# Flavor dependence of hadron spectrum in Technicolor theories

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(presented by George Fleming)
[for the LSD Collaboration]

Yale University

Lattice 2010

# Outline

# The Lattice Strong Dynamics Collaboration

| | |
|---|---|
| Tom Appelquist | Yale Univ. |
| Adam Avakian | Boston Univ. |
| Ron Babich | Boston Univ. |
| Rich Brower | Boston Univ. |
| Michael Buchoff | Univ. of Maryland |
| Michael Cheng | Lawrence Livermore Nat. Lab. |
| Mike Clark | Harvard Univ. |
| Saul Cohen | Boston Univ. |
| George Fleming | Yale Univ. |
| Fu-Jiun Jiang | MIT |
| Joe Kiskis | Univ. of California, Davis |
| Meifeng Lin | Yale Univ. |
| Ethan Neil | Yale Univ. |
| James Osborn | Argonne National Lab |
| Claudio Rebbi | Boston Univ. |
| David Schaich | Boston Univ. |
| Gennady Voronov | Yale Univ. |
| Pavlos Vranas | Lawrence Livermore Nat. Lab |

- 300 Million (and counting) BlueGene/L core-hours provided by LLNL on 40-rack unclassified machine. **THANKS!!!**
- Additional resources provided by U.S. NSF TeraGrid, and U.S. DOE resources dedicated to lattice gauge theory (USQCD).

# Origin of Mass?

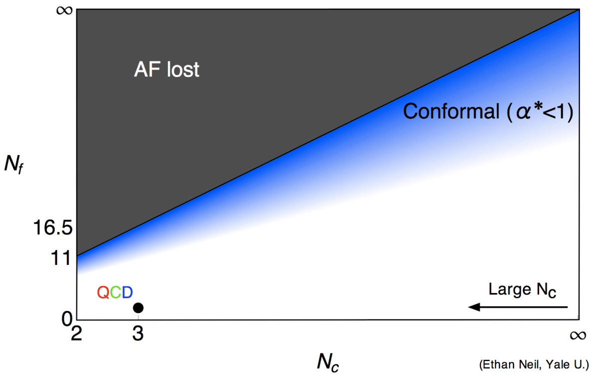
- Spontaneous electroweak symmetry breaking
  $SU(2)_L \otimes U(1)_Y \rightarrow U(1)_{EM} \Rightarrow$ masses of $W^{\pm}$ and $Z$ bosons

- What is the dynamics of EWSB?
  - Standard Model Higgs mechanism?
    - ◇ Of course this would be the most economical solution.
    - ◇ We are all waiting for the exciting results from LHC.
  - Other possibilities?
    - ◇ Dynamical electroweak symmetry breaking as a result of new strong interactions at the TeV scale and above.
    - ◇ Simple scaled-up version of QCD is not a viable option.
    - ◇ Technicolor theories with non-QCD behaviors (e.g., walking or strongly-coupled conformal) can be better candidates.

- We need to know more about strong interacting theories other than QCD.

# "Phase Diagram" of $SU(N)$ Gauge Theories?

For $N_f$ fundamental Dirac fermions




- QCD is just a single point in a large theory space; behaves very much like pure gauge theory, *i.e.* precociously free. Is it typical or exceptional?
- The location of the conformal window relevant to many model-buildings is poorly known.

# What are we looking for on the lattice?

- Non-perturbative exploration of the phase space of the SU(N) gauge theories.
- Understand novel features in different phases.
- Make predictions for LHC?

- How?
  - LGT allows us to change $N_f$, $N_c$ and representations without making over-simplified assumptions.
  - We can study the properties of
    - particle spectrum [this talk]
    - chiral condensate [talk by P. Vranas]
    - $S$ parameter[talk by D. Schaich]

    etc. from first principles.

- Current stage:
  - Exploratory studies of QCD-like theories.
  - Look for non-QCD behaviors in hadron spectrum, chiral condensate, etc.

# General Considerations

- In a slowly running, but confining, theory, one must push the UV cutoff higher than in QCD.
  - Lattice momentum cutoff $\sim 1/a$. $\Rightarrow$ Finer lattice spacings are needed.
  - We choose $1/a \approx 5M_\rho$.
- Chiral symmetry plays an important role.
  - Use domain wall fermions: *nearly exact chiral symmetry, not as computationally demanding as overlap fermions.*
- Start from something familiar on the lattice. Code is ready and well tested.
  - SU(3) in fundamental representation.
  - $N_f = 2$ as a starting point and a reference point.
- First focus on theories outside of the conformal window.
  - $N_f = 6$ as a test-bed: expected to be well away from the conformal window, QCD-like.
  - $N_f = 10$ more interesting: can be QCD-like, conformal or walking.
    In progress.

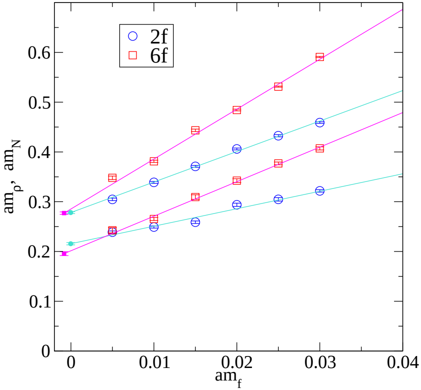
# Simulations Details

- SU(3) fundamental, $N_f = 2, 6,$ 10 (running)
- Domain wall fermions with Iwasaki gauge action
- $am_f = 0.005 \cdots 0.03$, lattice size $32^3 \times 64$
- Lattice cutoff tuned to $1/a \approx 5M_\rho$.
- $L_s = 16 \Rightarrow am_{\text{res}} \approx 2.5 \times 10^{-5}(2f), 8.2 \times 10^{-4}(6f), 1.7 \times 10^{-3}(10f)$

- Other Facts:
  - $N_f$-flavor simulations are much more expensive than QCD: Cost $\propto N_f^{3/2}$.
  - Because of the higher cost, runs for $N_f = 6$ are generally shorter. $\Rightarrow$ Statistical errors are large.
  - Binning size may not be large enough to account for autocorrelations in the simulations. $\Rightarrow$ Statistical errors may be underestimated.
- First $2f, 6f$ results were published in *Phys.Rev.Lett.104:071601,2010*
- Increased statistics since then. New results are PRELIMINARY.
- 10-flavor simulations are in progress. Results are VERY PRELIMINARY (no binning, short thermalization cuts...)

# Scale Matching

- The gauge couplings tuned so that $N_f = 2$ and $6$ have roughly the same UV cutoff.
- $aM_N$, $aM_\rho$ and $r_0/a$ all matched to 10%.
- Independent analysis agrees well with results shown by Vranas.



# Scale Matching

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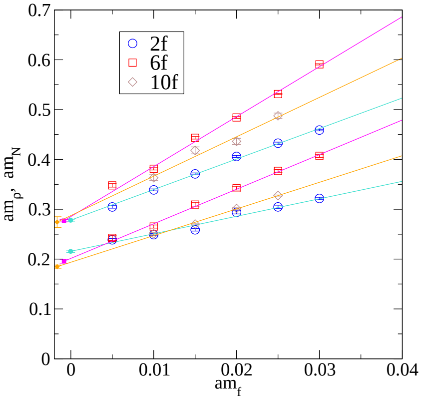
- $aM_N$, $aM_\rho$ and $r_0/a$ all matched to 10%.
- Adding the 10-flavor (VERY PRELIMINARY)....



# Hadron Masses and Decay Constants

- Simultaneous fit to wall-point (WP), point-point (PP), point-wall (PW) and wall-wall (WW) correlators to get a common mass and a separate amplitude for each correlator.
- Can use different combinations of the amplitudes to extract decay constants (*a la* RBC-UKQCD, PRD 78, 114508 (2008))
- Different determinations agree within errors. Use the final results from the WP and WW correlators to determine the decay constants.

OUR DEFINITIONS FOR THE DECAY CONSTANTS:

- pseudoscalar:

$$\langle 0 \,| A_4^a(x) |\, \pi^a \rangle \equiv \sqrt{2} F_\pi M_\pi \cdot Z_A$$

- vector:

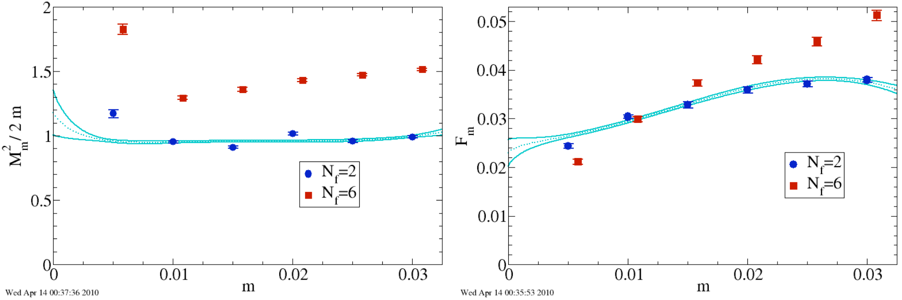
$$\langle 0 \,| V_i^a(x) |\, \rho^a \rangle \equiv \sqrt{2} F_\rho M_\rho \epsilon_i \cdot Z_V, \;\; i = 1, 2, 3$$

- axial-vector:

$$\langle 0 \,| A_i^a(x) |\, a_1^a \rangle \equiv \sqrt{2} F_{a_1} M_{a_1} \epsilon_i \cdot Z_A, \;\; i = 1, 2, 3$$

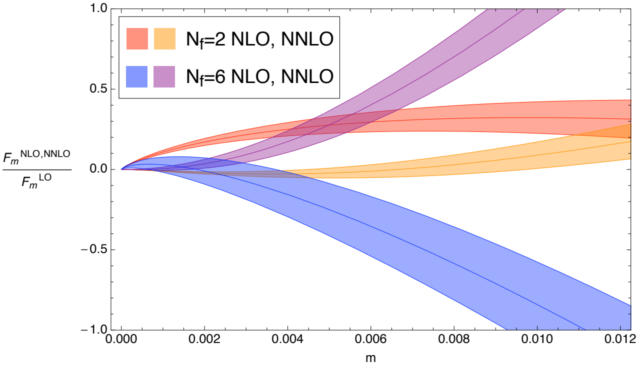
- $Z_A, Z_V$: axial and vector current renormalization constants. For DWF, $Z_A \approx Z_V$.

# Pseudoscalar Masses and Decay Constants




- Lightest points susceptible to finite volume effects.
- Simulations performed at finite quark masses. $\Rightarrow$ chiral extrapolations are needed to go to the chiral limit: $m_f + m_{\rm res} = 0$.
- Chiral fits to $N_f = 6$ are not reliable (explained next).

# Chiral Extrapolations?



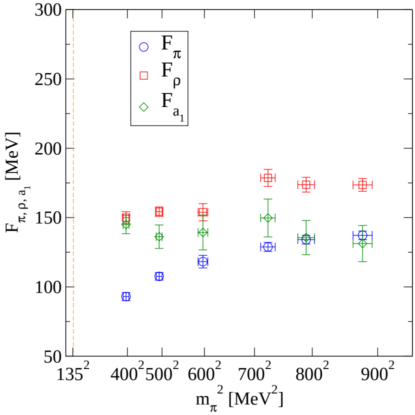

E. Neil, Chiral Dynamics 2009

- NLO has terms $\propto N_f$, NNLO has terms $\propto N_f^2$.
  J. Bijnens and J. Lu, *JHEP*, 11:116, 2009
- NNLO chiral fits work fine for $N_f = 2$.
- Sizes of NLO and NNLO terms are large for $N_f = 6$.
  Small quark masses are needed for reliable chiral extrapolations.

# How well do we reproduce 2-flavor QCD?

- Scale set by $m_\rho$: $a^{-1} \approx 3.60(4)$ GeV.
- Masses are too heavy for ChPT to work reliably.
- Simple linear extrapolations in $m_\pi^2$:

$$M(m_\pi) \text{ or } F(m_\pi) = A + Bm_\pi^2$$



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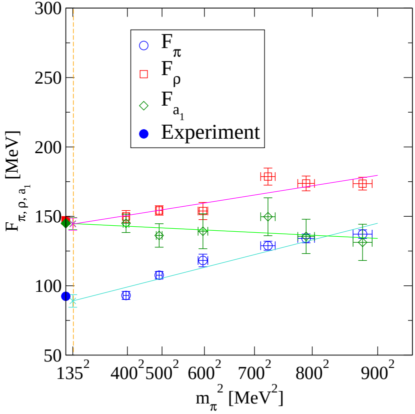
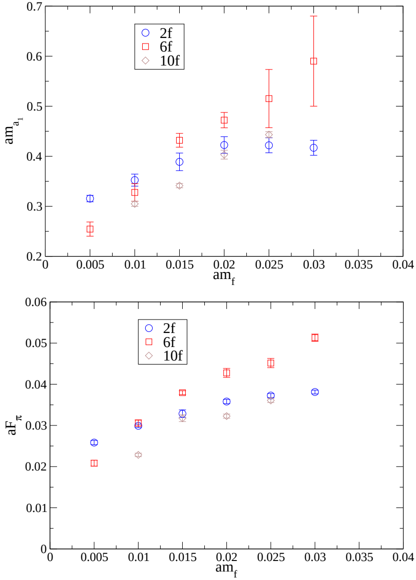



- Naive linear extrapolations give physical results consistent with experiments.
- Caveats:
  lack of sophisticated chiral extrapolations...
  possible finite volume effects at small masses...
- We are looking at HUGE effects, not percent-level precision...

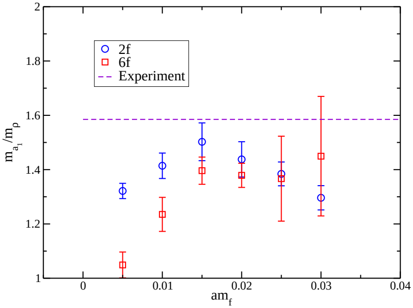
# Flavor Dependence...



# Parity Doubling?

- As the theory moves towards walking, chiral symmetry is less broken.
- Parity partners may acquire the same mass.
- We expect $N_f = 6$ to be far away from walking.

PRELIMINARY

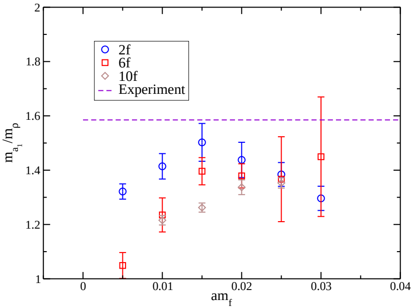



- Parity doubling at the chiral limit? Could be a finite volume effect.
- Earlier studies suggest parity doubling disappeared when the volume was increased. Cheng-zhong Sui, PhD thesis 2001

# Parity Doubling?

- As the theory moves towards walking, chiral symmetry is less broken.
- Parity partners may acquire the same mass.
- How about $N_f = 10$?

PRELIMINARY




- Trend: 10f-Masses of $a_1$ and $\rho$ become more degenerate in the chiral limit?
- Results are preliminary. Need to understand finite volume effects, etc...

# Parity Doubling and Electroweak $S$ Parameter

- The electroweak $S$ parameter is related to the spectral functions of the vector and axial-vector resonances, $R_V(s), R_A(s)$. Peskin and Takeuchi, PRD46, 381-409(1992)

$$S = -4\pi \left[\Pi'_{VV}(0) - \Pi'_{AA}(0)\right]$$

$$\Pi_{VV}(q^2) - \Pi_{AA}(q^2) = -\frac{q^2}{12\pi}\int_0^\infty \frac{ds}{\pi}\frac{R_V(s) - R_A(s)}{s - q^2} - F_\pi^2$$

Parity doubling can lead to smaller value for the $S$ parameter.

- In the chiral limit, $\Pi_{VV}(q^2) - \Pi_{AA}(q^2) \propto 1/q^4$, which leads to

First Weinberg Sum Rule:

$$\frac{1}{3\pi}\int_0^\infty ds\,[R_V(s) - R_A(s)] = 4\pi F_\pi^2$$

Second Weinberg sum rule

$$\frac{1}{3\pi}\int_0^\infty dss\,[R_V(s) - R_A(s)] = 0$$

# Parity Doubling and Electroweak $S$ Parameter

- However, at finite quark masses, we expect to have terms proportional to

$$\Pi_{VV}(q^2) - \Pi_{AA}(q^2) \sim m_f^2 \times (\cdots) + \frac{m_f \langle \overline{\psi}\psi \rangle}{q^2} \times (\cdots) + \mathcal{O}(1/q^4) + \cdots ,$$

  where $(\cdots)$ could involve logs of $q^2$.
- Thus at finite quark masses, Weinberg sum rules also receive mass-dependent corrections.
- Vector-pole dominance (VPD) is often assumed in model-building...

$$\begin{aligned} R_V(s) &= 12\pi^2 F_\rho^2 \delta(s - m_\rho^2) \\ R_A(s) &= 12\pi^2 F_{a_1}^2 \delta(s - m_{a_1}^2) \end{aligned}$$

# Weinberg's Sum Rules

With the VPD assumption, WSRs read, in the chiral limit:

$$\begin{aligned} F_\rho^2 - F_{a_1}^2 &= F_\pi^2, \\ F_\rho^2 M_\rho^2 - F_{a_1}^2 M_{a_1}^2 &= 0, \end{aligned}$$

and the $S$ parameter (*aka* Weinberg's zeroth sum rule):

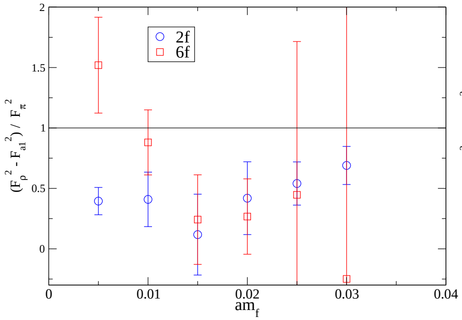
$$S = 4\pi \left[ \frac{F_\rho^2}{M_\rho^2} - \frac{F_{a_1}^2}{M_{a_1}^2} \right]$$

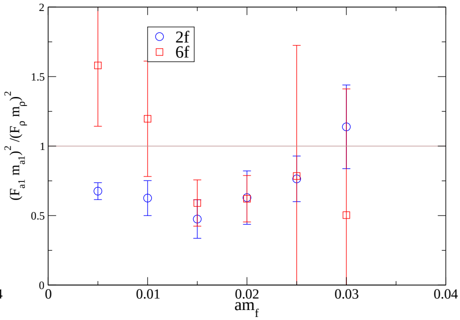
- These sum rules hold only in the chiral limit.
- At finite quark masses, we expect to see mass-dependent corrections, likely in the forms of

$$\begin{aligned} F_\rho^2 - F_{a_1}^2 &= F_\pi^2 + \mathcal{O}(m_f^2) \\ F_\rho^2 M_\rho^2 - F_{a_1}^2 M_{a_1}^2 &= \mathcal{O}(m_f \langle \overline{\psi}\psi \rangle_{\Lambda=1/a}) \end{aligned}$$

# Lattice Tests of WSRs

PRELIMINARY

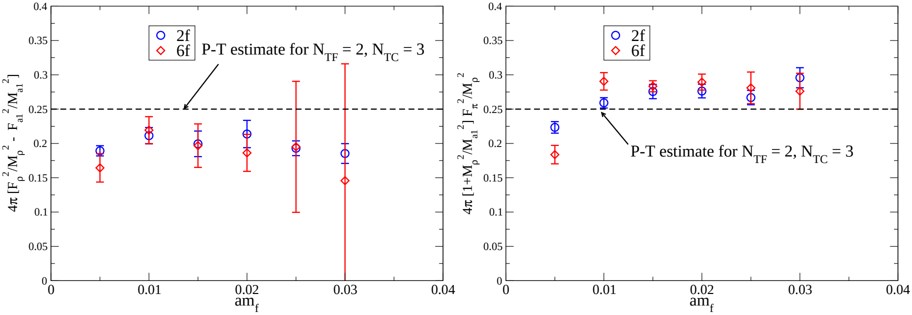



- Deviations from Weinberg's sum rules are seen. (Ratios should be 1 to satisfy).
- Not surprising. Even using phenomenological values cannot satisfy WSRs, since $F_\rho \approx F_{a_1}$.
- Can we quantify the corrections by the arguments given previously ? Work in progress...

# $S$ Parameter from Spectrum (before Standard Model subtraction)

- Zeroth Weinberg sum rule at the chiral limit, combined with the vector-pole dominance assumption, gives an estimate of the S parameter (LEFT).
- Making use of the Weinberg's first and second sum rules (RIGHT), Peskin and Takeuchi gave $S \approx 0.25 \frac{N_{TC}}{3} \frac{N_{TF}}{2}$




PRELIMINARY

◇ Our normalization eliminates the naive $N_{TF}/2$ scaling for $S$ so that $N_f = 2$ and 6 results can be compared directly.

# S Parameter from Direct Calculations

- Calculate $S$ parameter through

$$\Pi_{VV}^{\mu\nu}(q) = \sum_x e^{iq\cdot x}\langle V^\mu(x)V^\nu(0)\rangle,\ \Pi_{AA}^{\mu\nu}(q) = \sum_x e^{iq\cdot x}\langle A^\mu(x)A^\nu(0)\rangle$$

$$S = -4\pi\left[\Pi'_{VV}(0) - \Pi'_{AA}(0)\right],\ \Pi'(0) = \frac{d\Pi(q^2)}{dq^2}|_{q^2\to 0}$$

- Lattice calculations performed at finite $q^2$. Can use ChPT.

See talk by David Schaich, 15:10 Thursday

# Summary

1. We show exploratory studies of the $N_f = 6, 10$ SU(3) theory with domain wall fermions in the fundamental rep.
2. Such calculations are expensive. The results shown are preliminary and subject to change as more data become available and systematic errors are better controlled.
3. Small quark masses may suffer from large finite volume effects.
   Need larger volumes.
4. Chiral extrapolations for $N_f = 6, 10$ may not be reliable with current quark masses.
   Need smaller quark masses.

- Preliminary results are shown for the meson spectrum.
- We check Weinberg's spectral sum rules under the assumption of vector-pole dominance, and see deviations which may be from mass-dependent corrections.
- We also give a rough estimate for the $S$ parameter from the spectrum, which can be compared with direct calculations.