Preliminary study of the non perturbative renormalization of $K \to \pi(\pi)$ operators with $n_f = 2+1$ Domain Wall fermions

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Motivations

Direct and indirect CP violation occur in $K \to \pi\pi$ decays

▷ Direct CP violation well measured experimentally, but ϵ'/ϵ poorly determined theoretically
Large theoretical uncertainties coming from the non-perturbative part

▷ Constraint the SM model via the CKM matrix, room for BSM physics ?

▷ $\Delta I = 1/2$ rule

▷ Challenge for the lattice community,
chiral fermions have an important rôle to play

Computation of ϵ'/ϵ

Kaon decays to $(\pi\pi)_{\mathrm{I}=0,2}$ through an effective Hamiltonian

$$H^{\Delta s=1} = \frac{G_F}{\sqrt{2}} \left\{ \sum_{i=1}^{10} \left(V_{ud} V_{us}^* z_i(\mu) - V_{td} V_{ts}^* y_i(\mu) \right) Q_i(\mu) \right\}$$

$\Rightarrow$ 10 operators, which mix under renormalization

$\Rightarrow$ 2-pion state on the lattice (can one trust χPT at the kaon mass ?)

See eg [Norman Christ @ Kaon'09] for an overview of different strategies.

- Matrix element $\longrightarrow$ See talks by Matthew Lightman , and by Qi Liu
- Renormalization $\longrightarrow$ this talk

The Z factors have been already computed in the quenched case, this work follows [RBC'01]

Renormalization of 4-quark operators

- Mixing pattern given by the $SU(3)_L \otimes SU(3)_R$ decomposition of the operators
- Some Z factors of $\Delta I = 1/2$ operators can be obtained by from $\Delta I = 3/2$ operators
- Z factors of $\langle \pi | O^{\Delta S=1} | K \rangle$ can be related to those of $\langle \bar{K} | O^{\Delta S=2} | K \rangle$
- Dangerous mixing with lower dimension operators constrained by chiral symmetry
- ...

$\Rightarrow$ Important to work with good chiral-flavor symmetry

$\Rightarrow$ Domain Wall action is a natural candidate

4-quark operators (II)

Current-Current

$$Q_2 = (\bar{s}u)_{\mathrm{V-A}}(\bar{u}d)_{\mathrm{V-A}} \qquad Q_1 = \text{color mixed}$$

QCD penguins

$$Q_3 = (\bar{s}d)_{\mathrm{V-A}} \sum_{q=u,d,s} (\bar{q}q)_{\mathrm{V-A}} \qquad Q_4 = \text{color mixed}$$

$$Q_5 = (\bar{s}d)_{\mathrm{V-A}} \sum_{q=u,d,s} (\bar{q}q)_{\mathrm{V+A}} \qquad Q_6 = \text{color mixed}$$

EW penguins

$$Q_7 = \frac{3}{2}(\bar{s}d)_{\mathrm{V-A}} \sum_{q=u,d,s} e_q(\bar{q}q)_{\mathrm{V+A}} \qquad Q_8 = \text{color mixed}$$

$$Q_9 = \frac{3}{2}(\bar{s}d)_{\mathrm{V-A}} \sum_{q=u,d,s} e_q(\bar{q}q)_{\mathrm{V-A}} \qquad Q_{10} = \text{color mixed}$$

$SU(3)_L \otimes SU(3)_R$ decomposition

Irrep of $SU(3)_L \otimes SU(3)_R$

$$
\begin{aligned}
\bar{3} \otimes 3 &= 8 + 1 \\
\bar{8} \otimes 8 &= 27 + \overline{10} + 10 + 8 + 8 + 1
\end{aligned}
$$

Decomposition of the 4-quark operators gives

$$
\begin{aligned}
Q_{1,2} &= Q_{1,2}^{(27,1),\Delta I=3/2} + Q_{1,2}^{(27,1),\Delta I=1/2} + Q_{1,2}^{(8,8),\Delta I=1/2} \\
Q_{3,4} &= Q_{3,4}^{(8,1),\Delta I=1/2} \\
Q_{5,6} &= Q_{5,6}^{(8,1),\Delta I=1/2} \\
Q_{7,8} &= Q_{7,8}^{(8,8),\Delta I=3/2} + Q_{7,8}^{(8,8),\Delta I=1/2} \\
Q_{9,10} &= Q_{9,10}^{(27,1),\Delta I=3/2} + Q_{9,10}^{(27,1),\Delta I=1/2} + Q_{9,10}^{(8,8),\Delta I=1/2}
\end{aligned}
$$

see eg [Claude Bernard @ TASI'89] and [RBC'01]

Renormalization basis

We build a basis of seven 4-quark operatrors,

	(27,1)	(8,1)	(8,8)
Q'_1	3/2, 1/2		
$Q'_{2,3}$		1/2	
$Q'_{5,6}$		1/2	
$Q'_{7,8}$			3/2, 1/2

$\Rightarrow$ Renormalization matrix should be
$$\begin{pmatrix} Z_{11} & & & & & & \\ & Z_{22} & Z_{23} & Z_{24} & Z_{25} & & \\ & Z_{32} & Z_{33} & Z_{34} & Z_{45} & & \\ & Z_{52} & Z_{53} & Z_{54} & Z_{55} & & \\ & Z_{62} & Z_{63} & Z_{64} & Z_{65} & & \\ & & & & & Z_{77} & Z_{78} \\ & & & & & Z_{87} & Z_{88} \end{pmatrix}$$

$\Delta I = 3/2$ operators

Furthermore, different components of a chiral multiplet have the same renormalization factor

$\Rightarrow$ For Q'_1, Q'_7, Q'_8, it enough to compute the $\Delta I = 3/2$ part

$\Rightarrow$ Simplifies the computation since no disconnected parts are involved

These Z-factors are also involved in a compuation of B_K BSM

Non perturbative Renormalization

See e.g. Yasumichi Aoki @ lat'09 for a review,
Rome-Southampton method also discussed in the talks by Francesco di Renzo, Konstantin Petrov

Our choice :

- Momentum sources [QCDSF]

Solve $$\sum_y D(x,y) S_p(y) = \exp(ip.x)$$

Obtain $$S_p(x) = \sum_y D^{-1}(x,y) \exp(ip.y)$$

- Non exceptional kinematic [Yasumichi Aoki et al '08]

$$p_1^2 = p_2^2 = (p_1 - p_2)^2$$

- Twisted boundary conditions

See talk by [Rudy Arthur]

Connected part $\Delta I = 3/2$

We compute the amputed vertex function with gauge fixed momentum source of $(\bar{\Psi}_1 \Gamma \Psi_2)(\bar{\Psi}_3 \Gamma \Psi_4)$ (color unmixed) with the following Γ structure

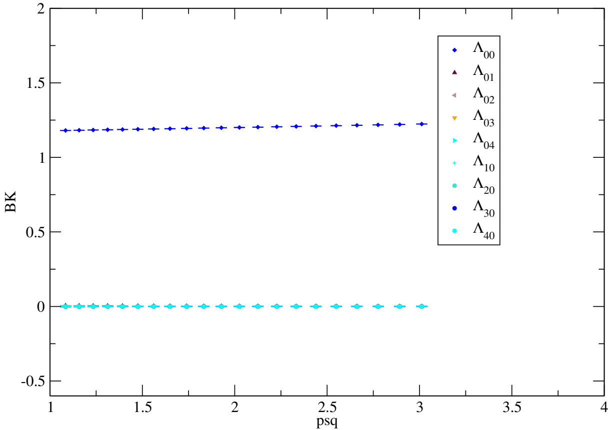
$$\gamma_\mu \times \gamma_\mu + \gamma_\mu\gamma_5 \times \gamma_\mu\gamma_5$$
$$\gamma_\mu \times \gamma_\mu - \gamma_\mu\gamma_5 \times \gamma_\mu\gamma_5$$
$$1 \times 1 + \gamma_5 \times \gamma_5$$
$$1 \times 1 - \gamma_5 \times \gamma_5$$
$$\sigma_{\mu\nu}$$

Related to $B_K^{\rm BSM}$, something that Jan Wennekers was working on [Jan Wennekers @lat'08]

Example of results for the connected part $\Delta I = 3/2$

$16^3 \times 32$, Λ_{VV+AA}

/home/Q01/ngarron/work/npr/BKsusy/mu0.02/BK.agr



Thanks to Rudy Arhtur !

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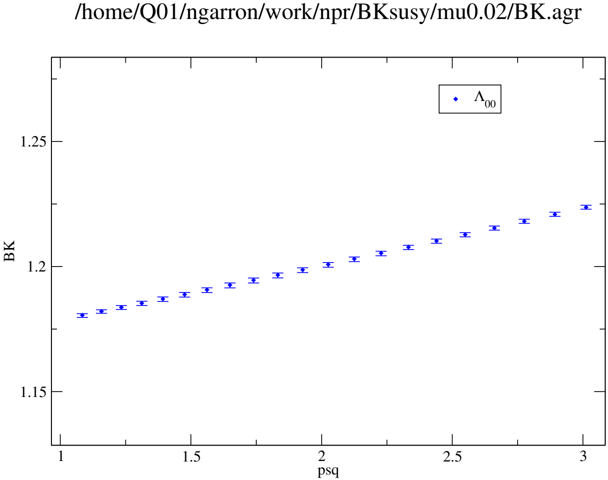

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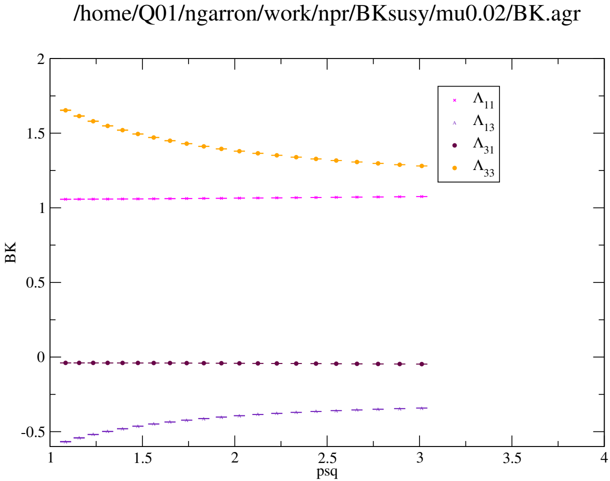
Example of results for the connected part $\Delta I = 3/2$

$16^3 \times 32$, Λ_{ij}/Λ_A^2 for Q_7, Q_8



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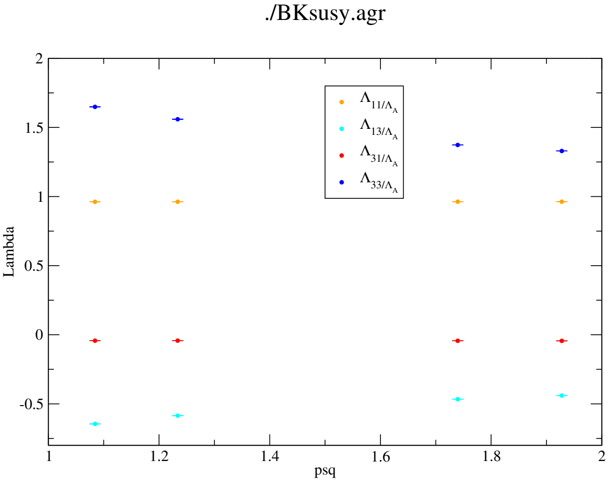
Example of results for the connected part $\Delta I = 3/2$

$32^3 \times 64$, Λ_{ij}/Λ_A^2 for Q_7, Q_8



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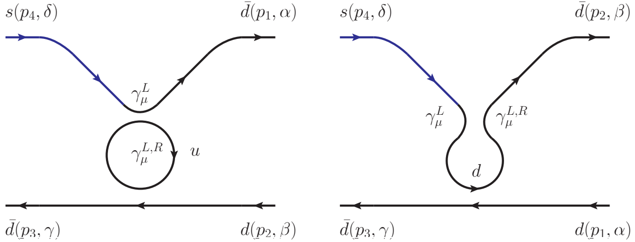
For the (8,8), we have to compute diagrams like



Strategy for the disconnected diagrams

- Invert the Dirac operator on a Z_2 stochastic source

$$D(x,y)G^{\rm eye}(y) = \eta(x) \quad \Rightarrow \quad G^{\rm eye}(x) = D^{-1}(x,y)\,\eta(y)$$

and compute (add the Γ matrices at the right place)

$$\sum_z \langle \left(\gamma_5 {S'}_{p_2}(z)^\dagger \gamma_5\right) \eta(z)\; G^{\rm eye}(z)\; {S'}_{p_3}(z) S(p_4,p_1)\rangle$$
$$= \sum_{z,x_1\ldots x_4} \langle e^{-ip_2(x_2-z)} D^{-1}(x_2,z)\; D^{-1}(z,z)\; D^{-1}(z,x_3) e^{ip_3(x_3-z)} e^{-ip_4x_4} D^{-1}(x_4,x_1) e^{ip_1x_1} \rangle$$

- For the spectator we just need

$$S(p,q) = \sum_x S_q(x) e^{-ip.x} = \sum_{xy} D^{-1}(x,y) e^{i(q.y-p.x)}$$

- on the way ...

Outlook

- We are currently doing the NPR of $K \to \pi\pi$
- Connected part far advanced
- Mom source + non-exceptional kinematic+ twisted boundary conditions, look very promising
- Implementation of the disconnected part in on the way
- We hope to check the results obtained with point source Shu Li phD thesis
- . . . More next year . . .

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