# Extracting resonance parameters from lattice data: Part II

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Work with Dr. Mike Peardon and Dr. Pietro Giudice

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# Outline

# The Problem of Resonances

- In lattice field theory masses are obtained from correlators $C(t)$, which behave for large $t$ as

$$C(t) \propto e^{-mt} \tag{1}$$

with $m$ being the lightest mass in the channel.

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# The Model

- We used the $O(4)$ model

$$\mathcal{L} = \frac{1}{2}\left(\partial\phi_i\partial\phi_i\right) + \lambda\left(\phi_i^2 - \nu^2\right)^2 - M_\pi^2\nu\phi_4 \tag{2}$$

- For the broken phase

$$\mathcal{L} = \frac{1}{2}\left(\partial\pi_i\partial\pi_i\right) + \frac{1}{2\nu}\sigma\left(\partial\pi_i\partial\pi_i\right) + \frac{1}{2}\left(\partial\sigma\partial\sigma\right) + \lambda\sigma^4 + 4\nu^2\lambda\sigma^2 + 4\nu\lambda\sigma^3 + \frac{M_{\pi^2}}{2}\left(\pi_i\pi_i\right) + \frac{M_\pi^2}{2\nu}\sigma\pi_i\pi_i \tag{3}$$

- Relevant vertices are $\frac{1}{2\nu}\sigma\left(\partial\pi_i\partial\pi_i\right)$ and $\frac{M_\pi^2}{2\nu}\sigma\pi_i\pi_i$

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# The Formula

- Lüscher's formula [Lüscher, 1991] provides a direct connection between the two-particle ($P = 0$) spectrum in finite volume and the scattering phase shift.

$$\delta(p) = -\phi(\kappa) + \pi n \tag{4}$$

$$Tan(\phi(\kappa)) = \left(\frac{\pi^{3/2}\kappa}{Z_{00}(1;\kappa^2)}\right), \kappa = \frac{pL}{2\pi} \tag{5}$$

p is the relative momentum.

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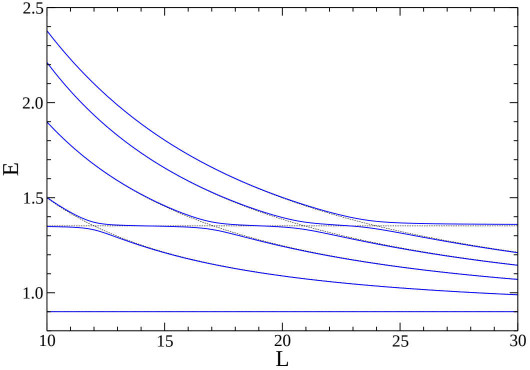


Avoided level crossing, plateau due to resonance.

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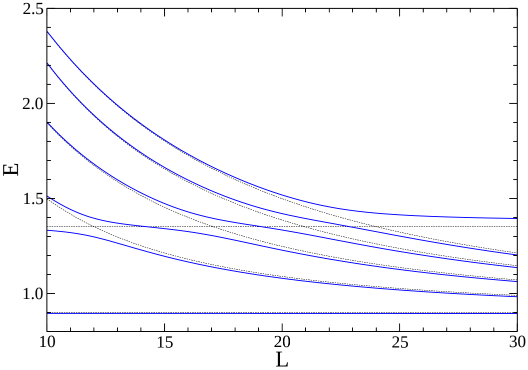


Broader as resonance width increases.

# Application to Data

- $E \longrightarrow p$, Dispersion relations. $p \longrightarrow \delta(p)$, Lüscher's formula.
- Resonance parameters can then be extracted by Breit-Wigner formula

$$\delta(p) \approx Tan^{-1}\left(\frac{4p^2 + 4M_\pi^2 - M_\sigma^2}{M_\sigma \Gamma_\sigma}\right) \tag{7}$$

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$$\phi(\kappa) \approx \pi\kappa^2$$

$$\begin{aligned}\phi(\kappa) \approx\ & (-0.09937)\kappa^8 + (0.47809)\kappa^6 \\ & +(-0.62064)\kappa^4 + (3.38974)\kappa^2 \end{aligned} \tag{8}$$

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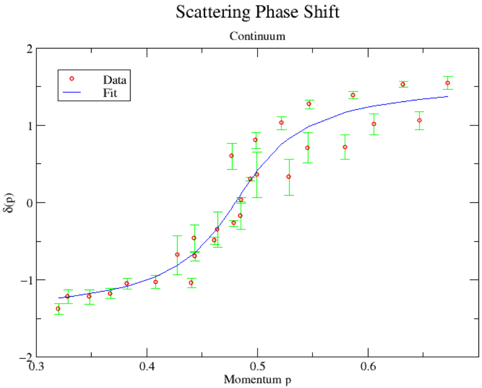


- Continuum Dispersion relations inaccurate, many points off the fit.

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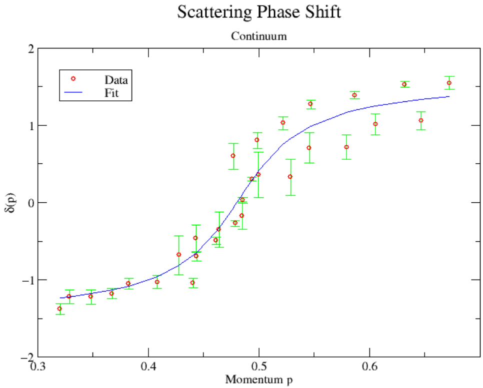


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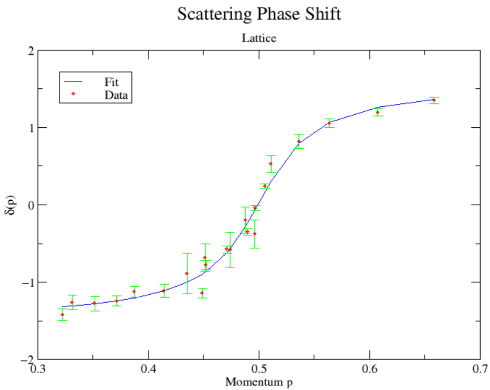


- Lattice dispersion relations: smaller errors and better fit.
- Also supresses lattice effects.

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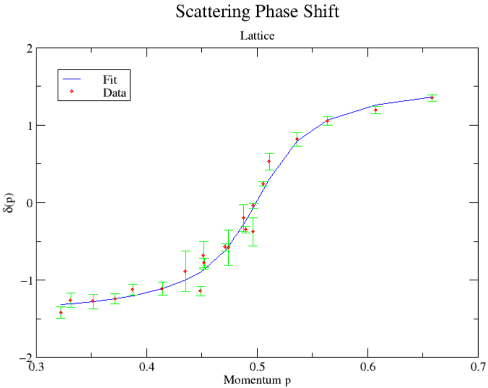


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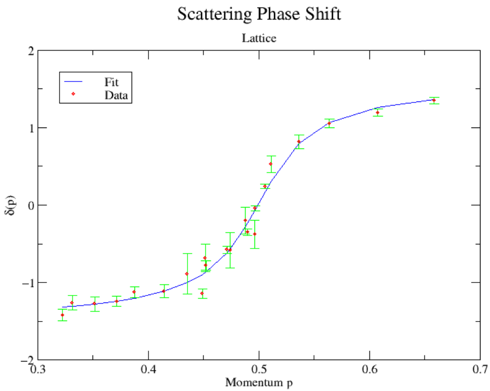


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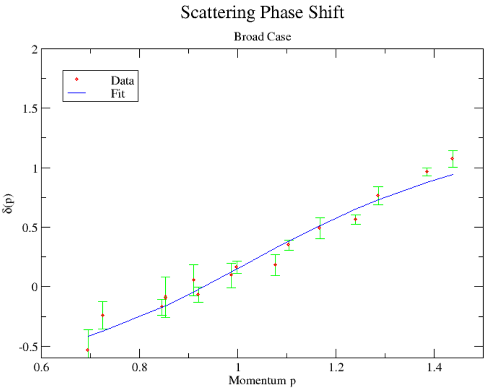
# Results

| Results | | |
|---|---|---|
| Parameters | $\phi(\kappa)$ | $\pi\kappa^2$ |
| $\nu = 1.0,\ \lambda = 1.4$ | $M_\sigma = 1.35(2)$<br>$\Gamma_\sigma = 0.115(8)$ | $M_\sigma = 1.36(4)$<br>$\Gamma_\sigma = 0.17(2)$ |
| $\nu = 1.0,\ \lambda = 4$ | $M_\sigma = 2.03(2)$<br>$\Gamma_\sigma = 0.35(2)$ | $M_\sigma = 2.2(2)$<br>$\Gamma_\sigma = 0.42(5)$ |
| $\nu = 1.0,\ \lambda = 200$ | $M_\sigma = 3.1(7)$<br>$\Gamma_\sigma = 1.2(5)$ | $M_\sigma = 3(1)$<br>$\Gamma_\sigma = 2(1)$ |

# Broad Resonance




- Profile almost linear. High precision data needed.
- Unless number of energy levels is increased (larger volumes), less data in elastic region.

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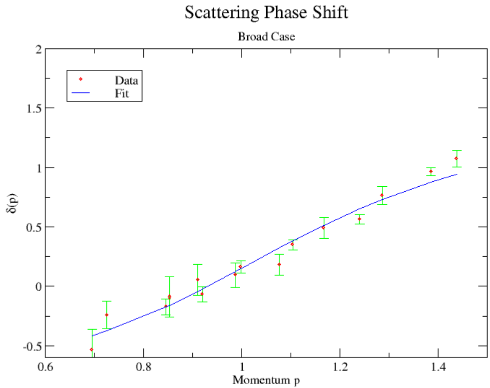


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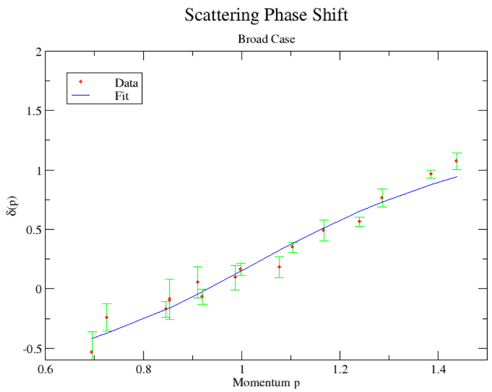


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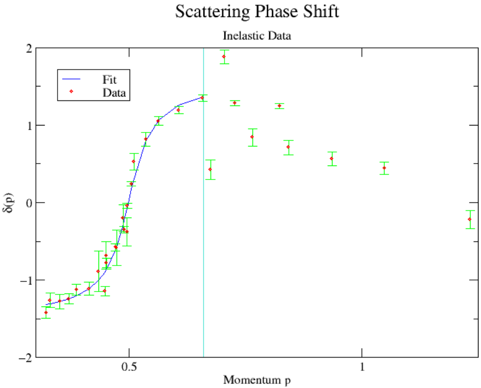


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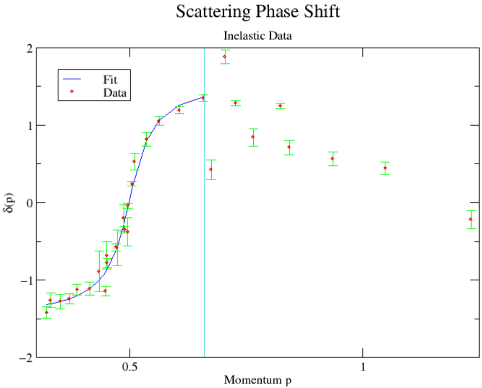


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# Outline

- An alternative method [Rusetsky et al. 2008] for analyzing resonances is the Histogram method.

- Here we construct a probability distribution $W(p) - W_{free}(p)$, the relative density of energy levels.

- Features of probability distribution should be related to resonance parameters as $N \to \infty$

$$C^{-1} W(p) - C_{free}^{-1} W_{free}(p) \approx \frac{1}{p} \left( \frac{\delta(p)}{p} - \delta'(p) \right) \qquad (9)$$

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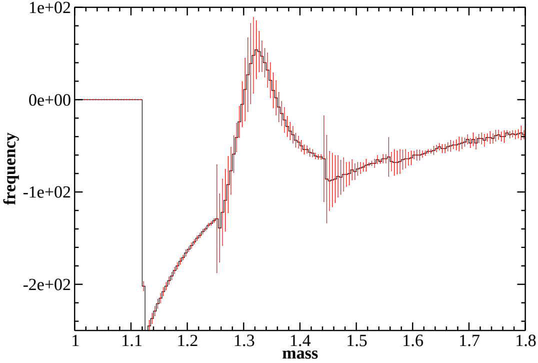

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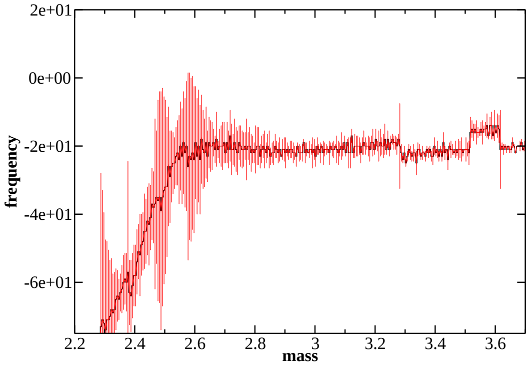


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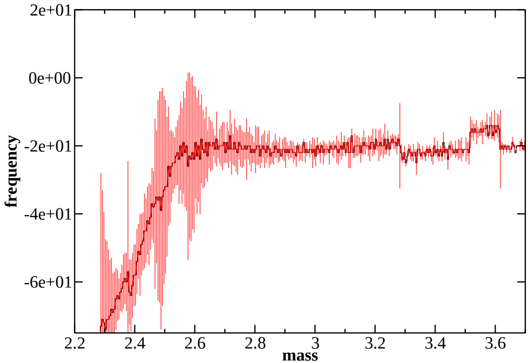


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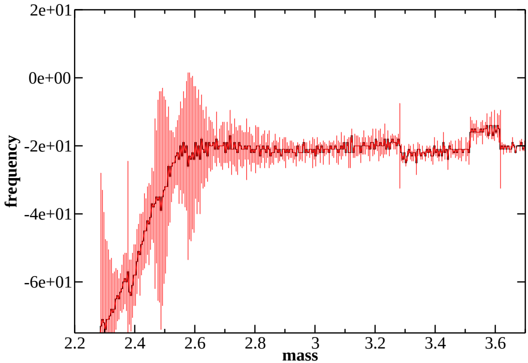


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# The Two Methods

- Both work well and produce similar results for narrow resonances.
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