

# MCRG Minimal Walking Technicolor

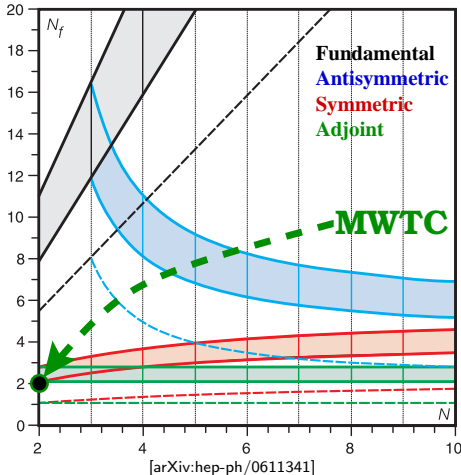
Liam Keegan

June 2010

Edinburgh University

Simon Catterall, Luigi Del Debbio, Joel Giedt

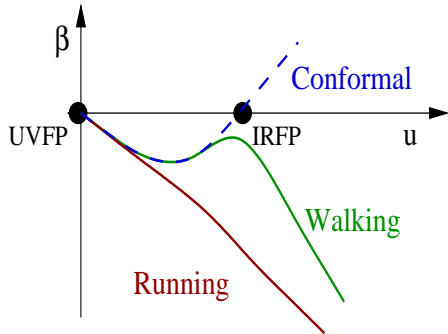
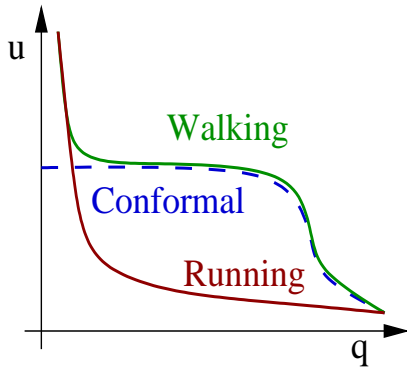
# Minimal Walking Technicolor



- Simplest interesting model: MWT
- 2 dirac fermions transforming under the adjoint representation of  $SU(2)$

Saninno, Tuominen  
[arXiv:hep-ph/0405209]

# Walking Technicolor Cartoon



# Scheme dependence

- Walking/Running of coupling is scheme dependent
- Want to measure physical, scheme independent quantities:
  - **Existence** of fixed point
  - **Anomalous mass dimension** at the fixed point

# Wilson Renormalisation Group



- Spatially average locally / integrate out UV modes
- Leaves IR physics intact
- Look at evolution of all couplings

$$\hat{\xi}^{(0)} , \{g_i^{(0)}\}$$

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$$\hat{\xi}^{(1)} = \hat{\xi}^{(0)} / 2 \quad , \quad \{g_i^{(1)}\}$$

# Wilson Renormalisation Group



- Spatially average locally / integrate out UV modes
- Leaves IR physics intact
- Look at evolution of all couplings

$$\hat{\xi}^{(2)} = \hat{\xi}^{(0)}/2^2 \quad , \quad \{g_i^{(2)}\}$$

# Wilson Renormalisation Group

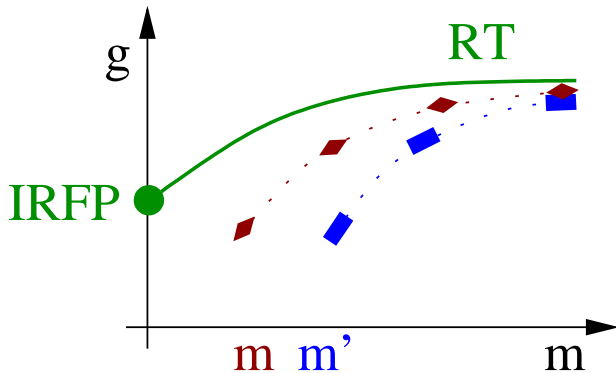


- Spatially average locally / integrate out UV modes
- Leaves IR physics intact
- Look at evolution of all couplings

$$\hat{\xi}^{(3)} = \hat{\xi}^{(0)} / 2^3 \quad , \quad \{g_i^{(3)}\}$$

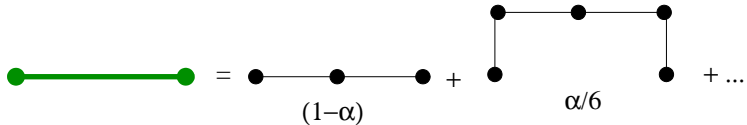


# Monte Carlo Renormalisation Group



- Match after  $n(n-1)$  steps
- $s = 2$  change in scale
- Step scaling of bare couplings

# Lattice Blocking Transform



- Free parameter  $\alpha$  adjusts RG blocking transform
- Optimise  $\alpha$  to approach RT quickly such that subsequent steps give the same matching

$$V_{n,\mu} = Proj \left[ (1 - \alpha) U_{n,\mu} U_{n+\mu,\mu} + \frac{\alpha}{6} \sum_{\nu \neq \mu} U_{n,\nu} U_{n+\nu,\mu} U_{n+\mu+\nu,\mu} U_{n+2\mu,\nu}^\dagger \right]$$

# MCRG Key Points

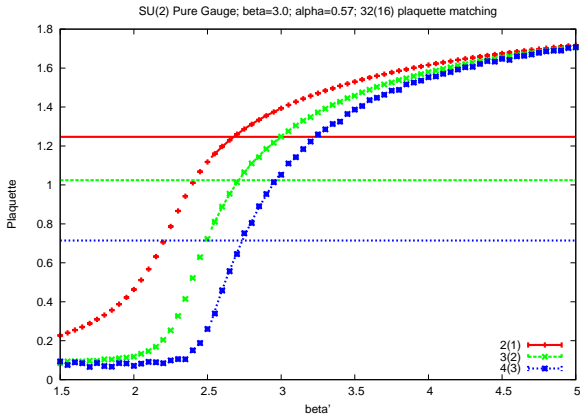
- Find pairs of couplings with identical actions, but whose correlation lengths differ by a factor 2
- Identify matching actions by comparing observables on blocked lattices (plaquette and 6-link loops)
- Always match between lattices with the same number of points to minimise finite size errors
- Optimise  $\alpha$  to approach RT quickly so that subsequent steps give the same matching

Hasenfratz [arXiv:hep-lat/0907.0919]

# Pure Gauge Simulation

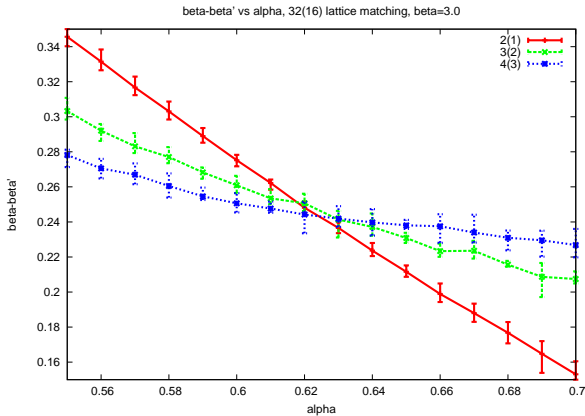
- Simulated on lattices of size  $L=32,16$
- Allows for 3 matchings; 2(1), 3(2), 4(3) steps on the  $32^4(16^4)$  lattices
- Optimise  $\alpha$  such that these steps predict the same matching coupling

# Plaquette Matching



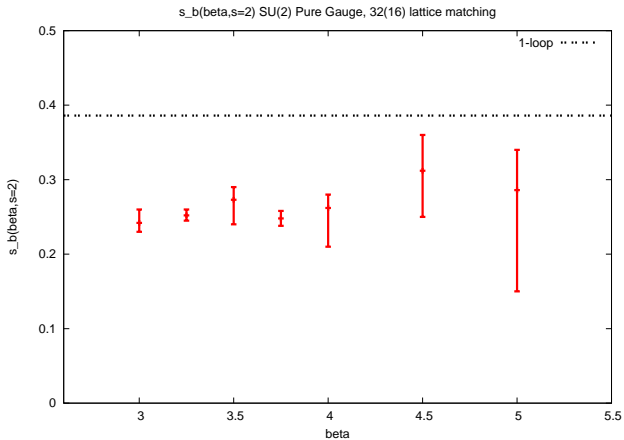
- 32(16) matching
- $\beta = 3.0$
- $\alpha = 0.57$

# Alpha Optimisation

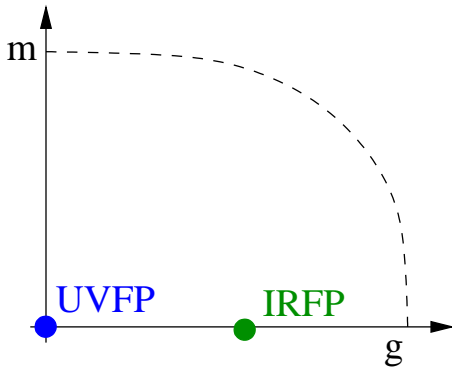


- $\beta = 3.0$
- $\alpha_{opt} \sim 0.63$
- $s_b = 0.24(2)$

# Pure Gauge Bare Step Scaling



# Phase diagram



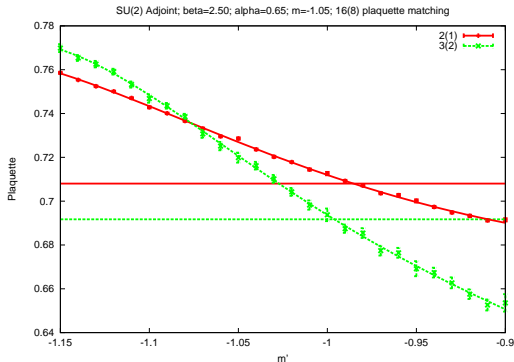
- UVFP: both  $m$  and  $g$  are relevant
- IRFP:  $m$  relevant,  $g$  irrelevant
- Near IRFP can match in  $m$ , value of  $g$  should be irrelevant



## Simulation details

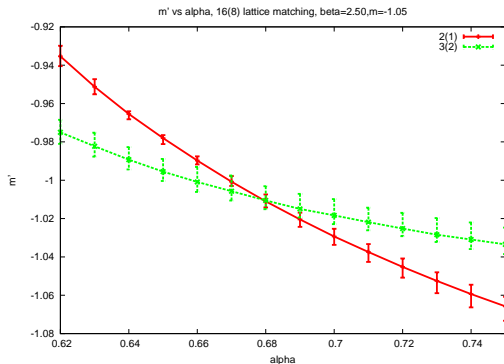
- Simulated on lattices of size  $L=16,8$
- Allows for 2 matchings; 2(1), 3(2) steps on the  $16^4(8^4)$  lattices
- Keep  $\beta$  constant, match in bare mass
- Optimise  $\alpha$  such that these all agree to find continuum physics

# Plaquette Matching



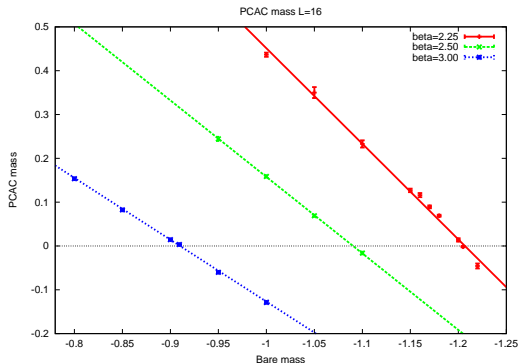
- $16^4$  blocked **two/three** times
- Single mass  $m = -1.05$
- $8^4$  blocked **one/two** times
- Many masses  $-1.15 < m' < -0.90$

# Alpha Optimisation



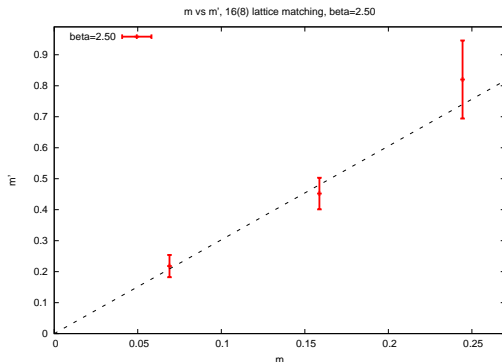
- $\alpha_{opt} \sim 0.68$
- $m = -1.05$
- $m' = -1.01(2)$

# PCAC Masses



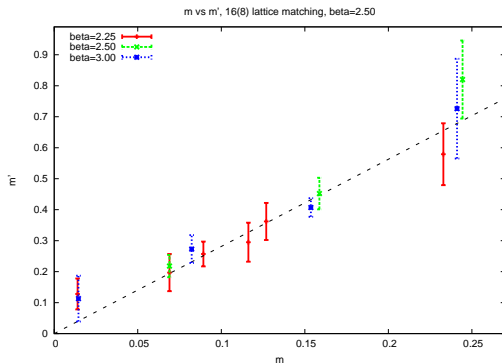
- Have matching bare masses, but additively renormalised quantities
- So need to convert to PCAC masses to be able to extract anomalous dimension

# Anomalous Dimension



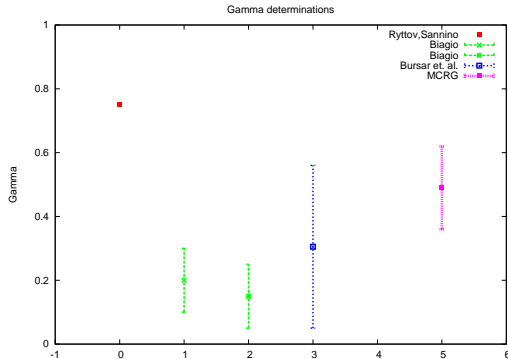
- Extract  $\gamma$  from ratio of masses:
- $m' = 2^{\gamma+1}m$
- To verify that beta is irrelevant, repeat at different beta...
- Linear fit gives  $\gamma = 0.49(13)$

# Anomalous Dimension



- Extract  $\gamma$  from ratio of masses:
- $m' = 2^{\gamma+1} m$
- To verify that beta is irrelevant, repeat at different beta...
- Linear fit gives  $\gamma = 0.49(13)$

# Summary



- We find  $\gamma = 0.49(13)$
- All-order prediction  $\gamma = 0.75$  has  $\chi^2/dof \sim 2.3$

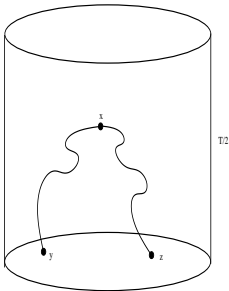
Rytov, Sannino [arXiv:hep-th/0711.3745], Biagio [arXiv:hep-ph/0911.0020], Bursar et. al. [arXiv:hep-ph/0910.4535]

# Future Plans

- Try different RG blocking transforms, look for universality.
- Use  $32^4$  lattices, would give 3 matching steps instead of 2.
- Match in more observables, including fermionic ones.
- Look for fixed point in coupling by matching in  $\beta$  at zero mass



# PCAC Mass



PCAC mass is defined using the Partially Conserved Axial Current:

## PCAC Mass

$$am(x_0) = \frac{\frac{1}{2}(\partial_0 + \partial_0^*)f_A(x_0)}{2f_P(x_0)}$$

$$f_A(x_0) = -1/12 \int d^3y d^3z \langle \bar{\psi}(x_0) \gamma_0 \gamma_5 \tau^a \psi(x_0) \bar{\zeta}(y) \gamma_5 \tau^a \zeta(z) \rangle$$

$$f_P(x_0) = -1/12 \int d^3y d^3z \langle \bar{\psi}(x_0) \gamma_5 \tau^a \psi(x_0) \bar{\zeta}(y) \gamma_5 \tau^a \zeta(z) \rangle$$

# Prediction for anomalous dimension

## Conjectured all orders beta function

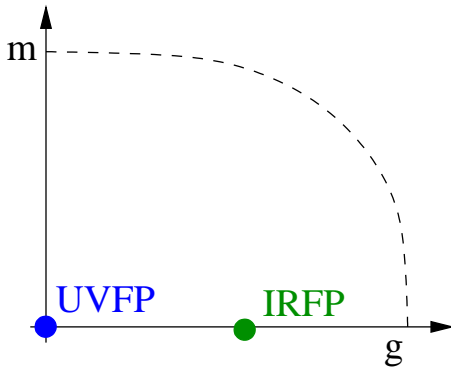
$$\beta(g) = \frac{g^3}{(4\pi)^2} \frac{\beta_0 - \frac{2}{3} T(r) N_f \gamma(g^2)}{1 - \frac{g^2}{8\pi^2} C_2(G) \left(1 + \frac{2\beta'_0}{\beta_0}\right)}$$

$$\beta_0 = \frac{11}{3} C_2(G) - \frac{4}{3} T(r) N_f, \quad \beta'_0 = C_2(G) - T(r) N_f$$

- For MWTC this predicts anomalous dimension  $\gamma = 3/4$  at fixed point
- This is a scheme-independent quantity at a fixed point

Ryttov, Sannino [arXiv:0711.3745]

# Phase diagram

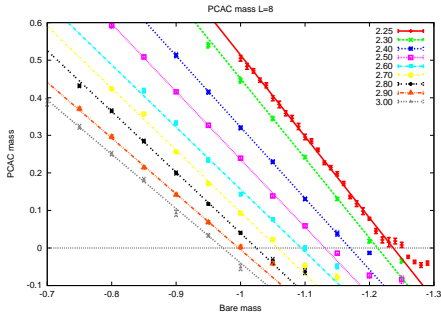


- UVFP: both  $m$  and  $g$  are relevant
- IRFP:  $m$  relevant,  $g$  irrelevant
- Can try tuning the mass to zero
- Then measure the scaling of the least irrelevant operator, hopefully  $g$

## Simulation details

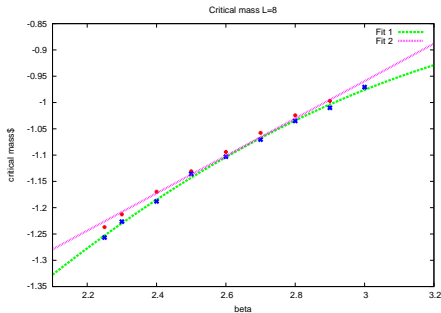
- Simulated on lattices of size  $L = 16, 8$
- Allows for 2 matchings; 2(1), 3(2) steps on the  $16^4(8^4)$  lattices
- Tune all runs to the critical  $m_{PCAC} = 0$  massless point
- Optimise  $\alpha$  such that these all agree to find continuum physics

# Mass tuning



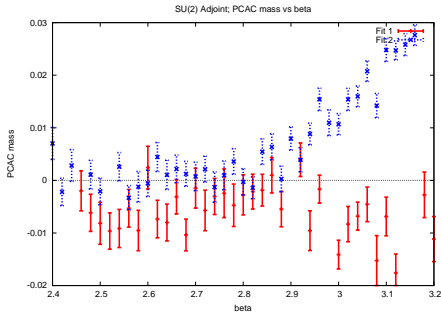
- Measure  $m_{PCAC}$  for a range of  $\beta, m$
- Interpolate to find the  $m_{crit}$  for each  $\beta$
- Simulate at  $m_{crit}$  for each  $\beta$
- Two different sets of runs to see mass dependence

# Mass tuning



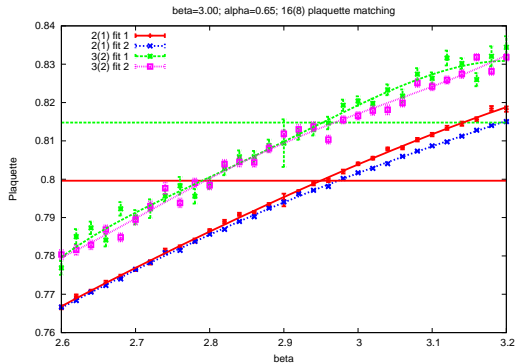
- Measure  $m_{PCAC}$  for a range of  $\beta, m$
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# Mass tuning



- Measure  $m_{PCAC}$  for a range of  $\beta, m$
- Interpolate to find the  $m_{crit}$  for each  $\beta$
- Simulate at  $m_{crit}$  for each  $\beta$
- Two different sets of runs to see mass dependence

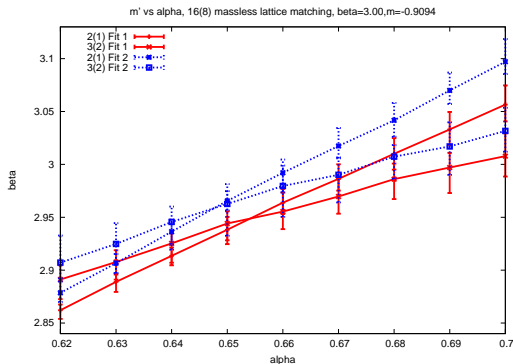
# Plaquette Matching



- Massless  $L = 16$
- $m_{PCAC} = 0.003(1)$
- Different fits show sensitivity to  $L=8$  mass tuning

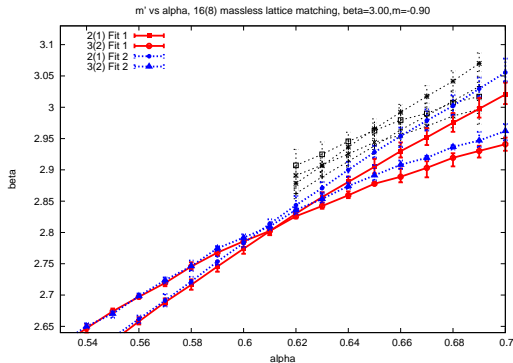


# Alpha Optimisation



- Looks less sensitive to L=8 mass tuning than expected.
- $s_b = 0.04(7)$
- $s_b = 0.05(7)$
- Step scaling consistent with zero within errors.

# Mass sensitivity



- Near-massless  $L = 16$
- $m_{PCAC} = 0.015(1)$
- $s_b = 0.20(6)$
- $s_b = 0.20(7)$
- Very sensitive to  $L=16$  mass tuning

# Particle content of MWT

- Fermionic content:
  - (U,D) techni-quark doublet
  - (N,E) new lepton doublet
  - Composite techniquark-technigluon doublet
- Composite Higgs from techni-pion

# MWT LHC Phenomenology

- Details depend on choice of ETC model
- Then construct low energy EFT for LHC

Frandsen, Sannino, et. al. [arXiv:0710.4333v1] [arXiv:0809.0793v1]

# MWT Dark Matter candidate

- Lightest technibaryon is a cold dark matter candidate
- TIMP: Technicolour Interacting Massive Particle
- iTIMP: lightest weak isotriplet technibaryon
- Prospects for discovery/exclusion from both dark matter experiments and LHC

Frandsen, Sannino [arXiv:0911.1570]