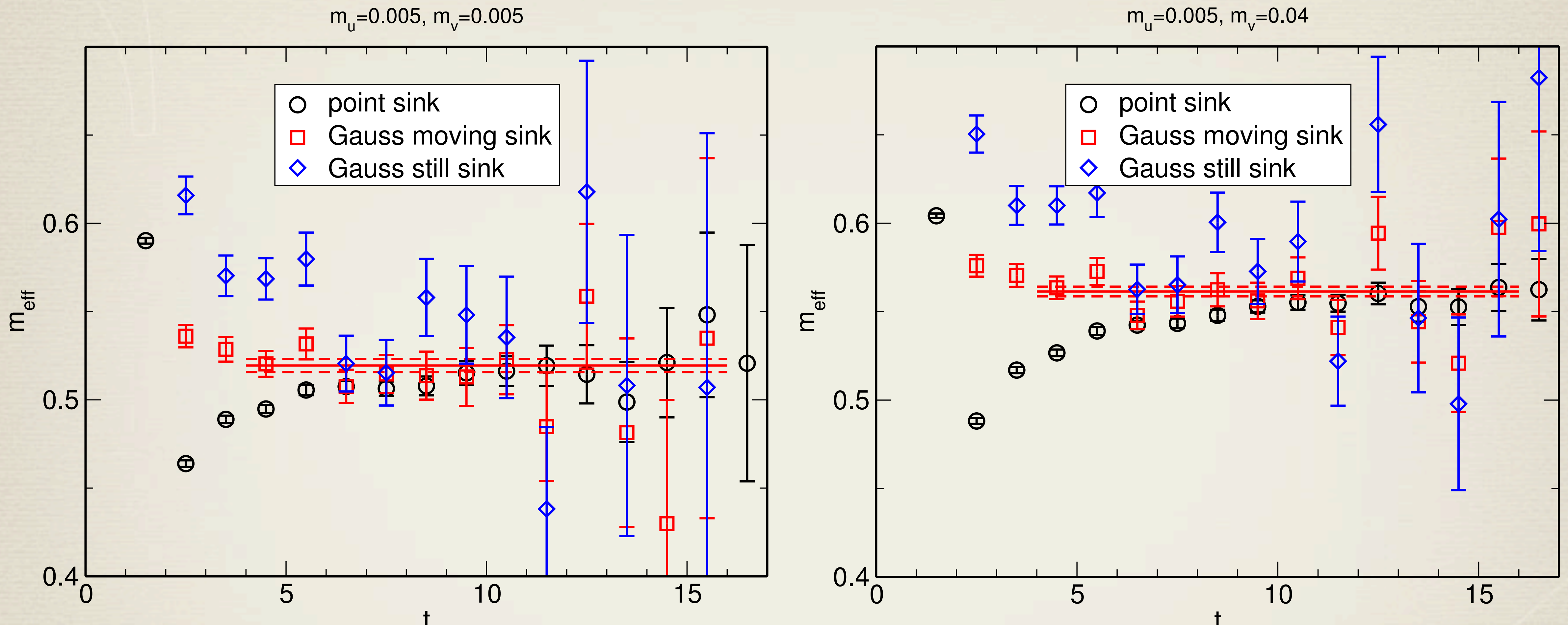


- $\sigma_s = 4$

heavy-light measurements

* ensemble: 2+1 f Iwasaki-DWF β=2.13, m_s=0.04, m_{ud}=0.005, 0.01, 0.02
* light valence: DWF m_s=0.03, 0.04 and m_{ud}=0.005, 0.01, 0.02
* static quark: Eichten-Hill with HYP smearing (Hasenfratz-Knechtli)
* both light and static quark source are smeared with the gauge-covariant Gaussian
    * with pre-smeared links (3d APE)
    * σ=4

heavy-light effective mass



$$C^{SL}(t) = \sum_{\vec{x}} \langle A_0^L(\vec{x},t) A_0^S(0)^\dagger \rangle,$$

$$C^{SSm}(t) = \sum_{\vec{x}} \langle A_0^S(\vec{x},t) A_0^S(0)^\dagger \rangle,$$

$$C^{SSs}(t) = \langle A_0^S(\vec{0},t) A_0^S(0)^\dagger \rangle$$

B meson decay constants

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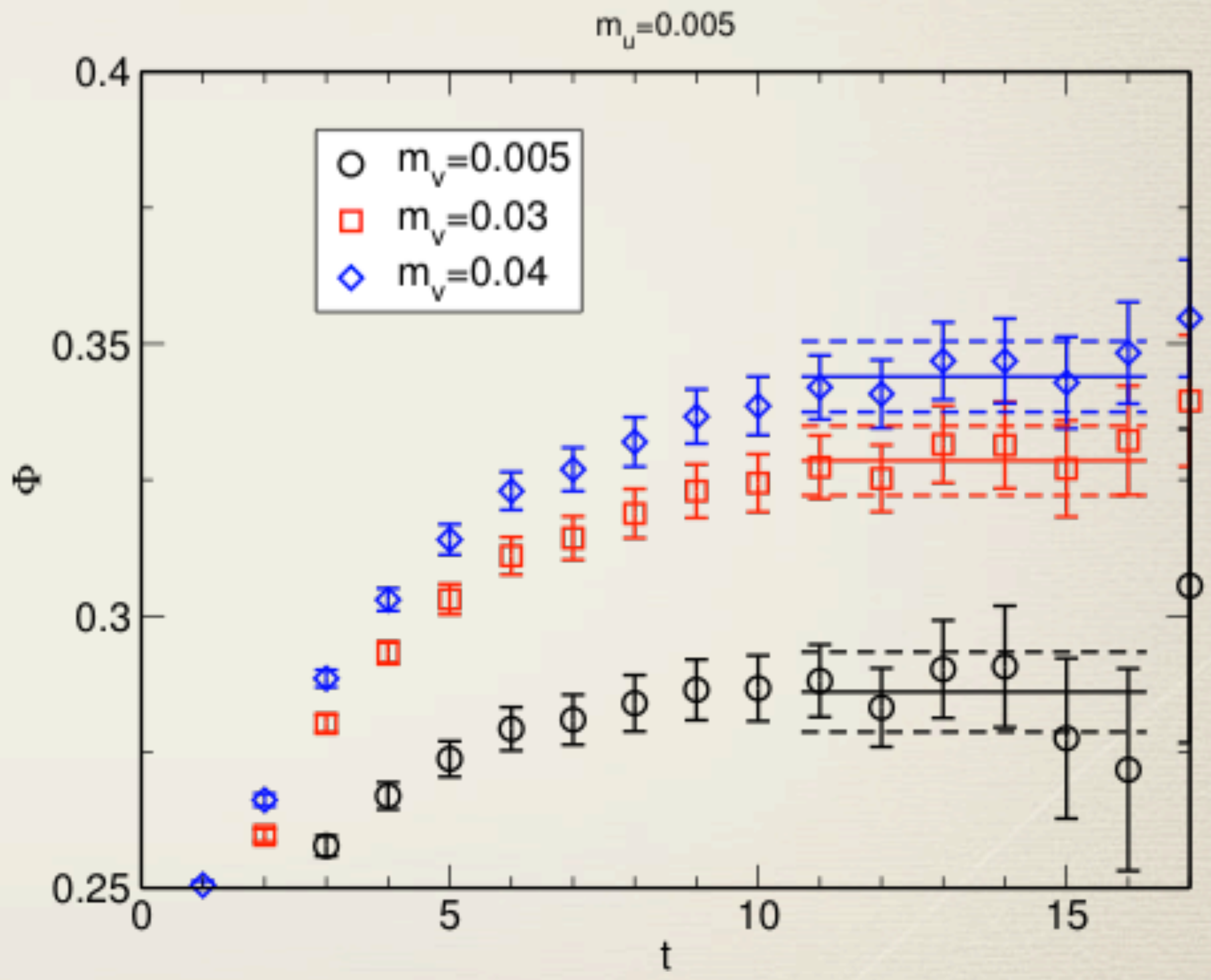

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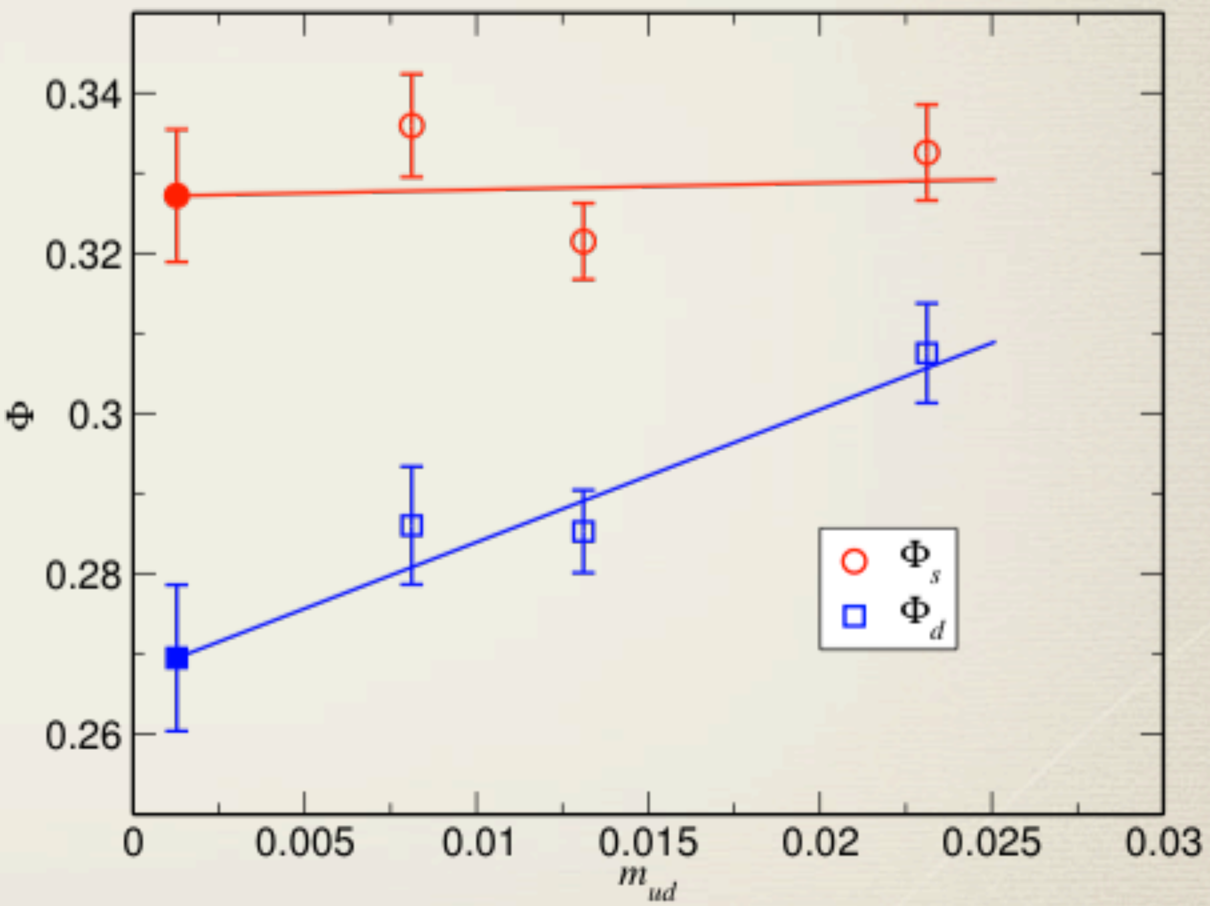
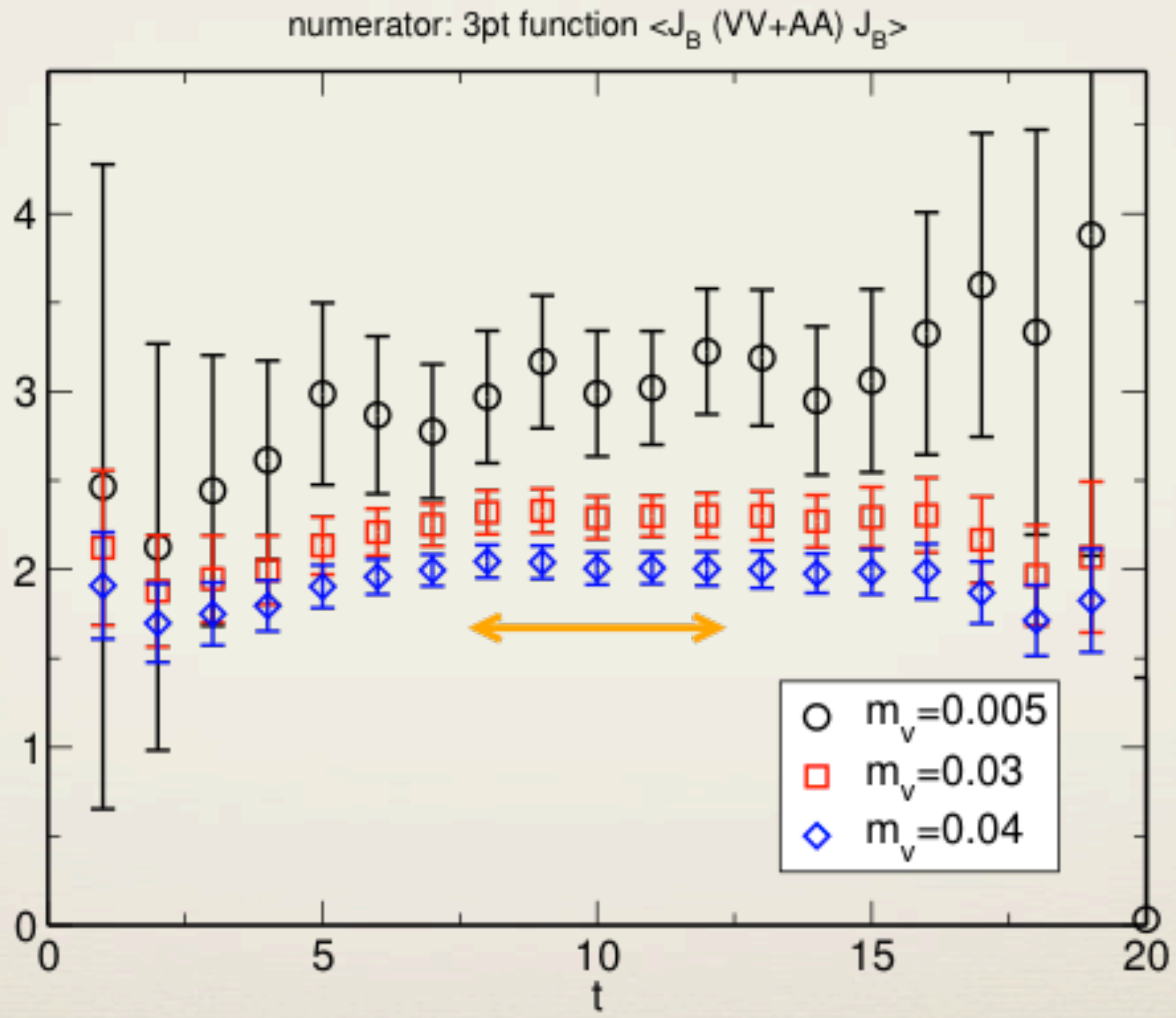


B-B mixing

- $\mathcal{M} = \langle \overline{B^0} | VV + AA | B^0 \rangle = \dfrac{\langle J^\dagger_{B^0}(t_{sink}) \cdot (VV + AA)(t) \cdot J^\dagger_{B^0}(t_{src}) \rangle}{\sqrt{\langle J^\dagger_{B^0}(t_{sink}) J_{B^0}(t) \rangle \cdot \langle J_{B^0}(t) J^\dagger_{B^0}(t_{src}) \rangle}} e^{m^*(t_{sink} - t_{src})/2}$

- with smeared source and smeared still sink



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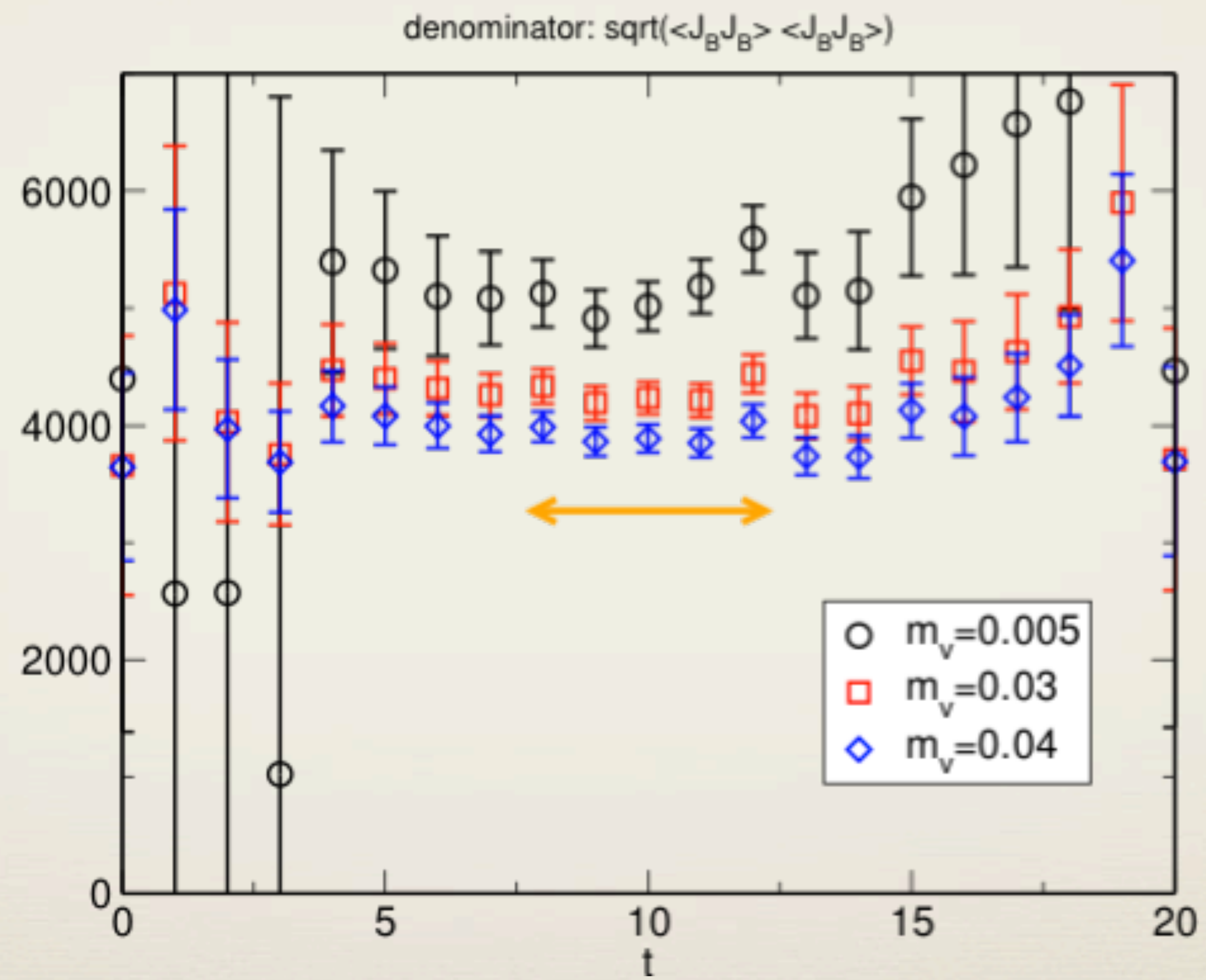

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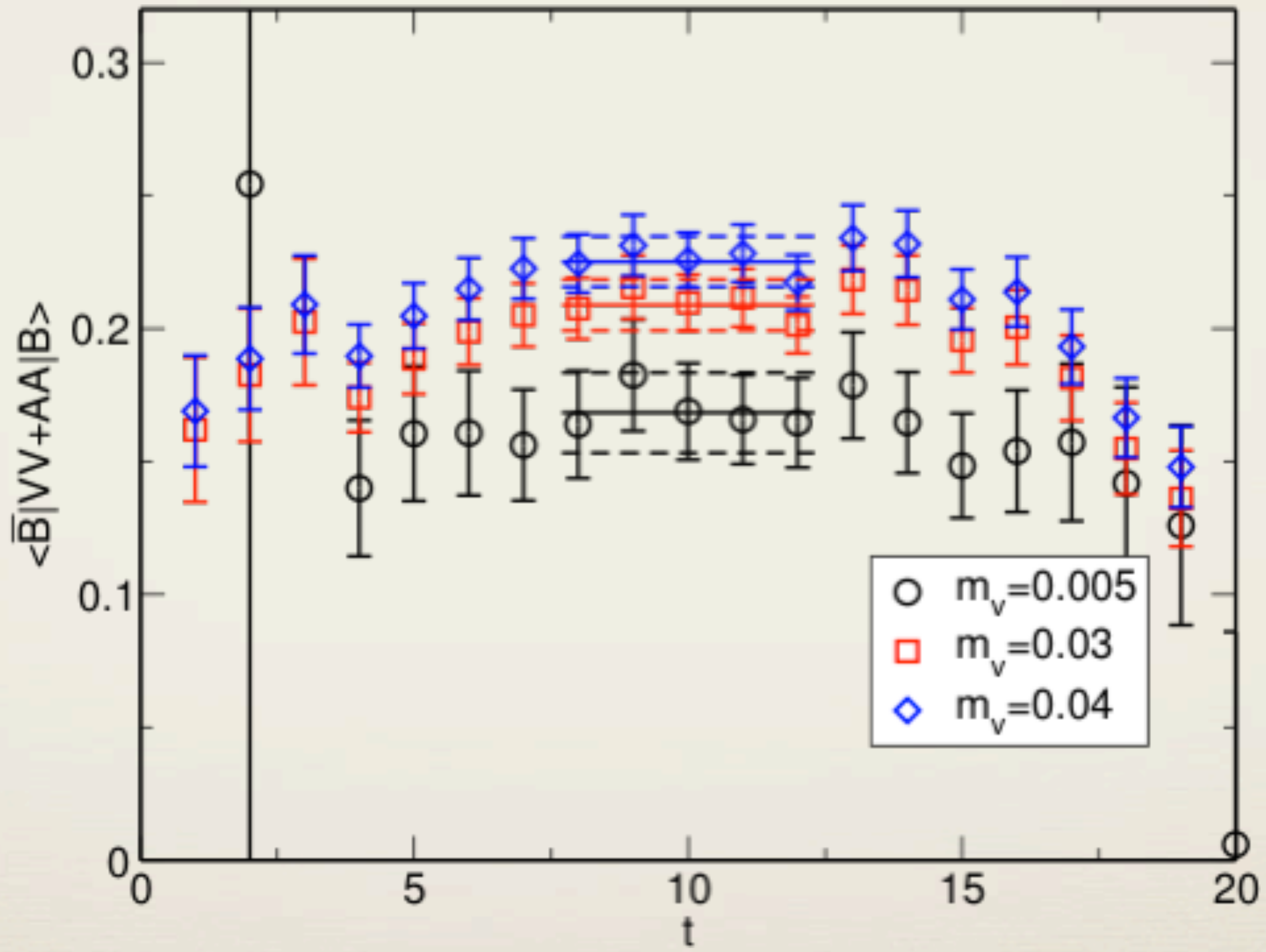

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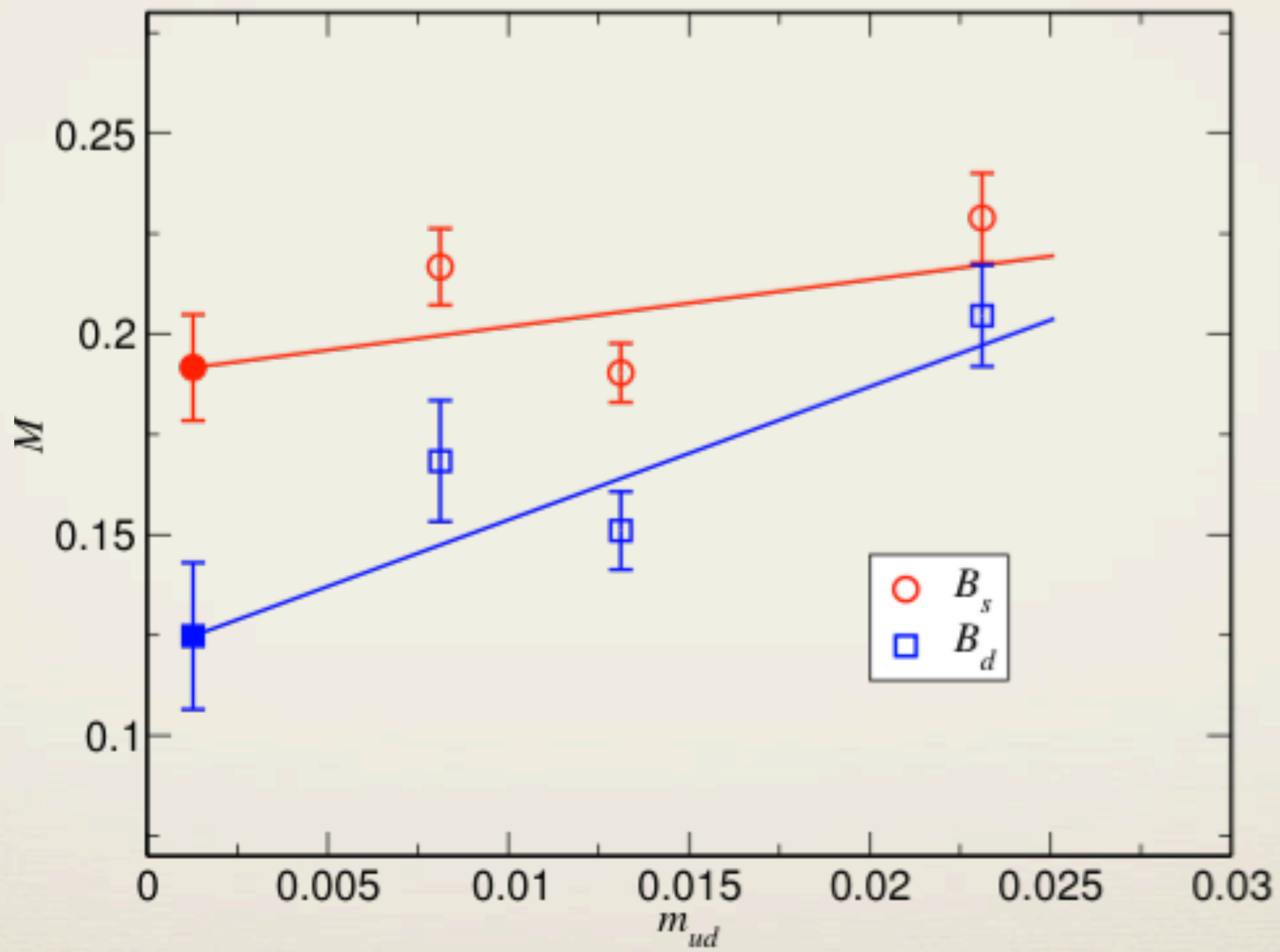
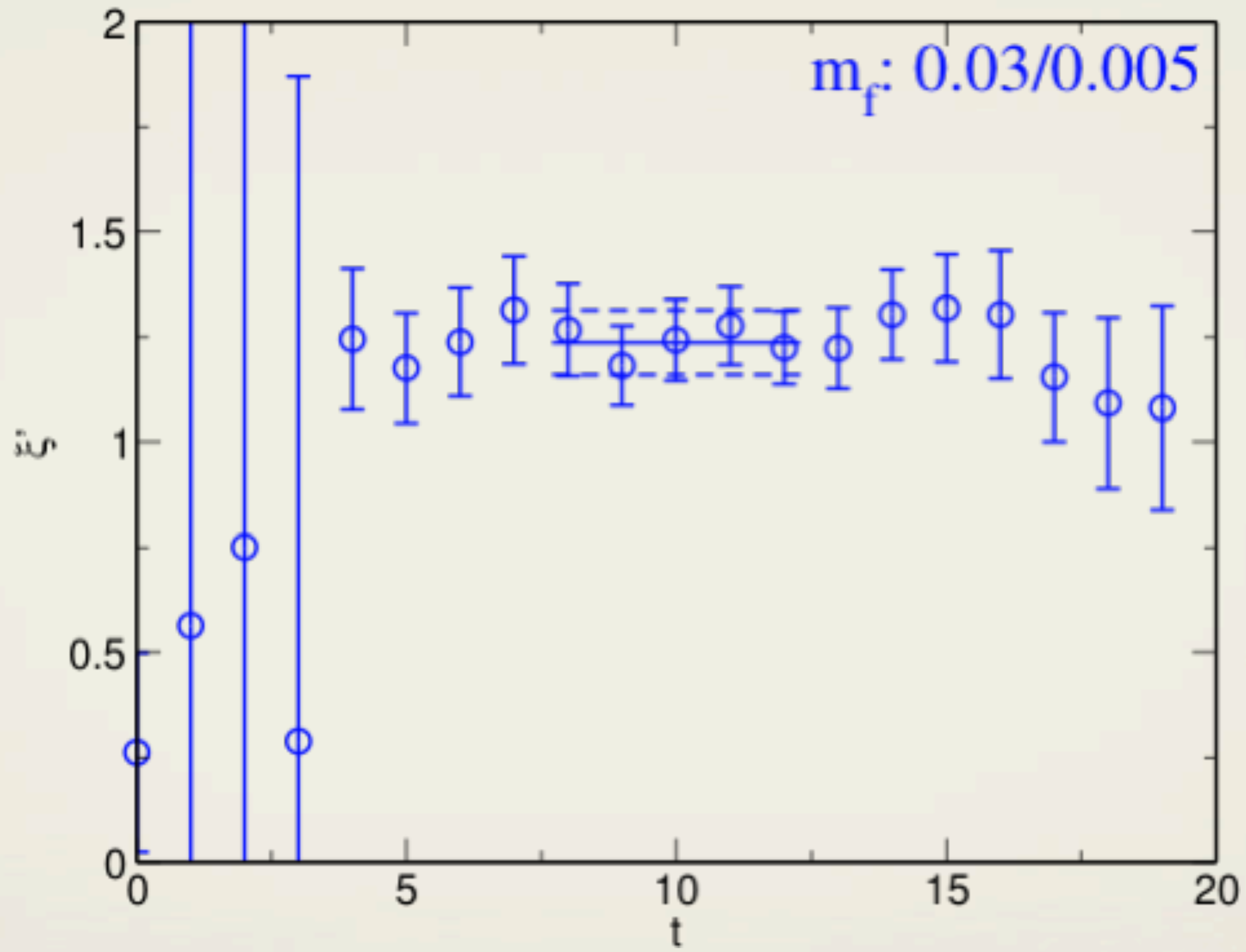


ξ from double ratio



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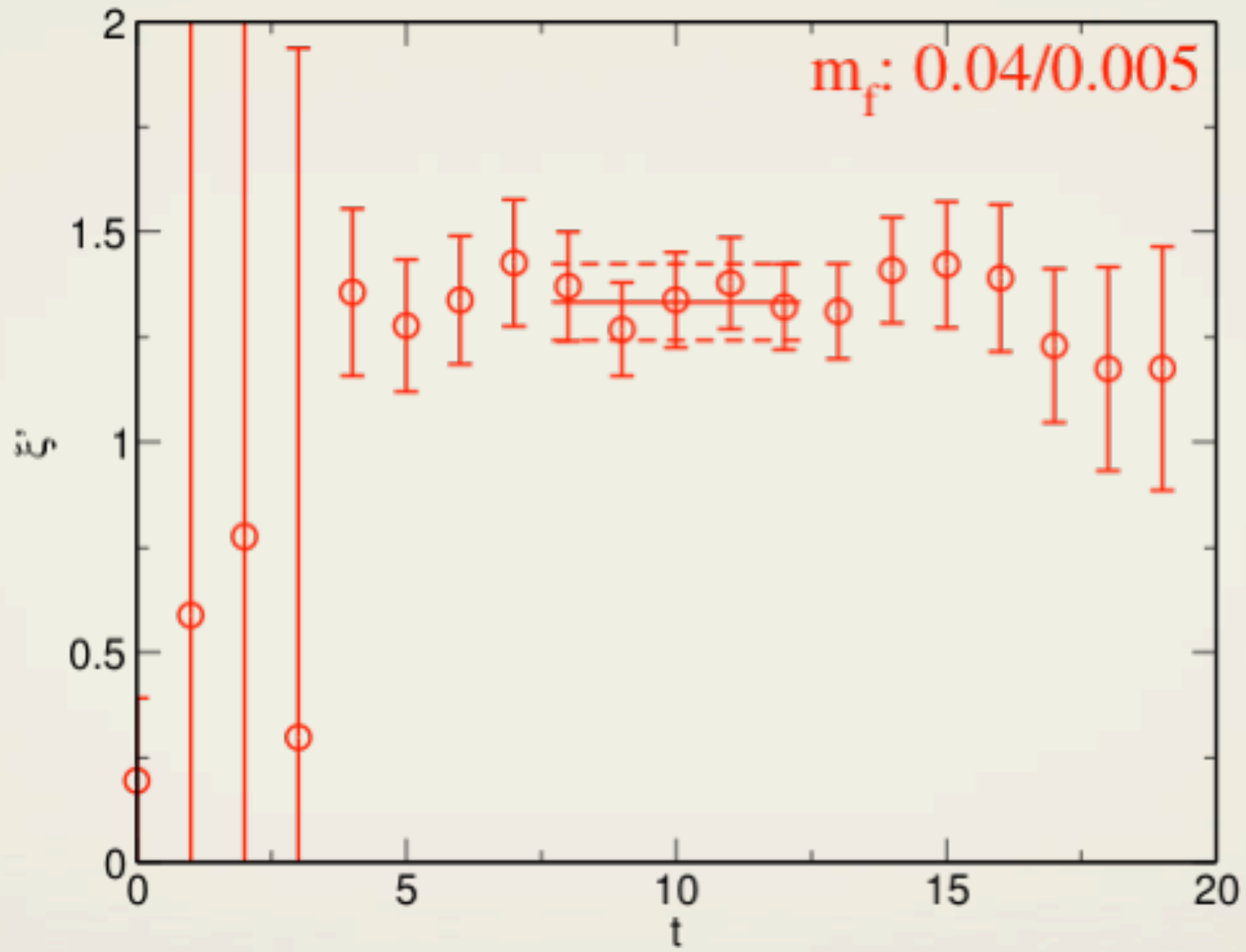

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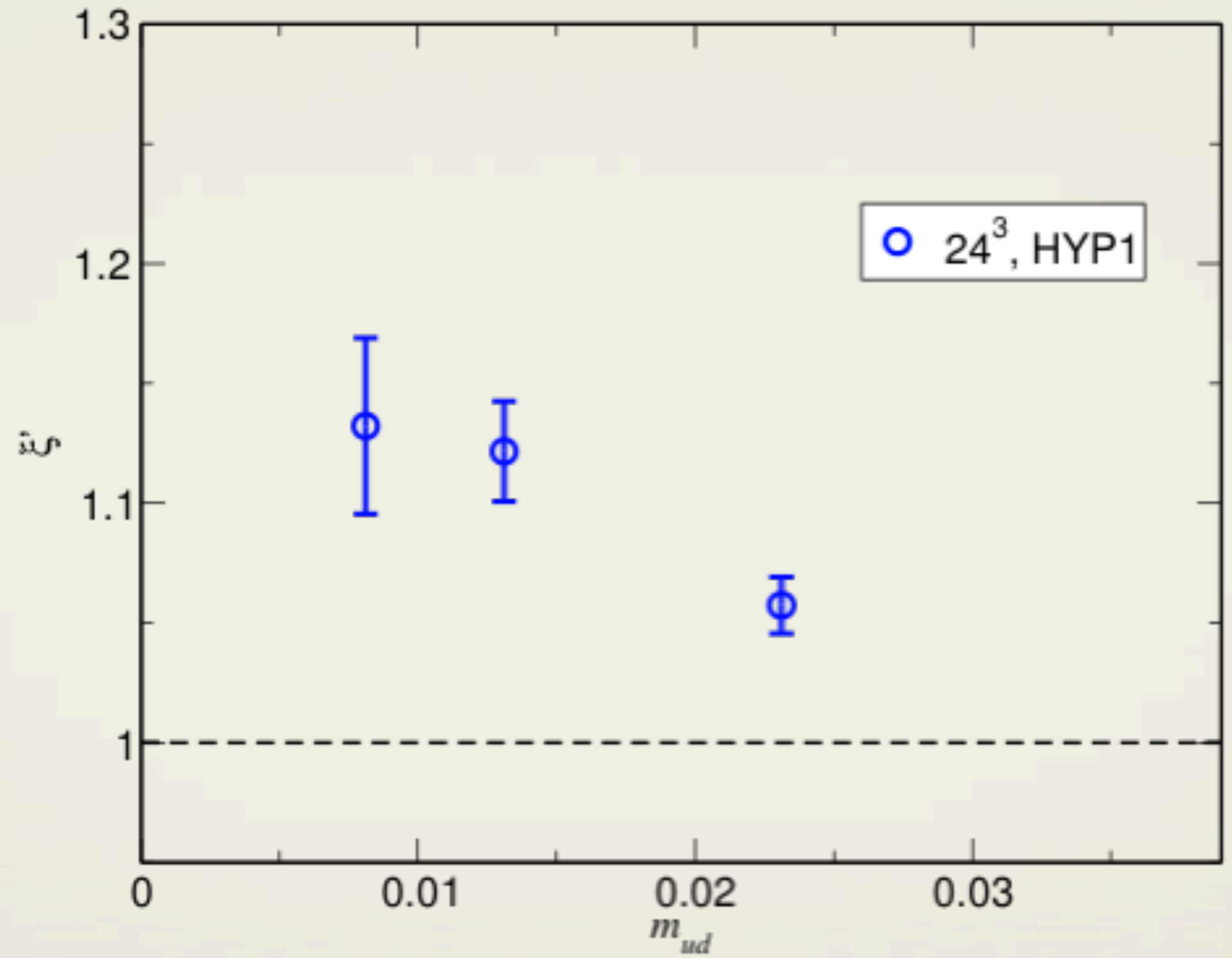

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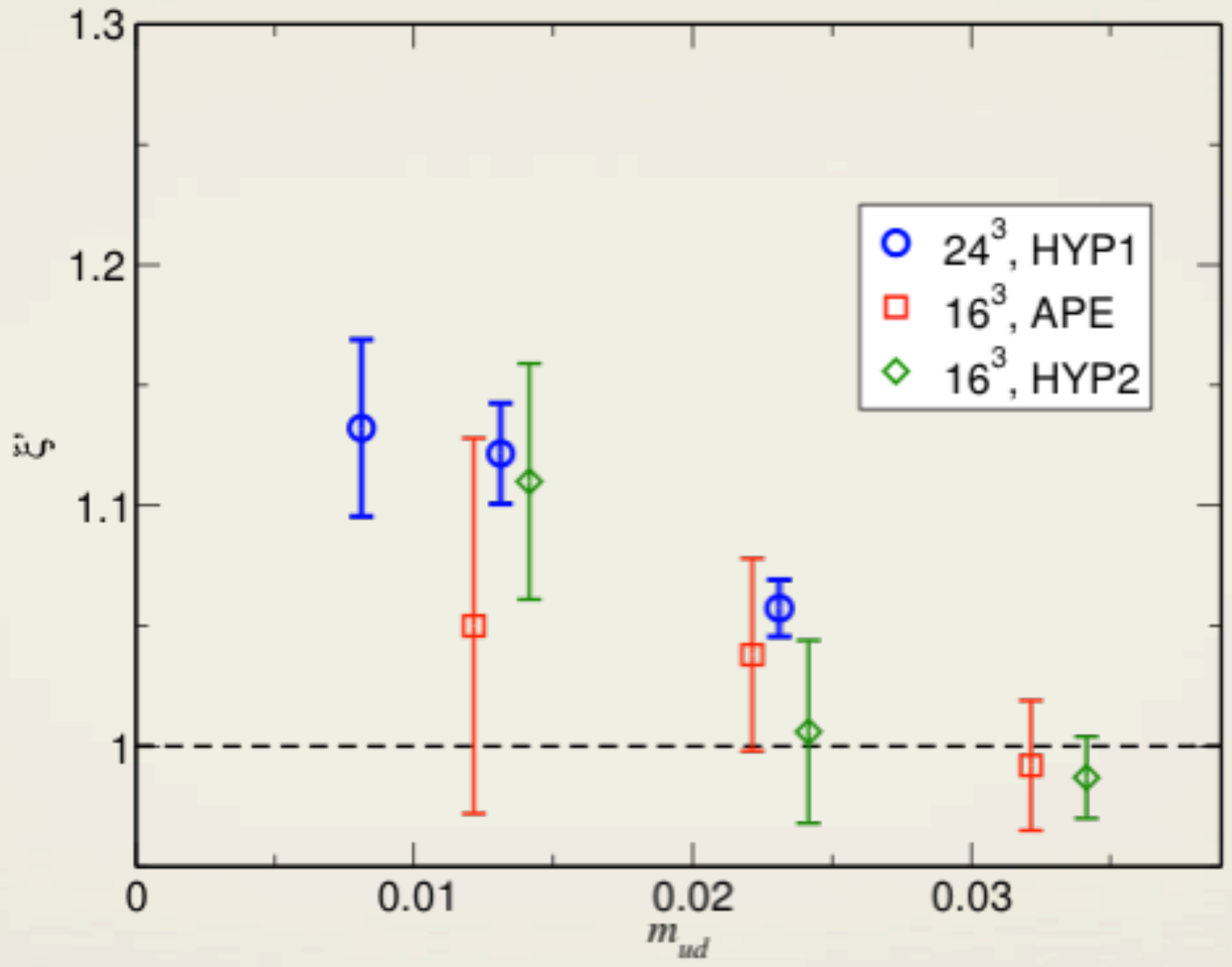

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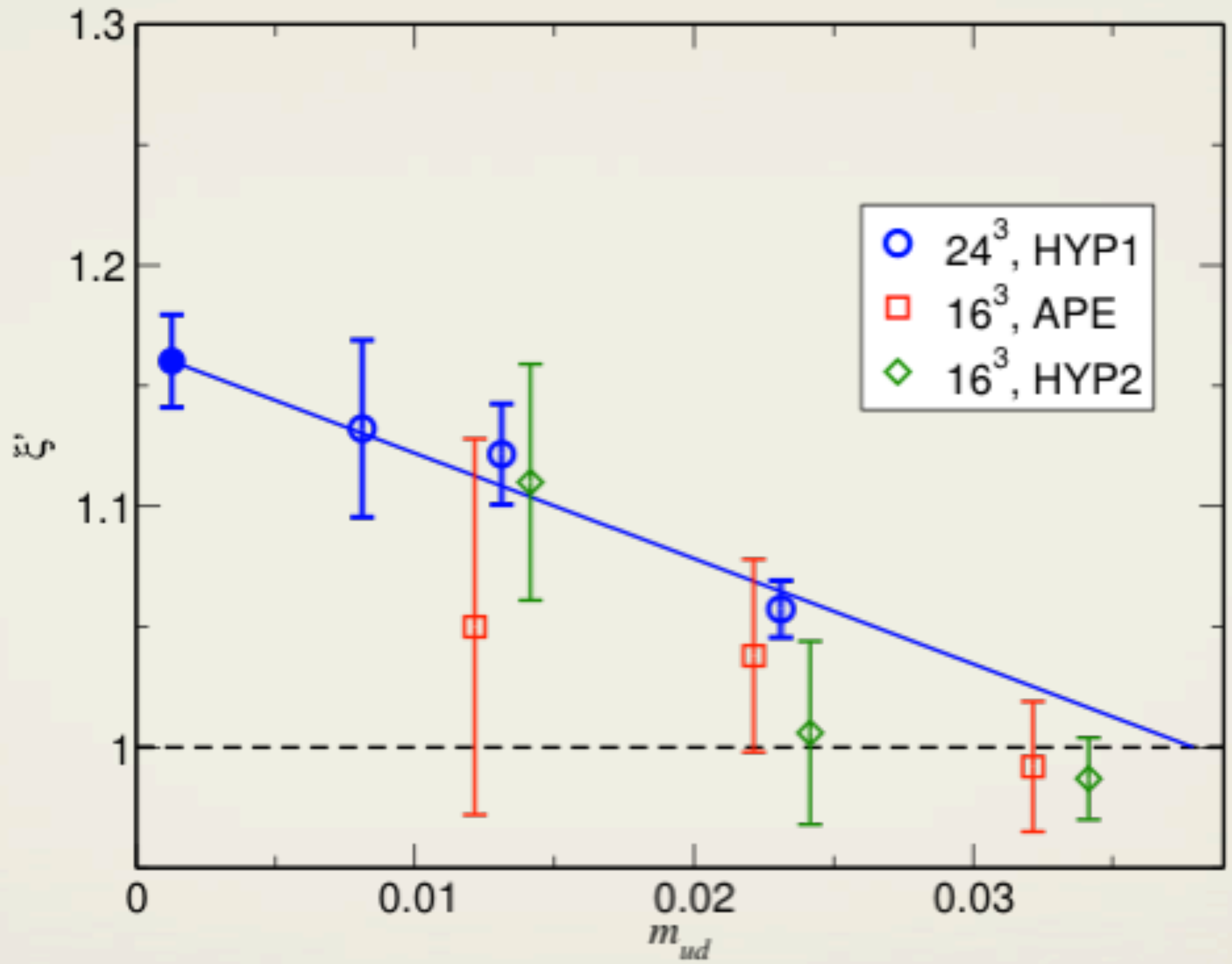

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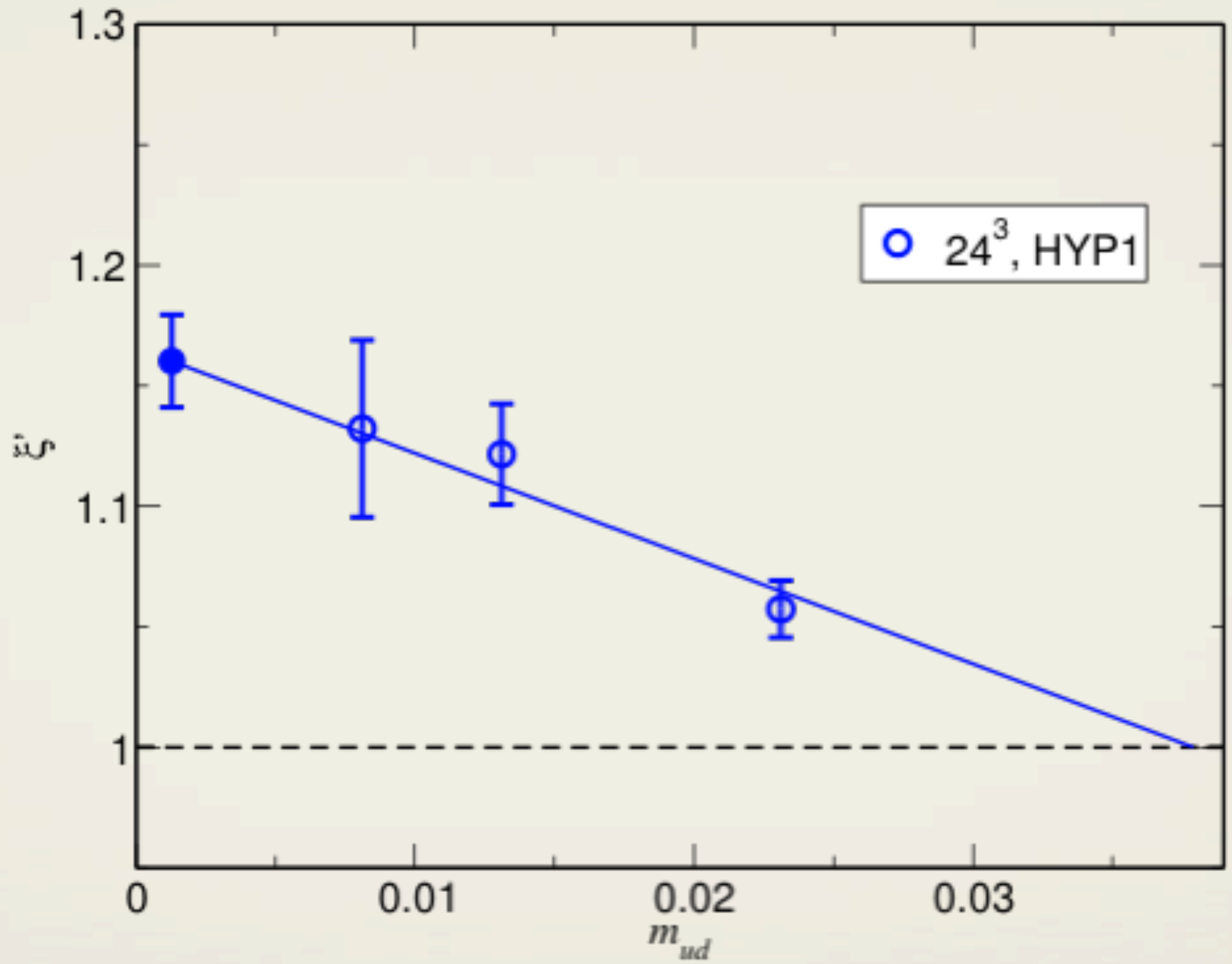

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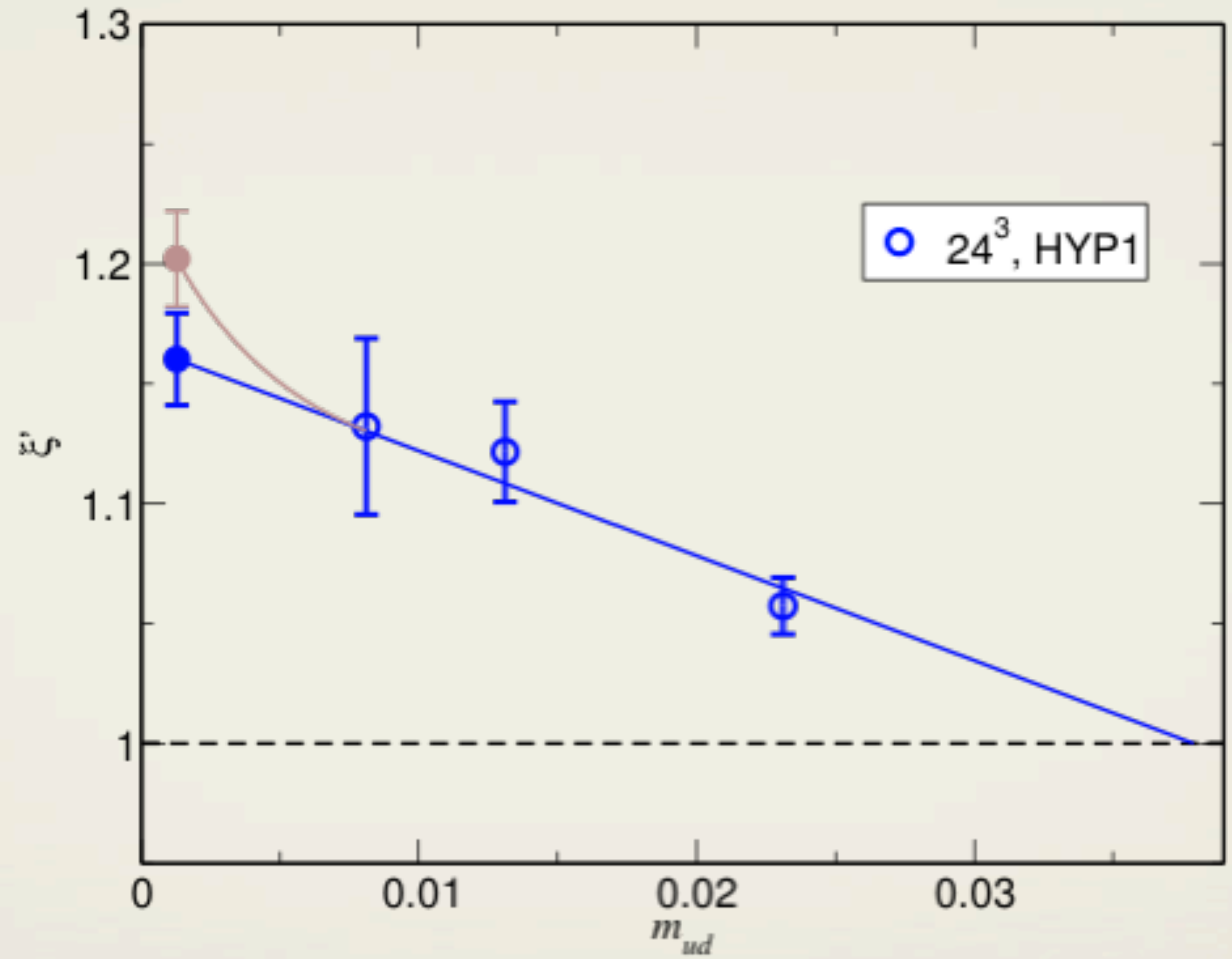

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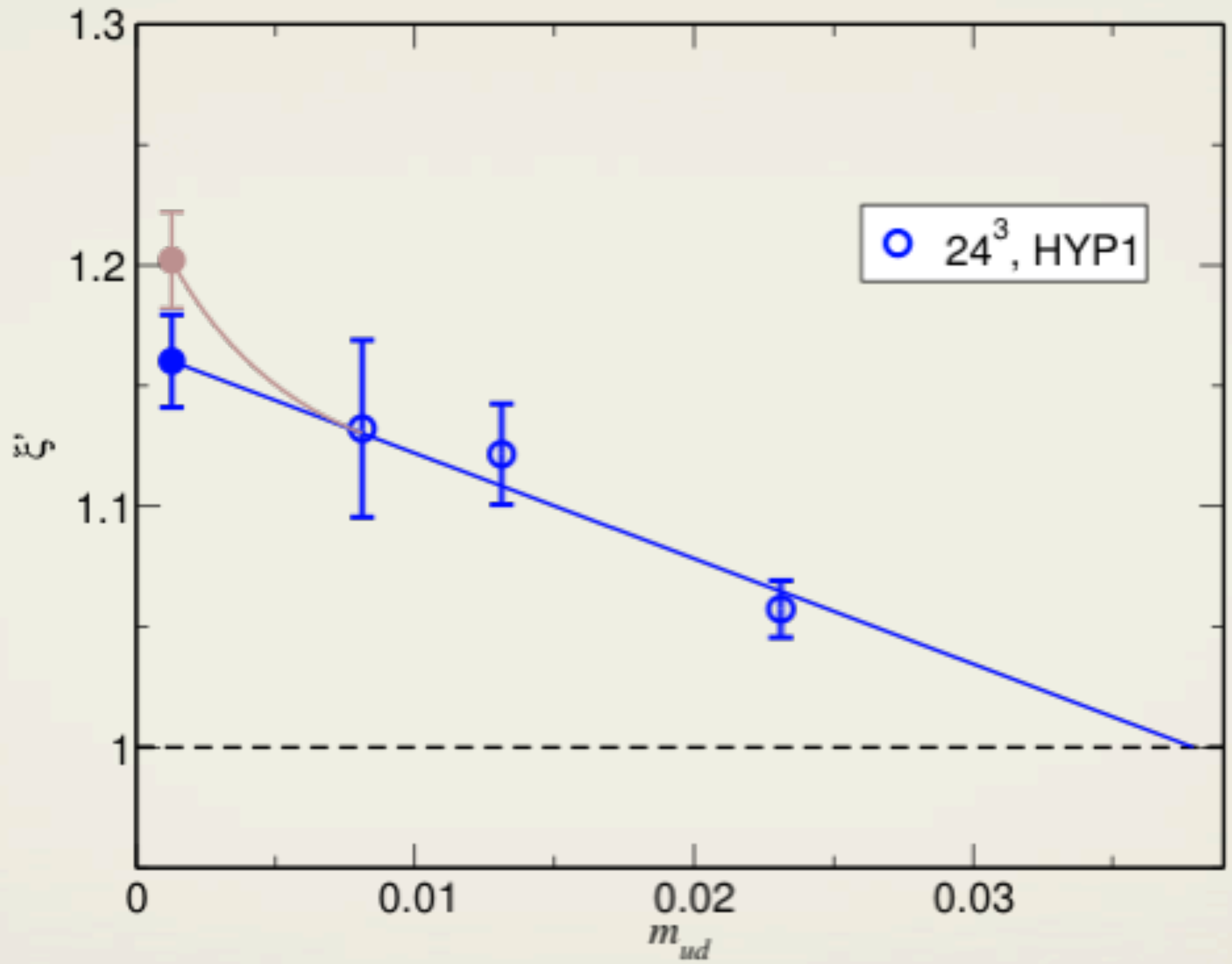

* O(a) yet to be improved

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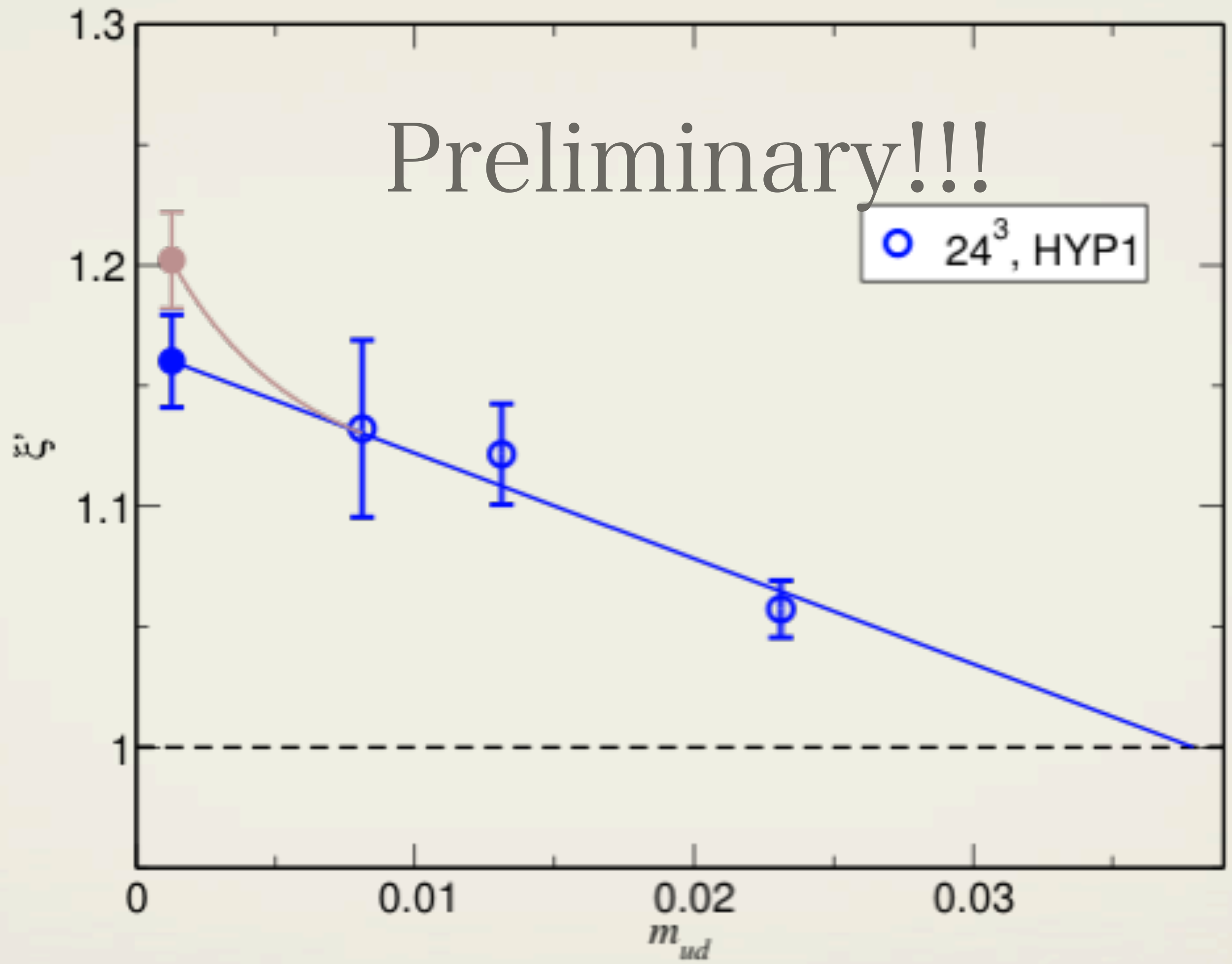

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error budget in ξ

error budget in ξ

	ξ (16^3) APE HYP2	ξ (24^3) HYP1 preliminary!!!
statistics	6% 5%	2%
chiral extrapolation	7%	4%
uncertainty in $g_{B^*B\pi}$	2%	
discretization error	4%	
renormalization factors	2%	
scale and quark mass uncertainties	1%	
finite volume error	1%	
$1/m_b$ corrections	2%	
total systematics	9%	

error budget in ξ

	ξ (16^3) APE HYP2	ξ (24^3) HYP1 preliminary!!!	ξ (24^3) HYP2 being generated	ξ (32^3) HYP1 in preparation	ξ (32^3) HYP2 in preparation
statistics	6% 5%	2%			
chiral extrapolation	7%	4%	make use of partially quenched points		
uncertainty in $g_{B^*B\pi}$	2%	calculate it or use $g_{B^*B\pi}$ from DWF+RHQ [P. Fritzsch (this morning)]			
discretization error	4%	O(a) improvement and continuum limit			
renormalization factors	2%				
scale and quark mass uncertainties	1%				
finite volume error	1%	1.5 x physical size: much smaller			
$1/m_b$ corrections	2%				
total systematics	9%	5% in our scope			

renormalization and O(a)

* all one loop renormalization factors and the coefficients of O(a) operators have been worked out
    * by matching matrix elements with on-shell light and heavy quark states
    * O(a): g^2pa, g^2ma: both had contribution of same order for ξ
    * T. Ishilawa @ Lattice 2008, T. Izubuchi @ Lattice 2009
    * will be incorporated in the analysis
* non-perturbative renormalization: RI/MOM type
    * T. Izubuchi @ Lattice 2009
    * will be investigated further
    * more important for individual matrix elements

Summary and outlook

- Summary
  - 2+1 flavor Iwasaki+DWF and static quarks were used for B mesons
  - tuning of the smearing and source-sink separation brought a dramatic improvement in the statistical error
  - lighter ud mass is helping reduce the systematic error of chiral extrapolation
- Outlook
  - partially quenched points will be incorporated in the chiral fits
    - various fits should be tested: SU(3), SU(2), analytic,..
  - $O(g^2a)$ improvement / 2 different heavy quark actions / continuum limit will help reduce the discretization errors.
  - total error of ξ = 5% is within reach
  - NPR will be investigated

THANK YOU