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Extracting resonance parameters from lattice data: Part II

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Work with Dr. Mike Peardon and Dr. Pietro Giudice

June 15, 2010

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- The Problem of Resonances
- The Model

2 Lüscher's Method

- The Formula
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- Application to Data
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- Lattice Relations
- Probability Distribution Method
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 - Histogram
 - Numerical Comparison
 - The Two Methods
 - Conclusion and Outlook

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• In lattice field theory masses are obtained from correlators C(t), which behave for large t as

$$C(t) \propto e^{-mt}$$
 (1)

with m being the lightest mass in the channel.

- This will not work for resonances.
- Never the lightest mass in their channel, mass exceeds many-particle-threshold, no clear signal.

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The Model			

• We used the O(4) model

$$\mathcal{L} = \frac{1}{2} \left(\partial \phi_i \partial \phi_i \right) + \lambda \left(\phi_i^2 - \nu^2 \right)^2 - M_\pi^2 \nu \phi_4 \tag{2}$$

For the broken phase

$$\mathcal{L} = \frac{1}{2} \left(\partial \pi_i \partial \pi_i \right) + \frac{1}{2\nu} \sigma \left(\partial \pi_i \partial \pi_i \right) +$$

$$\frac{1}{2} \left(\partial \sigma \partial \sigma \right) + \lambda \sigma^4 + 4\nu^2 \lambda \sigma^2 + 4\nu \lambda \sigma^3 +$$

$$\frac{M_{\pi^2}}{2} \left(\pi_i \pi_i \right) + \frac{M_{\pi}^2}{2\nu} \sigma \pi_i \pi_i$$
(3)

• Relevant vertices are $\frac{1}{2v}\sigma(\partial \pi_i\partial \pi_i)$ and $\frac{M_{\pi}^2}{2v}\sigma \pi_i\pi_i$

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• Lüscher's formula [Lüscher, 1991] provides a direct connection between the two-particle (P = 0) spectrum in finite volume and the scattering phase shift.

$$\delta(p) = -\phi(\kappa) + \pi n \tag{4}$$

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$$Tan(\phi(\kappa)) = \left(\frac{\pi^{3/2}\kappa}{Z_{00}(1;\kappa^2)}\right), \kappa = \frac{pL}{2\pi}$$
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p is the relative momentum.

$$\mathcal{Z}_{js}(1;q^2) = \sum_{\underline{n}\in\mathbb{Z}^3}rac{r^jY_{js}(heta,\phi)}{(\underline{n}^2-q^2)^s}$$

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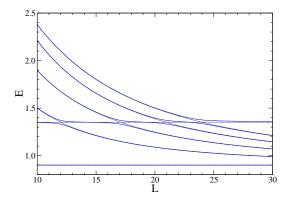
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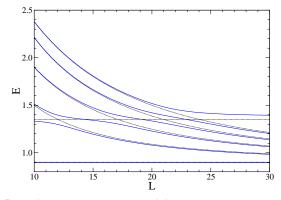
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Avoided level crossing, plateau due to resonance.

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Broader as resonance width increases.

Introduction 00	Lüscher's Method ○○○●○○○○○	Probability Distribution Method	Conclusion and Outlook
Application	to Data		

- $E \rightarrow p$, Dispersion relations. $p \rightarrow \delta(p)$, Lüscher's formula.
- Resonance parameters can then be extracted by Breit-Wigner formula

$$\delta(\rho) \approx Tan^{-1} \left(\frac{4\rho^2 + 4M_\pi^2 - M_\sigma^2}{M_\sigma \Gamma_\sigma} \right)$$
(7)

$$A(m) \sim (0.00027) m^8 + (0.00027) m^8$$

$$(\kappa) \approx (-0.09937)\kappa^{\circ} + (0.47809)\kappa^{\circ} + (-0.62064)\kappa^{4} + (3.38974)\kappa^{2}$$
 (8)

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• Which *p*?, Lattice or Continuum Dispersion Relations?

Application to Data

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Application to Data

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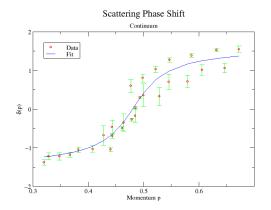
Lüscher's Method

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Continuum Relations



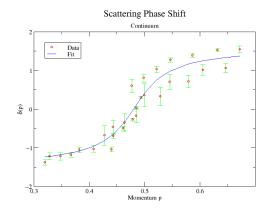
 Continuum Dispersion relations inaccurate, many points off the fit.

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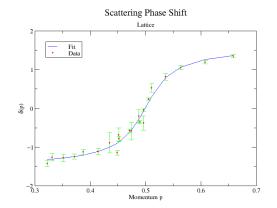


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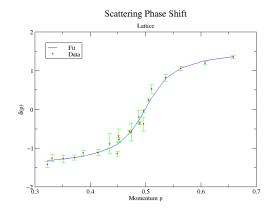
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Lattice dispersion relations: smaller errors and better fit.
Also supresses lattice effects.

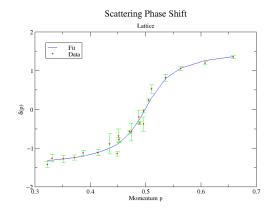
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Lattice R	elations		



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Results

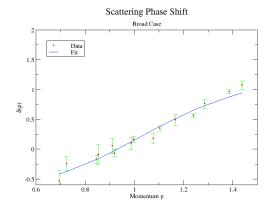
Results		
Parameters	$\phi(\kappa)$	$\pi\kappa^2$
$ u = 1.0, \ \lambda = 1.4 $	$M_{\sigma} = 1.35(2)$	$M_{\sigma} = 1.36(4)$
	$\Gamma_{\sigma} = 0.115(8)$	$\Gamma_{\sigma} = 0.17(2)$
$\nu = 1.0, \lambda = 4$	$M_{\sigma} = 2.03(2)$	$M_{\sigma} = 2.2(2)$
$\nu = 1.0, \ \lambda = 4$	$\Gamma_{\sigma} = 0.35(2)$	$\Gamma_{\sigma} = 0.42(5)$
$\nu = 1.0, \ \lambda = 200$	$M_{\sigma} = 3.1(7)$	$M_{\sigma}=3(1)$
$\nu = 1.0, \ \lambda = 200$	$\Gamma_{\sigma} = 1.2(5)$	$\Gamma_{\sigma}=2(1)$

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Conclusion and Outlook

Broad Resonance

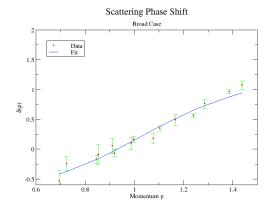


• Profile almost linear. High precision data needed.

 Unless number of energy levels is increased (larger volumes), less data in elastic region.

Conclusion and Outlook

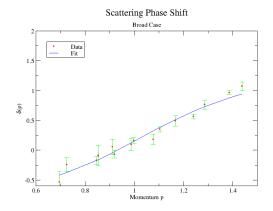
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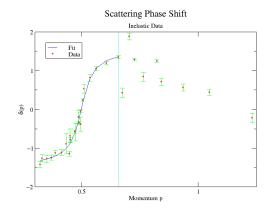
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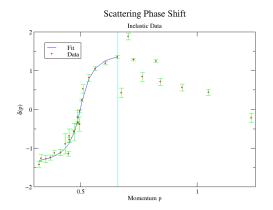
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Inelastic R	legion		



The formula cannot be extended to the inelastic region.

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Inelastic Re	gion		



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- An alternative method [Rusetsky et al. 2008] for analyzing resonances is the Histogram method.
- Here we construct a probability distribution W(p) W_{free}(p), the relative density of energy levels.
- Features of probability distribution should be related to resonance parameters as $N \to \infty$

$$C^{-1}W(p) - C_{free}^{-1}W_{free}(p) \approx \frac{1}{p} \left(\frac{\delta(p)}{p} - \delta'(p)\right)$$
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Lüscher's Method

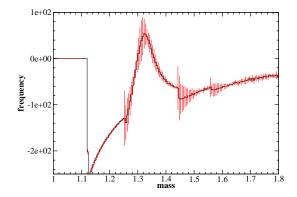
Probability Distribution Method

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Histogram



Lüscher's Method

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Numerical Comparison

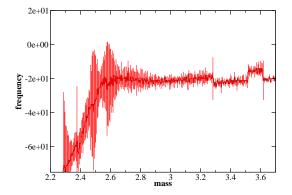
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$\nu = 1.0, \ \lambda = 200$	$M_{\sigma} = 3.1(7)$	$M_{\sigma} = N/A$	
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Lüscher's Method

Probability Distribution Method $\circ \circ \circ \circ \circ$

Conclusion and Outlook

Histogram for Broad Resonance



The distribution is so flat that it has no obvious peak or width.
Needs higher precision and more energy levels.

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Lüscher's Method

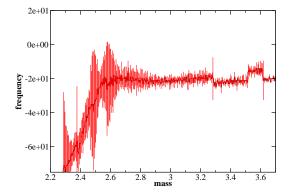
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Conclusion and Outlook

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Histogram for Broad Resonance



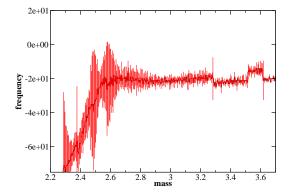
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- Both work well and produce similar results for narrow resonances.
- Lüscher's formula is less ambiguous, no problems with background.

- Errors clearer in Lüscher's formula.
- Difficulty with broad resonances for Histogram.



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• Does not apply in inelastic region however. Problem for QCD.

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