# Renormalized Polyakov loop in the Fixed Scale Approach

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# Introduction

- Polyakov loop $L(\vec{x})$ — Deconfinement Order Parameter (Spontaneous Breaking of $Z(N)$) (McLerran & Svetitsky, PRD 1981)

- One hopes to construct effective theories (Pisarski, PRD 2006) of $L$ for investigations of deconfinement phase transitions and many models employ $L$.

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- On an Euclidean $N_\sigma^3 \times N_\tau$ lattice $L(\vec{x})$ is defined at a site $\vec{x}$ as $L(\vec{x}) = \frac{1}{N_c}\ \mathrm{Tr}\ \Pi_{x_0=1}^{N_\tau}\ U^4(\vec{x}, x_0)$.

- No SSB on finite lattices/volumes. Usually one defines $\bar{L} = \sum_{\vec{x}} L(\vec{x})/N_\sigma^3$, and employs $\langle|\bar{L}|\rangle$, or its susceptibility, to locate the deonfinement phase transition.

- $\langle|\bar{L}|\rangle \rightarrow 0$ as 1/Volume in the confined phase, and $\langle|\bar{L}|\rangle \neq 0$ in the deconfined phase.

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- Like any Wilson loop, Polyakov loop needs to be renormalized.

- More so, since as an order parameter it seeks to label phases by being zero or nonzero.

# Earlier Work

♣ The physical interpretation of $L$ as related to the free energy of a single static quark offers a clue.

♠ The single quark free energy $F_b(N_\tau, a)$ is obtained from $\ln\langle|\bar{L}|\rangle = -F_b(T)/T = -aN_\tau F_b(N_\tau, a)$ .

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- Use of $N_\tau$-grids and fits to $L$ (Dumitru et al. PRD 2004)
- Use of renormalization group iteratively (S. Gupta et al. PRD 2008)

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♡ Let $\beta_c$, corresponding to the position of the peak of the $|L|$-susceptibility for some fixed $N_{\tau,c}$, be the choice of the fixed scale $a_c$.

♠ Further, let it lie in the scaling region, then in the fixed scale approach $T/T_c = N_{\tau,c}/N_\tau$.

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♡ Let $\beta_c$, corresponding to the position of the peak of the $|L|$-susceptibility for some fixed $N_{\tau,c}$, be the choice of the fixed scale $a_c$.

♠ Further, let it lie in the scaling region, then in the fixed scale approach $T/T_c = N_{\tau,c}/N_\tau$.

♠ Write the single quark free energy as a sum of a would-be divergent and a regular contribution,

$F_b(T, a_c) = F(T, a_c) - A(a_c)$,

where $A$ is the would-be divergent free energy in physical units.

♠ Since

$$\frac{T}{T_c}\ln\langle|\bar{L}|\rangle = -\frac{F(T,a_c)}{T_c} + \frac{A(a_c)}{T_c} \ ,$$

the free energy at any two different scales, $a_{c1}$ and $a_{c2}$, differs by the *same* constant at all $T$.

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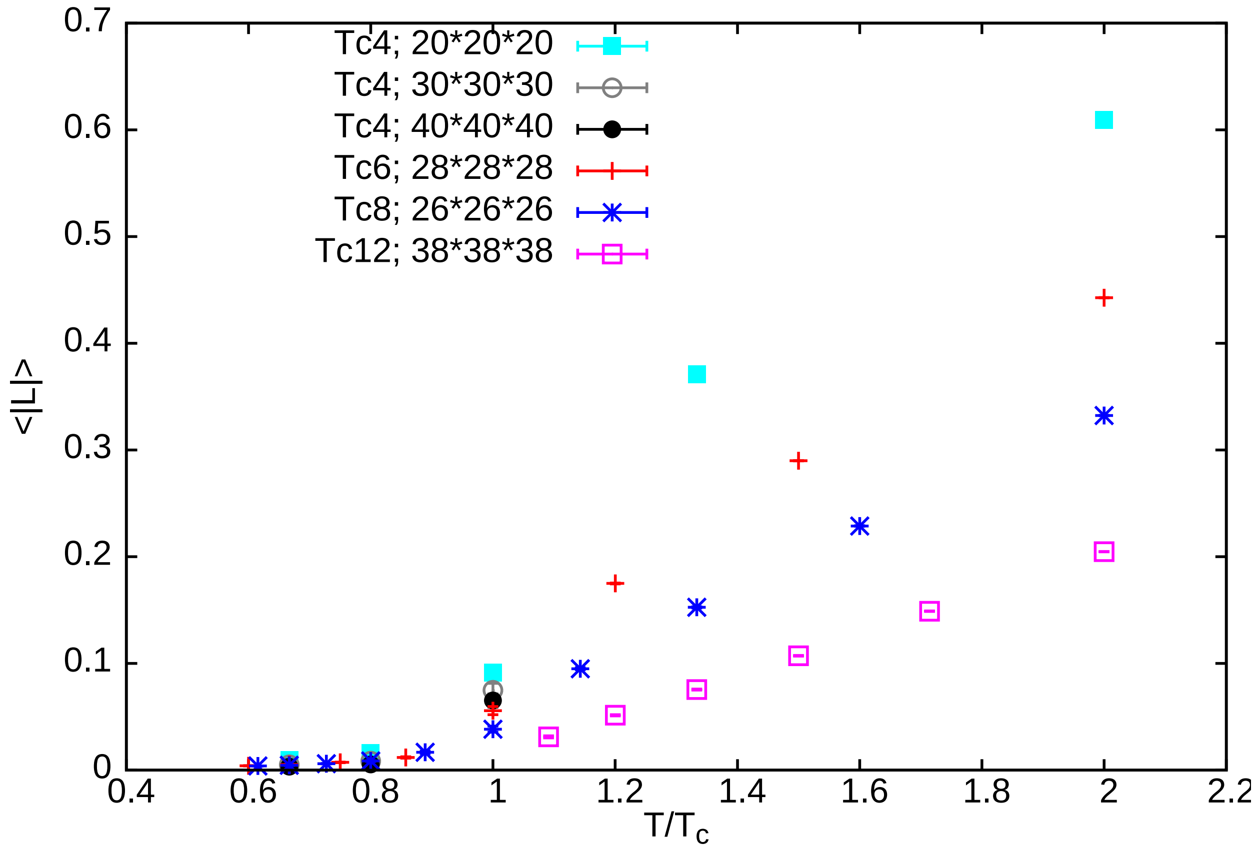
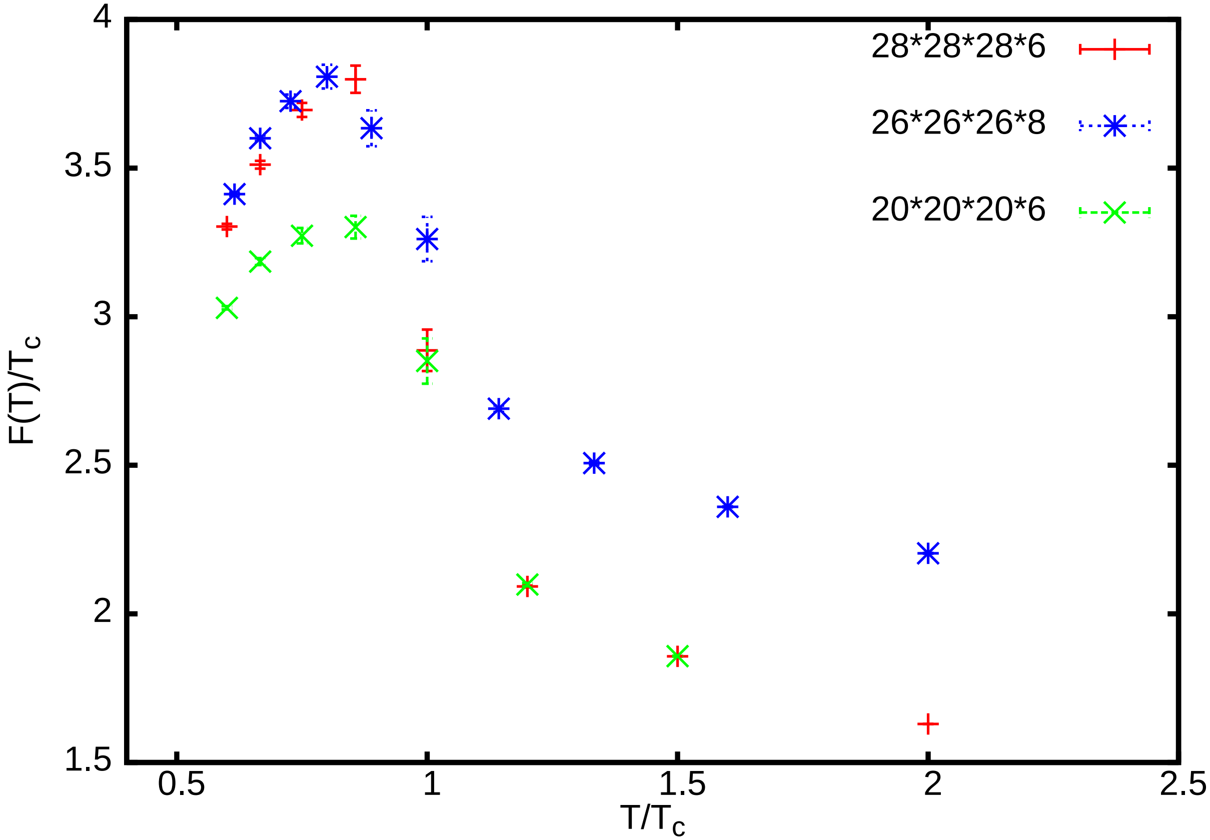
♣ In the following, I consider the simple case of $SU(2)$ to demonstrate how well it works. It should work similarly for any $N_c$ or QCD.

♡ I employ the critical $\beta$ for $N_\tau = 4$, 6, 8 and 12 from the table of Velytsky, IJMP C19, (2008), 1079, which agree with earlier results where available.

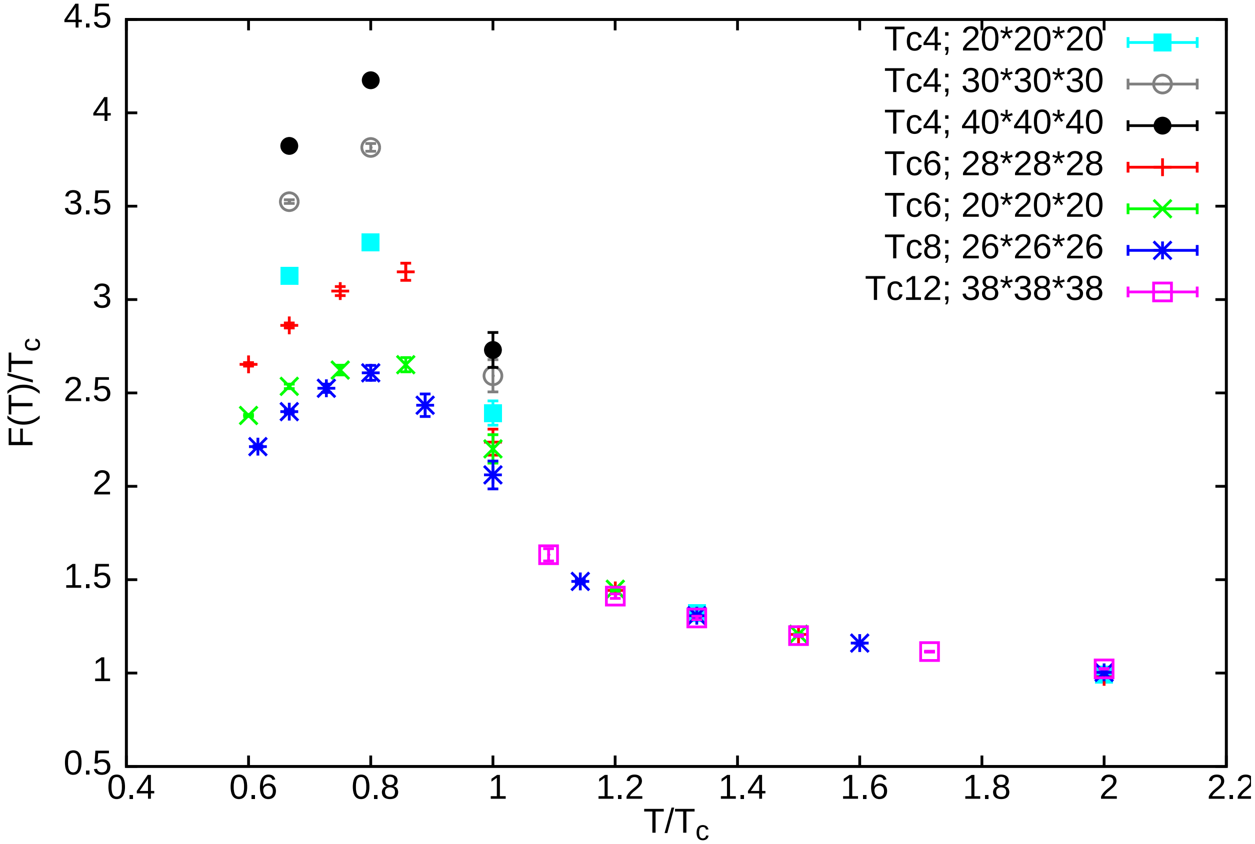
# Results for $SU(2)$



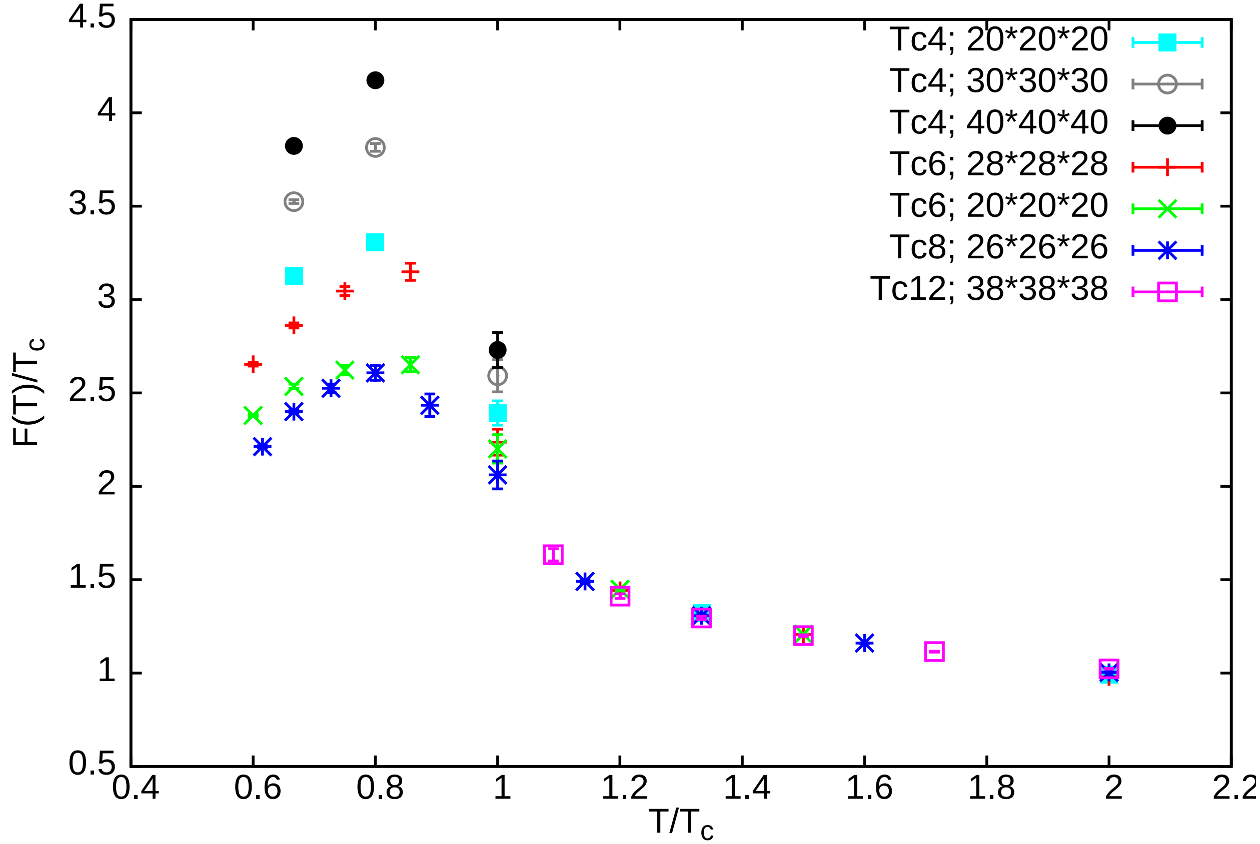

- 4 different scales : Tc4, Tc6, Tc8 and Tc12 with $a \to 0$ progressively. Increasing Spatial Volume leads to decrease in $L$ for $T < T_c$.




- Illustrate for two scales : Different behaviour in $T$ for the Free Energy. Shift $F$ by $\Delta F(2T_c)$.

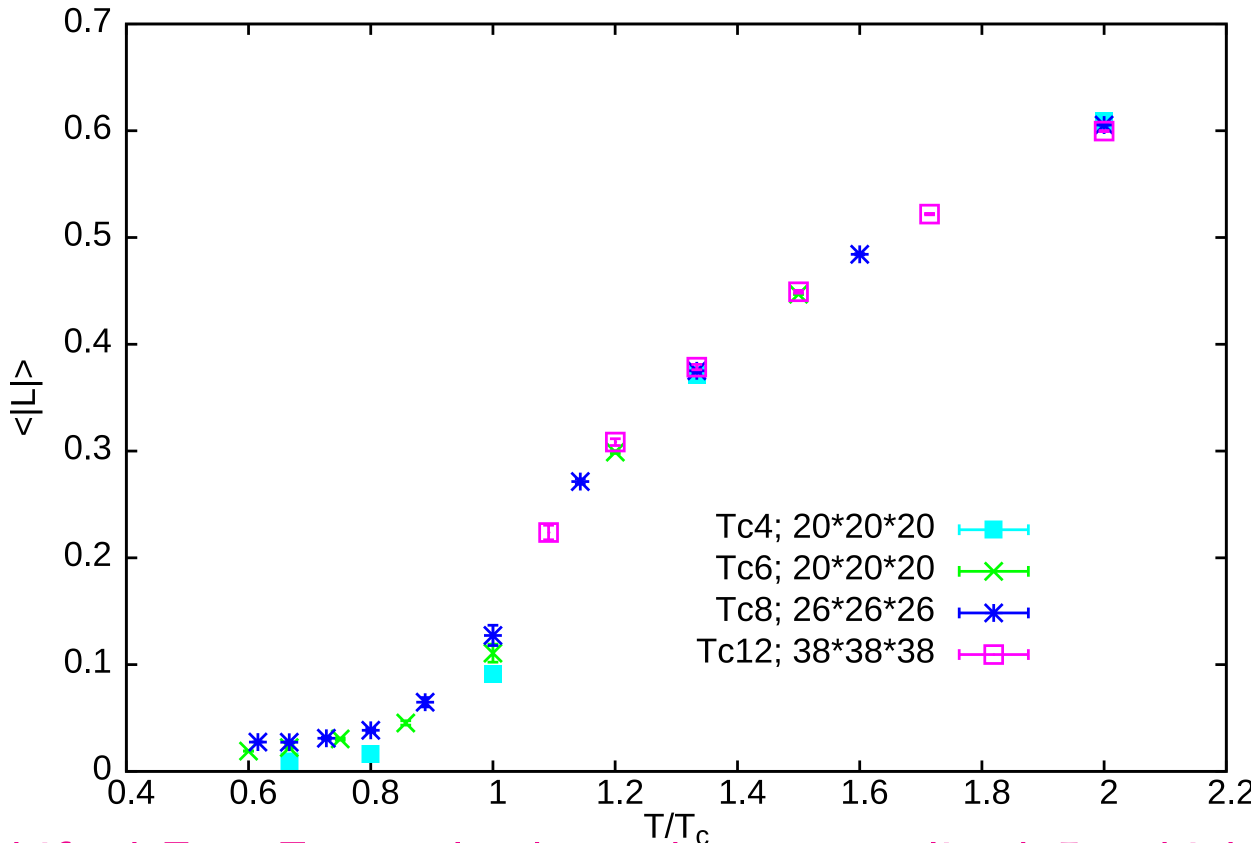



- Free Energy shifted by $\Delta F(2T_c)$ in each case: three constants for four scales.

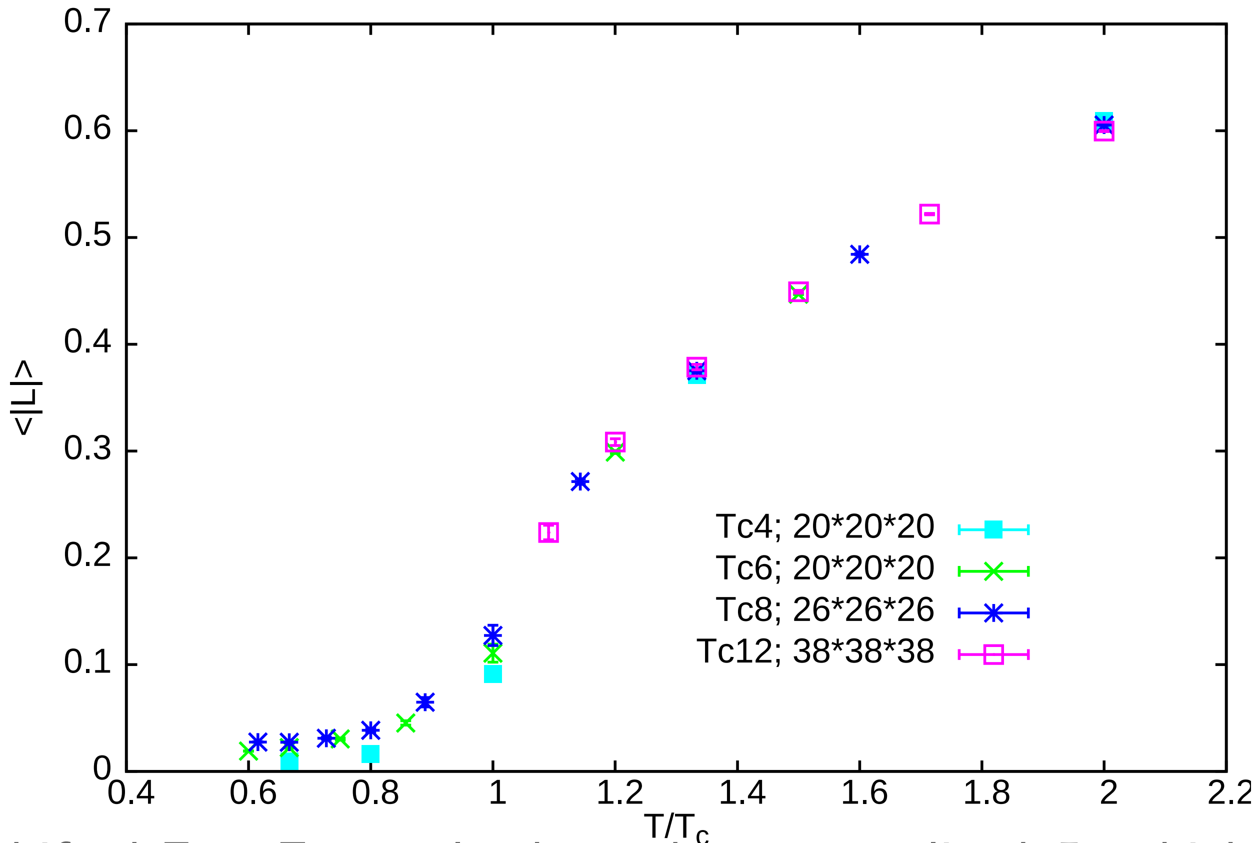



- Free Energy shifted by $\Delta F(2T_c)$ in each case: three constants for four scales.

- For $T \leq T_c$, $F$ increases with the spatial volume but scale-independent.




- The shifted Free Energy leads to the renormalized $L$, which is independent of cut-off for $\beta \geq 2.2991$.

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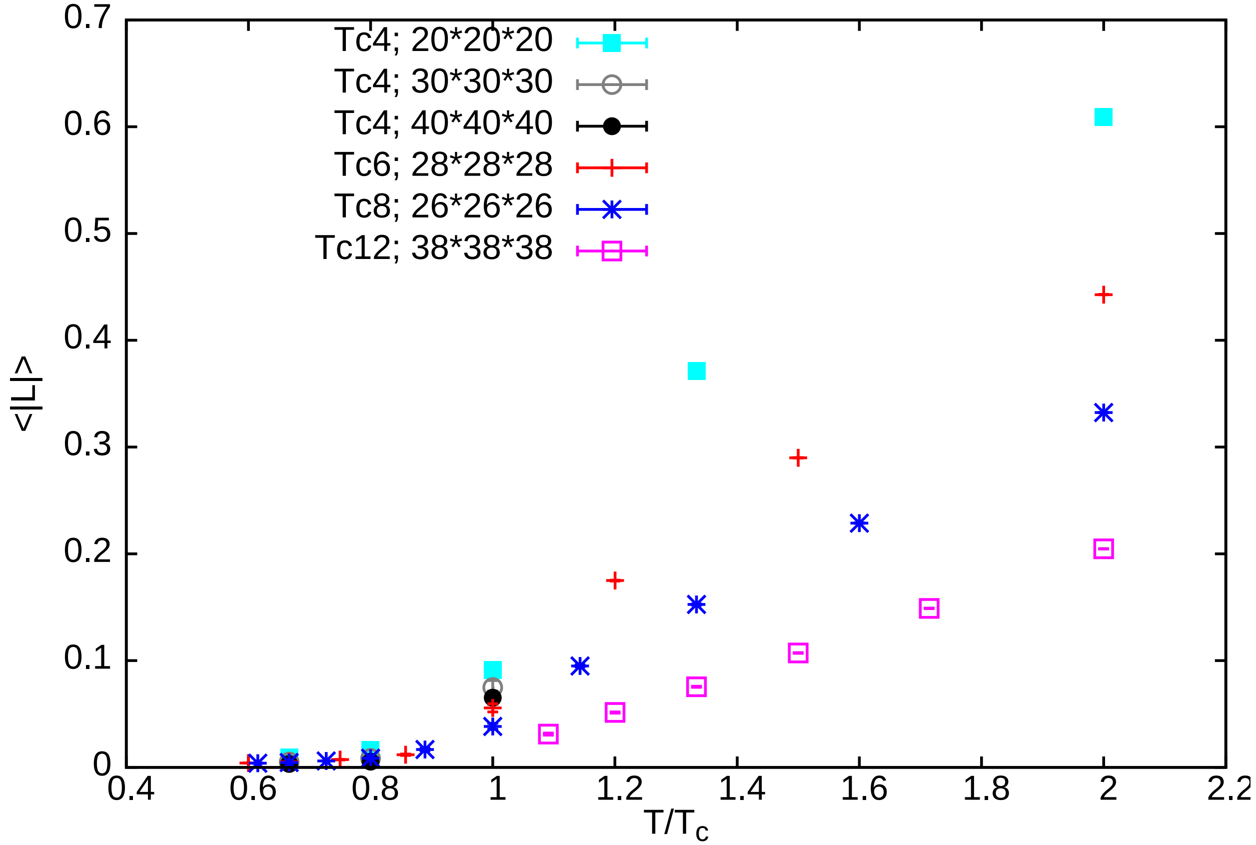


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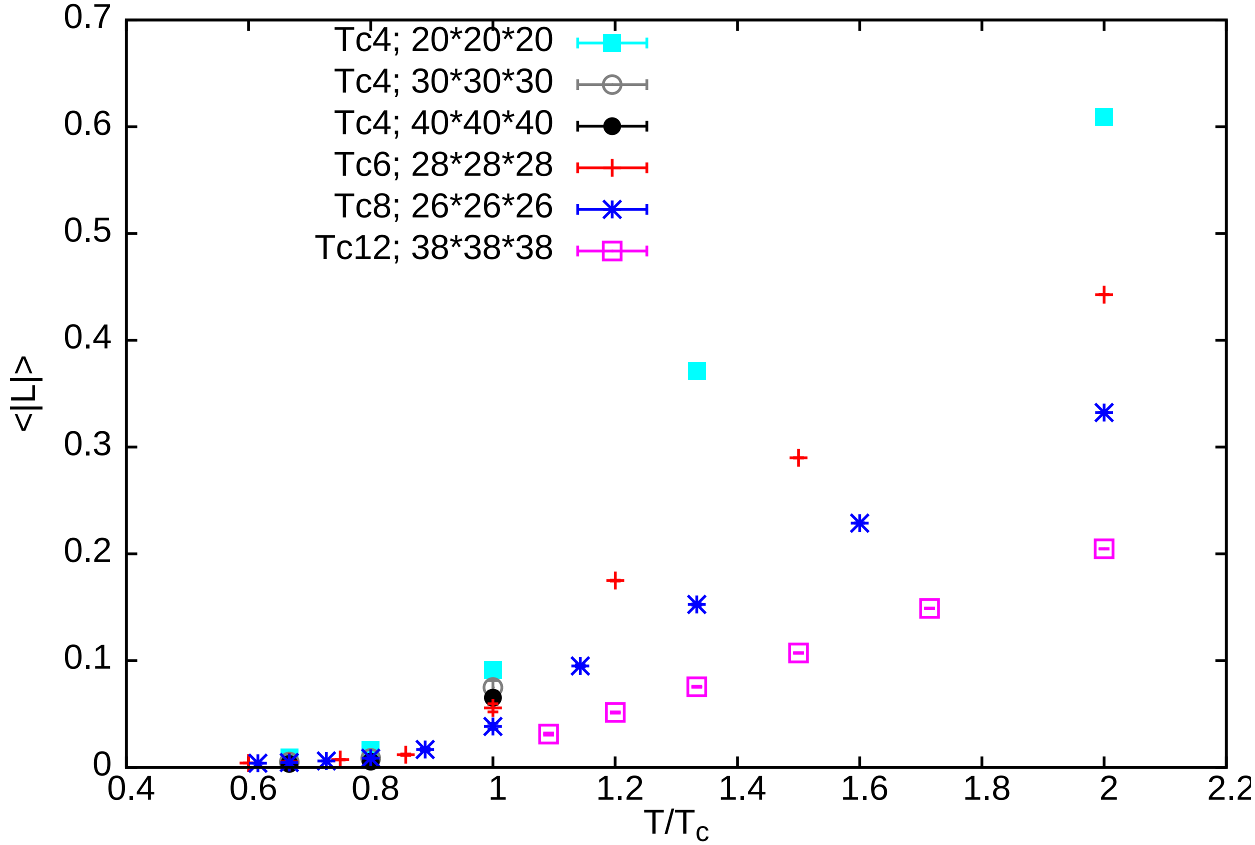


- I chose 3 constants to shift all the data to the Tc4 scale : The Tc6, Tc8, Tc12 results have simply jumped to their appropriate place on the $\langle |L| \rangle$ for it.
- Does the renormalized $L$ then climb to unity slowly?

- High Temperature Perturbation Theory (Gava-Jengo, PLB 1981) tells us that $L \longrightarrow 1$ from *above* at very large $T$ : $L = 1 + C_3 g^3 + \mathcal{O}(g^4)$, where $c_3(N_c) > 0$ is a constant.

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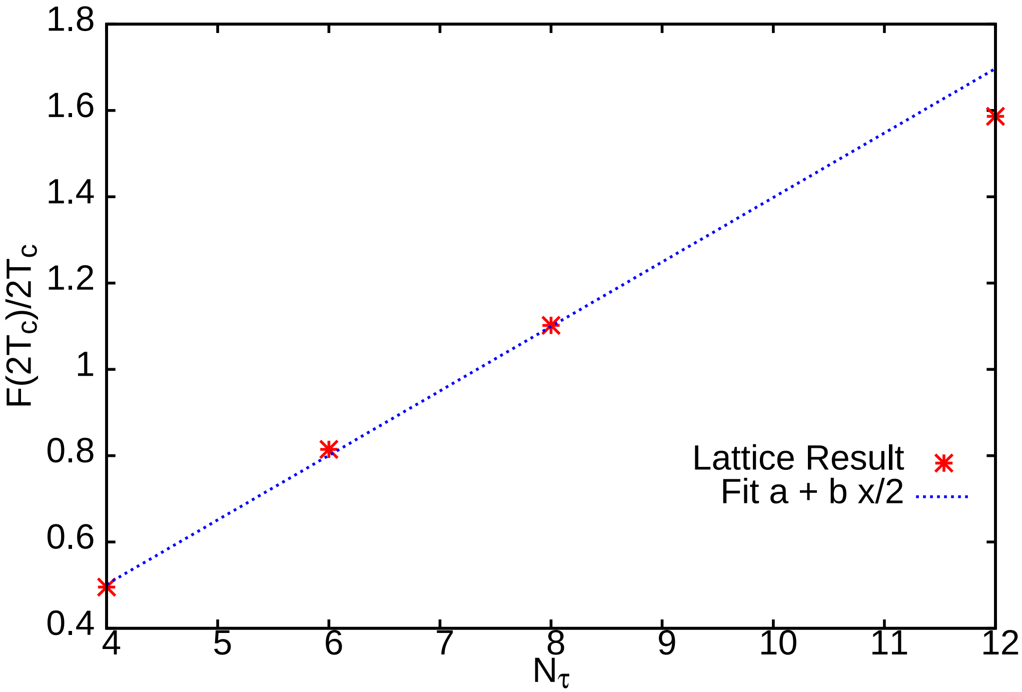
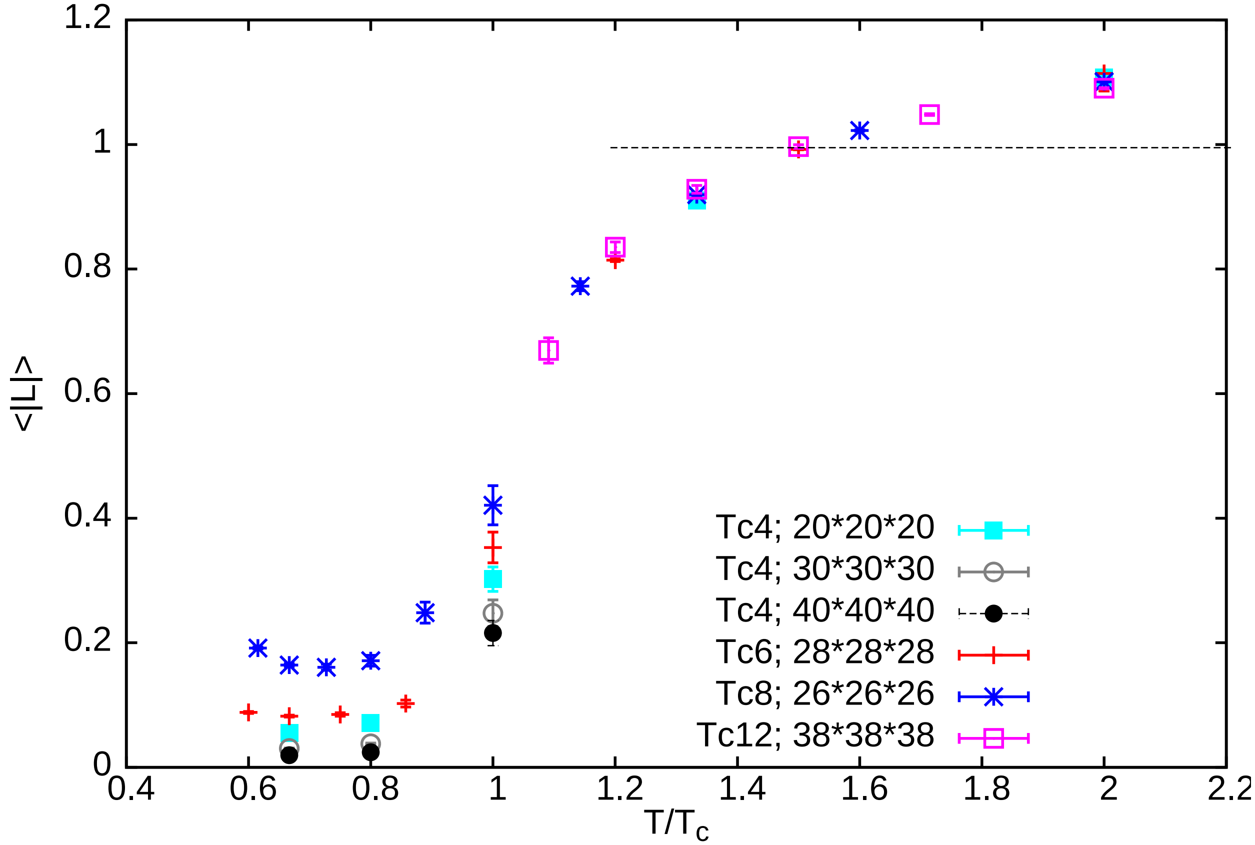
- In stead of shifts at $2T_c$ for varying scales, try a fit $-\ln\langle|\bar{L}_j|\rangle = F(2T_c)/2T_c + B \cdot N_{\tau j}/2$ .



♣ Eliminating the $B$-dependent divergent term for the Tc4-scale in addition to the shifts, one has,




♠ $L$ now does go to unity from above at large $T$. Large volumes, aspect ratio of $\sim 10$, needed for $L \simeq 0$ for low $T$.

# Summary

- I showed that the fixed scale approach leads to a natural definition of a physical, $N_\tau$-independent, order parameter which is defined in both the confined and the deconfined phases.

- It does not need two-point correlations, and works for even coarse lattices $(a \leq 1/4T_c)$.

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- I showed that the fixed scale approach leads to a natural definition of a physical, $N_\tau$-independent, order parameter which is defined in both the confined and the deconfined phases.

- It does not need two-point correlations, and works for even coarse lattices ($a \leq 1/4T_c$).

- The definition itself does not depend on any lattice artifacts or the lattice size in the deconfined phase.

- It displays the expected behaviour in both the phases, i.e., volume dependence in the low $T$-phase and approach to unity from above in high $T$-phase.