

COHERENT CENTER DOMAINS IN LOCAL POLYAKOV LOOPS

Julia Danzer¹

S. Borsanyi², Z. Fodor², C. Gattringer¹, S.D. Katz³, A. Schmidt¹, K.K. Szabo²

¹Institut of Physics, Karl-Franzens University Graz

²Department of Physics, University of Wuppertal

³Institut for Theoretical Physics, Eötvös University, Budapest

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CENTER SYMMETRY OF GLUODYNAMICS

- Local Polyakov loop:

$$L(\vec{x}) = \text{Tr} \left[\prod_{t=0}^{N_t-1} U_4(\vec{x}, t) \right]$$

- SU(3) gauge theory: Center elements $\mathbf{z} \in \{1, e^{2\pi i/3}, e^{-2\pi i/3}\}$
- Center transformation: Acts on temporal links at time slice $t = t_0$

$$U_4(\vec{x}, t_0) \longrightarrow \mathbf{z} U_4(\vec{x}, t_0)$$

- Action and gauge measure are invariant
- Polyakov loop transforms non-trivially under a center transformation

$$L(\vec{x}) \longrightarrow \mathbf{z} L(\vec{x})$$

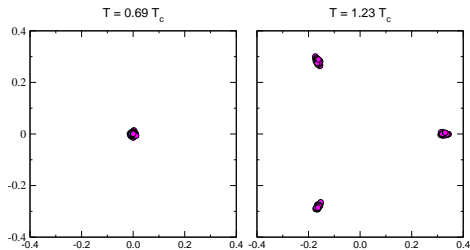
- Non-vanishing $\langle L(\vec{x}) \rangle$ signals spontaneous breaking of center symmetry

INCLUSION OF DYNAMICAL FERMIONS

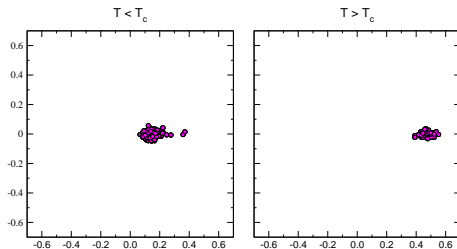
- When quarks are included the fermion determinant acts as an additional weight factor
- Fermions play the role of an external magnetic field which break the center symmetry explicitly
- Although the Polyakov loop P is no true order parameter anymore it signals the crossover to deconfinement

SCATTER PLOTS OF THE POLYAKOV LOOP

- In pure gauge theory:



- Dynamical case:



SVETITSKY-YAFFE CONJECTURE (1981):

- At T_c the critical behavior of $SU(N)$ gauge theory in $d + 1$ dimensions can be described by a d - dimensional spin system with a \mathbb{Z}_N - invariant effective action
- The spins are related to the local loops $L(\vec{x})$
- In spin systems one can define clusters of parallel spins (e.g. Fortuin-Kasteleyn clusters in Ising systems)
- At T_c these clusters start to percolate
- Can we identify characteristic **properties of such clusters** directly in QCD?
- Here we focus on clusters with coherent phases of the Polyakov loop and their percolation properties near T_c .

SETTINGS AND GOAL IN OUR ANALYSIS

- We study clusters and critical percolation directly in quenched and dynamical $SU(3)$ gauge theory
- For that purpose we analyze properties of the local loops $L(\vec{x})$
- Technicalities – Quenched case:
 - ▶ Lüscher-Weisz gauge action
 - ▶ Lattice sizes: $20^3 \times 6 \dots 40^3 \times 12$
 - ▶ Temperatures: $T \in [0.63 T_c, 1.32 T_c]$
 - ▶ [arXiv:1004.2200](#)
- Technicalities – Dynamical case:
 - ▶ Symanzik improved gauge and stout-link improved staggered action
 - ▶ 2 + 1 flavors with physical quark masses
 - ▶ Lattice sizes: $18^3 \times 6, 36^3 \times 6, 24^3 \times 8$
 - ▶ Temperatures: $T \in [50 \text{ MeV} \dots, 1000 \text{ MeV}]$
 - ▶ [Phys. Lett. B643 \(2006\) 46](#), [JHEP 0906:088 \(2009\)](#)

THE LOCAL POLYAKOV LOOP $L(\vec{x})$

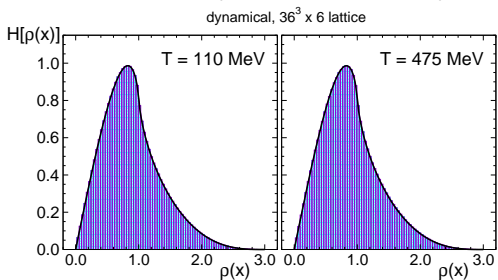
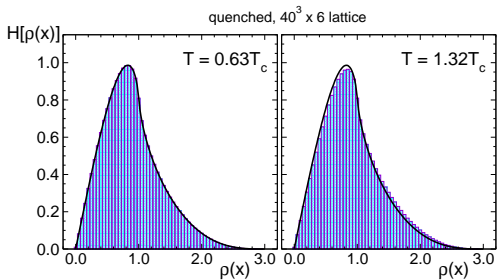
- For the analysis of local properties of $L(\vec{x})$ we define:

$$L(\vec{x}) = \rho(\vec{x}) e^{i\varphi(\vec{x})}$$

- We analyze properties of the modulus $\rho(\vec{x})$ and the phase $\varphi(\vec{x})$

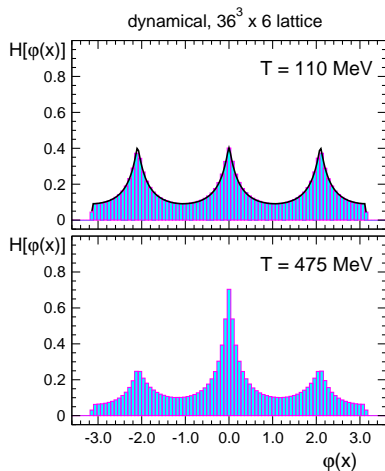
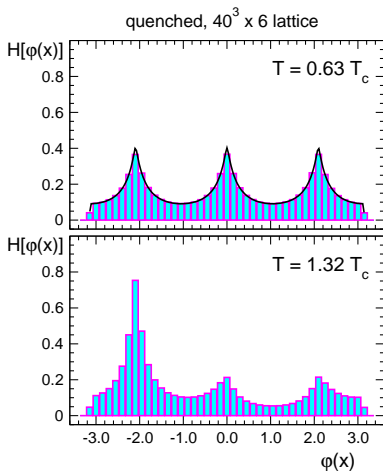
MODULUS $\rho(\vec{X})$:

(full curve = Haar measure distribution)



PHASE $\varphi(\vec{X})$:

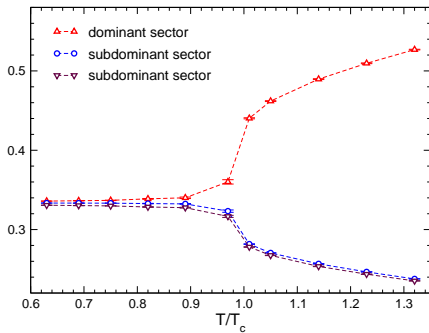
(full curve = Haar measure distribution)



CENTER SECTORS ACROSS THE PHASE TRANSITION

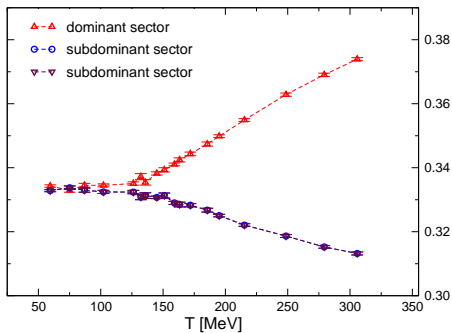
quenched

Abundance of lattice sites in center sectors



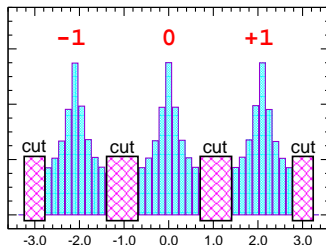
dynamical

Abundance of lattice sites in center sectors



CONSTRUCTION OF CLUSTERS

- Assign the sector numbers $-1, 0, 1$ to the three phases of the Polyakov loop
- Neighboring sites with same sector number are put in the same cluster
- In 3 dimensions the critical site percolation probability $p_c = 0.3116$
- Below T_c thus we always find percolating clusters
- **Idea:** Introduce a cut for sites far from center elements

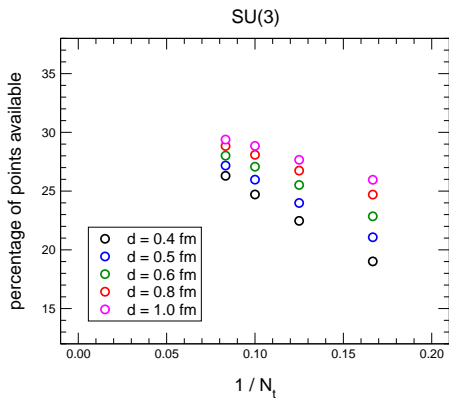


- A similar cut is necessary for percolating clusters in spin systems

- We determine the **physical diameter** d of the cluster
- The diameter clearly depends on the value of the cut parameter
- For a given value of d (e.g. 0.5 fm, 0.8 fm, 1.0 fm) we **compare the cut parameters** on lattices with different lattice spacing a
- The number of lattice points which are not cut and thus are available for the clusters is

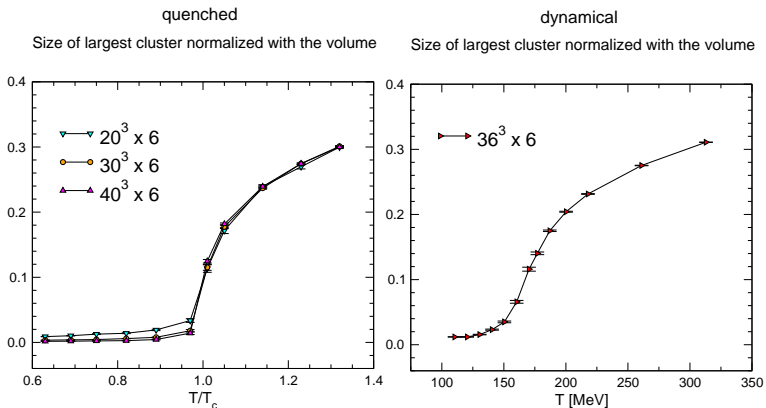
$$A = \frac{100\% - \text{cut}\%}{N_c}$$

- A scales linearly to a value very **close to the critical percolation density**
 $\rho_c = 0.3116$

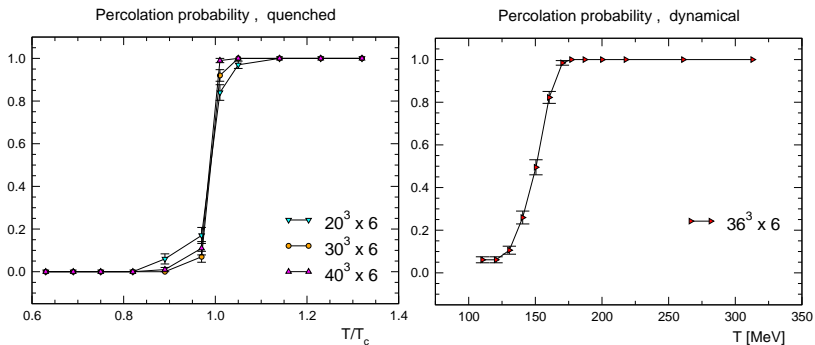


- fixed spatial volume
- fixed temperature $T < T_c$

LARGEST CLUSTER



PERCOLATION PROBABILITY



SUMMARY AND OUTLOOK

- We analyze the behavior of local Polyakov loops $L(\vec{x})$
- We find that below T_c they are distributed according to Haar measure
- The phases always have preferred values near the center angles $0, \pm i2\pi/3$
- The phases form spatially localized clusters
- The clusters can be shown to have a continuum limit
- For pure gauge theory the deconfinement transition can be characterized by the onset of percolation of these clusters
- For the full theory, due to the crossover nature of the transition, this cannot be expected a priori
- Related analysis of this behavior is in progress