

# Orientifold Planar Equivalence: the Quenched Meson Spectrum

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# Orientifold planar equivalence

## Meson Spectrum

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## Motivations

Correlation Functions

Proof of the quenched equivalence

The numerical calculation

Large- $N$  extrapolation

Conclusions and perspectives

- The antisymmetric and the antifundamental representations coincide for  $SU(3)$  (but not in general for  $SU(N)$ )  $\Rightarrow$  different  $SU(N)$  generalizations of QCD.
- In the planar limit, the (anti)symmetric representation is equivalent to another gauge theory with the same number of Majorana fermions in the adjoint representation (in a common sector). In particular, QCD with one massless fermion in the antisymmetric representation is equivalent to  $\mathcal{N} = 1$  SYM in the planar limit  $\Rightarrow$  copy analytical predictions from SUSY to QCD.
- The orientifold planar equivalence holds if and only if the  $\mathcal{C}$ -symmetry is not spontaneously broken in both theories  $\Rightarrow$  a calculation from first principles is mandatory.
- Assuming that planar equivalence works, how large are the  $1/N$  corrections?

A. Armoni, M. Shifman and G. Veneziano. *SUSY relics in one-flavor QCD from a new  $1/N$  expansion*. Phys. Rev. Lett. 91, 191601, 2003.

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M. Unsal and L. G. Yaffe. *(In)validity of large  $N$  orientifold equivalence*. Phys. Rev. D74:105019, 2006.

A. Armoni, M. Shifman and G. Veneziano. *A note on  $C$ -parity conservation and the validity of orientifold planar equivalence*. Phys.Lett.B647:515-518,2007.

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Dynamical fermions difficult to simulate  $\Rightarrow$  start with the quenched theory

A. Armoni, B. Lucini, A. Patella and C. Pica. *Lattice Study of Planar Equivalence: The Quark Condensate*. Phys.Rev.D78:045019,2008.

# The Quenched Chiral Condensate

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$$\frac{\langle \lambda \lambda \rangle_{\text{Adj}}(m = 0.012)}{N^2} = 0.23050(22) - \frac{0.3134(72)}{N^2}$$

$$\frac{\langle \bar{\psi} \psi \rangle_{\text{AS}}(m = 0.012)}{N^2} = 0.23050(22) - \frac{0.4242(11)}{N} - \frac{0.612(43)}{N^2} - \frac{0.811(25)}{N^3}$$

$$\frac{\langle \bar{\psi} \psi \rangle_{\text{S}}(m = 0.012)}{N^2} = 0.23050(22) + \frac{0.4242(11)}{N} - \frac{0.612(43)}{N^2} + \frac{0.811(25)}{N^3}$$

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# Mesonic two-point functions on the lattice

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To measure the mesonic two-point functions with Wilson fermions in the two-index representations of the gauge group, in the quenched lattice theory.

- Wilson action.
- Wilson Dirac operator.
- The two-index representations.
- The mesonic two-point correlation functions.

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$$S_{YM} = -\frac{2N}{\lambda} \sum_p \Re \text{tr} U(p)$$

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$$D_{xy;\alpha\beta} = (m + 4r)\delta_{xy}\delta_{\alpha\beta} - K_{xy;\alpha\beta}$$

$$K_{xy;\alpha\beta} = -\frac{1}{2} \left[ (r - \gamma\mu)_{\alpha\beta} R \left[ U_{\mu}(x) \right] \delta_{y,x+\hat{\mu}} + (r + \gamma\mu)_{\alpha\beta} R \left[ U_{\mu}^{\dagger}(y) \right] \delta_{y,x-\hat{\mu}} \right]$$

# Mesonic two-point functions on the lattice

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To measure the mesonic two-point functions with Wilson fermions in the **two-index representations** of the gauge group, in the quenched lattice theory.

- Wilson action.
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$$\text{tr Adj}[U] = |\text{tr } U|^2 - 1$$

$$\text{tr S/AS}[U] = \frac{(\text{tr } U)^2 \pm \text{tr}(U^2)}{2}$$

# Mesonic two-point functions on the lattice

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$$\begin{aligned} C_{\Gamma_1 \Gamma_2}^R(x, y) &= r_R \left\langle \bar{\psi}_a^R(x) \Gamma_1^\dagger \psi_b^R(x) \bar{\psi}_b^R(y) \Gamma_2 \psi_a^R(y) \right\rangle_{YM} \\ &= r_R \left\langle \text{tr}_R \left( D_{yx; \alpha\beta}^{-1} \Gamma_1^{\gamma\beta\star} D_{xy; \gamma\delta}^{-1} \Gamma_2^{\delta\alpha} \right) \right\rangle_{YM} \end{aligned}$$

$$\begin{cases} r_R = 1 & R = S/AS \\ r_R = 1/2 & R = \text{Adj} \end{cases}$$

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# Heuristic proof of the quenched equivalence

## Equivalence

$$\lim_{N \rightarrow \infty} \frac{1}{N^2} C_{\Gamma_1 \Gamma_2}^{S/AS}(x, y) = \lim_{N \rightarrow \infty} \frac{1}{N^2} C_{\Gamma_1 \Gamma_2}^{\text{Adj}}(x, y)$$

- Expand in Wilson loops.
- Replace the two-index representations.
- Take the large- $N$  limit.
- Use invariance under charge conjugation.



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$$\frac{1}{N^2} C_{\Gamma_1 \Gamma_2}^R(x, y) = \frac{r_R}{N^2} \sum_{\mathcal{C} \supset (x, y)} \alpha_{\mathcal{C}} \langle \text{tr}_R W_{\mathcal{C}} \rangle$$

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$$\langle \text{tr } W_C^\dagger \rangle = \langle \text{tr } W_C \rangle \quad \Rightarrow \quad \lim_{N \rightarrow \infty} \frac{1}{N^2} C_{\Gamma_1 \Gamma_2}^{S/AS}(x, y) = \lim_{N \rightarrow \infty} \frac{1}{N^2} C_{\Gamma_1 \Gamma_2}^{\text{Adj}}(x, y)$$

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- Expand in Wilson loops.
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A more formal proof of the equivalence exists which does not use the expansion in Wilson loops, but is much more involved

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### ● Simulations performed for $N = 2, 3, 4, 6$

- $\beta(N)$  chosen in such a way that  $(aT_c)^{-1} = 5$  ( $a \simeq 0.145$  fm)
- Calculations on a  $32 \times 16^3$  lattice, which corresponds to  $L \simeq 2.3$  fm
- $C_{\Gamma_1 \Gamma_2}^R$  determined for  $\Gamma_1 = \Gamma_2 = \gamma_5$  ( $\pi$  channel) and  $\Gamma_1 = \Gamma_2 = \gamma_i$  ( $\rho$  channel)
- Mass extracted from the ansatz  $C_{\Gamma_1 \Gamma_2}^R(t) = A \cosh(m(t - T/2))$
- Chiral extrapolation of  $m_\rho$  using  $m_\rho(m_\pi) = cm_\pi^2 + m_\rho(m_\pi = 0)$
- Extrapolation to large  $N$

The calculation has been performed using the **HiRep** code  
(L. Del Debbio, A. Patella, C. Pica, arXiv:0805.2058)



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# $m_\rho$ vs. $m_\pi$ in SU(3)

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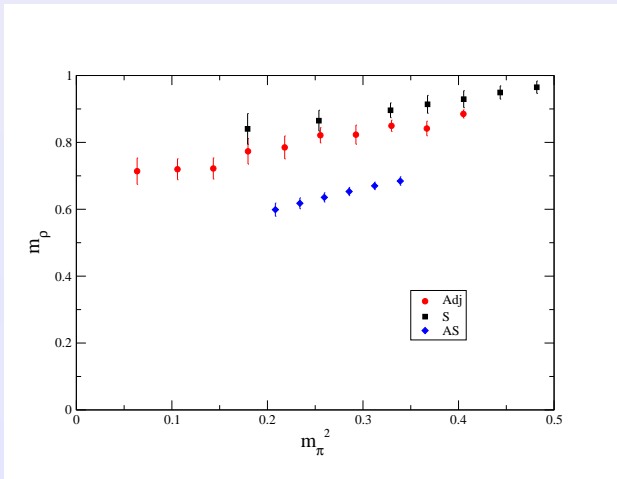
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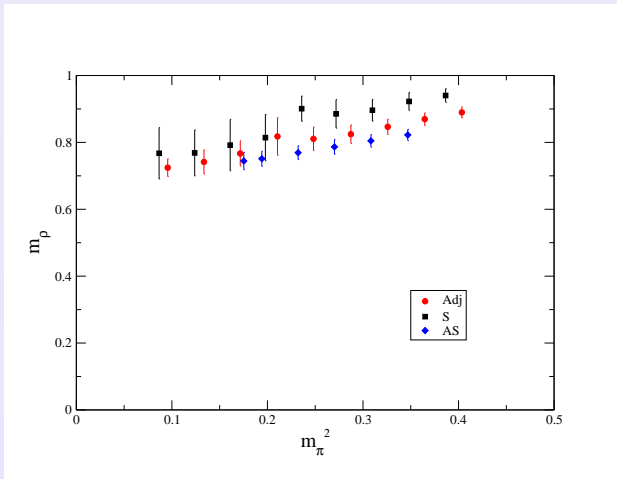
Correlation Functions

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The numerical calculation

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# $m_\rho$ vs. $m_\pi$ (Antisymmetric)

## Meson Spectrum

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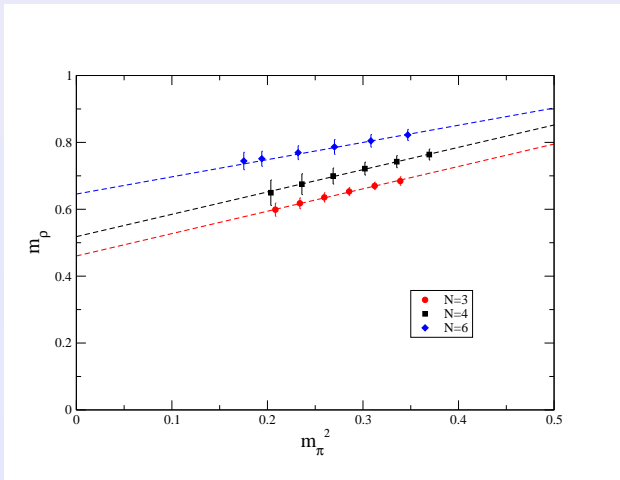
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# $m_\rho$ vs. $m_\pi$ (Symmetric)

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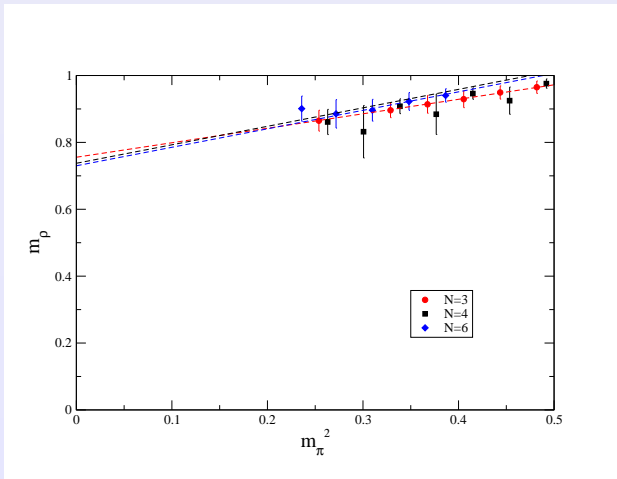
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# $m_\rho$ vs. $m_\pi$ (Adjoint)

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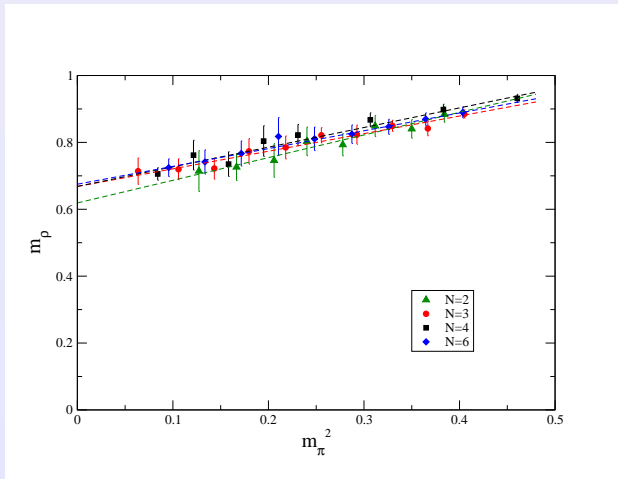
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# Chiral extrapolation of $m_\rho$

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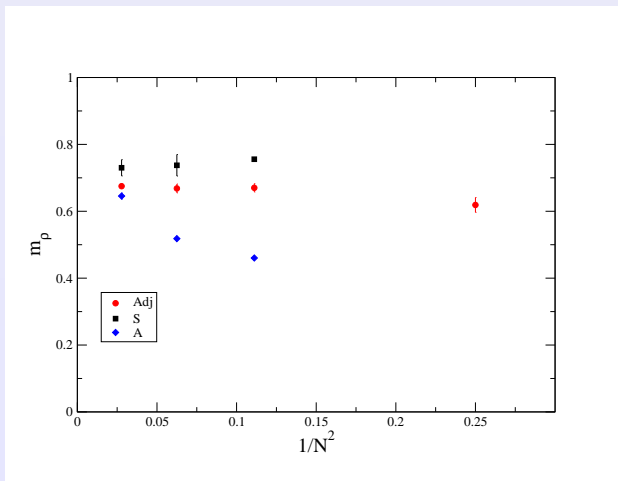
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# Outline

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# Order of corrections

The correlator in the adjoint representation decays with a mass  $m_\rho^{\text{Adj}}$  that can be expressed as a power series in  $1/N^2$ , while  $m_\rho^{\text{AS}}$  and  $m_\rho^{\text{S}}$  have  $1/N$  corrections that are related:

$$\begin{aligned}m_\rho^{\text{Adj}}(N) &= F\left(\frac{1}{N^2}\right) ; \\m_\rho^{\text{S}}(N) &= M\left(\frac{1}{N^2}\right) + \frac{1}{N}\mu\left(\frac{1}{N^2}\right) ; \\m_\rho^{\text{AS}}(N) &= M\left(\frac{1}{N^2}\right) - \frac{1}{N}\mu\left(\frac{1}{N^2}\right) .\end{aligned}$$

$M = (m_\rho^{\text{S}} + m_\rho^{\text{AS}}) / 2$  and  $\mu = N (m_\rho^{\text{S}} - m_\rho^{\text{AS}}) / 2$  can be expressed as a power series in  $1/N^2$

Orientifold planar equivalence is the statement  $F(N = \infty) = M(N = \infty)$

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# Chiral extrapolation of $m_\rho$

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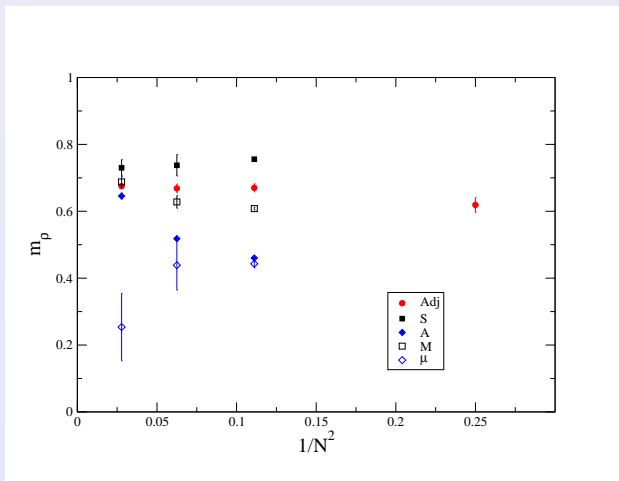
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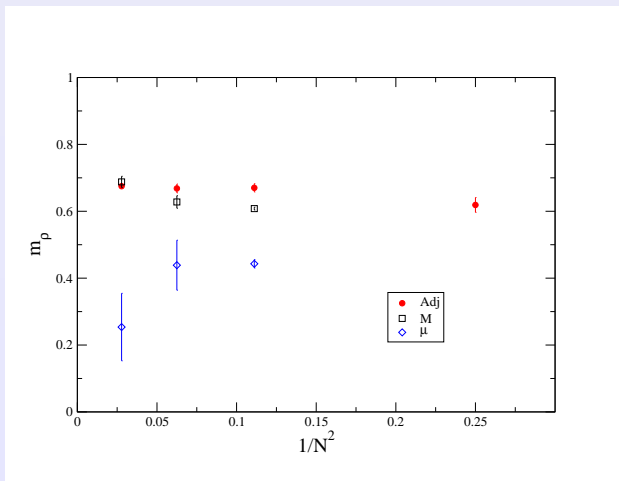
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# Large- $N$ fits

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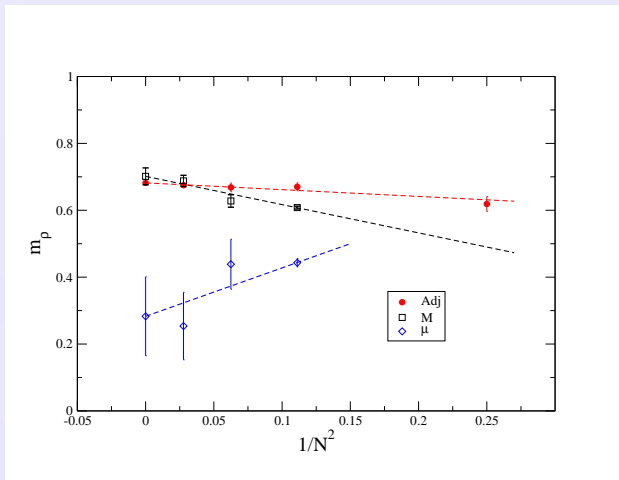
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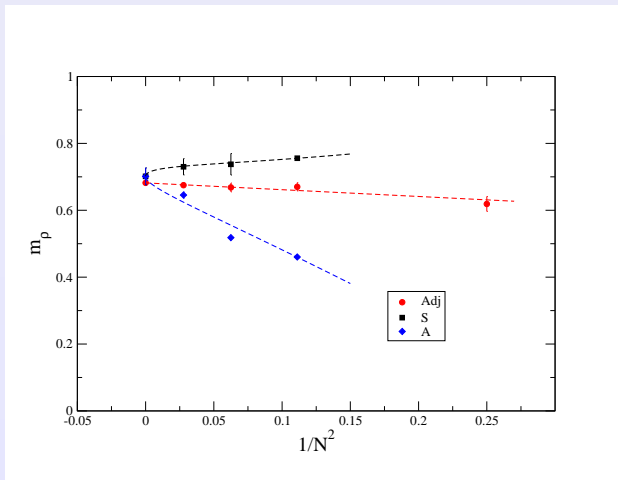
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# Fit results

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$$m_{\rho}^{\text{Adj}} = 0.6819(51) - \frac{0.202(67)}{N^2} ;$$

$$m_{\rho}^{\text{S}} = 0.701(25) + \frac{0.28(12)}{N} - \frac{0.85(24)}{N^2} + \frac{1.4(1.0)}{N^3} ;$$

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Orientifold planar equivalence verified within 3.5%

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- Check of the orientifold planar equivalence in a simple case
  - Computation of the  $\rho$  and  $\pi$  masses for two-index irreducible representations and evaluation of the corrections in  $1/N$
  - SU(3) AS is numerically far from the large  $N$  limit of the adjoint representation, but it is obtainable from it with a controlled power expansion
  - Corrections up to  $1/N^3$  describe SU(3) within the accuracy of the numerical results
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